Estimating gluon saturation in dijet photoproduction in UPC

Piotr Kotko

Penn State University

based on:

P.K., K. Kutak, S. Sapeta,
A. Stasto, M. Strikman, arXiv:1702.03063

A. van Hameren, P.K., K. Kutak,
C. Marquet, E. Petreska, S. Sapeta,
JHEP 1612 (2016) 034, JHEP 1509 (2015) 106

supported by:
DEC-2011/01/B/ST2/03643
DE-FG02-93ER40771

PennState
Eberly College of Science

Motivation

Two basic small-x Transverse Momentum Dependent (TMD) gluon distributions:

- Weizsacker-Williams (WW) xG_1 – gluon number density
- Dipole xG_2 – does not have gluon number interpretation

Motivation

Two basic small-x Transverse Momentum Dependent (TMD) gluon distributions:

- Weizsacker-Williams (WW) xG_1 – gluon number density
- Dipole xG_2 – does not have gluon number interpretation

xG_2	xG_1
inclusive DIS*	inclusive dijet in $\gamma A^\star$
inclusive jet in $pA^\star$	inclusive dijet in pA
inclusive dijet in pA	

* – couples directly to a given small-x TMD gluon distribution

Motivation

Two basic small-x Transverse Momentum Dependent (TMD) gluon distributions:

- Weizsacker-Williams (WW) xG_1 – gluon number density
- Dipole xG_2 – does not have gluon number interpretation

inclusive DIS*	inclusive dijet in $\gamma A^\star$
inclusive jet in $pA^\star$	inclusive dijet in pA
inclusive dijet in pA	
xG_2	xG_1

* – couples directly to a given small-x TMD gluon distribution

Constraints from data

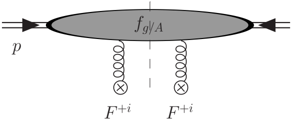
- xG_2 is well constrained from data
- xG_1 is loosely known from Color Glass Condensate (CGC)
- Dijets in Ultrapheripheral Collisions (UPC) probes xG_1 directly.

Plan

1. Introduction
   - Collinear vs TMD gluon distributions
2. Formalism for hard jets in pA (dilute-dense) collisions
   - Generalized factorization approach
   - Proliferation of TMD gluon distributions
   - Small-x TMD gluon distributions from Gaussian approximation
3. Application and results for forward dijets in UPC at LHC
   - Factorization formula with xG_1
   - Nuclear modification ratios for γA
4. Summary

Gluon distributions

Operator definition of the collinear gluon distribution



$$f_{g/H}(x) = \int \frac{dz^-}{2\pi p^+} e^{-ixp^+z^-} \langle p| \,\mathrm{Tr}\{F^{+i}\left(0,\vec{0}_T,z^-\right) U\left(z^-,0;\vec{0}_T\right) F^{+i}(0)\}\,|p\rangle$$

$F^{+i}(x) = F_a^{+i}(x)\,t^a$ – gluon strength tensor in fundamental representation

$U\left(z^-,0;\vec{0}_T\right) = \mathcal{P}\exp\left[ig\int_0^{z^-} dy^- A_a^+(0,\vec{0}_T,y^-)t^a\right]$ – the Wilson line

Gluon distributions

In saturation regime k_T of gluons $\sim Q_s$

Q_s can be large (a few GeV) $\Rightarrow$ we don't neglect the transverse momentum.

Gluon distributions

In saturation regime k_T of gluons $\sim Q_s$

Q_s can be large (a few GeV) $\Rightarrow$ we don't neglect the transverse momentum.

Transverse momentum dependent (TMD) gluon distributions

The position of one of the gluon operators is off the light-cone:

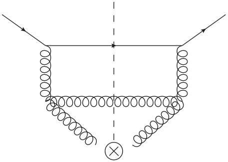
$$\mathcal{F}_{g^*/H}^{(C_1,C_2)}(x,k_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}}{(2\pi)^3 p^+} e^{ixp^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \langle p|\, \mathrm{Tr}\left\{F^{+i}\left(0,\xi_T,\xi^-\right)[\xi,0]_{C_1} F^{+i}(0)\,[0,\xi]_{C_2}\right\}|p\rangle$$

where $[\xi,0]_{C_i}$ are again Wilson lines which lie along some paths C_1 and C_2.

The structure of Wilson lines depends on the particular hard process attached to the gluon distribution, more precisely on its color structure.

Gluon distributions

Example: TMD distribution for a particular diagram[1]



$$\langle p|\,\mathrm{Tr}\{F^{+i}(\xi)\,\mathcal{U}^{[+]\dagger}F^{+i}(0)\left[\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]\dagger}}{N_c}\mathcal{U}^{[+]}+\mathcal{U}^{[-]}\right]\}\,|p\rangle$$

where the Wilson lines (and loops) are defined as

$$\mathcal{U}^{[\pm]}=U(0,\pm\infty;0_T)U_T(\pm\infty;0_T,\xi_T)U(\pm\infty,\xi^-;\xi_T)$$

$$\mathcal{U}^{[\square]}=\mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger}=\mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$$

[1] C.J. Bomhof, P.J. Mulders, F. Pijlman, Eur.Phys.J.C. 47, 147 (2006)

Improved small x TMD factorization (ITMD)

Factorization formula for forward dijets in pA ($\mu = \bar{p}_T \gg Q_s$)

[P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106]

$$\frac{d\sigma_{AB\to 2j+X}}{dy_1 d^2p_{T1} dy_2 d^2p_{T2}} \sim \sum_{a,c,d} f_{a/B}\left(x_B,\mu^2\right) \sum_{i=1,2} \Phi^{(i)}_{ag\to cd}\left(x_A, k_T^2, \mu^2\right) K^{(i)}_{ag\to cd}\left(k_T^2,\mu^2\right)$$

$\Phi^{(i)}_{ag\to cd}$ – small-x TMD Gluon Distributions
$K^{(i)}$ – hard factors calculated from off-shell gauge invariant amplitudes

