Discovery probability of next-generation $0\nu\beta\beta$ decay experiments

M. Agostini, G. Benato, J. A. Detwiler, arXiv:1705.02996

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$0\nu\beta\beta$ Decay Searches: Motivation and Status

Open questions to which $0\nu\beta\beta$ decay could answer

- What mechanism for ν masses?
- $\nu = \bar{\nu}$?
- Is L conserved?
- Are we here thanks to leptogenesis?

O(10^2 M$) are going to be invested for an answer

- What return can be expected on this investment?
- What are we capable of, and what should we aim for as a community?

Goals of this work

- What is the discovery probability (DP) of future experiments?
- What sensitivity is of interest?
- On which parameters should optimize and compare experimental designs?

How to Allow the Comparison of Experiments?

Proposal for a Common Approach

- Energy is the only observable that is necessary and sufficient for discovery
- Fold additional information (topology, pulse shape, ...) into efficiency
- Poisson counting in Region Of Interest (ROI) with (effective) fiducial volume and background level

Formalism

- Sensitive exposure: $\mathcal{E} = m_{iso} \cdot \varepsilon_{\text{ROI}} \cdot \varepsilon_{FV} \cdot \varepsilon_{sig} \cdot t$
- Sensitive background: $\mathcal{B} = \frac{n_{bkg}}{\mathcal{E}}$
- Number of signal events: $n_{0\nu\beta\beta} = \frac{\ln 2}{T^{0\nu}_{1/2}} \cdot \frac{N_A \cdot \mathcal{E}}{m_a}$
- Number of background events: $n_{bkg} = \mathcal{B} \cdot \mathcal{E}$

- m_{iso} = isotope mass
- t = live time
- N_A = Avogadro number
- m_a = isotope's molar mass

$\Rightarrow$ $\mathcal{B}$ and $\mathcal{E}$ provide full characterization of $0\nu\beta\beta$ decay experiments and are naturally suited for comparisons

~~Exclusion~~ or Discovery Sensitivity?

From the "Report to NSAC, NLDBD 2015":

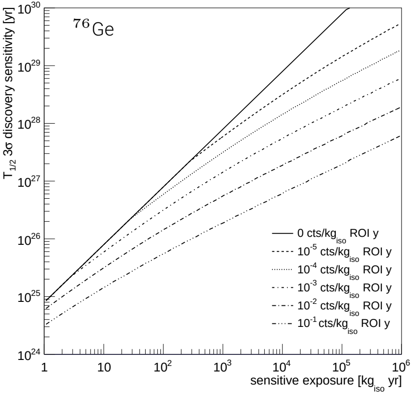
"In particular, we advocated using the smallest of the available nuclear matrix elements for each isotope and quoting a lifetime for a $3\,\sigma$ discovery in the complete inverted hierarchy region."

Discovery sensitivity

- Value of $\mathrm{T}^{0\nu}_{1/2}$ or $m_{\beta\beta}$ for which an experiment has a 50% chance to measure a signal above background with $\geq 3\sigma$ significance:

$$\hat{T}^{0\nu}_{1/2} = \ln 2 \frac{N_A \mathcal{E}}{m_a S_{3\sigma}(n_{bkg})} \quad \text{with } n_{bkg} = \mathcal{B}\mathcal{E}$$

- $S_{3\sigma}(n_{bkg}) =$ Poisson signal expectation at which 50% of experiments report a 3σ fluctuation above n_{bkg}

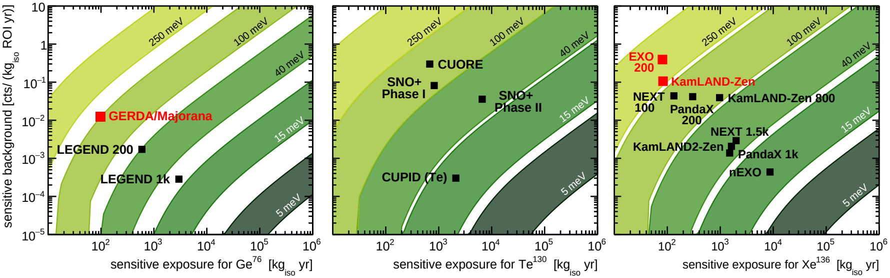


Experiments

- Experiments listed by “Report to NSAC, NLDBD 2015”

