CoEPP
ARC Centre of Excellence for Particle Physics at the Terascale

# Standard Model Extensions

coepp.org.au

School of Chemistry & Physics
The University of Adelaide
South Australia 5005
Australia

School of Physics
The University of Sydney
New South Wales 2006
Australia

School of Physics
The University of Melbourne
Victoria 3010
Australia

School of Physics
Monash University
Victoria 3800
Australia

# How to Extend the Standard Model

## Add new matter multiplets

Lorentz scalar or fermion?

Representation under SM gauge symmetries?

- $U(1)$ charge?
- $SU(2)$ representation?
- $SU(3)$ representation?

## Extend gauge group

Unify or contain $G_{SM}$ as factor?

How do SM field transform?

Do we need new matter for anomally cancellation?

- Extra $U(1)$ - sepcify all charges
- Extra $SU(N)$ - specify representation?
- GUTs $SU(5), SO(10), E_6, E_8 \ldots$


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New South Wales 2006
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Victoria 3010
Australia

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# (Real) scalar singlet model

Add new matter field: $S$

Scalar under Lorentz
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Add all renormalisable interactions involving $S$ to SM Lagrangian

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\begin{aligned}
\mathcal{L}_{SSM} &= \mathcal{L}_{SM} + \mathcal{L}_{Z_2} + \mathcal{L}_{\not{Z}_2} \\
\mathcal{L}_{Z_2} &= \partial^\mu S \partial_\mu S + \mu_S^2 S^2 - \lambda_2 S^4 - \lambda_3 S^2 \Phi^\dagger \Phi \\
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$\rightarrow$ scalar potential

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\begin{aligned}
V_{SSM} = & -\mu^2 \Phi \Phi^\dagger + \lambda (\Phi \Phi^\dagger)^2 - \mu_S^2 S^2 + \lambda_2 S^4 + \lambda_3 S^2 \Phi^\dagger \Phi \\
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EWSB: $\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \ \langle S \rangle = \frac{1}{\sqrt{2}} v_s$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}} \left[ v + (\phi_r^0) + (\phi_I^0) \right] \end{pmatrix}, \ S = \frac{v_s + S_r}{\sqrt{2}}$$

# (Real) scalar singlet model $(Z_2)$

$$V_{SSM} \quad = \quad -\mu^2\Phi\Phi^\dagger + \lambda(\Phi\Phi^\dagger)^2 - \mu_S^2 S^2 + \lambda_2 S^4 + \lambda_3 S^2 \Phi^\dagger \Phi$$

$$\left\langle \frac{\partial V_{SSM}}{\partial \mathrm{R}e(\phi^0)} \right\rangle = -\sqrt{2}\mu^2 v + \sqrt{2}\lambda v^3 + \frac{\lambda_3}{\sqrt{2}} v_s^2 v \quad \left\langle \frac{\partial V_{SSM}}{\partial S^R} \right\rangle = -\sqrt{2}\mu_s^2 v_s + \sqrt{2}\lambda_2 v_s^3 - \frac{\lambda_3}{\sqrt{2}} v_s v^2$$

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$$\left\langle \frac{\partial V_{SSM}}{\partial \mathrm{Re}(\phi^0)} \right\rangle = -\sqrt{2}\mu^2 v + \sqrt{2}\lambda v^3 + \frac{\lambda_3}{\sqrt{2}} v_s^2 v \qquad \left\langle \frac{\partial V_{SSM}}{\partial S^R} \right\rangle = -\sqrt{2}\mu_s^2 v_s + \sqrt{2}\lambda_2 v_s^3 - \frac{\lambda_3}{\sqrt{2}} v_s v^2$$

$$\boxed{\mu^2 = \lambda v^2 + \frac{\lambda_3}{2} v_s^2}$$

**EWSB Conditions**

$$\boxed{\mu_s^2 = \lambda_2 v_s^2 + \frac{\lambda_3}{2} v^2}$$

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Mass matrix in basis $(\phi^0, S)$:

$$M^2_{h^0,S} = \frac{1}{2}\begin{pmatrix} \frac{\partial^2 V_{SSM}}{(\partial \mathrm{Re}(\phi^0))^2} & \frac{\partial^2 V_{SSM}}{\partial \mathrm{Re}(\phi^0)\partial S} \\ \frac{\partial^2 V_{SSM}}{\partial S \partial \mathrm{Re}(\phi^0)} & \frac{\partial^2 V_{SSM}}{(\partial S)^2} \end{pmatrix},$$

$$= \begin{pmatrix} -\mu^2 + 3\lambda v^2 + \frac{\lambda_3}{2} v_s^2 & \lambda_3 v v_s \\ \lambda_3 v v_s & -\mu_s^2 + 3\lambda_2 v_s^2 + \frac{\lambda_3}{2} v^2 \end{pmatrix},$$