Formula contains essential limiting cases:

1. $Q_s \ll k_T \sim \overline{p}_T$ – High Energy Factorization (HEF)/k_T-factorization
   - All $\Phi^{(i)}_{ag\to cd}$ become equal to xG_2.
   - Off-shell factors combine to appropriate Lipatov vertex.
   - Correct collinear limit (if collinear PDF is defined as integrated xG_2)
2. $k_T \sim Q_s \ll \overline{p}_T$ – leading power limit of CGC[1]
   - Off-shell factors become on-shell
   - TMD gluon distributions can be identified with color averages in CGC

[1] F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan Phys.Rev. D 83 (2011) 105005

Improved small x TMD factorization (ITMD)

Factorization formula for forward dijets in pA ($\mu = \bar{p}_T \gg Q_s$)

[P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106]

$$\frac{d\sigma_{AB\to 2j+X}}{dy_1 d^2p_{T1} dy_2 d^2p_{T2}} \sim \sum_{a,c,d} f_{a/B}\left(x_B,\mu^2\right) \sum_{i=1,2} \Phi^{(i)}_{ag\to cd}\left(x_A,k_T^2,\mu^2\right) K^{(i)}_{ag\to cd}\left(k_T^2,\mu^2\right)$$

TMD gluon distributions:

$$\Phi^{(1)}_{qg\to gq} = \mathcal{F}^{(1)}_{qg},\ \Phi^{(2)}_{qg\to gq} = \frac{1}{N_c^2-1}\left(N_c^2\mathcal{F}^{(2)}_{qg} - \mathcal{F}^{(1)}_{qg}\right),\ \Phi^{(1)}_{gg\to q\bar{q}} = \frac{1}{N_c^2-1}\left(N_c^2\mathcal{F}^{(1)}_{gg} - \mathcal{F}^{(3)}_{gg}\right),\ \Phi^{(2)}_{gg\to q\bar{q}} = \mathcal{F}^{(3)}_{gg} - N_c^2\mathcal{F}^{(2)}_{gg}$$

$$\Phi^{(1)}_{gg\to gg} = \frac{1}{2N_c^2}\left(N_c^2\mathcal{F}^{(1)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg}\right),\ \Phi^{(2)}_{gg\to gg} = \frac{1}{N_c^2}\left(N_c^2\mathcal{F}^{(2)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg}\right)$$

$$\mathcal{F}^{(1)}_{qg} \sim \langle p_A|\,\mathrm{Tr}\{F^{+i}(\xi)\,\mathcal{U}^{[-]\dagger}F^{+i}(0)\,\mathcal{U}^{[+]}\}\,|p_A\rangle, \quad \mathcal{F}^{(2)}_{qg} \sim \langle p_A|\,\mathrm{Tr}\{F^{+i}(\xi)\,\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]}}{N_c}\mathcal{U}^{[+]\dagger}F^{+i}(0)\,\mathcal{U}^{[+]}\}\,|p_A\rangle,$$

$$\mathcal{F}^{(1)}_{gg} \sim \langle p_A|\,\mathrm{Tr}\{F^{+i}(\xi)\,\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]}}{N_c}\mathcal{U}^{[-]\dagger}F^{+i}(0)\,\mathcal{U}^{[+]}\}\,|p_A\rangle, \quad \mathcal{F}^{(2)}_{gg} \sim \frac{1}{N_c}\,\langle p_A|\,\mathrm{Tr}\left\{F^{+i}(\xi)\,\mathcal{U}^{[\square]\dagger}\right\}\mathrm{Tr}\left\{F^{+i}(0)\,\mathcal{U}^{[\square]}\right\}|p_A\rangle,$$

$$\mathcal{F}^{(3)}_{gg} \sim \langle p_A|\,\mathrm{Tr}\left\{F^{+i}(\xi)\,\mathcal{U}^{[+]\dagger}F^{+i}(0)\,\mathcal{U}^{[+]}\right\}|p_A\rangle, \quad \mathcal{F}^{(4)}_{gg} \sim \langle p_A|\,\mathrm{Tr}\left\{F^{+i}(\xi)\,\mathcal{U}^{[-]\dagger}F^{+i}(0)\,\mathcal{U}^{[-]}\right\}|p_A\rangle,$$

$$\mathcal{F}^{(5)}_{gg} \sim \langle p_A|\,\mathrm{Tr}\left\{F^{+i}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}F^{+i}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right\}|p_A\rangle, \quad \mathcal{F}^{(6)}_{gg} \sim \langle p_A|\,\mathrm{Tr}\left\{F^{+i}(\xi)\,\mathcal{U}^{[+]\dagger}F^{+i}(0)\,\mathcal{U}^{[+]}\right\}\left(\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]}}{N_c}\right)^2|p_A\rangle$$

Improved small x TMD factorization (ITMD)

Factorization formula for forward dijets in pA ($\mu = \bar{p}_T \gg Q_s$)

[P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106]

$$\frac{d\sigma_{AB\to 2j+X}}{dy_1 d^2p_{T1} dy_2 d^2p_{T2}} \sim \sum_{a,c,d} f_{a/B}\left(x_B, \mu^2\right) \sum_{i=1,2} \Phi^{(i)}_{ag\to cd}\left(x_A, k_T^2, \mu^2\right) K^{(i)}_{ag\to cd}\left(k_T^2, \mu^2\right)$$