Experiment	Iso.	Iso. Mass	σ	ROI	ϵ_{FV}	ϵ_{sig}	$\mathcal{E}$	$\mathcal{B}$	3σ disc. sens. $\hat{T}_{1/2}$	3σ disc. sens. $\hat{m}_{\beta\beta}$	Required Improvement Bkg	Required Improvement σ	Required Improvement Iso. Mass
		[kg$_{iso}$]	[keV]	[σ]	[%]	[%]	$\left[\frac{\text{kg}_{iso}\text{ yr}}{\text{yr}}\right]$	$\left[\frac{\text{cts}}{\text{kg}_{iso}\text{ ROI yr}}\right]$	[yr]	[meV]			
LEGEND 200	^{76}Ge	175	1.3	[-2, 2]	93	77	119	$1.7 \cdot 10^{-3}$	$8.4 \cdot 10^{26}$	40−73	3	1	5.7
LEGEND 1k	^{76}Ge	873	1.3	[-2, 2]	93	77	593	$2.8 \cdot 10^{-4}$	$4.5 \cdot 10^{27}$	17−31	18	1	29
SuperNEMO	^{82}Se	100	51	[-4, 2]	100	16	16.5	$4.9 \cdot 10^{-2}$	$6.1 \cdot 10^{25}$	82−138	49	2	14
CUPID	^{82}Se	336	2.1	[-2, 2]	100	69	221	$5.2 \cdot 10^{-4}$	$1.8 \cdot 10^{27}$	15−25	n/a	6	n/a
CUORE	^{130}Te	206	2.1	[-1.4, 1.4]	100	81	141	$3.1 \cdot 10^{-1}$	$5.4 \cdot 10^{25}$	66−164	6	1	19
CUPID	^{130}Te	543	2.1	[-2, 2]	100	81	422	$3.0 \cdot 10^{-4}$	$2.1 \cdot 10^{27}$	11−26	3000	1	50
SNO+ Phase I	^{130}Te	1357	82	[-0.5, 1.5]	20	97	164	$8.2 \cdot 10^{-2}$	$1.1 \cdot 10^{26}$	46−115	n/a	n/a	n/a
SNO+ Phase II	^{130}Te	7960	57	[-0.5, 1.5]	28	97	1326	$3.6 \cdot 10^{-2}$	$4.8 \cdot 10^{26}$	22−54	n/a	n/a	n/a
KamLAND-Zen 800	^{136}Xe	750	114	[0, 1.4]	64	97	194	$3.9 \cdot 10^{-2}$	$1.6 \cdot 10^{26}$	47−108	1.5	1	2.1
KamLAND2-Zen	^{136}Xe	1000	60	[0, 1.4]	80	97	325	$2.1 \cdot 10^{-3}$	$8.0 \cdot 10^{26}$	21−49	15	2	2.9
nEXO	^{136}Xe	4507	25	[-1.2, 1.2]	60	85	1741	$4.4 \cdot 10^{-4}$	$4.1 \cdot 10^{27}$	9−22	400	1.2	30
NEXT 100	^{136}Xe	91	7.8	[-1.3, 2.4]	88	37	26.5	$4.4 \cdot 10^{-2}$	$5.3 \cdot 10^{25}$	82−189	n/a	1	20
NEXT 1.5k	^{136}Xe	1367	5.2	[-1.3, 2.4]	88	37	398	$2.9 \cdot 10^{-3}$	$7.9 \cdot 10^{26}$	21−49	n/a	1	300
PandaX-III 200	^{136}Xe	180	31	[-2, 2]	100	35	60.2	$4.2 \cdot 10^{-2}$	$8.3 \cdot 10^{25}$	65−150	n/a	n/a	n/a
PandaX-III 1k	^{136}Xe	901	10	[-2, 2]	100	35	301	$1.4 \cdot 10^{-3}$	$9.0 \cdot 10^{26}$	20−46	n/a	n/a	n/a