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&\xrightarrow[\text{EWSB}]{\text{tree-level}} \begin{pmatrix} 2\lambda v^2 & \lambda_3 v_s v \\ \lambda_3 v_s v & 2\lambda_2 v_s^2 \end{pmatrix},
\end{aligned}$$

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$$\xrightarrow[\text{EWSB}]{\text{tree-level}} \begin{pmatrix} 2\lambda v^2 & \lambda_3 v_s v \\ \lambda_3 v_s v & 2\lambda_2 v_s^2 \end{pmatrix},$$

$$m_{h,H} = \lambda v^2 + \lambda_2 v_s^2 \pm \sqrt{(\lambda v^2 - \lambda_2^2 v_s^2)^2 + \lambda_3^2 v_s^2 v^2}$$

Exercise: redo including $Z_2$ violating terms

# scalar singlet + VL quarks (Toy Model)

$$\mathcal{L}_{SSM+VLq} = \mathcal{L}_{SSM} + \overline{\psi}_{FL} \not{D} \psi_{FL} + \overline{\psi}_{FR} \not{D} \psi_{FR} + \mu_F \overline{\psi}_{FL} \psi_{FR} + \kappa S_F \overline{\psi}_{FL} \psi_{FR}$$

| Fields | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $\mathbb{Z}_2$ |
|---|---|---|---|---|
| $\Psi_{FL_1}, \Psi_{FR_1}$ | $\mathbf{3}$ | $\mathbf{2}$ | $\frac{7}{6}$ | $-$ |
| $\Psi_{FL_2}, \Psi_{FR_2}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\frac{2}{3}$ | $-$ |
| $\Psi_{FL_3}, \Psi_{FR_3}$ | $\mathbf{3}$ | $\mathbf{2}$ | $-\frac{5}{6}$ | $-$ |
| $\Psi_{FL_4}, \Psi_{FR_4}$ | $\mathbf{3}$ | $\mathbf{3}$ | $-\frac{1}{3}$ | $-$ |
| $\Psi_{FL_5}, \Psi_{FR_5}$ | $\mathbf{3}$ | $1$ | $\frac{2}{3}$ | $-$ |
| $\Psi_{FL_6}, \Psi_{FR_6}$ | $\mathbf{3}$ | $\mathbf{2}$ | $\frac{1}{6}$ | $-$ |
| $\Psi_{FL_7}, \Psi_{FR_7}$ | $\mathbf{3}$ | $1$ | $-\frac{1}{3}$ | $-$ |
| $\Psi_{FL_8}, \Psi_{FR_8}$ | $\mathbf{3}$ | $1$ | $\frac{5}{3}$ | $-$ |

left handed $\psi_{FL}$

transforms same way as

right handed $\psi_{FR}$

VL = Vector-like - name comes from having vector couplings only (no axial)

$$ig(g_V \gamma^\mu - g_A \gamma^\mu \gamma^5)$$

Vector coupling: $g_V = Q_{\psi_{FL}} + Q_{\psi_{FR}}$ Axial coupling: $g_A = Q_{\psi_{FL}} - Q_{\psi_{FR}}$

The discrete symmetry forbids decays – Toy models

Adding Yukawa interactions for decays to SM matter can reveal strong constraints

# General Two Higgs Doublet model

Extend SM by having two Higgs doublets instead of one

i.e. replace $\Phi$ with $H_1$ and $H_2$

Transform trivially under Lorentz group
Transform as **2** representation of $SU(2)$
Transform under $U(1)_Y$ with hypercharge $Y = \frac{1}{2}$

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$$\mathcal{L}_{\text{Higgs}} = (D_\mu H_1)^\dagger (D^\mu H_1) + (D_\mu H_2)^\dagger (D^\mu H_2) - V(H_1, H_2)$$

$$\begin{aligned} V(H_1, H_2) \quad = \quad & m_1^2 (H_1^\dagger H_1) + m_2^2 (H_2^\dagger H_2) - (m_3^2 H_1^\dagger H_2 + h.c.) + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 \\ & + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + \frac{1}{2}[\lambda_5 (H_1^\dagger H_2)^2 + h.c.] \\ & + [\lambda_6 (H_1^\dagger H_1)(H_1^\dagger H_2) + \lambda_7 (H_2^\dagger H_2)(H_1^\dagger H_2) + h.c.] \end{aligned}$$

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$$\begin{aligned} \mathcal{L}_Y \quad &= \quad Y_u^{ij} Q_i \overline{u}_j H_1 + Y_d^{ij} Q_i \overline{d}_j (i\sigma_2 H_1^*) + Y_e^{ij} L_i \overline{e}_j (i\sigma_2 H_1^*) \\ &\quad + \tilde{Y}_u^{ij} Q_i \overline{u}_j H_2 + \tilde{Y}_d^{ij} Q_i \overline{d}_j (i\sigma_2 H_2^*) + \tilde{Y}_e^{ij} L_i \overline{e}_j (i\sigma_2 H_2^*) \end{aligned}$$

Exercise: rewrite for $H_1$ and $H_2$ with opposite hypercharge

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Cannot simultaneously transform $Y_u$ and $\tilde{Y}_u$ etc

$\Rightarrow$ Flavor Changing Neutral Currents

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Imposing symmetries can help us again

# Type I 2HDM

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Introduce $Z_2$ symmetry :

- Type I: $H_1 \to -H_1$ (odd); all other fields even.

