Dark Matter Searches at the LHC

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on behalf of the ATLAS & CMS Collaborations

Topics in Astroparticle and Underground Physics
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Complementary capabilities for observing DM interactions

Collider Production  Direct Detection (DD)  Indirect Detection (ID)
DM-Nucleon Scattering  DM annihilation

WIMP DM falls under the LHC “lamppost” ...
- DM as a thermal relic implies weak-ish mass/interaction scales

Some of the benefits of collider searches:
- Excellent control of systematic uncertainties
- No velocity / p-wave suppression
- Possible resonant enhancement
- Possibility to characterize DM particle properties
Many BSM models provide DM candidates

- Eg: SUSY (nMSSM, cMSSM, ...), UED/ADD, Little Higgs ...
- Large numbers of parameters, wide range of phenomenology

Can we take a more general approach?

- Perform broad searches based on general DM phenomenology
- Use “models” that are simple as possible, even if they are incomplete (eg: EFT → )
- Turn as large a stone as possible ... mono-X!
Simplest mode of DM production *unobservable* @ LHC

Dark Matter is **DARK**

- Leaves no activity in the detector
- Nothing to trigger on / reconstruct above

“Mono-X” (or “MET+X”) includes “X” for viable detection

- X: quarks/gluons, photons, W/Z …

DM must instead recoil against *something* to become “visible”
Non-interacting particles escape the detector

- Their presence inferred from energy/momentum imbalance

**(Transverse) analog of nuclear recoil in DD …**

- Transverse → because final state particles can be lost in the beampipe

- \( E_T^{\text{miss}} = \) Negative vector sum of all visible pT

A well understood collider observable

- Wide use in SM measurements
Models used in the design and interpretation of DM searches

Need to balance model complexity with predictive accuracy ...

EFTs

UV-complete Models

Validity issues @ LHC ..
cf: 1307.2253, 1308.6799

To specific?
Theory baggage?
Models used in the design and interpretation of DM searches

Need to balance model complexity with predictive accuracy ...

- EFTs
- Simplified Models
- UV-complete Models

Validity issues @ LHC ..
cf: 1307.2253, 1308.6799

Just right?

Too specific?
Theory baggage?
Models used in the design and interpretation of DM searches

Need to balance model complexity with predictive accuracy ...

**Simplified models**: capture kinematics, lack completion

- Pair-produced DM Dirac fermions, $\chi$
- Massive DM $\leftrightarrow$ SM mediator, on/off-shell production
- Couplings: vector/axial/scalar/pseudo
- Minimal flavor violation
- Minimal mediator width: couples only to SM and $\chi$

Only four parameters:
$$g_q, g_{DM}, m_{\chi}, M_{med}$$

LHC DM searches using simplified models/benchmarks from the LHC Dark Matter Forum: 1507.00966
Extraction of potential DM signals ...

In absence of excess: limit setting, model constraints

- NB: 95% CLs limits are standard in collider world

$m(Med)-m(DM)$ plane: provides natural representation of collider results

- Results shown as limit on signal cross section or on signal strength ($\mu = \sigma_{\text{obs}}/\sigma_{\text{th}}$)
- Fixed $g_{DM}$ & $g_{SM}$
- All model assumptions (eg: mediator & DM type) specified
Comparison of collider results with (in)direct detection

- Recent focus of LHC Dark Matter Working Group (DMWG)
- Developed recommendations for collider/non-collider comparison

Translate collider limits to $\sigma_{\text{DM-N}}$ & $\sigma_{\text{rel}}$, rather than reverse

- Avoid subtleties and assumptions involved in mapping DD/ID to collider
- DD: vector/scalar (SI) axial (SD) mediators
- ID: pseudoscalar mediators

Recommendations on presenting LHC searches for missing transverse energy signals using simplified s-channel models of dark matter