Results



- 15 meV band corresponds to the end of IO region
- Red dots: published limits
- Black dots: 3σ discovery sensitivities with 5 yr live time
- Discovery sensitivity after 10 yr is $\sim\sqrt{2}$ higher for all experiments
- Bands represent NME spread

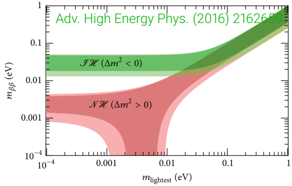
Discovery Probability with Light Neutrino Exchange

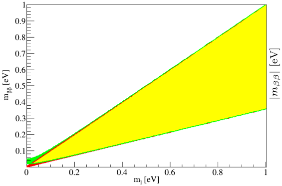
Bayesian method offers natural way to approach the problem:

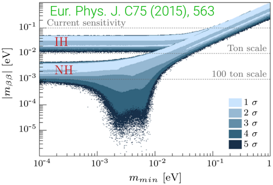
$$\text{DP} = \frac{\text{Region of parameter space for which a} \geq 3\,\sigma \text{ discovery is possible}}{\text{Total parameter space volume}}$$

$$= \int_0^\infty \frac{dP(m_{\beta\beta})}{dm_{\beta\beta}} \cdot \overline{\text{CDF}}_{Poisson}(C_{3\sigma}|S(m_{\beta\beta}) + B)\, dm_{\beta\beta},$$

How to compute the total parameter space volume?







- *we cannot even invoke 'naturalness' to argue that $m_{\beta\beta}$ should not be much smaller than its individual m_i-contributions. [...] some flavour symmetry could easily force a small $m_{\beta\beta}$, giving rise to apparently unnatural cancellations [...]**
- On the other hand, several mass models do NOT predict specific values of $m_{\beta\beta}$

* F. Feruglio, A. Strumia and F. Vissani, Nucl. Phys. B637 (2002) 345-377

Bayesian Global Fit of Neutrino Data

Characteristics and limitations of our approach

- Use all information available to date to extract PDF of $m_{\beta\beta}$
  $\Rightarrow$ Need to interpret plot of $m_{\beta\beta}$ vs m_l as a 2-dim PDF
- Assumptions on neutrino masses folded in parameter basis and priors
- Need well-behaved (=normalizable) variables

Parameter basis:

$$\left\{\Sigma, \Delta m_{21}^2, \left|\Delta m_{3l}^2\right|, \theta_{12}, \theta_{13}, \alpha_{21}, (\alpha_{31} - \delta_{CP})\right\}$$

- Natural to characterize by some scale which is then split $\Rightarrow$ *quasi-degenerate* ν masses
- Σ would characterize the mass scale
  $\Rightarrow$ Does not capture hierarchical neutrinos
- Hierarchical neutrinos case can be studied setting $m_l = 0^*$

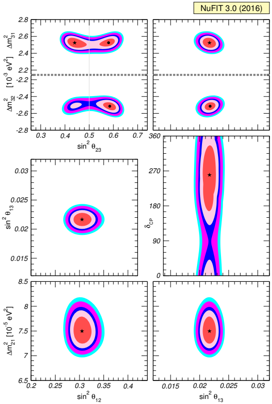
Priors

- Masses: neutrino mass scale unknown
  $\Rightarrow$ log-flat priors
  $\Rightarrow$ Non-normalizable: need upper and lower cut-off. OK if data provide them.
- Angles: flat in $[0, 2\pi[$
- Phases: flat in $[0, 2\pi[$
  $\Rightarrow$ Not non-informative
  $\Rightarrow$ Invoke naturalness!