# Type II 2HDM

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$$+ \tilde{Y}_u^{ij} Q_i \overline{u}_j H_2 + \cancel{\tilde{Y}_d^{ij} Q_i \overline{d}_j (i\sigma_2 H_2^*)} + \cancel{\tilde{Y}_e^{ij} L_i \overline{e}_j (i\sigma_2 H_2^*)}$$

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# Flipped 2HDM

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# Lepton Specific 2HDM

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- Lepton specific: $H_1, e_j$ odd; all other fields even

# Two Higgs Doublet models

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Each option suppresses three of the Yukawa matrices

# Two Higgs Doublet models

So the 2HDM is not automatically killed by FCNC

What does the Higgs sector look like?

In SM we have one doublet $\longrightarrow$ 4 degrees of freedom
$\longrightarrow$ 3 Goldstone bosons, 1 Higgs boson

In 2HDM we have two doublets $\longrightarrow$ 8 degrees of freedom
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$$\begin{aligned} V(H_1,H_2) \;=\;& m_1^2(H_1^\dagger H_1) + m_2^2(H_2^\dagger H_2) - (m_3^2 H_1^\dagger H_2 + h.c.) + \frac{1}{2}\lambda_1(H_1^\dagger H_1)^2 + \frac{1}{2}\lambda_2(H_2^\dagger H_2)^2 \\ & + \lambda_3(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) + \frac{1}{2}[\lambda_5(H_1^\dagger H_2)^2 + h.c.] \\ & + [\lambda_6(H_1^\dagger H_1)(H_1^\dagger H_2) + \lambda_7(H_2^\dagger H_2)(H_1^\dagger H_2) + h.c.] \end{aligned}$$

# Two Higgs Doublet models

So the 2HDM is not automatically killed by FCNC

What does the Higgs sector look like?

In SM we have one doublet ⟶ 4 degrees of freedom
⟶ 3 Goldstone bosons, 1 Higgs boson

In 2HDM we have two doublets ⟶ 8 degrees of freedom
⟶ 3 Goldstone bosons, 5 Higgs bosons

$$\begin{aligned} V(H_1, H_2) &= m_1^2 (H_1^\dagger H_1) + m_2^2 (H_2^\dagger H_2) - (m_3^2 H_1^\dagger H_2 + h.c.) + \frac{1}{2}\lambda_1 (H_1^\dagger H_1)^2 + \frac{1}{2}\lambda_2 (H_2^\dagger H_2)^2 \\ &+ \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + \frac{1}{2}[\lambda_5 (H_1^\dagger H_2)^2 + h.c.] \\ &+ [\lambda_6 (H_1^\dagger H_1)(H_1^\dagger H_2) + \lambda_7 (H_2^\dagger H_2)(H_1^\dagger H_2) + h.c.] \end{aligned}$$

$$\textbf{EWSB:}\ \langle H_1 \rangle = \tfrac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix},\ \langle H_2 \rangle = \tfrac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \qquad H_i = \begin{pmatrix} H_i^+ \\ \frac{1}{\sqrt{2}} \left[ v_i + \mathrm{R}e(H_i^0) + \mathrm{I}m(H_i^0) \right] \end{pmatrix},$$

$$\textbf{Define:}\ v^2 = v_1^2 + v_2^2,\ \ \tan\beta = \tfrac{v_2}{v_1} \Rightarrow v_1 = v c_\beta,\ \ v_2 = v s_\beta$$

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$$\textbf{Define:}\ v^2=v_1^2+v_2^2,\ \ \tan\beta=\tfrac{v_2}{v_1}\Rightarrow v_1=vc_\beta,\ \ v_2=vs_\beta$$

$$\textbf{Also:}\ m_A^2=\frac{\mathrm{Re}(m_3^2)}{s_\beta c_\beta},\qquad \lambda_j^R=\mathrm{Re}(\lambda_j),\ \ \lambda_j^I=\mathrm{Im}(\lambda_j),\ \ \Sigma_\lambda=\lambda_3+\lambda_4+\lambda_5^R$$