Antonio Boveia,1,* Oliver Buchmueller,2,* Giorgio Busoni,3 Francesco D’Eramo,4 Albert De Roeck,1,5 Andrea De Simone,6 Caterina Doglioni,7,* Matthew J. Dolan,3 Marie-Helene Genest,8 Kristian Hahn,9,* Ulrich Haisch,10,11,* Philip C. Harris,1 Jan Heisig,12 Valerio Ippolito,13 Felix Kahlhoefer,14,* Valentin V. Khoze,15 Suchita Kulkarni,16 Greg Landsberg,17 Steven Lowette,18 Sarah Malik,2 Michelangelo Mangano,11,* Christopher McCabe,19,* Stephen Mrenna,20 Priscilla Pani,21 Tristan du Pree,1 Antonio Riotta,11 David Salek,19,22 Kai Schmidt-Hoberg,14 William Shepherd,23 Tim M.P. Tait,24,* Lian-Tao Wang,25 Steven Worm26 and Kathryn Zurek27

Interpretation
Recent ATLAS & CMS DM Results

Focusing on the hadronic search channels

- Monojet, tt/bb + DM, dijet
- In the simplified model framework, these provide most of the DM reach at the LHC

Complete list of recent results in the backups
A generic & powerful DM search strategy at the LHC

- Assumes only that DM couples in someway to incoming quarks
- Require energetic recoiling jet to trigger detector

But no need to limit to a single recoiling jet ...

- The “monojet” search actually targets multijet + $E_{T}^{\text{miss}}$ !
DM + hadronic decays of EWK bosons can also produce a multijet + $E_T^{\text{miss}}$ signature … mono-V

- W/Z decay products will be **boosted** when DM recoil is significant
- Reconstruction algorithms can merge these into a ~small radius jet
- But can use jet grooming / substructure techniques to identify the underlying 2-prong nature
Monojet : general strategy

At least one central (|η| < 2.4), good-quality, high-pT (eg >250 GeV) jet

Require minimum Δφ separation between jets and $E_T^{\text{miss}}$ to suppress misreconstruction BGs.

Veto additional objects: electrons, muons, tau leptons, photons, bjets ...

Significant $E_T^{\text{miss}}$ (eg >200 GeV)

Dominant backgrounds from SM processes with real $E_T^{\text{miss}}$ and/or leptons out of detector acceptance

- $Z(\nu\nu) + \text{jets}$, $W(\tau[qq'] \nu) + \text{jets}$, $W(l\nu) + \text{jets}$
- Bread & butter EWK processes @ the LHC
- Wealth of precise calculations & simulation tools available
Selections define signal enriched regions (SR) in data

- Residual backgrounds in these regions from events in tails of $E_T^{\text{miss}}$ kinematic distributions
- Associated SM theory uncertainties are typically large here ...

BG dominated control regions (CR) help constrain SM rates & kinematics in the SRs

- Augment precise calculations of EW processes with measurements!
SR selection: large $E_T^{\text{miss}}$, 
$\geq 1$ high-$p_T$ jet, $\Delta \phi > 0.5$ radian

- Mono-V: $p_{T}^{\text{AK8}}, E_T^{\text{miss}} > 250$ GeV, $m_{jj}$ 65-105 GeV, $\tau_{12} < 0.6$ (“n-subjettiness”)
- Mono-jet: remaining events, $p_{T}^{\text{AK4}} > 100$ GeV, $E_T^{\text{miss}} > 250$ GeV

5 (categorized) SM control regions to constrain high-$E_T^{\text{miss}}$ BGs

- Use observable analogues of the invisible SM processes
  - $Z(\mu\mu), Z(ee), W(\mu\nu), W(e\nu) + \text{jets}$, high-stat $\gamma + \text{jet}$
- Subtract visible signatures $\rightarrow$ hadronic recoil, a proxy for $E_T^{\text{miss}}$
- Use NLO QCD + NLO EWK calculations to translate rates + distributions in CRs into SR predictions!

