A *brief* guide to Supersymmetry

More extensive lectures can be found at:
http://www.physics.adelaide.edu.au/cssm/seminars/SUSY/

coepp.org.au

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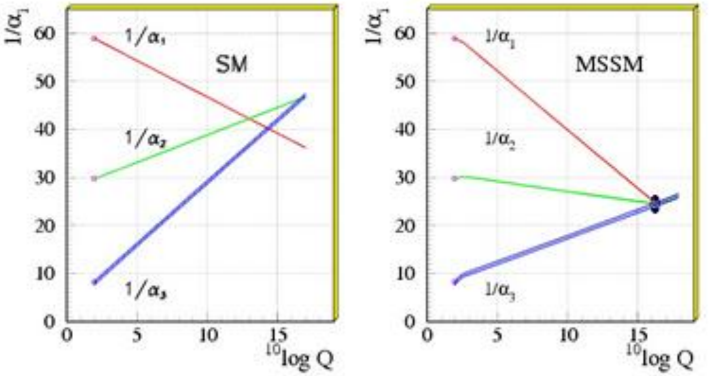
Motivation for SUSY at the LHC

Hierarchy Problem

$$m_h^2 = m_0^2 - \frac{\lambda_t^2}{8\pi^2}(\Lambda^2) \quad \Rightarrow \quad \text{Huge Fine tuning!}$$

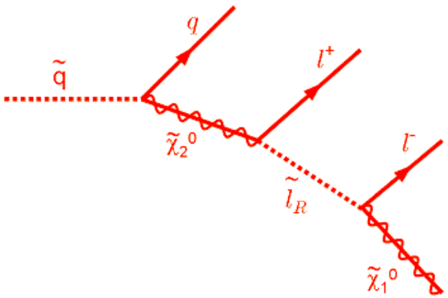
$$\text{SUSY:} \quad +\frac{\lambda_{\tilde{t}}}{16\pi^2}(2\Lambda^2) \quad \lambda_{\tilde{t}} = \lambda_t^2 \quad \Rightarrow \quad \text{Quadratic divergences cancel}$$

Gauge Coupling Unification



Dark-Matter / R-parity

- R-Parity: SM particles even
  SUSY partners odd



Stable LSP
Dark Matter candidate

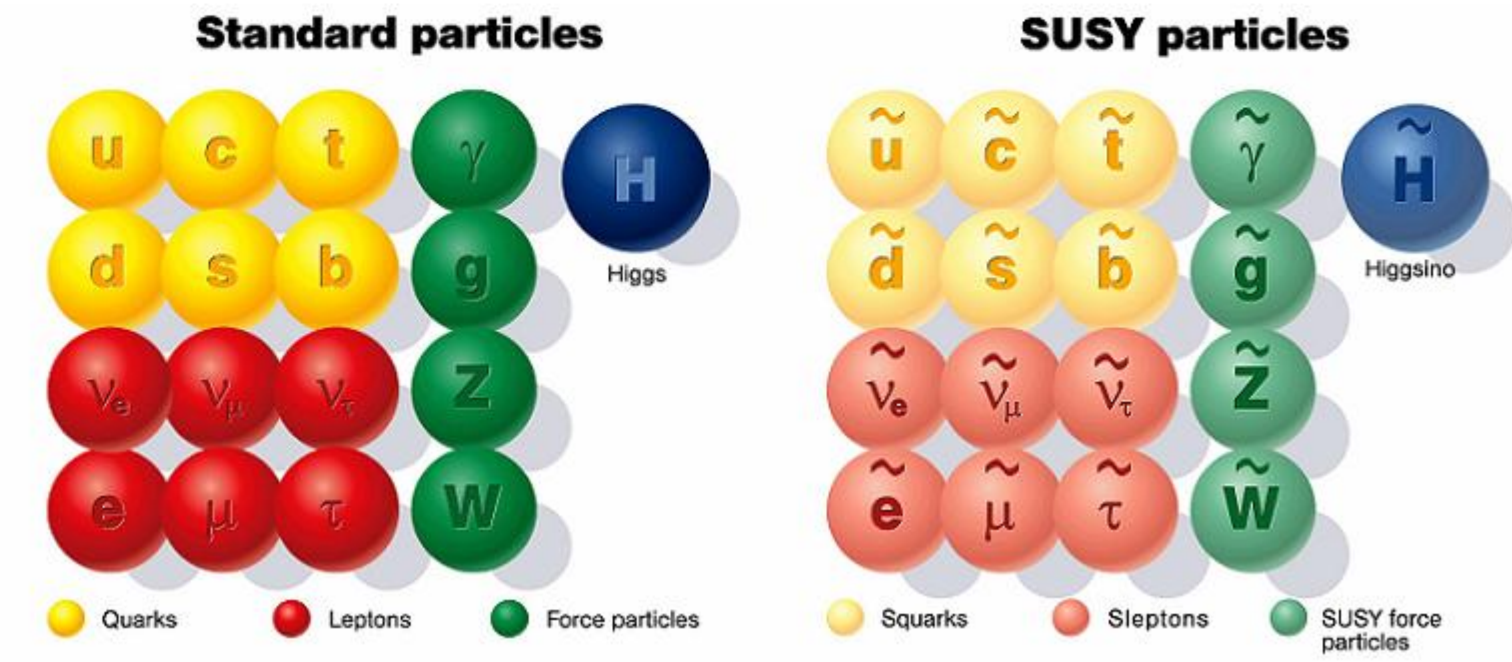
One More Reason

Ideas for new physics for experimentalists/ phenomenologists to work with!

Minimal Supersymmetric Standard Model (MSSM)

The MSSM = minimal particle content compatible with known physics, i.e Standard Model particles and properties.

Basic idea: take SM and supersymmetrise:



Warning: Image not entirely accurate.