* A. Caldwell et al., arXiv:1705.01945

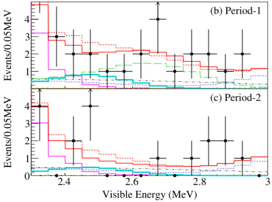
Available Data

Oscillation data

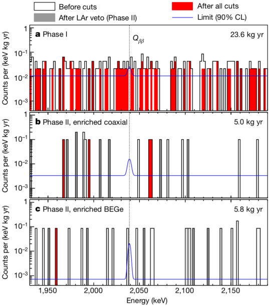


I. Esteban et al., JHEP 01 (2017) 087

$0\nu\beta\beta$ decay data

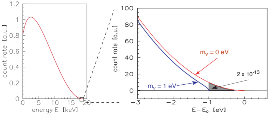


A. Gando et al., PRL 117 (2016) 082503



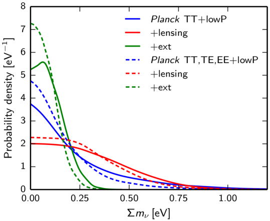
GERDA Coll., Nature 544 (2017) 47-52

β decay end-point

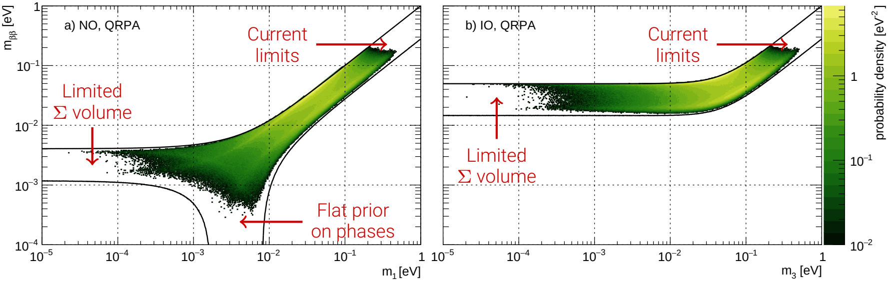


V. N. Aseev et al., Phys. Rev. D 84 (2011) 112003
Ch. Kraus et al., Eur. Phys. J. C 40 (2005) 447-468

(Cosmology)



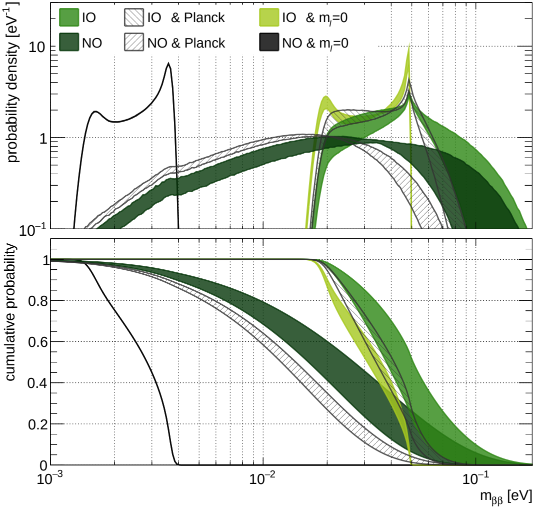
Planck Coll., Astron. Astrophys. 594 (2016) A13



Caveats

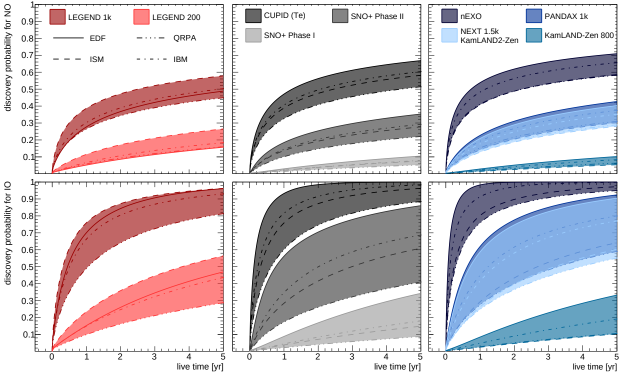
- In NO, a flavor symmetry could induce an apparent fine tuning of the Majorana phases and a vanishing $m_{\beta\beta}$
- Mass mechanisms that drive m_l to zero are not considered
  $\Rightarrow$ Just extrapolate down the horizontal bands

Results



- Bands show NME variation
- Top: $P\left(m_{\beta\beta}|\mathrm{D}\right)$
- Bottom: $\int_{m_{\beta\beta}}^{+\infty} P\left(m_{\beta\beta}|\mathcal{D}\right) dm_{\beta\beta}$
- Planck limit reduces cumulative probability by $\sim 20\%$ for NO and $\sim 10\%$ for IO
- With $m_l = 0$, $m_{\beta\beta}$ pushed to horizontal bands
  $\Rightarrow$ Small impact on DP for IO, huge for NO

Results



- Fold $m_{\beta\beta}$ PDF with discovery sensitivity
- Bands represent NME maximal deviations
- Multi-isotope approach suggested!