# Two Higgs Doublet models

$$\begin{aligned} V(H_1,H_2) &= m_1^2(H_1^\dagger H_1) + m_2^2(H_2^\dagger H_2) - (m_3^2 H_1^\dagger H_2 + h.c.) + \frac{1}{2}\lambda_1(H_1^\dagger H_1)^2 + \frac{1}{2}\lambda_2(H_2^\dagger H_2)^2 \\ &+ \lambda_3(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) + \frac{1}{2}[\lambda_5(H_1^\dagger H_2)^2 + h.c.] \\ &+ [\lambda_6(H_1^\dagger H_1)(H_1^\dagger H_2) + \lambda_7(H_2^\dagger H_2)(H_1^\dagger H_2) + h.c.] \end{aligned}$$

$$\left\langle \frac{\partial V}{\partial \mathrm{Re}(H_1^0)} \right\rangle = v c_\beta \left[ m_1^2 - m_A^2 s_\beta^2 + \frac{v^2}{2}(\lambda_1^2 c_\beta^2 + \Sigma_\lambda s_\beta^2 + 3\lambda_6^R s_\beta c_\beta + \lambda_7^R \frac{s_\beta^3}{c_\beta}) \right]$$

$$\left\langle \frac{\partial V}{\partial \mathrm{Re}(H_2^0)} \right\rangle = v s_\beta \left[ m_2^2 - m_A^2 c_\beta^2 + \frac{v^2}{2}(\lambda_2^2 s_\beta^2 + \Sigma_\lambda c_\beta^2 + 3\lambda_6^R \frac{c_\beta^3}{s_\beta} + \lambda_7^R s_\beta c_\beta) \right]$$

$$\left\langle \frac{\partial V}{\partial \mathrm{I}m(H_1^0)} \right\rangle = v s_\beta \left[ \mathbf{Im}(m_3^2) + \frac{v^2}{2}(\lambda_5^I s_\beta c_\beta + \lambda_6^I c_\beta^2 + \lambda_7^I s_\beta^2) \right]$$

$$\left\langle \frac{\partial V}{\partial \mathrm{I}m(H_1^0)} \right\rangle = v c_\beta \left[ \mathbf{Im}(m_3^2) + \frac{v^2}{2}(\lambda_5^I s_\beta c_\beta + \lambda_6^I c_\beta^2 + \lambda_7^I s_\beta^2) \right]$$

Note: if there are no imaginary components to parameters: $\left\langle \frac{\partial^2 V}{\partial \mathrm{I}m(H_i^0)\mathrm{Re}(H_j^0)} \right\rangle = 0$

# Two Higgs Doublet models

$$
\begin{aligned}
V(H_1, H_2) \quad = \quad & m_1^2(H_1^\dagger H_1) + m_2^2(H_2^\dagger H_2) - (m_3^2 H_1^\dagger H_2 + h.c.) + \frac{1}{2}\lambda_1(H_1^\dagger H_1)^2 + \frac{1}{2}\lambda_2(H_2^\dagger H_2)^2 \\
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\end{aligned}
$$

$$
\begin{pmatrix} \mathrm{Re}(H_2^0) \\ \mathrm{Re}(H_1^0) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} c_\alpha & s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}
$$

$$
\begin{pmatrix} \mathrm{Im}(H_2^0) \\ \mathrm{Im}(H_1^0) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_{\beta^0} & s_{\beta^0} \\ s_{\beta^0} & c_{\beta^0} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}
$$

$$
\begin{pmatrix} H_2^+ \\ H_1^+ \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_{\beta^\pm} & s_{\beta^\pm} \\ s_{\beta^\pm} & c_{\beta^\pm} \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}
$$

Note: if there are no imaginary components to parameters: $\left\langle \frac{\partial^2 V}{\partial \mathrm{Im}(H_i^0)\mathrm{Re}(H_j^0)} \right\rangle = 0$

# Two Higgs Doublet models

$$
\begin{aligned}
V(H_1,H_2) \quad = \quad & m_1^2(H_1^\dagger H_1) + m_2^2(H_2^\dagger H_2) - (m_3^2 H_1^\dagger H_2 + h.c.) + \frac{1}{2}\lambda_1(H_1^\dagger H_1)^2 + \frac{1}{2}\lambda_2(H_2^\dagger H_2)^2 \\
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\end{aligned}
$$

$$
\begin{aligned}
M^2_{(H^+)^\dagger H^+} \quad & = \quad \begin{pmatrix} \dfrac{\partial^2 V}{\partial(H_1^+)^\dagger)\partial H_1^+)} & \dfrac{\partial^2 V}{\partial(H_1^+)^\dagger)\partial H_2^+)} \\ \dfrac{\partial^2 V}{\partial(H_2^+)^\dagger)\partial H_1^+)} & \dfrac{\partial^2 V}{\partial(H_2^+)^\dagger)\partial H_2^+)} \end{pmatrix} \\
& \xrightarrow[\text{EWSB}]{\text{tree-level}} \quad \left[m_3^2 - (\lambda_4 + \lambda_5^R - \frac{\lambda_6^R}{t_\beta} - \lambda_7^R t_\beta) v^2 s_\beta c_\beta\right] \begin{pmatrix} t_\beta & -1 \\ -1 & \frac{1}{t_\beta} \end{pmatrix},
\end{aligned}
$$