Superfield content of the MSSM

Gauge group is that of SM: $G_{SM} \equiv \underset{\text{Strong}}{SU(3)} \times \underset{\text{Weak}}{SU(2)} \times \underset{\text{hypercharge}}{U(1)_Y}$

Vector superfields of the MSSM

Supermultiplet	Gauge	spin 1/2	spin 1	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\hat{G}$	$SU(3)_C$	$\widetilde{g}$	g	$\mathbf{8}$	$\mathbf{1}$	0
$\hat{W}$	$SU(2)_W$	$\widetilde{W}^{\pm}\ \ \widetilde{W}^0$	$W^{\pm}\ \ W^0$	$\mathbf{1}$	$\mathbf{3}$	0
$\hat{B}$	$U(1)_Y$	$\widetilde{B}^0$	B^0	$\mathbf{1}$	$\mathbf{1}$	0

Gauge supermultiplets of the MSSM, and gauge group representations.

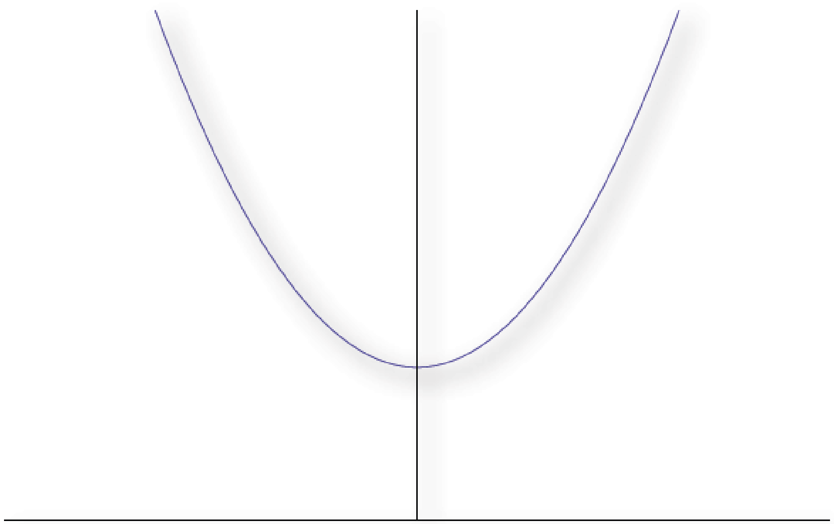
MSSM Chiral Superfield Content

Supermultiplet	spin 0	spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\hat{Q}_i$	$(\widetilde{u}_L \;\; \widetilde{d}_L)_i$	$(u_L \;\; d_L)_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$
$\overline{u}_i$	$\widetilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$
$\overline{d}_i$	$\widetilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$
$\hat{L}_i$	$(\widetilde{\nu} \;\; \widetilde{e}_L)_i$	$(\nu \;\; e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
$\overline{e}_i$	$\widetilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1
$\hat{H}_u$	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$
$\hat{H}_d$	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$

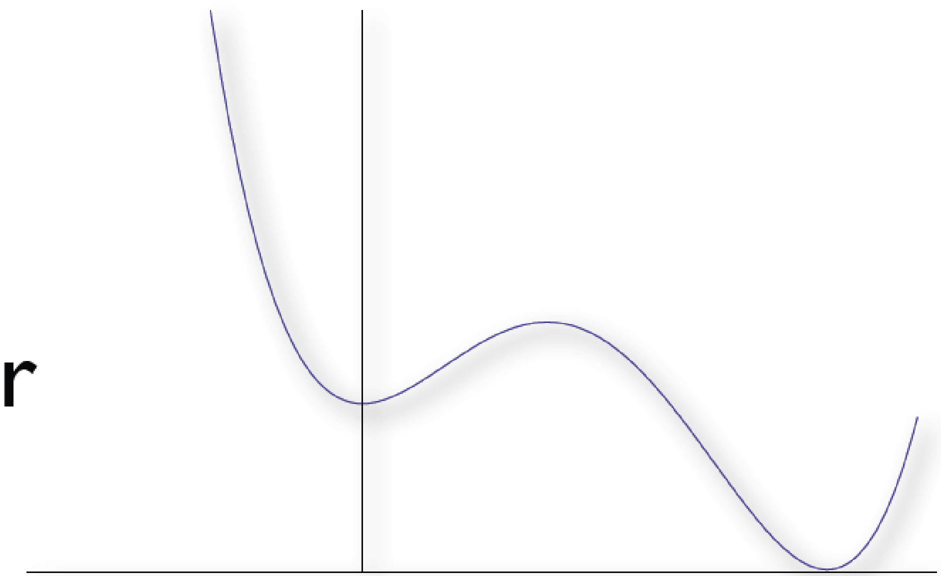
Spontaneous SUSY breaking

Recall: $$H = P^0 = \frac{1}{4}(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2)$$

$Q_\alpha|0\rangle \neq 0 \Rightarrow \langle 0|H|0\rangle > 0$ OR $Q_\alpha|0\rangle = 0 \Rightarrow \langle 0|H|0\rangle = 0$