Off-shell hard factors:

$$K^{(1)}_{qg\to gq} = -\frac{\bar{u}\left(\bar{s}^2+\bar{u}^2\right)}{2\bar{t}\hat{t}\hat{s}}\left(1+\frac{\bar{s}\hat{s}-\bar{t}\hat{t}}{N_c^2\,\bar{u}\hat{u}}\right),\ K^{(2)}_{qg\to gq} = -\frac{C_F}{N_c}\,\frac{\bar{s}\left(\bar{s}^2+\bar{u}^2\right)}{\bar{t}\hat{t}\hat{u}},\ K^{(1)}_{gg\to q\bar{q}} = \frac{1}{2N_c}\,\frac{\left(\bar{t}^2+\bar{u}^2\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}\right)}{\bar{s}\hat{s}\hat{t}\hat{u}}$$

$$K^{(2)}_{gg\to q\bar{q}} = \frac{1}{4N_c^2 C_F}\,\frac{\left(\bar{t}^2+\bar{u}^2\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s}\right)}{\bar{s}\hat{s}\hat{t}\hat{u}},\ K^{(2)}_{gg\to q\bar{q}} = \frac{1}{4N_c^2 C_F}\,\frac{\left(\bar{t}^2+\bar{u}^2\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s}\right)}{\bar{s}\hat{s}\hat{t}\hat{u}},$$

$$K^{(1)}_{gg\to gg} = \frac{N_c}{C_F}\,\frac{\left(\bar{s}^4+\bar{t}^4+\bar{u}^4\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}\right)}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}},\ K^{(2)}_{gg\to gg} = -\frac{N_c}{2C_F}\,\frac{\left(\bar{s}^4+\bar{t}^4+\bar{u}^4\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s}\right)}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$$

$\hat{s}, \hat{t}, \hat{u}$ – ordinary Mandelstam variables, $\hat{s}+\hat{t}+\hat{u} = k_T^2$.
$\bar{s}, \bar{t}, \bar{u}$ off-shell momentum is replaced by its longitudinal component of off-shell momentum, $\bar{s}+\bar{u}+\bar{t} = 0$
.

Improved small x TMD factorization (ITMD)

Two most basic TMD distributions:

1. $$\langle p|\,\mathrm{Tr}\left\{F\left(\xi\right)\mathcal{U}^{[+]\dagger}F\left(0\right)\mathcal{U}^{[+]}\right\}|p\rangle \sim xG_1$$
2. $$\langle p|\,\mathrm{Tr}\left\{F\left(\xi\right)\mathcal{U}^{[-]\dagger}F\left(0\right)\mathcal{U}^{[+]}\right\}|p\rangle \sim xG_2$$

It is possible to choose a gauge to eliminate Wilson lines in xG_1 so that it has an interpretation as a gluon number density. This is not possible for xG_2.

Only xG_2 is restricted from data (inclusive DIS)

Improved small x TMD factorization (ITMD)

Two most basic TMD distributions:

1. $$\langle p|\,\mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[+]\dagger}F(0)\,\mathcal{U}^{[+]}\right\}|p\rangle \sim xG_1$$
2. $$\langle p|\,\mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[-]\dagger}F(0)\,\mathcal{U}^{[+]}\right\}|p\rangle \sim xG_2$$

It is possible to choose a gauge to eliminate Wilson lines in xG_1 so that it has an interpretation as a gluon number density. This is not possible for xG_2.

Only xG_2 is restricted from data (inclusive DIS)

How to obtain the rest?

- Full evolution equations of the hierarchy of the operators[1,2]
- Approximations:
  - At large N_c some TMD gluon distributions are suppressed $\Rightarrow$ 5 left.
  - In the leading power limit we recover CGC and thus we may assume the Gaussian distribution of color sources known from CGC

$\Rightarrow$ All TMD gluons can be calculated from xG_2

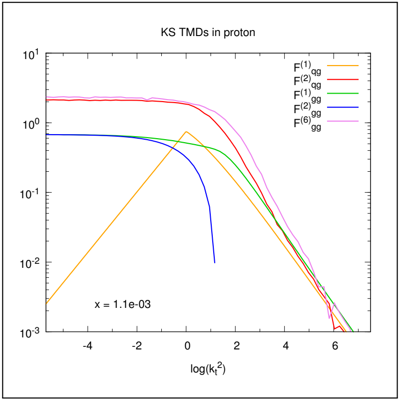
[1] I. Balitsky, A. Tarasov, JHEP 1510 (2015) 017
[2] C. Marquet, E. Petreska, C. Roiesnel, JHEP 1610 (2016) 065

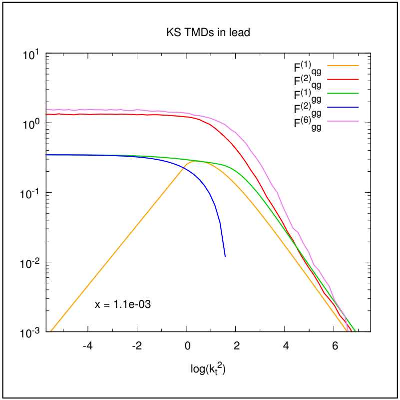
Gluon distributions for ITMD

Small x TMD gluon distributions from data

[A. van Hameren, P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, JHEP 1612 (2016) 034]

The dipole gluon xG_2 was fitted to HERA data by Kutak and Sapeta[1] (KS) using nonlinear extension[2] of Kwiecinski-Martin-Stasto[3] (KMS) evolution equation.





All gluons merge for large k_T (except $\mathcal{F}_{gg}^{(2)}$ which vanishes) $\Rightarrow$ correct HEF limit.