DP for most promising experiments

- Reference analysis (quasi-degenerate neutrinos): $\sim 100\%$ for IO, $> 50\%$ for NO
- Hierarchical neutrinos ($m_l = 0$): $> 90\%$ for IO, $< 2\%$ for NO

Join the game: quantify your prejudice and extract your own DP!

- Example: 50:50 for IO:NO, 50:50 for H:QD $\Rightarrow$ DP=60%

Alternative Analyses

Cosmological limit

- For NO, discovery probability degrades by $\sim 30\%$
- For IO, difference at percent level

Quenching of g_A

- Quenching degrades discovery sensitivity as well as current limits
  $\Rightarrow$ Effect on discovery potential smaller than on sensitivity
- 30% quenching reduces discovery potential by 15% for IO
- 30% quenching reduces discovery potential by 25% for NO

Cosmological limit AND quenching of g_A

- Region at high $m_{\beta\beta}$ stays disfavored $\Rightarrow$ Reduced experimental reach
- Discovery power $\geq 50\%$ for IO
- Discovery power $O(10\%)$ for NO

What if KATRIN sees a signal?

- DP$= 100\%$ regardless of ordering, mass model, NME, quenching, cosmology

Summary and Outlook

On which parameters should optimize and compare experimental designs?

- Sensitive background and exposure appear to be the appropriate parameters

What sensitivity is of interest?

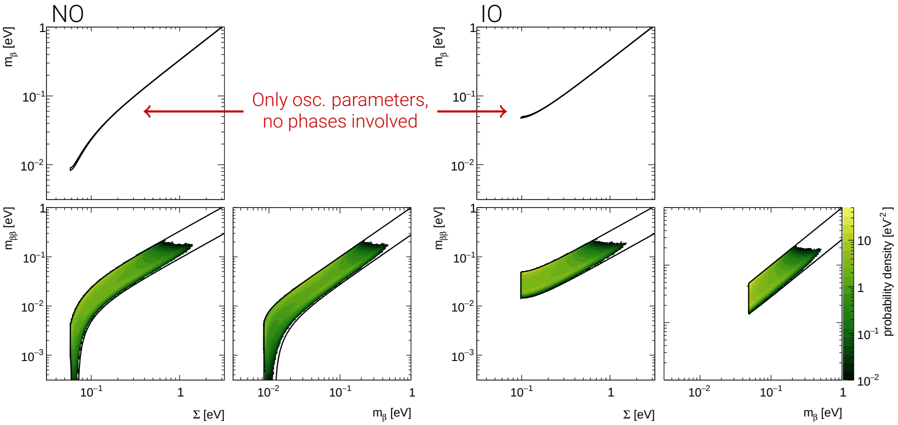
- $3\,\sigma$ discovery $\Rightarrow$ Computable without toy-MC

What is the DP of future experiments?