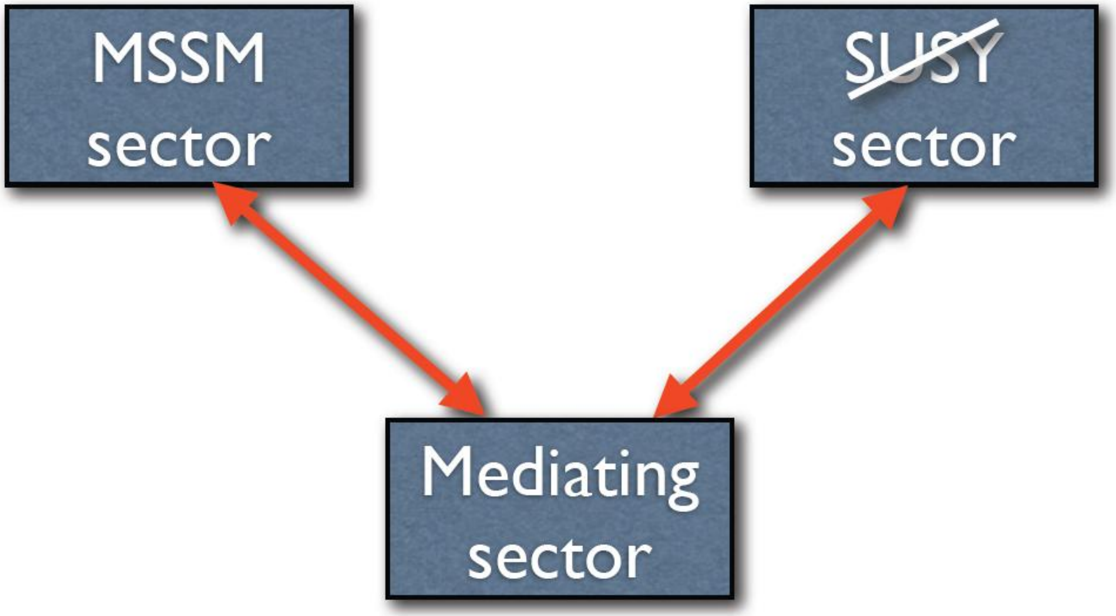


or



$$\boxed{V(\phi, \phi^*) = F^{*i} F_i + \frac{1}{2} \sum_a D^a D^a}$$

Hidden Sector



- Gaugino Masses: $fF\lambda\lambda/M_{Pl} \to M\lambda\lambda$ where $M_a = f_a\langle F\rangle/M_{Pl}$
- soft scalar trilinears: $hF\phi\phi\phi \to a\phi\phi\phi$ where $a = h\langle F\rangle/M_{Pl}$
- soft scalar bilinear $\mu F\phi\phi/M_{Pl} \to b\phi\phi$ where $b = \mu\langle F\rangle/M_{Pl}$
- soft scalar masses $FF^*\kappa\phi^*\phi/M_{Pl}^2 \to (m^2)\phi^*\phi$ where $(m^2) = \kappa\langle F\rangle^2/M_{Pl}^2$

Soft SUSY breaking

- Soft = doesn't break dimensionless coupling relations

  $\Rightarrow$ Maintain solutions to Hierarchy problem + gauge coupling unification

Dimension 3 or less operators only

Soft SUSY breaking

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Dimension 3 or less operators only

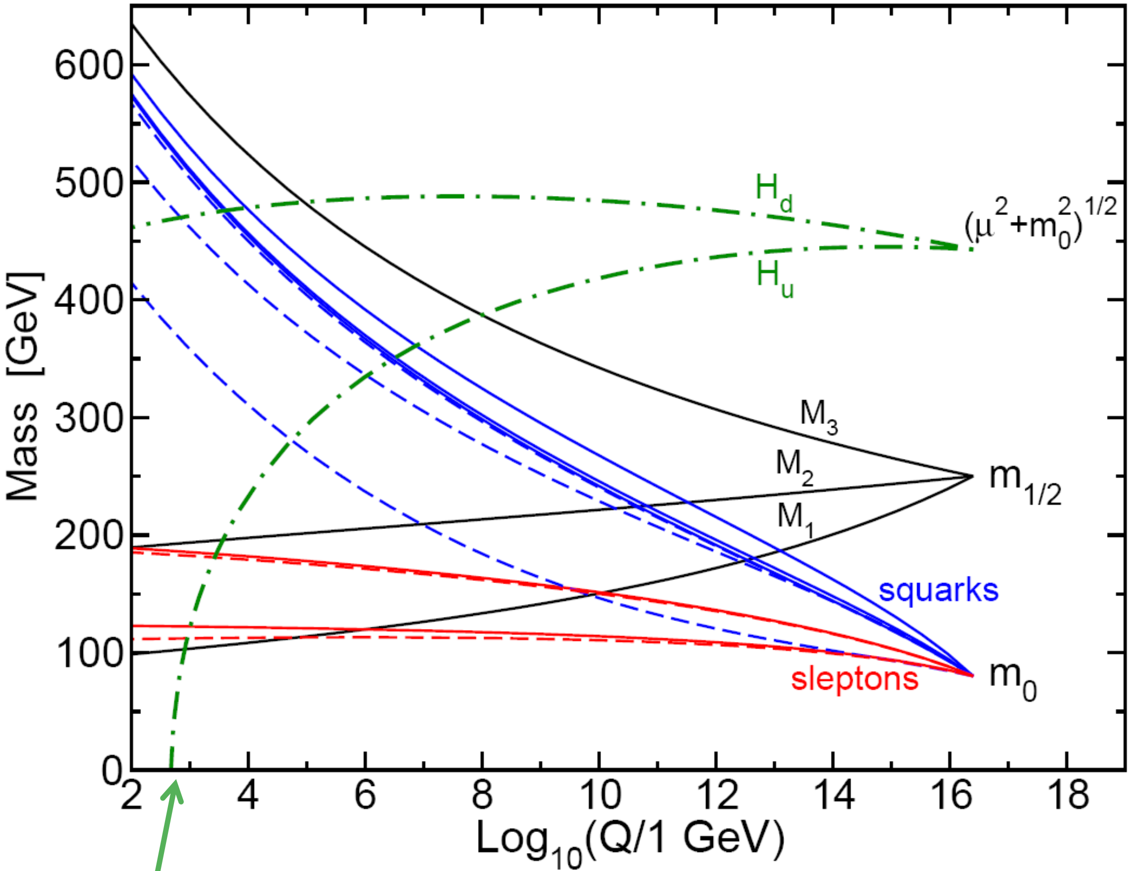
$$
\begin{aligned}
-\mathcal{L}_{soft}^{MSSM} &= \frac{1}{2}\left[M_3\lambda_{\widetilde{g}}\lambda_{\widetilde{g}} + M_2\widetilde{W}^a\widetilde{W}^a + M_1\widetilde{B}\widetilde{B} + \mathrm{h.c.}\right] \\
&+ \epsilon_{\alpha\beta}[B\mu H_d^\alpha H_u^\beta - a_{u_{ij}} H_u^\alpha\,\widetilde{\overline{u}}_i\,\widetilde{Q}_j^\beta + a_{d_{ij}} H_d^\alpha\,\widetilde{\overline{d}}_i\,\widetilde{Q}_j^\beta + a_{e_{ij}} H_d^\alpha\,\widetilde{\overline{e}}_i\,\widetilde{L}_j^\beta + \mathrm{h.c.}] \\
&+ m_{H_d}^2|H_d|^2 + m_{H_u}^2|H_u|^2 + \widetilde{Q}_i^\alpha m_{Q_{ij}}^2\widetilde{Q}_j^{\alpha*} \\
&+ \widetilde{L}_i^\alpha m_{L_{ij}}^2\widetilde{L}_j^{\alpha*} + \widetilde{\overline{u}}_{Ri}^* m_{u_{ij}}^2\widetilde{\overline{u}}_j + \widetilde{\overline{d}}_i^* m_{d_{ij}}^2\widetilde{\overline{d}}_j + \widetilde{\overline{e}}_i^* m_{e_{ij}}^2\widetilde{\overline{e}}_j.
\end{aligned}
$$