# Two Higgs Doublet models

$$V(H_1,H_2) = m_1^2(H_1^\dagger H_1) + m_2^2(H_2^\dagger H_2) - (m_3^2 H_1^\dagger H_2 + h.c.) + \frac{1}{2}\lambda_1(H_1^\dagger H_1)^2 + \frac{1}{2}\lambda_2(H_2^\dagger H_2)^2$$
$$+\lambda_3(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) + \frac{1}{2}[\lambda_5(H_1^\dagger H_2)^2 + h.c.]$$
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$$M^2_{(H^+)^\dagger H^+} = \begin{pmatrix} \frac{\partial^2 V}{\partial (H_1^+)^\dagger)\partial H_1^+)} & \frac{\partial^2 V}{\partial (H_1^+)^\dagger)\partial H_2^+)} \\ \frac{\partial^2 V}{\partial (H_2^+)^\dagger)\partial H_1^+)} & \frac{\partial^2 V}{\partial (H_2^+)^\dagger)\partial H_2^+)} \end{pmatrix}$$
$$\xrightarrow[\text{EWSB}]{\text{tree-level}} \left[m_3^2 - (\lambda_4 + \lambda_5^R - \frac{\lambda_6^R}{t_\beta} - \lambda_7^R t_\beta)v^2 s_\beta c_\beta\right] \begin{pmatrix} t_\beta & -1 \\ -1 & \frac{1}{t_\beta} \end{pmatrix},$$

Note: linear dependence between rows, one eigenvalue is zero

As per Goldstone's theorem we have two charged Goldstone degrees of freedom

And two charged Higgs boson degrees of freedom, with mass:

$$m_{H^\pm}^2 = m_A^2 - \frac{v^2}{2}(\lambda_4 + \lambda_5^R - \frac{\lambda_6^R}{t_\beta} - \lambda_7 t_\beta)$$

# Two Higgs Doublet models

$$V(H_1,H_2) = m_1^2(H_1^\dagger H_1) + m_2^2(H_2^\dagger H_2) - (m_3^2 H_1^\dagger H_2 + h.c.) + \frac{1}{2}\lambda_1(H_1^\dagger H_1)^2 + \frac{1}{2}\lambda_2(H_2^\dagger H_2)^2$$
$$+\lambda_3(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) + \frac{1}{2}[\lambda_5(H_1^\dagger H_2)^2 + h.c.]$$
$$+[\lambda_6(H_1^\dagger H_1)(H_1^\dagger H_2) + \lambda_7(H_2^\dagger H_2)(H_1^\dagger H_2) + h.c.]$$

Setting all imaginary components of parameters to zero

$$M^2_{\mathrm{Im}(H_1^0)\mathrm{Im}(H_2^0)} = \frac{1}{2}\begin{pmatrix} \frac{\partial^2 V}{\partial \mathrm{Im}(H_1^0)\partial \mathrm{Im}(H_1^0)} & \frac{\partial^2 V}{\partial \mathrm{Im}(H_1^0)\partial \mathrm{Im}(H_2^0)} \\ \frac{\partial^2 V}{\partial \mathrm{Im}(H_2^0)\partial \mathrm{Im}(H_1^0)} & \frac{\partial^2 V}{\partial \mathrm{Im}(H_2^0)\partial \mathrm{Im}(H_2^0)} \end{pmatrix}$$
$$\xrightarrow[\text{EWSB}]{\text{tree-level}} m_3^2 \begin{pmatrix} t_\beta & -1 \\ -1 & \frac{1}{t_\beta} \end{pmatrix},$$

Note: linear dependence between rows, one eigenvalue is zero

As per Goldstone's theorem 1 neutral CP-odd Goldstone degree of freedom (longitudinal mode of Z boson )

One physical pseudoscalar boson, with mass: $\mathrm{m}_A^2 = \frac{\mathrm{Re}(m_3^2)}{s_\beta c_\beta}$

# Two Higgs Doublet models

$$
\begin{aligned}
V(H_1,H_2) \quad = \quad & m_1^2(H_1^\dagger H_1)+m_2^2(H_2^\dagger H_2)-(m_3^2 H_1^\dagger H_2+h.c.)+\frac{1}{2}\lambda_1(H_1^\dagger H_1)^2+\frac{1}{2}\lambda_2(H_2^\dagger H_2)^2 \\
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$$

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$$
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& \xrightarrow[\text{EWSB}]{\text{tree-level}} \begin{pmatrix} -m_3^2 t_\beta - \lambda_1 v^2 c_\beta^2 & m_3^2 - \Sigma_\lambda v^2 c_\beta s_\beta \\ m_3^2 - \Sigma_\lambda v^2 c_\beta s_\beta & -\frac{m_3^2}{t_\beta} - \lambda_2 v^2 s_\beta^2 \end{pmatrix},
\end{aligned}
$$

Rows are linearly independent, two massive CP even Higgs bosons

$$
\begin{pmatrix} \mathrm{Re}(H_2^0) \\ \mathrm{Re}(H_1^0) \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} c_\alpha & s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}\begin{pmatrix} h \\ H \end{pmatrix}
$$

# More models...

- Minimal Dark Matter: Add single $SU(2)$ multiplet (fermion or scalar) minimally couple to SM through gauge interactions fixed by specifying reprsentation and hypercharge
- Higgs Portal Dark Matter: Dark matter candidate couples to SM through quartic Higgs intecations.
- Manohar-Wise: adds scalar $SU(3)$ Octets
- $SU(2)$ triplet models: e.g. Georgi-Machacek model, add new matter in form of triplet representain of $SU(2)$
- 4th Generation, extend SM by a 4th generation (heavily constrained)
- 2HDM + singlet
- ...