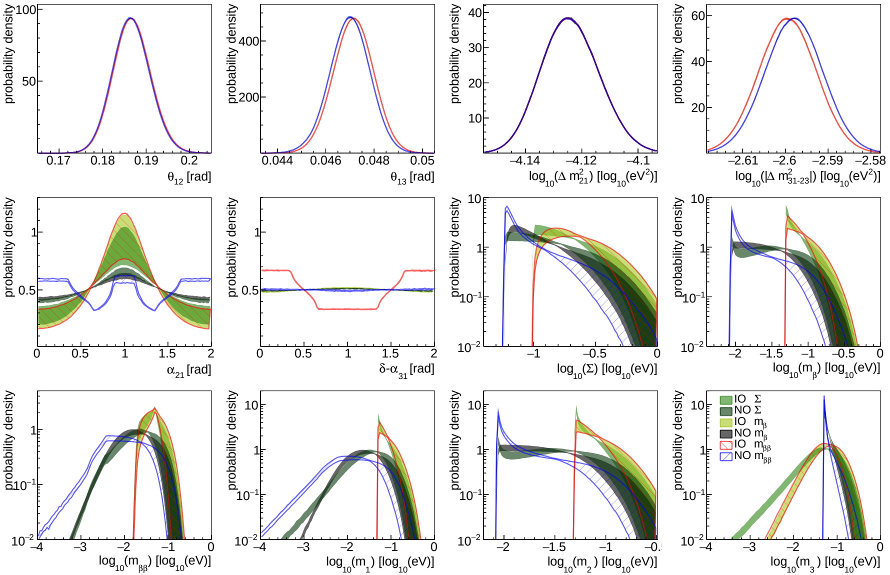
- DP for IO is as high as has been advertised by the experimental collaborations
- Surprisingly high DP for NO relative to the recent community perception, module the mass model bias $\Rightarrow$ Highlights the importance of what we believe about how neutrinos get their mass
- Surprisingly robust against many of the “damning” impacts of IO/NO discrimination, g_A quenching, or cosmological limits
- We believe these argument make a strong case for the value for investing in a global multi-isotope campaign to search for $0\nu\beta\beta$ decay at the IO scale

M. Agostini, G. Benato, J. A. Detwiler, arXiv:1705.02996
in press at Phys. Rev. D

Bonus: $m_{\beta\beta}$ vs m_β and Σ



Bonus: All Posteriors



Bonus: $0\nu\beta\beta$ Decay with Light Neutrino Exchange

Assumptions

0) Flavor and mass neutrino eigenstates: $\nu_l = \sum_i U_{li}\nu_i$.

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\frac{\delta_{CP}}{2}} \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix} =$$

PMNS mixing matrix with Majorana phases

1) Only 3 known neutrinos are involved

2) Neutrinos have a Majorana mass term in the SM Lagrangian. We don't consider an additional Dirac term or additional fields.

Physical observable: the effective Majorana mass

$$m_{\beta\beta} = \left|\sum_{i=1}^{3} U_{ei}^2 m_i\right| = \left|m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{1(\alpha_{31}-\delta_{CP})}\right|$$

$0\nu\beta\beta$ decay half life

$$\frac{1}{T_{1/2}^{0\nu}} = G_{0\nu} \cdot |M_{0\nu}|^2 \cdot \frac{m_{\beta\beta}^2}{m_e^2}$$

- $G_{0\nu}$ = phase space (known)
- $M_{0\nu}$ = nuclear matrix element (NME) $\Rightarrow$ factor ~ 2 uncertainty from models

Bonus: Available Data

Oscillation measurements

- $\left|\Delta m^2_{31}\right|$, δm^2_{21}, θ_{12} and θ_{13} known well and with negligible correlation
- Global Bayesian fit of oscillation exp.: I. Esteban et al., JHEP 01 (2017) 087
- Treat NO and IO separately
  $\Rightarrow$ Can always re-weight DP when more information becomes available

β decay end-point measurements

- Sensitive to $m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 \, m_i^2}$
- Troitsk (Bayesian):
  $m_\beta < 2.12$ eV (95% C.I.)*
- Mainz (frequentist):
  $m_\beta < 2.3$ eV (95% C.L.)†
- Include Troitsk even if impact is minimal

* V. N. Aseev et al., Phys. Rev. D 84 (2011) 112003
† Ch. Kraus et al., Eur. Phys. J. C 40 (2005) 447-468

Recent $0\nu\beta\beta$ decay searches

- Power constrained limits on 90% C.L. exclusion sensitivity
- GERDA exclusion sensitivity*:
  $\hat{T}^{0\nu}_{1/2}(^{76}\text{Ge}) = 4.0 \cdot 10^{25}$ yr
  $\Rightarrow \hat{m}_{\beta\beta} = 200 - 430$ meV
- KamLAND-Zen exclusion sensitivity†:
  $\hat{T}^{0\nu}_{1/2}(^{136}\text{Xe}) = 5.6 \cdot 10^{25}$ yr
  $\Rightarrow \hat{m}_{\beta\beta} = 84 - 230$ meV