[1] K. Kutak, S. Sapeta, Phys. Rev. D 86 (2012) 094043 [2] K. Kutak, K. Kwiecinski, Eur. Phys. J. C 29 (2003) 521
[2] K. Kwiecinski, A. Martin, A. Stasto, Phys.Rev. D59 (1999) 093002

ITMD for dijets in γA

Factorization formula

[P.K., K. Kutak, S. Sapeta, A. Stasto, M. Strikman, arXiv:1702.03063]

$$\frac{d\sigma_{\gamma A \to 2j+X}}{dy_1 d^2 p_{T1} dy_2 d^2 p_{T2}} \sim x_A G_1 \left(x_A, k_T^2, \mu^2\right) \otimes K_{\gamma g^* \to q\bar{q}} \left(k_T, \mu^2\right)$$

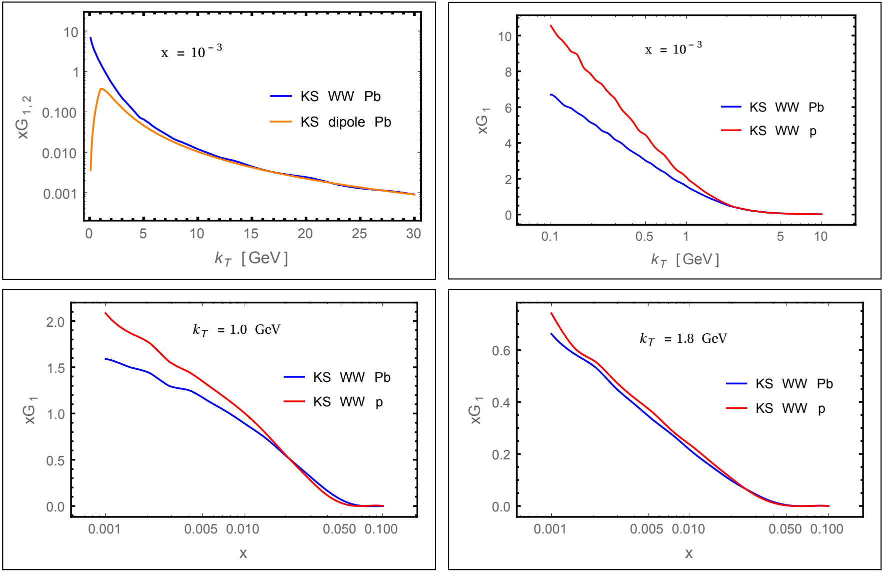
xG_1 – the Weizsacker-Williams gluon distribution
$K_{\gamma g^* \to q\bar{q}}$ – off-shell gauge invariant hard factor for the $\gamma g^* \to q\bar{q}$ process

- Similar to inclusive DIS, but probes xG_1 instead of xG_2.
- For UPC one needs to convolute this with the photon flux from nucleus.
- With the current LHC setup the problem with UPC is that the photon flux dies out very fast above $x_\gamma \sim 0.03$ for Pb, so there is not much phase space for the asymmetric kinematics $x_A \ll x_\gamma$ which guarantees that xG_1 is probed at small $x \Rightarrow$ jets with rather small p_T are required.

Weizsacker-Williams xG_1 from KS fit

Results for $Pb = 208$

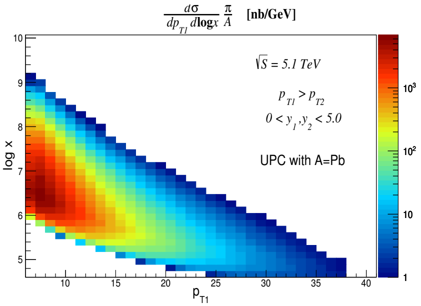
[P.K., K. Kutak, S. Sapeta, A. Stasto, M. Strikman, arXiv:1702.03063]

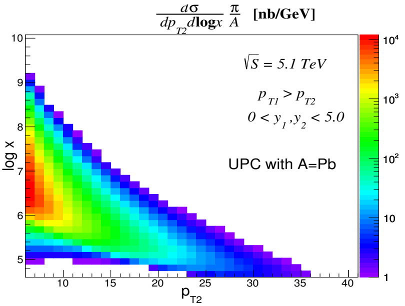


Results for dijets in UPC at LHC

Kinematic cuts

CM energy: 5.1 TeV	rapidity: $0 < y_1, y_2 < 5$
transverse momenta: $p_{T1} > p_{T2} > p_{T0}$, $p_{T0} = 6 \div 25\,\mathrm{GeV}$	jet algorithm: $R = 0.5$

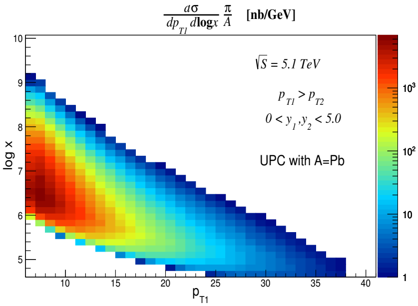


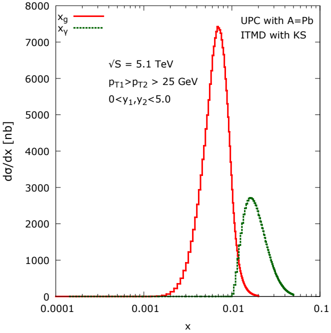


Results for dijets in UPC at LHC

Kinematic cuts

CM energy: 5.1 TeV	rapidity: $0 < y_1, y_2 < 5$
transverse momenta: $p_{T1} > p_{T2} > p_{T0}$, $p_{T0} = 6 \div 25\,\mathrm{GeV}$	jet algorithm: $R = 0.5$

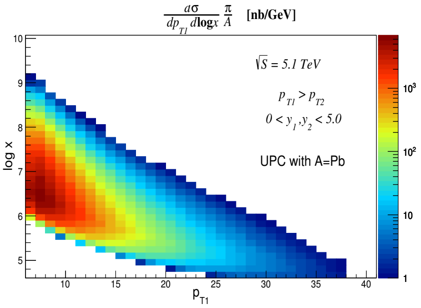


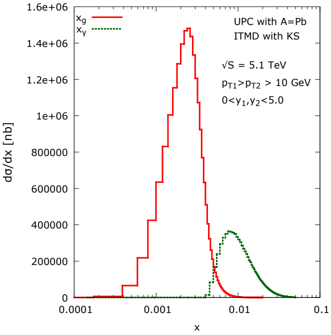


Results for dijets in UPC at LHC

Kinematic cuts

CM energy: 5.1 TeV	rapidity: $0 < y_1, y_2 < 5$
transverse momenta: $p_{T1} > p_{T2} > p_{T0}$, $p_{T0} = 6 \div 25\,\mathrm{GeV}$	jet algorithm: $R = 0.5$