There are many ways to add matter multiplets
a bewildering array of possibilities
Look for a guiding principal!

# Gauge Extensions

Adding new matter multiplets is not very constrained… many possibilities

Maybe new gauge extensions will be simpler...

## Extra $U(1)$ gauge symmetry

Gauge group: $G_{USM} = G_{SM} \times U(1)' = SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)'$

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The two $U(1)$ symmetries can mix.
If $Y_i Q'_i \neq 0, \Rightarrow$ mixing generated through loops

$$\mathcal{L}^{\text{abelian}}_{\text{gauge}} = F^{\mu\nu} F_{\mu\nu} + F'^{\mu\nu} F'_{\mu\nu} - \frac{\sin\chi}{2} F^{\mu\nu} F'_{\mu\nu}$$

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ F'_{\mu\nu} &= \partial_\mu B'_\nu - \partial_\nu B'_\mu \end{aligned}$$

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$$\mathcal{L}_{\text{gauge}}^{\text{abelian}} = F_1^{\mu\nu} F_{1\mu\nu} + F_2^{\mu\nu} F_{2\mu\nu}$$

Diagonalise with non-unitary transformation

$$\begin{aligned} B_\mu &= B_{1\mu} - B_{2\mu} \tan\chi \\ B'_\mu &= \frac{B_{2\mu}}{\cos\chi} \end{aligned}$$

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$$\mathcal{L}^{\text{abelian}}_{\text{gauge}} = F_1^{\mu\nu}F_{1\mu\nu} + F_2^{\mu\nu}F_{2\mu\nu}$$

$$\begin{aligned} D_\mu &= \partial_\mu + igW^A_\mu\frac{\sigma^A}{2} + ig_Y Y B_\mu + ig_{Q'}Q'B'_\mu \\ &= \partial_\mu + igW^A_\mu\frac{\sigma^A}{2} + ig_Y Y B_{1\mu} + i\left(\frac{g_{Q'}}{\cos\chi}Q' - g_Y\tan\chi Y_i\right)B_{2\mu} \end{aligned}$$

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Extra gauge symmetry must be broken to avoid massless gauge boson

$\rightarrow$ Add extra Higgs

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$$\mathcal{L}_{USM} = \mathcal{L}_{\text{gauge}}^{\text{abelian}} + \mathcal{L}_{\text{gauge}}^{\text{non-abelian}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{fermion}}$$

$$\mathcal{L}_{\text{Higgs}} = |D_\mu \Phi|^2 + |D_\mu S|^2 + \mu^2 \Phi\Phi^\dagger - \lambda(\Phi\Phi^\dagger)^2 + \mu_S^2 |S|^2 - \lambda_2 |S|^4 - \lambda_3 |S|^2 \Phi^\dagger \Phi$$

$S$: complex scalar
singlet under $G_{SM}$
charged under $U(1)$

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$$\mathcal{L}_{\text{Higgs}} \quad = \quad |D_\mu \Phi|^2 + |D_\mu S|^2 + \mu^2 \Phi\Phi^\dagger - \lambda(\Phi\Phi^\dagger)^2 + \mu_S^2 |S|^2 - \lambda_2 |S|^4 - \lambda_3 |S|^2 \Phi^\dagger \Phi$$

$S$: complex scalar
singlet under $G_{SM}$
charged under $U(1)$

$$\langle |D_\mu S|^2 \rangle \quad \sim \quad (g_1' Q_S v_s)^2 B_{2\mu}^2$$

$$\begin{aligned} A_\mu &= s_{\theta_W} W_\mu^3 + c_{\theta_W} B_{1\mu} \\ Z_\mu &= c_{\theta_W} W_\mu^3 - s_{\theta_W} B_{1\mu} \\ Z'_\mu &= B_{2\mu} \end{aligned}$$

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$\rightarrow$ Add extra Higgs

$$\mathcal{L}_{USM} = \mathcal{L}_{\text{gauge}}^{\text{abelian}} + \mathcal{L}_{\text{gauge}}^{\text{non-abelian}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{fermion}}$$

$$\mathcal{L}_{\text{Higgs}} = |D_\mu \Phi|^2 + |D_\mu S|^2 + \mu^2 \Phi\Phi^\dagger - \lambda(\Phi\Phi^\dagger)^2 + \mu_S^2 |S|^2 - \lambda_2 |S|^4 - \lambda_3 |S|^2 \Phi^\dagger \Phi$$