* GERDA Collaboration, Nature 544 (2017) 47-52
† A. Gando et al., Phys. Rev. Lett. 117 (2016) 082503

Cosmological observations

- Sensitive to: $\Sigma = m_1 + m_2 + m_3$
- Several limits in 10–100 meV range
  $\Rightarrow$ Results depend on considered models and data sets
- We evaluate the DP with/without the cosmological limit

Bonus: Statistical Formulation

Likelihood

$$\mathcal{L} = \mathcal{L}\left(\mathcal{D}_{osc}|\Delta m^2_{21}\right) \cdot \mathcal{L}\left(\mathcal{D}_{osc}|\Delta m^2_{31}/\Delta m_{23}\right) \cdot \mathcal{L}\left(\mathcal{D}_{osc}|s^2_{12}\right) \cdot \mathcal{L}\left(\mathcal{D}_{osc}|s^2_{13}\right) \cdot \mathcal{L}\left(\mathcal{D}_{Troitsk}|m_\beta\right) \cdot \mathcal{L}\left(\mathcal{D}_{0\nu\beta\beta}|m_{\beta\beta}\right)$$

- $\mathcal{D}_{osc}$ = posteriors from nu-fit
- $\mathcal{D}_{Troitsk}$ = Bayesian limit from Troitsk experiment
- $\mathcal{D}_{0\nu\beta\beta}$ = Power-constrained combination* of GERDA and KZ sensitivities due to presence of background under-fluctuations

* G. Cowan et al., arXiv:1105.2166

Priors

- Masses: neutrino mass scale unknown ⇒ log-flat priors
  - Insensitive to changes in units
  - Non-normalizable: need upper and lower cut-off. OK if experimental data provide them.
- Angles: flat in $[0, 2\pi[$
  - Angles are well known, prior has no effect
- Phases: flat in $[0, 2\pi[$
  - OK for angles
  - Not non-informative
  - Some flavor symmetry model can predict specific values of the Majorana phases!

Bonus: Experimental Sensitivity

Efficiencies

- ε_{ROI} = fraction of $0\nu\beta\beta$ decay events falling in the ROI

$$\varepsilon_{\text{ROI}} = \int_{\text{ROI}} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(E - Q_{\beta\beta})^2}{2\sigma^2}\right) dE$$

- ε_{FV} = active volume / fiducial folume fraction
- $\varepsilon_{sig} = \varepsilon_{MC} \cdot \varepsilon_{cut}$
- ε_{MC} = containment efficiency (fraction of detected $0\nu\beta\beta$ events) from MC
- ε_{cut} = selection efficiency (analysis cuts, escluding FV)

ROI

- Chosen based on **(a)** bkg or **(b)** available publications/slides
- If **(a)**, we obtain an optimal ROI of $\pm 2\ \sigma$ for quasi-background-free experiments, and $\pm 1.4\ \sigma$ for background-dominated experiments
- Special situation for SuperNEMO (see later)

Bonus: Heuristic Counting Analysis

Discovery sensitivity

- Value of T$^{0\nu}_{1/2}$ or $m_{\beta\beta}$ for which an experiment has a 50% change to measure a signal above background with $\geq 3\sigma$ significance
- Computed for T$^{0\nu}_{1/2}$, then converted to a range of $m_{\beta\beta}$ for different NMEs
- Computed as:

$$\hat{T}^{0\nu}_{1/2} = \ln 2 \frac{N_A \mathcal{E}}{m_a S_{3\sigma}(B)} \qquad \text{with } B = \mathcal{B}\mathcal{E}$$

- $S_{3\sigma}(B)$ = Poisson signal expectation at which 50% of identical experiments report a 3σ upwards fluctuation above B
- If B large $\Rightarrow$ $S_{3\sigma}(B) \propto \sqrt{B}$
- If $B \ll 1$ $\Rightarrow$ $S_{3\sigma}(B)$ = constant
- Find number of counts $C_{3\sigma}$ such that $CDF\left(C_{3\sigma}|B\right) = 3\sigma$, with CDF the cumulative of a Poisson distribution of mean B
  $\Rightarrow$ Solve $1 - CDF\left(C_{3\sigma}|S_{3\sigma} + B\right) = 50\%$ to find $S_{3\sigma}$