Extract signal from combined likelihood fit to $E_T^{\text{miss}}$ distributions
Uncertainties & correlations on transfer factors (see 1705.04664)

- Incorporated as nuisance parameters in the fit
- Pure QCD effects: scale/normalization, recoil shape pT dependence, cross section ratios
- Pure EWK effects: missing NNLO, unknown Sudakov logs, NLL Sudakov approximation
- Combined multiplicatively, nuisance added for possible non-factorization

Control regions fit simultaneously with the signal regions

- Excellent post-fit agreement in CRs
Data in signal region consistent w/ post-fit SM expectations …
Limits on both spin-1 and spin-0 mediators

- Vector/Axial exclusion (this slide) up to 1.8 TeV
- Pseudoscalar (backup) up to 400 GeV
Reinterpret as invisible Higgs: BR( h → inv.) < 0.53 (0.4 exp.)

And recast as limits on SI/SD DM-nucleon cross section (1603.04156)

Low-mDM reach complementary to direct detection!
Similar monojet search strategy pursued in ATLAS:

- \( p_T^{AK4} , E_T^{\text{miss}} > 250 \text{ GeV}, \ \Delta \phi > 0.4 \text{ radian}, \ \text{vetos} \)
- Simultaneous binned likelihood fit to \( E_T^{\text{miss}} \)
- No mono-V category, dedicated mono-W search
- No Z(\text{ee}) + jets, \( \gamma + \text{jets} \) CRs, adds ttbar CR

Good agreement in Z(ll)+jets & W(l\nu)+jets control regions
And good agreement in the signal region ...
Limits on both spin-1 and spin-0 mediators

- Axial-vector exclusion up to 1.55 TeV
- Not yet sensitive to pseudoscalars
Monojet drives sensitivity to spin-1 mediator scenarios

- Picture more nuanced for spin-0 models ...
  - MFV → mediator has Yukawa coupling
  - Monojet through heavy quark loops

- Implies tree-level couplings to top and bottom
  - Same mediator as in monojet
  - Yukawa enhancement → tt+DM competitive with monojet at low mMed!

- Can also anticipate a “monotop” signature ...
  - Assumes specialized signal model (see backup)

DM+ heavy quarks = rich signatures!

- tt final states: all-hadronic, semileptonic, dileptonic
  - Produces leptons, high-pT jets, b jets, $E_{T}^{\text{miss}}$
- Many experimental handles → many viable DM search strategies

Backgrounds: mostly SM ttbar (with a lost lepton), single top, ttV
SUSY stop searches also looking for the \( \text{tt} + E_T^{\text{miss}} \) signature

- These generally involve many SRs & CRs to explore wide range of SUSY scenarios
- Leverage SUSY observables (eg: \( mT^2 \)) optimized for selecting \( E_T^{\text{miss}} \) from decays of heavy particles
- Extend SUSY search with regions that target DM production, add DM interpretation

\[ \text{ATLAS tt/bb + DM} \]

\[ \text{ATLAS-CONF-2016-076 (13.3 fb}^{-1}) \]

\[ \text{ttbar (dilepton) + } E_T^{\text{miss}} \]
Dedicated $bb+E_{T}^{miss}$ search

- Sensitive to models (eg: 2HDM w/ large $\tan\beta$) in which coupling to down-type quarks enhanced
- Select events with large pT imbalance between 2 high-pT b-tagged jets
- 3 CRs to control Z+jets, W+jets and ttbar

Update: $tt$(semileptonic)$+E_{T}^{miss}$ search

- DM categories provide sensitivity to low (~20 GeV) and high (~300 GeV) mass DM mediators
- New SRs use boosted top-tagging discriminant to identify hadronic decays of high-pT top quarks
- ttbar normalized via CR fit, signal extraction from 3 bin cut & count analysis
ATLAS tt/bb + DM Limits

**Scalar**

- **ATLAS Preliminary**
- 

**Pseudoscalar**

- **ATLAS Preliminary**
  - Observed 95% CL
  - Expected 95% CL
  - Theory unc. on $\sigma(g=1.0)$