$\longrightarrow$ These soft breaking masses describe all possible ways the MSSM could be broken softly without any assumptions about the breaking mechanism

$\longrightarrow$ Different breaking mechanisms can constrain the phenomenology further

Renormalisation Group flow

Renormalisation group equations (RGEs)
connect soft masses (at M_X)
to the EW scale.



$m_2^2 < 0 \Rightarrow$ EWSB conditions satisfied!

Mass eigenstates of the MSSM (and SUSY jargon)

	Gauge Eigenstates	Mass Eigenstates
up squarks down squarks	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{s}_L\ \widetilde{s}_R\ \widetilde{t}_L\ \widetilde{t}_R$ $\widetilde{d}_L\ \widetilde{d}_R\ \widetilde{c}_L\ \widetilde{c}_R\ \widetilde{b}_L\ \widetilde{b}_R$	$\widetilde{u}_1\ \widetilde{u}_2\ \widetilde{c}_1\ \widetilde{c}_2\ \widetilde{t}_1\ \widetilde{t}_2$ $\widetilde{d}_1\ \widetilde{d}_2\ \widetilde{s}_1\ \widetilde{s}_2\ \widetilde{b}_1\ \widetilde{b}_2$
charged sleptons sneutrinos	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\tau}_L\ \widetilde{\tau}_R$ $\widetilde{\nu}_e\ \widetilde{\nu}_\mu\ \widetilde{\nu}_\tau$	$\widetilde{e}_1\ \widetilde{e}_2\ \widetilde{\mu}_1\ \widetilde{\mu}_2\ \widetilde{\tau}_1\ \widetilde{\tau}_2$ $\widetilde{\nu}_e\ \widetilde{\nu}_\mu\ \widetilde{\nu}_\tau$
Higgs bosons	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
neutralinos	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$	$\widetilde{\chi}_1^0\ \widetilde{\chi}_2^0\ \widetilde{\chi}_3^0\ \widetilde{\chi}_4^0$
charginos	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$	$\widetilde{\chi}_1^\pm\ \widetilde{\chi}_2^\pm$
gluino	$\widetilde{g}$	$\widetilde{g}$

SUSY partners of SM particles are "sparticles".

Scalar partners of SM fermions are "sfermions" $\longrightarrow$ "squarks" and "sleptons".

Wino, Bino, Higgsinos $\longrightarrow$ Neutralinos

(charged) Wino, Higgsinos $\longrightarrow$ Charginos

How do we understand
the phenomenology of a SUSY model?

The physics of a SUSY model is given by the

Superpotential

+

Gauge structure

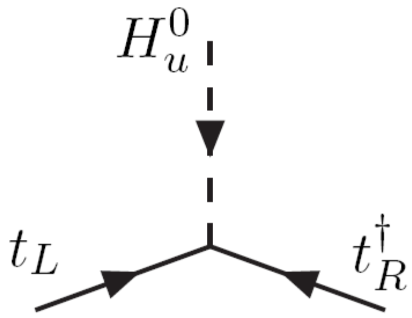
+

Soft breaking

MSSM Superpotential

$$\boxed{\mathcal{W}_{MSSM} = (Y_{\rm u}^{ij}\hat{H}_{\rm u}\hat{\bar{u}}_i\hat{Q} - Y_{\rm d}^{ij}\hat{H}_{\rm d}\hat{\bar{d}}_i\hat{Q}_j - Y_{\rm e}^{ij}\hat{H}_{\rm d}\hat{\bar{e}}_i\hat{L}_j + \mu\hat{H}_{\rm u}\hat{H}_{\rm d})}$$

$$\mathcal{L}_{\mathcal{W}} = \sum_i - \left|\frac{\partial\mathcal{W}(\phi)}{\partial\phi_i}\right|^2 - \sum_{i,j}\psi_i\psi_j\frac{\partial^2\mathcal{W}(\phi)}{\partial\phi_i\partial\phi_j} + h.c.$$



SM-like Yukawa coupling H-f-f

MSSM Superpotential

$$\boxed{\mathcal{W}_{MSSM} = (Y_{\mathrm{u}}^{ij}\hat{H}_{\mathrm{u}}\hat{\bar{u}}_i\hat{Q} - Y_{\mathrm{d}}^{ij}\hat{H}_{\mathrm{d}}\hat{\bar{d}}_i\hat{Q}_j - Y_{\mathrm{e}}^{ij}\hat{H}_{\mathrm{d}}\hat{\bar{e}}_i\hat{L}_j + \mu\hat{H}_{\mathrm{u}}\hat{H}_{\mathrm{d}})}$$