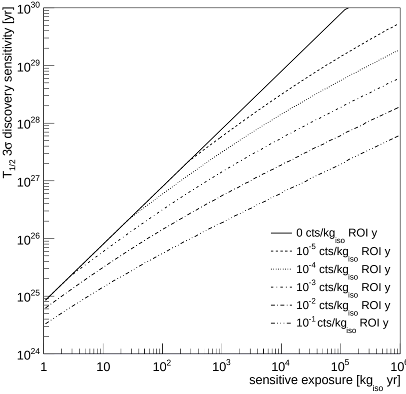
Bonus: Heuristic Counting Analysis

Avoiding discrete jumps

- $C_{3\sigma}$ is integer
  $\Rightarrow$ $S_{3\sigma}$ has discrete jumps
- Define $CDF_{Poisson}$ with normalized upper incomplete gamma function to make it continuous:

$$CDF_{Poisson}(C|\mu) = \frac{\Gamma(C+1|\mu)}{\Gamma(C+1)}$$

- $S_{3\sigma}(B) =$ constant for small B
- $S_{3\sigma}(B)$ smooth and monotonical for increasing B
- Greatly improves computation speed (no toy-MC needed)
- Combination of data sets with different $\mathcal{E}$ and $\mathcal{B}$ results in smooth curve well represented by $S_{3\sigma}(B)$



- Example: ^{76}Ge
- For other isotopes, just rescale by molar mass

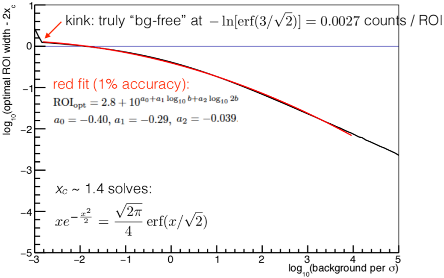
Bonus: Heuristic Counting Analysis

ROI optimization

- For high resolution and flat background around $Q_{\beta\beta}$, maximize figure-of-merit:

$$F.O.M. = \frac{\mathrm{erf}\,(n/2)}{S_{3\sigma}(bn)}$$

- $n =$ ROI half-width in units of σ
- $b =$ background counts per unit σ in 5 years of live time
- Large background: $S_{3\sigma}(bn) \propto \sqrt{(bn)}$ $\Rightarrow$ Optimal ROI: 2.8σ
- Background-free case: numerical solution $\Rightarrow$ Optimal ROI: 4σ



Bonus: NME and Quenching of g_A

Nuclear Matrix Elements

- Consider different nuclear models (QRPA, ISM, IBM-2, EDF)*
- Use average among independent results obtained with the same method
- Run fit for each set of NME separately, then quote max and min

Quenching of g_A

- Axial-vector coupling enters NME as g_A^2
- For some nuclear models, g_A seems to be quenched by up to 30% as a function of Z $\Rightarrow$ Effect on $m_{\beta\beta}$ potentially huge!
- Repeat fit with 30% quenching and quote difference

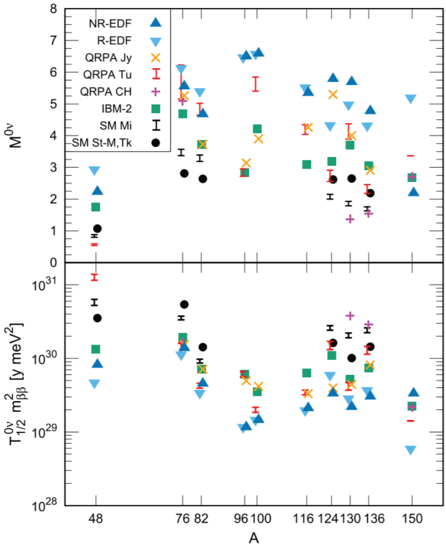


Image from J. Engel and J. Menendéz, Rept. Prog. Phys. 80 (2017) 046301