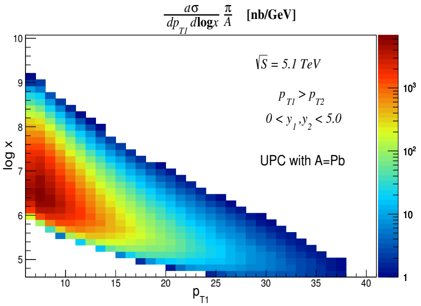


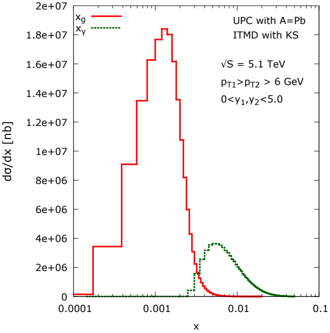


Results for dijets in UPC at LHC

Kinematic cuts

CM energy: 5.1 TeV	rapidity: $0 < y_1, y_2 < 5$
transverse momenta: $p_{T1} > p_{T2} > p_{T0}$, $p_{T0} = 6 \div 25\,\mathrm{GeV}$	jet algorithm: $R = 0.5$





Results for dijets in UPC

Nuclear modification factor $R_{\gamma A}$

For UPC collisions we define the nuclear modification ratio as

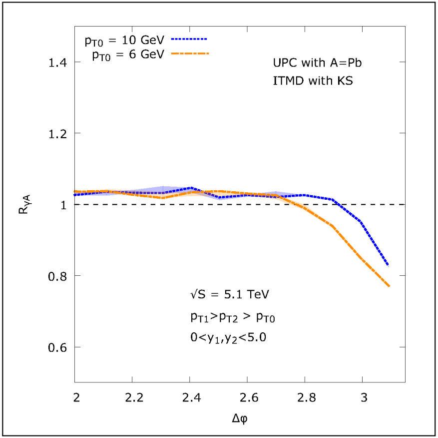
$$R_{\gamma A} = \frac{d\sigma_{AA}^{\mathrm{UPC}}}{A d\sigma_{Ap}^{\mathrm{UPC}}}$$

where $A = \mathrm{Pb}$ and the $d\sigma_{Ap}^{\mathrm{UPC}}$ is with jets going in the nucleus direction.

Results for dijets in UPC

Azimuthal decorrelations

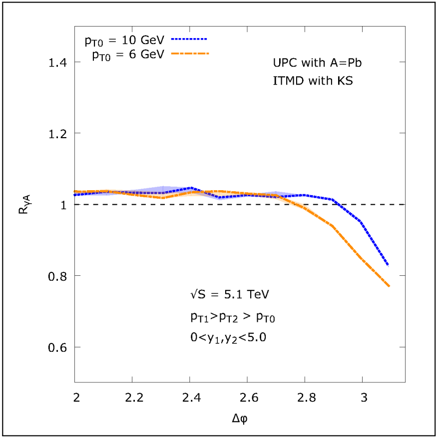
[P.K., K. Kutak, S. Sapeta, A. Stasto, M. Strikman, arXiv:1702.03063]

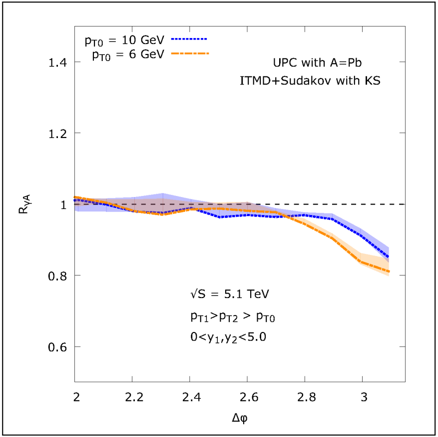


Results for dijets in UPC

Azimuthal decorrelations

[P.K., K. Kutak, S. Sapeta, A. Stasto, M. Strikman, arXiv:1702.03063]

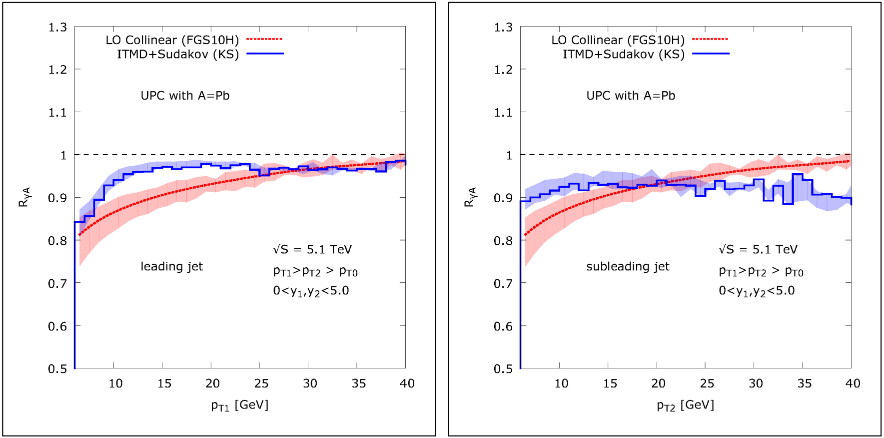




Results for dijets in UPC

Jet p_T spectra

[P.K., K. Kutak, S. Sapeta, A. Stasto, M. Strikman, arXiv:1702.03063]