**bb**

- **ATLAS Preliminary**
  - Expected $\pm 1\sigma$
  - Expected $\pm 2\sigma$

**tt semileptonic**

- **ATLAS Preliminary**
  - Observed 95% CL
  - Expected 95% CL

---

**ATLAS tt/bb + DM Limits**

**Scalar**

- **ATLAS Preliminary**
  - Observed limit, $g=3.5$
  - Expected limit, $g=3.5 (\pm 1\sigma_{\exp})$
  - Scalar mediator

**Pseudoscalar**

- **ATLAS Preliminary**
  - Observed limit, $g=3.5$
  - Expected limit, $g=3.5 (\pm 1\sigma_{\exp})$
  - Scalar mediator
**ATLAS tt/bb + DM Limits**

**Pseudoscalar exclusion for mMed < 220 GeV**
Combined search using all \( \text{tt} + E_T^{\text{miss}} \) and \( \text{bb} + E_T^{\text{miss}} \) channels

- \( E_T^{\text{miss}} > 200 \) for bb & all-hadronic tt, \( E_T^{\text{miss}} > 160 \) GeV for semileptonic tt, \( E_T^{\text{miss}} > 50 \) GeV for dileptonic tt
- Employs novel resolved top quark tagger to reconstruct low/moderate pT hadronic decays
  - Top pT is soft in for mediator masses for which there is LHC sensitivity
  - BG from SM tt with missing lepton
  - Categorize signal and bkg according to number of top tags
- Simultaneous \( E_T^{\text{miss}} \) fit using 8 SRs + 19 CRs

Search uses just 2.2 fb-1 from Run2
- Analysis of full 35.9 fb-1 in progress
CMS tt/bb + DM Limits

2.2 fb⁻¹ (13 TeV)

Scalar, Dirac, \( g_\chi = 1 \), \( m_\chi = 1 \) GeV

- Observed limit 95% CL
- Expected limit 95% CL
- \( b\bar{b} + p_{T}^{miss} \) (\( b\bar{b} + \chi \) only)
- \( b\bar{b} + p_{T}^{miss} \)
- Dileptonic \( t\bar{t} + p_{T}^{miss} \)
- \( 1+\text{jets} + p_{T}^{miss} \)
- All-hadronic \( t\bar{t} + p_{T}^{miss} \)

CMS

Upper limit on \( \mu = \sigma / \sigma_{TH} \)

2.2 fb⁻¹ (13 TeV)

Pseudoscalar, Dirac, \( g_\chi = 1 \), \( m_\chi = 1 \) GeV

- Observed limit 95% CL
- Expected limit 95% CL
- \( b\bar{b} + p_{T}^{miss} \) (\( b\bar{b} + \chi \) only)
- \( b\bar{b} + p_{T}^{miss} \)
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- All-hadronic \( t\bar{t} + p_{T}^{miss} \)

CMS

Upper limit on \( \mu = \sigma / \sigma_{TH} \)

Full combination, scalar

Pseudoscalar

Full combination, pseudoscalar
Direct Mediator Searches

If mediator couples to quarks, then also decay to SM particles

- Search for the DM mediators directly via traditional LHC “bump hunts”
  - Dijet (+ISR), dilepton, di-bjet, etc … eg:

  Dijet : 15.7 fb⁻¹ ATLAS-CONF-2016-069, 27 & 36 fb⁻¹ CMS-PAS-EXO-16-056
  Dijet angular, 3.6 fb⁻¹ (ATLAS) PLB 754 (2016) 302-322, 36 fb⁻¹ CMS-PAS-EXO-16-046
  Boosted dijet : 3.2 fb⁻¹ (bjets) ATLAS-CONF-2016-031, 36 fb⁻¹ CMS-PAS-EXO-17-001
  Dilepton :: 36 fb⁻¹ (ATLAS) 1707.02424, 2.9+19.7 fb⁻¹ (CMS) PLB 768 (2017) 57

- New techniques (data scouting [CMS], Trigger Level Analysis [ATLAS]) allows searches to now push to lower mediator masses