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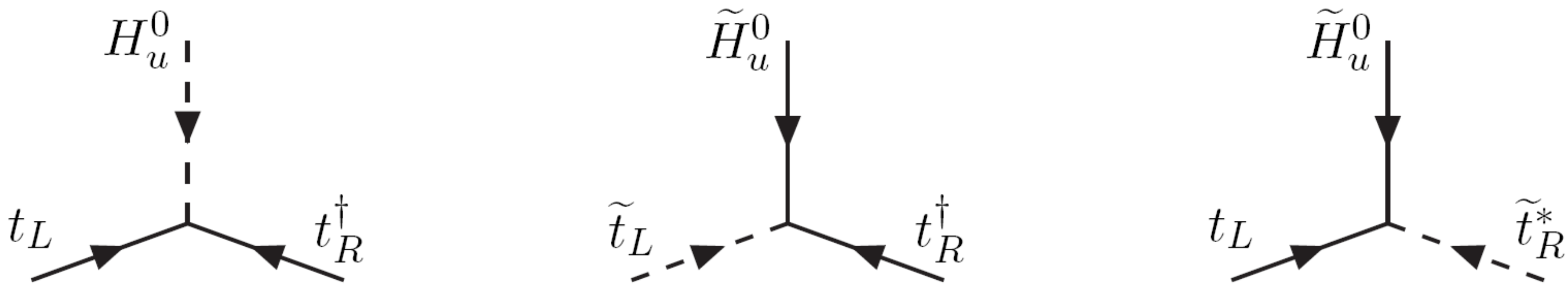
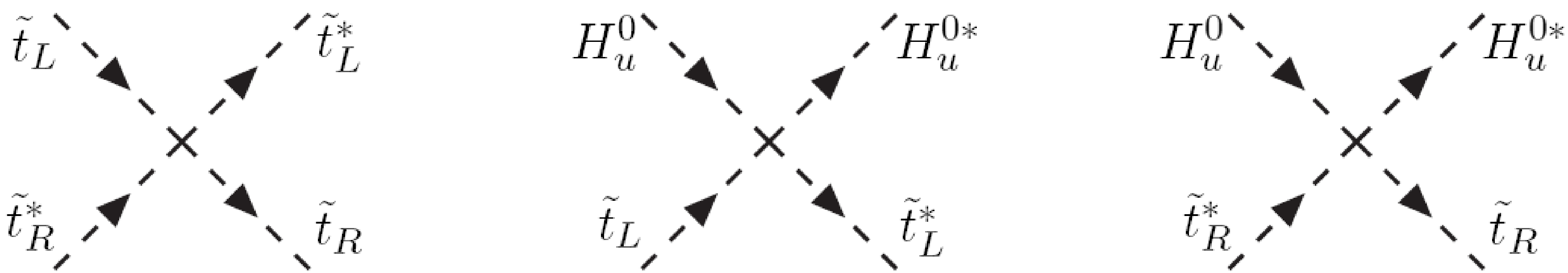


SM-like Yukawa coupling H-f-f

Higgs-squark-quark couplings with same Yukawa coupling!

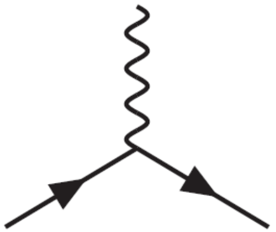


Quartic scalar couplings from same Yukawa coupling

SUSY Gauge interactions

For gauge invariant Lagrangian : in usual kinetic terms, giving:

derivatives ⟶ covariant derivatives $\sum |D_\mu \phi_i|^2 + i\overline{\psi}_i \not{D} \psi_i$



$q,\ \ell,\ \widetilde{H}_u,\ \widetilde{H}_d$ $q,\ \ell,\ \widetilde{H}_u,\ \widetilde{H}_d$

gauge-fermion-fermion vertex

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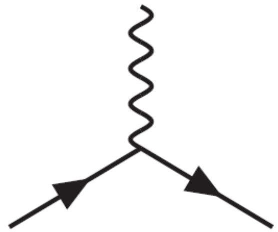
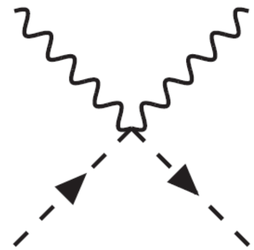
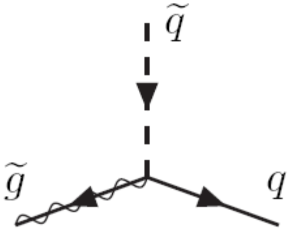


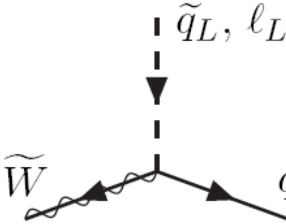


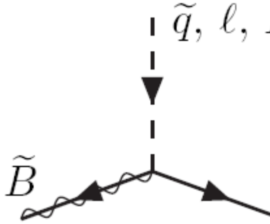
gauge-fermion-fermion vertex

Scalar-gauge vertices

Add Supersymmetric terms to keep SUSY invariant $ig\sqrt{2}(\phi^* \lambda \psi - \overline{\lambda \psi} \phi)$