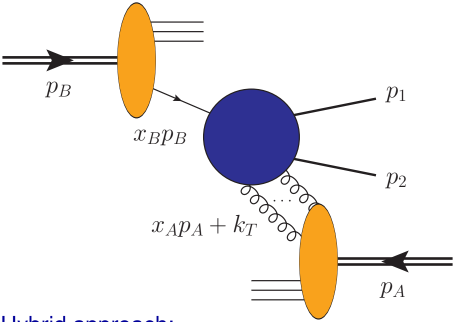


Summary

- When $P_T \gg Q_s$ the CGC expressions for dijet production can be reinterpret in terms of small-x TMD gluon distributions.
  - They are not universal, but can be calculated within certain approximations
  - The language of gluon distribution has practical advantages
- Direct component of dijet production in UPC is directly sensitive to Weizsacker-Williams (WW) gluon distribution; this is the only 'true' gluon distribution at small x
- Present calculations use WW obtained from the 'dipole' gluon distribution fitted to data; no experimental information about WW is available
- Within the kinematics allowed by the present formalism the suppression factor is 10-20% depending on the transverse momentum cut
- Similar suppression is visible for leading twist shadowing, but it vanishes faster with increase of x

BACKUP

Dijets in *pA* collisions



forward dijets with transverse momentum imbalance:

$$\left|\vec{p}_{T1} + \vec{p}_{T2}\right| = \left|\vec{k}_T\right| = k_T$$

asymmetric kinematics:

$$x_B \gg x_A$$

Hybrid approach:

- large-x parton in hadron B is treated as 'collinear' with standard PDFs
- small-x partons within hadron A have internal transverse momentum k_T

Three-scale problem

1. hard scale P_T (of the order of the average transverse momentum of jets)
2. transverse momentum imbalance k_T
3. saturation scale $\Lambda_{\rm QCD} \ll Q_s$ (increasing with energy)

Forward dijets in *pA* collisions within CGC

Example: $qA \to qg$ channel

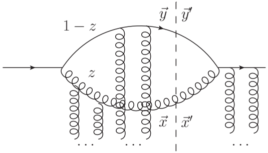
[C. Marquet, Nucl. Phys. A 796 (2007) 41]

$$\frac{d\sigma_{qA\to 2j}}{d^3p_1 d^3p_2} \sim \int \frac{d^2x}{(2\pi)^2}\frac{d^2x'}{(2\pi)^2}\frac{d^2y}{(2\pi)^2}\frac{d^2y'}{(2\pi)^2} e^{-i\vec{p}_{T1}\cdot\left(\vec{x}_T-\vec{x}'_T\right)} e^{-i\vec{p}_{T2}\cdot\left(\vec{y}_T-\vec{y}'_T\right)} \psi_z^*\left(\vec{x}'_T-\vec{y}'_T\right)\psi_z\left(\vec{x}_T-\vec{y}_T\right)$$

$$\Big\{S^{(6)}_{x_g}\left(\vec{y}_T,\vec{x}_T,\vec{y}'_T,\vec{x}'_T\right) - S^{(3)}_{x_g}\left(\vec{y}_T,\vec{x}_T,(1-z)\,\vec{y}'_T + z\vec{x}'_T\right) - S^{(3)}_{x_g}\left((1-z)\,\vec{y}_T + z\vec{x}_T,\vec{y}'_T,\vec{x}'_T\right)$$
$$-S^{(2)}_{x_g}\left((1-z)\,\vec{y}_T + z\vec{x}_T,(1-z)\,\vec{y}'_T + z\vec{x}'_T\right)\Big\}$$

$\psi_z\left(\vec{x}_T\right)$ – quark wave function

$S^{(i)}_{x_g}$ – correlators of Wilson line operators, e.g.



$$S^{(2)}_{x_g}\left(\vec{y}_T,\vec{x}_T\right) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[U\left(\vec{y}_T\right)U^\dagger\left(\vec{y}'_T\right)\right]\right\rangle_{x_g}$$

$$S^{(3)}_{x_g}\left(\vec{z}_T,\vec{y}_T,\vec{x}_T\right) = \frac{1}{2C_FN_c}\left\langle \mathrm{Tr}\left[U\left(\vec{z}_T\right)U^\dagger\left(\vec{y}_T\right)\right]\mathrm{Tr}\left[U\left(\vec{y}_T\right)U^\dagger\left(\vec{x}_T\right)\right]\right\rangle_{x_g} - S^{(2)}_{x_g}\left(\vec{z}_T,\vec{x}_T\right) \text{ etc.}$$

where $U(\vec{x}_T) = U(-\infty,+\infty;\vec{x}_T)$ and $\langle\ldots\rangle_{x_g}$ denotes the average over color sources.

The WW gluon distribution from data

Relation between xG_1 and xG_2 in gaussian approximation

$$\nabla^2_{k_T} G^{(1)}(x,k_T) = \frac{4\pi^2}{N_c S_\perp} \int \frac{d^2 q_T}{q_T^2} \frac{\alpha_s}{(k_T - q_T)^2} G^{(2)}(x,q_T)\, G^{(2)}(x,|k_T - q_T|)$$

Realistic evolution equation for xG_2

Nonlinear extension of the Kwiecinski-Martin-Stasto (KMS) evolution equation (below $xG_2 \equiv \mathcal{F}$):

[K. Kutak, K. Kwiecinski, Eur. Phys. J. C 29 (2003) 521]
[J. Kwiecinski, Alan D. Martin, A.M. Stasto, Phys.Rev. D56 (1997) 3991-4006]