- Dijet search results below …
Comprehensive picture of LHC sensitivity to DM simplified models

- Axial-vector mediator shown here (see ATLAS Exotica Summaries)

Axial-vector mediator, Dirac DM

\[ g_q = 0.25, \ g_\perp = 0, \ g_{DM} = 1 \]

All limits at 95% CL
Collider DM Summaries

Comprehensive picture of LHC sensitivity to DM simplified models

- Axial-vector mediator shown here (see CMS DM Summaries)

![Graph showing mediator mass and dark matter mass](image)

**CMS Preliminary**

**LHCP 2017**

- $M_{\text{Med}} = 2 \times m_{\text{DM}}$
- $\Omega_c \, h^2 \geq 0.12$

**Exclusion at 95% CL**

- **Observed**
- **Expected**

- Dijet (35.9 fb$^{-1}$) [EXO-16-056]
- Boosted dijet (35.9 fb$^{-1}$) [EXO-17-001]
- DM + $j/V(qq)$ (35.9 fb$^{-1}$) [EXO-16-048]
- DM + $\gamma$ (12.9 fb$^{-1}$) [EXO-16-039]
- DM + $Z(\ell\ell)$ (35.9 fb$^{-1}$) [EXO-16-052]
Robust program of $E_T^{\text{miss}} + X$ DM searches at the LHC

Run 2 results pushing into new territory, limits on

- Multi-Tev spin-1 mediators
- Low-mass spin-0 mediators

Complementary strengths vs direct/indirect detection

On the horizon:

- Large bump in stats for several searches
- Stronger interplay between DM channels
- New methods for treating SM systematics (eg: arxiv: 1705.04664)
- Interpretations with somewhat-less-simplified models (eg: 1701.07427)
“SOMEBODY CALL FOR BACKUP!”
## CMS mono-X Searches

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<td>36.1 fb-1</td>
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<td>Z(ll)</td>
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<td>Higgs (yy)</td>
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<td>Higgs (bb), with yy</td>
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<td>t hadronic</td>
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<th>Direct Mediator Production</th>
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## ATLAS Mono-X Searches

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### Direct Mediator Production

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<td>dilepton</td>
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<td>ATLAS-CONF-2017-027</td>
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</table>
Monojet candidate at $\sqrt{s} = 13$ TeV
More CMS summary plots
Vector mediator
Dirac DM
\( g_{DM} = 1.0 \)
\( g_q = 0.25 \)
\( g_l = 0 \)
CMS Preliminary

Axial-vector mediator
Dirac DM
$g_{DM} = 1.0$
$g_q = 0.25$
$g_l = 0$

LHCP 2017

$M_{Med} = 2 \times m_{DM}$

$\Omega_c h^2 \geq 0.12$

Exclusion at 95% CL
- Observed
- Expected

- Dijet (35.9 fb$^{-1}$) [EXO-16-056]
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- DM + $\gamma$ (12.9 fb$^{-1}$) [EXO-16-039]
- DM + Z(II) (35.9 fb$^{-1}$) [EXO-16-052]
More ATLAS summary plots
**DM Simplified Model Exclusions**

**ATLAS Preliminary July 2017**

- **Dijet**
  - $\sqrt{s} = 13$ TeV, 37.0 fb$^{-1}$

- **Dijet 8 TeV**
  - $\sqrt{s} = 8$ TeV, 20.3 fb$^{-1}$

- **Dijet TLA**
  - $\sqrt{s} = 13$ TeV, 3.4 fb$^{-1}$
  - ATLAS-CONF-2016-030

- **Dijet + ISR**
  - $\sqrt{s} = 13$ TeV, 15.5 fb$^{-1}$
  - ATLAS-CONF-2016-070

- **$E_T^{miss} + \gamma$**
  - $\sqrt{s} = 13$ TeV, 36.1 fb$^{-1}$

- **$E_T^{miss} + \text{jet}$**
  - $\sqrt{s} = 13$ TeV, 36.1 fb$^{-1}$
  - ATLAS-CONF-2017-060