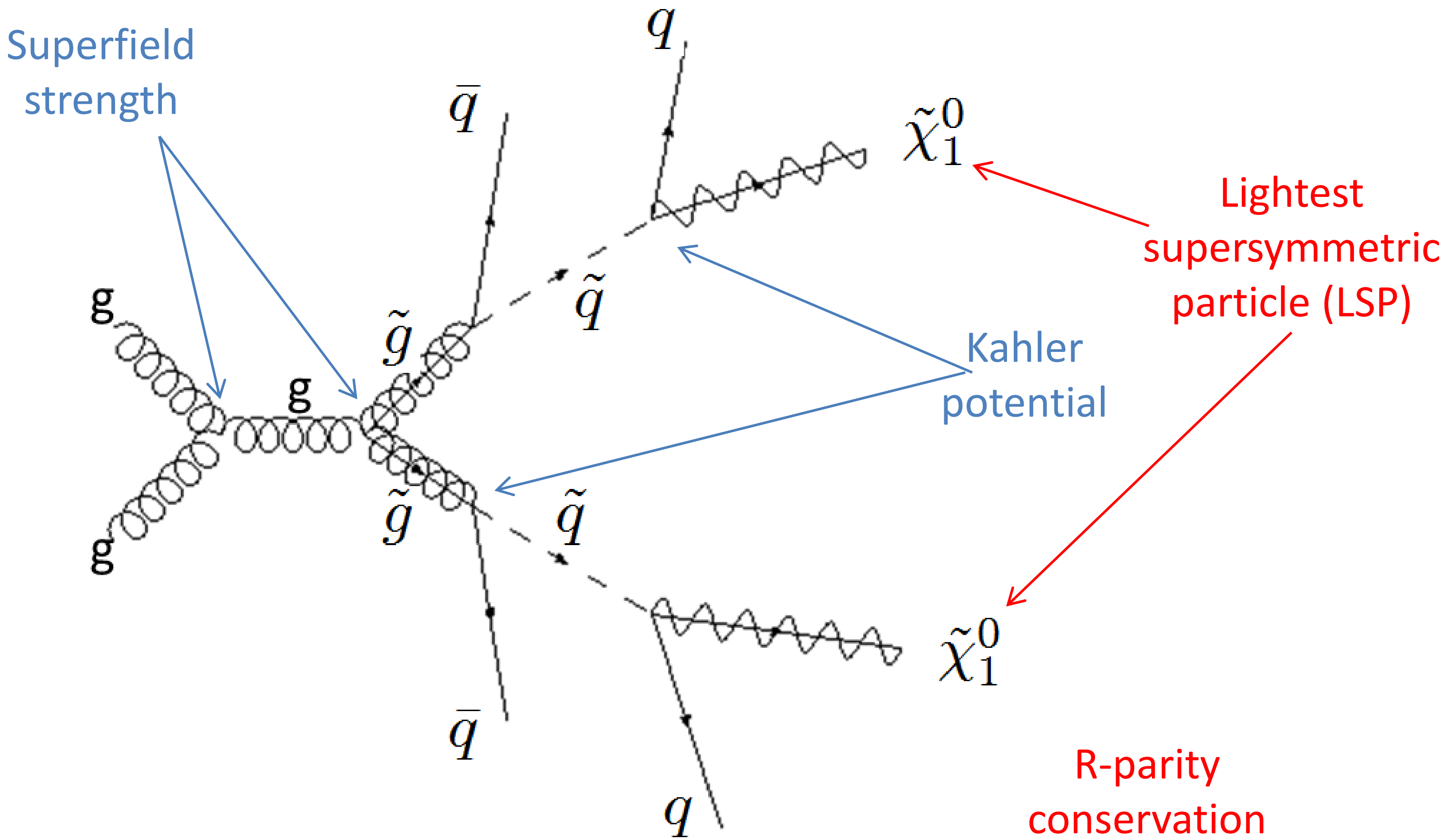


Gaugino interactions from Kahler potential

All arise from something called the "Kahler Potential" $K = \sum_i \Phi_i^\dagger e^{2gV} \Phi_i$

A SUSY signature at the LHC

Gluino pair production → Cascade Decay



R-parity conservation signal

Contributes to: $pp \to q\bar{q}q\bar{q} + E_T^{Miss} + X$

MSSM Higgs Sector (Type II 2HDM)

Two complex Higgs doublets $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix},\ H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$ $\Rightarrow$ 8 degrees of freedom

Gauge bosons Z^0, W^+, W^- Swallow 3 degrees of freedom

$\longrightarrow$ 5 physical Higgs bosons $h,\quad H,\quad A^0\quad H^{\pm}$

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Light Higgs Upper bound: $m_h \leq M_Z |\cos 2\beta|$

Consequence of quartic coupling fixed in terms of gauge couplings
(compare with free λ parameter in SM)

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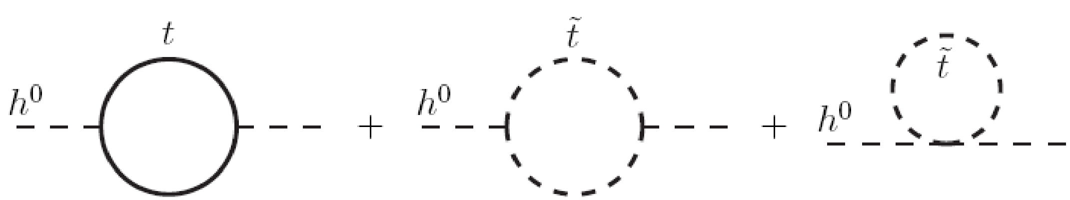
(compare with free λ parameter in SM)

Radiative corrections significantly raise this

Including radiative corrections $m_{h^0} \lesssim 135\ \mathrm{GeV}$

$$\Delta(m_{h^0}^2) = \frac{3}{4\pi^2}\cos^2\alpha\ y_t^2 m_t^2 \ln\left(m_{\tilde{t}_1} m_{\tilde{t}_2}/m_t^2\right).$$

$\Delta(m_{h^0}^2) =$ 

Bonus Material
Beyond the MSSM

Beyond the MSSM

Non-minimal Supersymmetry

The fundamental motivations for Supersymmetry are:

- The hierarchy problem (fine tuning)
- Gauge Coupling Unification
- Dark matter

None of these require Supersymmetry to be realised in a minimal form.

MSSM is not the only model we can consider.