$$\mathcal{F}\left(x,k_T^2\right) = \mathcal{F}_0\left(x,k_T^2\right) + \frac{\alpha_s N_c}{\pi}\int_x^1 \frac{dz}{z}\int_{k_{T0}^2}^{\infty}\frac{dq_T^2}{q_T^2}\left\{\frac{q_T^2\mathcal{F}\left(\frac{x}{z},q_T^2\right)\theta\left(\frac{k_T^2}{z}-q_T^2\right) - k_T^2\mathcal{F}\left(\frac{x}{z},k_T^2\right)}{\left|q_T^2-k_T^2\right|} + \frac{k_T^2\mathcal{F}\left(\frac{x}{z},k_T^2\right)}{\sqrt{4q_T^4+k_T^4}}\right\}$$
$$+\frac{\alpha_s}{2\pi k_T^2}\int_x^1 dz\left\{\left(P_{gg}(z)-\frac{2N_c}{z}\right)\int_{k_{T0}^2}^{k_T^2} dq_T^2\,\mathcal{F}\left(\frac{x}{z},q_T^2\right) + zP_{gq}(z)\,\Sigma\left(\frac{x}{z},k_T^2\right)\right\}$$
$$-\frac{2\alpha_s^2}{R^2}\left\{\left[\int_{k_T^2}^{\infty}\frac{dq_T^2}{q_T^2}\mathcal{F}\left(x,q_T^2\right)\right]^2 + \mathcal{F}\left(x,k_T^2\right)\int_{k_T^2}^{\infty}\frac{dq_T^2}{q_T^2}\ln\left(\frac{q_T^2}{k_T^2}\right)\mathcal{F}\left(x,q_T^2\right)\right\}$$

This equation was fitted to HERA data for proton by Kutak-Sapeta (KS).

[K. Kutak, S. Sapeta, Phys. Rev. D 86 (2012) 094043]

For nucleus $R_A = RA^{1/3}/\sqrt{d}$ is used so the nonlinear term is enhanced by $dA^{1/3}$.

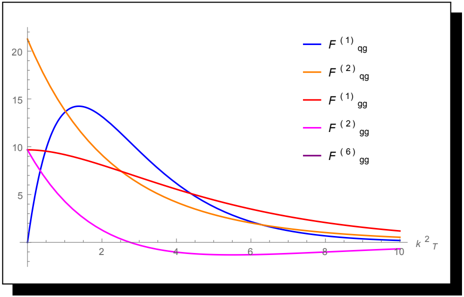
Gluon distributions: GBW model

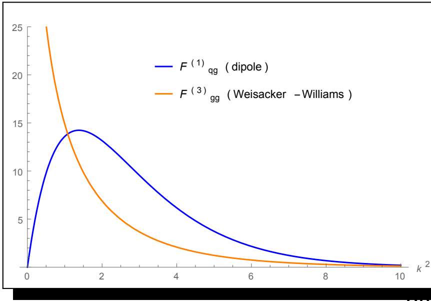
How to obtain 5 gluon distributions?

To start, we take the Golec-Biernat-Wusthoff (GBW) model:

$$xG_2\left(x,k_T^2\right)=\mathcal{F}_{qg}^{(1)}\left(x,k_T^2\right)=\frac{N_cS_\perp}{2\pi^3\alpha_s}\,\frac{k_T^2}{Q_s^2\left(x\right)}\,\exp\left(-\frac{k_T^2}{Q_s^2\left(x\right)}\right),\;\;Q_s\left(x\right)=Q_{s0}^2\left(\frac{x}{x_0}\right)^\lambda$$

Assuming Gaussian distribution of colour sources, the WW gluon $xG_1\left(x,k_T^2\right)$ can be related to $xG_2\left(x,k_T^2\right)$, hence all five gluons can be calculated analytically[1]





Improved small x TMD factorization (ITMD): $P_T \gg Q_s$

Color-ordered off-shell helicity amplitudes

Color decomposition of gluon amplitudes:

$$\mathcal{M}^{a_1 \ldots a_N}\left(\varepsilon_1^{\lambda_1}, \ldots, \varepsilon_N^{\lambda_N}\right) = \sum_{\sigma \in S_N} \mathrm{Tr}\left(t^{a_{\sigma_1}} t^{a_{\sigma_2}} \ldots t^{a_{\sigma_N}}\right) \mathcal{M}\left(\sigma_1^{\lambda_{\sigma_1}}, \sigma_2^{\lambda_{\sigma_2}} \ldots, \sigma_N^{\lambda_{\sigma_N}}\right)$$

a_i- color indices, $\varepsilon_i^{\lambda_i}$ - polarization vectors with helicity λ_i, S_N - set of noncyclic permutations.

In spinor formalism, the non-zero off-shell helicity amplitudes have the form of the MHV amplitudes with certain modification of spinor products:

[A. van Hameren, PK, K. Kutak, JHEP 1212 (2012) 029]

$$\mathcal{M}_{g^* g \to gg}\left(1^*, 2^-, 3^+, 4^+\right) = 2g^2 \rho_1 \frac{\langle 1^* 2\rangle^4}{\langle 1^* 2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

$$\mathcal{M}_{g^* g \to gg}\left(1^*, 2^+, 3^-, 4^+\right) = 2g^2 \rho_1 \frac{\langle 1^* 3\rangle^4}{\langle 1^* 2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

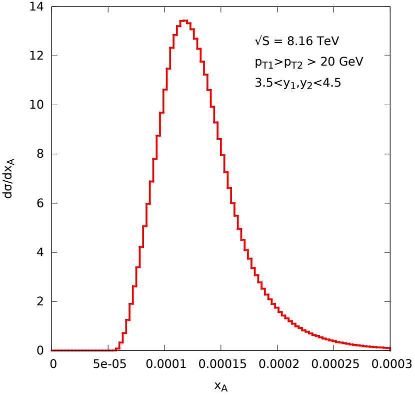
$$\mathcal{M}_{g^* g \to gg}\left(1^*, 2^+, 3^+, 4^-\right) = 2g^2 \rho_1 \frac{\langle 1^* 4\rangle^4}{\langle 1^* 2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

where $\langle ij\rangle = \langle k_i - | k_j + \rangle$ with spinors defined as $|k_i \pm\rangle = \frac{1}{2}\left(1 \pm \gamma_5\right) u\left(k_i\right)$.
Spinor products for off-shell states involve only longitudinal component of the off-shell momentum $\langle 1^* i\rangle = \langle p_1 i\rangle$, where $k_1 = p_1 + k_{T1}$, $k_1^2 \neq 0$, $p_1^2 = 0$.