$S$: complex scalar
singlet under $G_{SM}$
charged under $U(1)$

$$\begin{aligned} \langle |D_\mu S|^2 \rangle &\sim (g_1' Q_S v_s)^2 B_{2\mu}^2 \\ \langle |D_\mu \Phi|^2 \rangle &\sim \begin{pmatrix} 0 & v \end{pmatrix} (g_1' \tilde{Q}_\Phi B_{2\mu} + g_Y Y_\Phi B_{1\mu} \\ &\quad + g \sigma^A W_\mu^A)^2 v^2 \begin{pmatrix} 0 \\ v \end{pmatrix} \end{aligned}$$

$$\begin{aligned} A_\mu &= s_{\theta_W} W_\mu^3 + c_{\theta_W} B_{1\mu} \\ Z_\mu &= c_{\theta_W} W_\mu^3 - s_{\theta_W} B_{1\mu} \\ Z'_\mu &= B_{2\mu} \end{aligned}$$

$Z - Z'$ mixing

# U(1) Gauge Extension

Gauge group: $G_{USM} = G_{SM} \times U(1)' = SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)'$

Must specify the charges, $Q'_\psi$, for every matter field $\psi$

| | $Q_i$ | $\overline{u}_i$ | $\overline{d}_i$ | $L_i$ | $\overline{e}_i$ | $S$ | $\Phi$ |
|---|---|---|---|---|---|---|---|
| $SU(3)_C$ | $3$ | $\overline{3}$ | $\overline{3}$ | $1$ | $1$ | $1$ | $1$ |
| $SU(2)_L$ | $2$ | $1$ | $1$ | $2$ | $1$ | $1$ | $2$ |
| $Y_i$ | $\frac{1}{6}$ | $-\frac{2}{3}$ | $\frac{1}{3}$ | $-\frac{1}{2}$ | $1$ | $0$ | $\frac{1}{2}$ |
| $Q'_i$ | $Q'_Q$ | $Q'_u$ | $Q'_d$ | $Q'_L$ | $Q'_e$ | $Q'_S$ | $Q'_\Phi$ |

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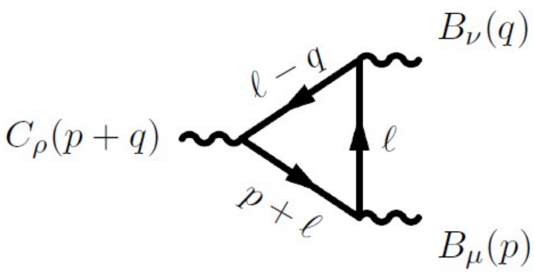
| | $Q_i$ | $\overline{u}_i$ | $\overline{d}_i$ | $L_i$ | $\overline{e}_i$ | $S$ | $\Phi$ |
|---|---|---|---|---|---|---|---|
| $SU(3)_C$ | 3 | $\overline{3}$ | $\overline{3}$ | 1 | 1 | 1 | 1 |
| $SU(2)_L$ | 2 | 1 | 1 | 2 | 1 | 1 | 2 |
| $Y_i$ | $\frac{1}{6}$ | $-\frac{2}{3}$ | $\frac{1}{3}$ | $-\frac{1}{2}$ | 1 | 0 | $\frac{1}{2}$ |
| $Q'_i$ | $Q'_Q$ | $Q'_u$ | $Q'_d$ | $Q'_L$ | $Q'_e$ | $Q'_S$ | $Q'_\Phi$ |

Charges constrained by Lagrangian and cancellation of gauge anomalies

$$\begin{aligned} Q'_\Phi + Q'_Q + Q'_u &= 0 \\ Q'_\Phi + Q'_Q + Q'_s &= 0 \\ Q'_\Phi + Q'_L + Q'_e &= 0 \end{aligned}$$




$$\begin{aligned} U(1)' - U(1)' - U(1)' : \Sigma_f Q_f^3 &= 0 \\ U(1)' - U(1)' - U(1)_Y : \Sigma_f Q_f^2 Y_f &= 0 \\ U(1)' - U(1)_Y - U(1)_Y : \Sigma_f Q_f Y_f^2 &= 0 \\ U(1)' - SU(2) - SU(2) : \Sigma_f Q_f &= 0 \\ U(1)' - SU(3) - SU(3) : \Sigma_f Q_f &= 0 \end{aligned}$$

$f$ sums over fermions in the loop

# U(1) Gauge Extension

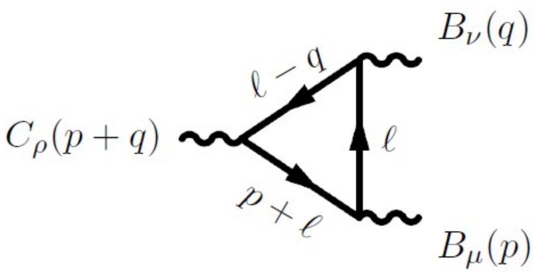
Gauge group: $G_{USM} = G_{SM} \times U(1)' = SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)'$