The μ problem and singlet extensions

- The MSSM superpotential contains a mass scale, μ

$$W_{MSSM} = Y_u \hat{\bar{Q}}_L \hat{H}_u \hat{u}_R - Y_d \hat{\bar{Q}}_L \cdot \hat{H}_d \hat{d}_R - Y_e \hat{\bar{E}} \cdot \hat{H}_d \hat{d}_R - \mu \hat{H}_u \hat{H}_d$$

apriori this is not connected to SUSY breaking or EWSB

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- TeV scale soft SUSY breaking masses are generated

$\Rightarrow$ EWSB naturally driven by radiative correc tions.

Requires

$$\mu \approx 0.1 - 1 \text{ TeV}$$

Why should μ be related to this scale?

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Add $\hat{S}$ - SM-gauge singlet , $\Rightarrow \lambda S H_u H_d$ is allowed

$$\langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad \langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \qquad \langle S \rangle = \frac{s}{\sqrt{2}}$$

$\Longrightarrow$ $\mu_{\text{eff}} H_u H_d$ with $\mu_{\text{eff}} = \lambda \langle S \rangle$

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$\Rightarrow$ $\mu_{\text{eff}} H_u H_d$ with $\mu_{\text{eff}} = \lambda \langle S \rangle$

If μ not generated or forbidden, and radiative corrections $\Rightarrow s = \mathcal{O}(TeV)$

Problem solved!

So our superpotential so far is $$\mathcal{W} = \underbrace{Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R}_{\text{Yukawa terms}} - \underbrace{\lambda S H_u H_d}_{\text{effective } \mu\text{–term}}$$

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But this has a (global) Peccei-Quinn symmetry $\hat{\Psi}_i \to e^{iQ_i^{PQ}\theta} \hat{\Psi}_i$

Supermultiplet	spin 0	spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{PQ}$
$\hat{Q}_i$	$(\widetilde{u}_L \;\; \widetilde{d}_L)_i$	$(u_L \;\; d_L)_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	-1
$\overline{u}_i$	$\widetilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	0
$\overline{d}_i$	$\widetilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	0
$\hat{L}_i$	$(\widetilde{\nu} \;\; \widetilde{e}_L)_i$	$(\nu \;\; e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	-1
$\overline{e}_i$	$\widetilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1	0
$\hat{H}_u$	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	1
$\hat{H}_d$	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	1
$\hat{S}$	S	$\widetilde{S}$	$\mathbf{1}$	$\mathbf{1}$	0	-2

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effective μ–term

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$\overline{u}_i$	$\widetilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	0
$\overline{d}_i$	$\widetilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	0
$\hat{L}_i$	$(\widetilde{\nu} \;\; \widetilde{e}_L)_i$	$(\nu \;\; e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	-1
$\overline{e}_i$	$\widetilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1	0
$\hat{H}_u$	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	1
$\hat{H}_d$	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	1
$\hat{S}$	S	$\widetilde{S}$	$\mathbf{1}$	$\mathbf{1}$	0	-2

$S \to \langle S \rangle \Rightarrow \mu_{eff} H_u H_d$ and breaks $U(1)_{PQ}$.

$\longrightarrow$ **massless axion!**

$$\mathcal{W} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \lambda S H_u H_d \longrightarrow$$ **massless axion!**

NMSSM Chiral Superfield Content

[Dine, Fischler and Srednicki] [Ellis, Gunion, Haber, Roszkowski, Zwirner]

Supermultiplet	spin 0	spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{PQ}$
$\hat{Q}_i$	$(\widetilde{u}_L \;\; \widetilde{d}_L)_i$	$(u_L \;\; d_L)_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	-1
$\overline{u}_i$	$\widetilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	0
$\overline{d}_i$	$\widetilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	0
$\hat{L}_i$	$(\widetilde{\nu} \;\; \widetilde{e}_L)_i$	$(\nu \;\; e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	-1
$\overline{e}_i$	$\widetilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1	0
$\hat{H}_u$	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	1
$\hat{H}_d$	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	1
$\hat{S}$	S	$\widetilde{S}$	$\mathbf{1}$	$\mathbf{1}$	0	-2

$$\mathcal{W}_{NMSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \lambda S H_u H_d + \frac{1}{3}\kappa S^3$$

μ_{eff}

PQ breaking

The superpotential of the Next-to-Minimal Supersymmetric Standard Model (NMSSM) is

[Dine, Fischler and Srednicki]
[Ellis, Gunion, Haber, Roszkowski, Zwirner]

$$\mathcal{W}_{NMSSM} = \overbrace{Y_u \widehat{\bar{Q}}_L \widehat{H}_u \widehat{u}_R - Y_d \widehat{\bar{Q}}_L \cdot \widehat{H}_d \widehat{d}_R - Y_e \widehat{\bar{E}} \cdot \widehat{H}_d \widehat{d}_R}^{\text{Yukawa terms as in MSSM}} + \underbrace{\lambda \widehat{S} \widehat{H}_u \widehat{H}_d}_{\text{effective } \mu\text{–term}} + \underbrace{\frac{1}{3}\kappa \widehat{S}^3}_{\text{PQ breaking term}}$$

$$+\mu \widehat{H}_u \widehat{H}_d + \xi_F \widehat{S} + \mu' \widehat{S}^2 \quad (Z_3 \text{ violating})$$

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$$+\mu \hat{H}_u \hat{H}_d + \xi_F \hat{S} + \mu' \hat{S}^2 \quad (Z_3 \text{ violating})$$

We also need new soft supersymmetry breaking terms in the Lagrangian:

$$-\mathcal{L}_{\text{soft}} \supset m_S^2 |S|^2 + [\lambda A_\lambda S H_u H_d + \frac{1}{3}\kappa A_\kappa S^3 + \text{h.c.}]$$

$$[+m_3^2 H_u H_d + \frac{1}{2} m_S'^2 S^2 + \xi_S S + \text{h.c.}] \; (Z_3 \text{ violating})$$