- **$E_T^{miss} + Z$**
  - $\sqrt{s} = 13$ TeV, 36.1 fb$^{-1}$
  - ATLAS-CONF-2017-040

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**Vector mediator, Dirac DM**

- $g_q = 0.25$, $g_l = 0$, $g_{DM} = 1$
- All limits at 95% CL
CMS Mono-top

**non-resonant model**

\[ g_V = 0.25 \quad g_\chi = 1 \]

**resonant model**

\[ a_q = b_q = 0.1 \quad a_{1/2} = b_{1/2} = 0.2 \]
CMS tt/bb + DM Limits

Better sensitivity vs monojet for light spin-0 mediators!
SI/SD Translation

\[ \sigma_{SI}^0 = \frac{9 \, g_{DM}^2 \, g_q^2 \, \mu_{n\chi}^2}{\pi \, M_{med}^4} \]

\[ \approx 1.1 \times 10^{-39} \, \text{cm}^2 \cdot \left( \frac{g_{DM} \, g_q}{1} \right)^2 \left( \frac{1 \, \text{TeV}}{M_{med}} \right)^4 \left( \frac{\mu_{n\chi}}{1 \, \text{GeV}} \right)^2 . \]

\[ \sigma_{SD}^0 = \frac{3 \, g_{DM}^2 \, g_q^2 (\Delta_u + \Delta_d + \Delta_s)^2 \, \mu_{n\chi}^2}{\pi \, M_{med}^4} \]

\[ \approx 4.6 \times 10^{-41} \, \text{cm}^2 \cdot \left( \frac{g_{DM} \, g_q}{1} \right)^2 \left( \frac{1 \, \text{TeV}}{M_{med}} \right)^4 \left( \frac{\mu_{n\chi}}{1 \, \text{GeV}} \right)^2 . \]
\[ \Gamma_{\text{min}}^{V} = \frac{g_{\chi}^{2} M_{\text{med}}}{12\pi} \left( 1 + \frac{2m^{2}_{\chi}}{M^{2}_{\text{med}}} \right) \beta_{DM} \theta(M_{\text{med}} - 2m_{\chi}) \]

\[ + \sum_{q} \frac{3g_{q}^{2} M_{\text{med}}}{12\pi} \left( 1 + \frac{2m^{2}_{q}}{M^{2}_{\text{med}}} \right) \beta_{q} \theta(M_{\text{med}} - 2m_{q}) , \]

\[ \Gamma_{\text{min}}^{A} = \frac{g_{\chi}^{2} M_{\text{med}}}{12\pi} \beta_{DM}^{3} \theta(M_{\text{med}} - 2m_{\chi}) \]

\[ + \sum_{q} \frac{3g_{q}^{2} M_{\text{med}}}{12\pi} \beta_{q}^{3} \theta(M_{\text{med}} - 2m_{q}) . \]

\[ \Gamma_{\phi,a} = \sum_{f} N_{c} \frac{y_{f}^{2} g_{q}^{2} m_{\phi,a}}{16\pi} \left( 1 - \frac{4m^{2}_{f}}{m^{2}_{\phi,a}} \right)^{x/2} + \frac{g_{\chi}^{2} m_{\phi,a}}{8\pi} \left( 1 - \frac{4m^{2}_{\chi}}{m^{2}_{\phi,a}} \right)^{x/2} \]

\[ + \frac{\alpha_{s}^{2} y_{i}^{2} g_{q}^{2} m_{\phi,a}^{3}}{32\pi^{3} v^{2}} \left| f_{\phi,a} \left( \frac{4m^{2}_{i}}{m^{2}_{\phi,a}} \right) \right|^{2} \]

\[ f_{\phi}(\tau) = \tau \left[ 1 + (1 - \tau) \arctan^{2} \left( \frac{1}{\sqrt{\tau - 1}} \right) \right] , \]

\[ f_{a}(\tau) = \tau \arctan^{2} \left( \frac{1}{\sqrt{\tau - 1}} \right) \]