**Example: $U(1)^{B-L}$**

| | $Q_i$ | $\overline{u}_i$ | $\overline{d}_i$ | $L_i$ | $\overline{e}_i$ | $S$ | $\Phi$ | $\nu_R$ |
|---|---|---|---|---|---|---|---|---|
| $SU(3)_C$ | $3$ | $\overline{3}$ | $\overline{3}$ | $1$ | $1$ | $1$ | $1$ | $1$ |
| $SU(2)_L$ | $2$ | $1$ | $1$ | $2$ | $1$ | $1$ | $2$ | $1$ |
| $Y_i$ | $\frac{1}{6}$ | $-\frac{2}{3}$ | $\frac{1}{3}$ | $-\frac{1}{2}$ | $1$ | $0$ | $\frac{1}{2}$ | $0$ |
| $Q'_i$ | $\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $-1$ | $-1$ | $2$ | $0$ | $-1$ |

**Charges constrained by Lagrangian and cancellation of gauge anomalies**

$$\begin{aligned} Q'_\Phi + Q'_Q + Q'_u &= 0 \\ Q'_\Phi + Q'_Q + Q'_s &= 0 \\ Q'_\Phi + Q'_L + Q'_e &= 0 \end{aligned}$$

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$f$ sums over fermions in the loop

# U(1) Gauge Extension

Gauge group: $G_{USM} = G_{SM} \times U(1)' = SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)'$

**Example: $E_6$ inspired $U(1)'$**

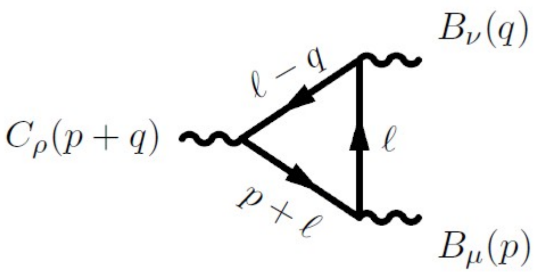
| | $\hat{Q}_i$ | $\hat{u}_i^c$ | $\hat{d}_i^c$ | $\hat{L}_i$ | $\hat{e}_i^c$ | $\hat{D}_i$ | $\hat{\overline{D}}_i$ | $\hat{S}_i$ | $\hat{H}_i^u$ | $\hat{H}_i^d$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $SU(3)_C$ | $3$ | $\overline{3}$ | $\overline{3}$ | $1$ | $1$ | $3$ | $\overline{3}$ | $1$ | $1$ | $1$ |
| $SU(2)_L$ | $2$ | $1$ | $1$ | $2$ | $1$ | $1$ | $1$ | $1$ | $2$ | $2$ |
| $Y_i$ | $\frac{1}{6}$ | $-\frac{2}{3}$ | $\frac{1}{3}$ | $-\frac{1}{2}$ | $1$ | $Y_D$ | $Y_{\overline{D}}$ | $0$ | $Y_{H^u}$ | $Y_{H^d}$ |
| $Q'_i$ | $Q'_Q$ | $Q'_u$ | $Q'_d$ | $Q'_L$ | $Q'_e$ | $Q'_D$ | $Q'_{\overline{D}}$ | $Q'_S$ | $Q'_{H^u}$ | $Q'_{H^d}$ |

**complete $E_6$ 27-plets ensure anomally cancellation automatically**

$\mathrm{E}_6 \to SO(10) \times U(1)_\psi$

$\mathrm{SO}(10) \to SU(5) \times U(1)_\chi$

$\mathrm{U}(1)' = \cos\theta U(1)_\chi + \sin\theta U(1)_\psi$




$$
\begin{aligned}
U(1)' - U(1)' - U(1)' : \Sigma_f Q_f^3 &= 0 \\
U(1)' - U(1)' - U(1)_Y : \Sigma_f Q_f^2 Y_f &= 0 \\
U(1)' - U(1)_Y - U(1)_Y : \Sigma_f Q_f Y_f^2 &= 0 \\
U(1)' - SU(2) - SU(2) : \Sigma_f Q_f &= 0 \\
U(1)' - SU(3) - SU(3) : \Sigma_f Q_f &= 0
\end{aligned}
$$

$f$ sums over fermions in the loop

# More Gauge Extensions..

- add non-abelian extension to SM gauge group. Advantage: no free charges. Disadvantage: still need to break this more complicated gauge structure. Why make gauge sector more complicated?
- Left -right symmetric models: embed SM gauge group in group with $SU(2)_R \times SU(2)_L$ factor.
- Unify SM gauge group into GUT ($SU(5), SO(10), E_6$). Advantage: beutiful, reductionist. Disadvantage : tough constraints from proton decay. New physics scale very heavy. Gauge couplings don't unify.

Even when model building to fit experimental anomalies or discovery there are many options
Need a guiding principal...

Now back to Sujeet …