R_{pA} for dijet azimuthal imbalance in ITMD

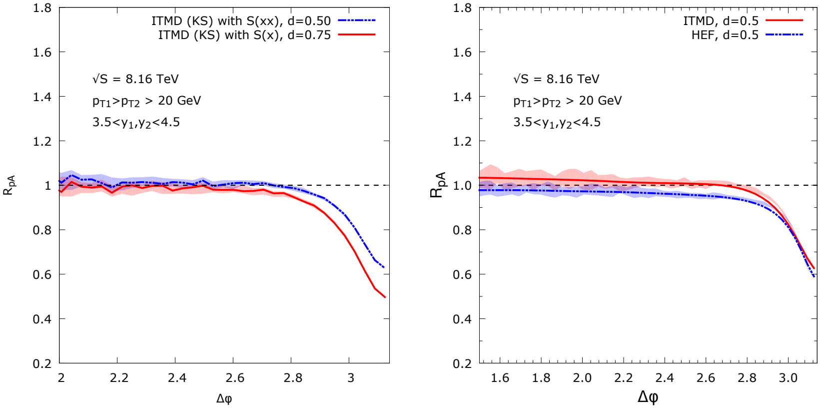
Cuts: $p_{T1} > p_{T2} > 20\,\mathrm{GeV}$, $3.5 < y_1, y_2 < 4.5$, $\sqrt{S} = 8.16\,\mathrm{TeV}$

x fractions probed:



R_{pA} for dijet azimuthal imbalance in ITMD

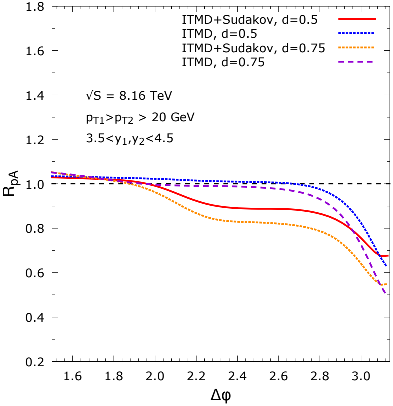
Cuts: $p_{T1} > p_{T2} > 20\,\mathrm{GeV}$, $3.5 < y_1, y_2 < 4.5$, $\sqrt{S} = 8.16\,\mathrm{TeV}$



R_{pA} for dijet azimuthal imbalance in ITMD

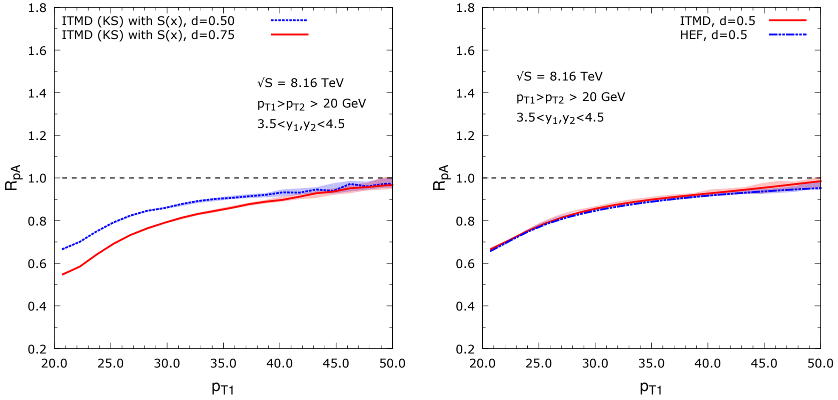
Cuts: $p_{T1} > p_{T2} > 20\,\mathrm{GeV}$, $3.5 < y_1, y_2 < 4.5$, $\sqrt{S} = 8.16\,\mathrm{TeV}$

Including Sudakov resummation (a model of):



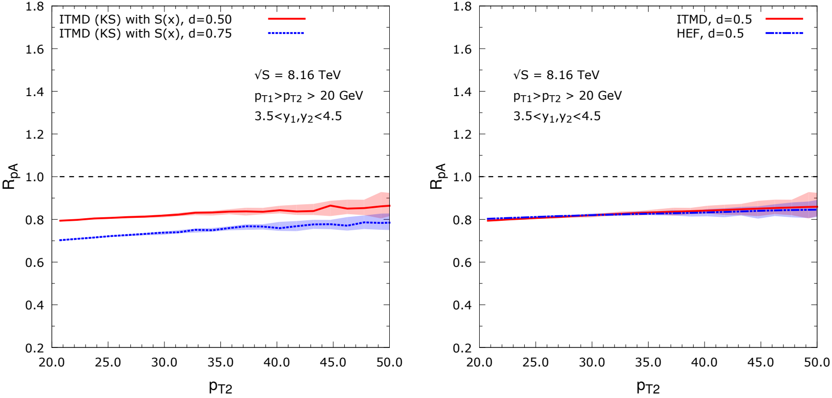
R_{pA} for p_T dijet spectra in ITMD

leading jet spectrum:



R_{pA} for p_T dijet spectra in ITMD

sub-leading jet spectrum:

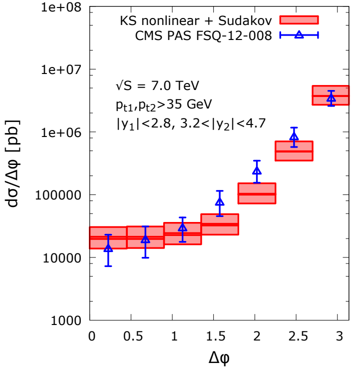


High energy factorization (HEF): $k_T \sim P_T \gg Q_s$

Comparison with data

[A. van Hameren, PK, K. Kutak, S. Sapeta, Phys.Lett. B737 (2014) 335-340]
[A. van Hameren, PK, K. Kutak, Phys.Rev. D92 (2015) 054007]

- central-forward dijet production



- forward Z_0+jet production