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$$[+m_3^2 H_u H_d + \frac{1}{2} m_S'^2 S^2 + \xi_S S + \text{h.c.}] \ (Z_3 \text{ violating})$$

Extended Higgs sector: 3 CP-even Higgs, 2 CP-odd Higgs (new real and imagnary scalar S)

Extended Neutralino sector: 5 neutralinos (singlino, the new fermion component of S)

Supersymmetric Models

- Minimal Supersymmetric Standard Model (MSSM)

$$\mathcal{W}_{MSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \mu H_u H_d$$

- Next to Minimal Supersymmetric Standard Model (NMSSM)

[Dine, Fischler and Srednicki] [Ellis, Gunion, Haber, Roszkowski, Zwirner]

$$\mathcal{W}_{NMSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \lambda S H_u H_d + \frac{1}{3}\kappa S^3$$

$$\mu_{eff} = \lambda \langle S \rangle$$

NMSSM is minimal matter extension.

We can also extend the gauge group of the SM.

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- Minimal Supersymmetric Standard Model (MSSM)

$$\mathcal{W}_{MSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \mu H_u H_d$$

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What if the $U(1)_{PQ}$ was a local rather global symmetry?

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- **M**inimal **S**upersymmetric **S**tandard **M**odel (**MSSM**)

$$\mathcal{W}_{MSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \mu H_u H_d$$

- **N**ext to **M**inimal **S**upersymmetric **S**tandard **M**odel (**NMSSM**)

[Dine, Fischler and Srednicki] [Ellis, Gunion, Haber, Roszkowski, Zwirner]

$$\mathcal{W}_{NMSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \lambda S H_u H_d + \frac{1}{3}\kappa S^3$$

$$\mu_{eff} = \lambda \langle S \rangle$$

NMSSM is minimal matter extension.

We can also extend the gauge group of the SM.

What if the $U(1)_{PQ}$ was a local rather global symmetry?

When a local $U(1)'$ is broken
Z′ eats the massless axion to become massive vector boson!

USSM Chiral Superfield Content

Supermultiplet	spin 0	spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{PQ}$
$\hat{Q}_i$	$(\widetilde{u}_L \;\; \widetilde{d}_L)_i$	$(u_L \;\; d_L)_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	-1
$\overline{u}_i$	$\widetilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	0
$\overline{d}_i$	$\widetilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	0
$\hat{L}_i$	$(\widetilde{\nu} \;\; \widetilde{e}_L)_i$	$(\nu \;\; e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	-1
$\overline{e}_i$	$\widetilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1	0
$\hat{H}_u$	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	1
$\hat{H}_d$	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	1
$\hat{S}$	S	$\widetilde{S}$	$\mathbf{1}$	$\mathbf{1}$	0	-2

$$\mathcal{W}_{USSM} = \underbrace{Y_u\bar{Q}_L H_u u_R - Y_d\,\bar{Q}_L\cdot H_d\,d_R - Y_e\,\bar{E}\cdot H_d\,d_R}_{\text{Yukawa terms}} - \lambda S H_u H_d$$

effective μ–term

Problem: to avoid gauge anomalies $\sum_i Q_i^{U(1)} = 0$

USSM Chiral Superfield Content

Supermultiplet	spin 0	spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
$\hat{Q}_i$	$(\widetilde{u}_L \;\; \widetilde{d}_L)_i$	$(u_L \;\; d_L)_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	Q'_Q
$\overline{u}_i$	$\widetilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	Q'_u
$\overline{d}_i$	$\widetilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	Q'_d
$\hat{L}_i$	$(\widetilde{\nu} \;\; \widetilde{e}_L)_i$	$(\nu \;\; e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	Q'_L
$\overline{e}_i$	$\widetilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1	Q'_e
$\hat{H}_u$	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	Q'_{H_u}
$\hat{H}_d$	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	Q'_{H_d}
$\hat{S}$	S	$\widetilde{S}$	$\mathbf{1}$	$\mathbf{1}$	0	Q'_S

$$\mathcal{W}_{USSM} = \underbrace{Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R}_{\text{Yukawa terms}} - \lambda S H_u H_d$$

effective μ–term

Problem: to avoid gauge anomalies $\sum_i Q_i^{U(1)} = 0$

Charges not specified in the definition of the USSM

U(1) extended Supersymmetric Standard Model (USSM)

$$\mathcal{W}_{USSM} = \underbrace{Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R}_{\text{Yukawa terms}} - \lambda S H_u H_d$$

effective μ–term

Modified Gauge sector, new Z'

Modified Higgs sector: 3 CP-even Higgs,
2 CP-odd Higgs (new real and imaginary scalar S)

Modified Neutralino sector: 6 neutralinos:
(new singlino + Zprimino)

Supersymmetric Models

- **M**inimal **S**upersymmetric **S**tandard **M**odel (MSSM)

$$\mathcal{W}_{MSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \mu H_u H_d$$

[Dine, Fischler and Srednicki]
[Ellis, Gunion, Haber, Roszkowski, Zwirner]

- **N**ext to **M**inimal **S**upersymmetric **S**tandard **M**odel (NMSSM)

$$\mathcal{W}_{NMSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \lambda S H_u H_d + \frac{1}{3} \kappa S^3$$

$$\mu_{eff} = \lambda \langle S \rangle$$

- **U**(1) extended **S**upersymmetric **S**tandard **M**odel (USSM)

E_6 inspired models

For anomaly cancelation, one can use complete E_6 matter multiplets

$$\text{New } U(1)' \text{ from } E_6$$

E_6 inspired models

For anomaly cancelation, one can use complete E_6 matter multiplets

New $U(1)'$ from E_6

$$E_6 \to SO(10) \times U(1)_\psi$$

$$\hookrightarrow SU(5) \times U(1)_\chi$$

$$\hookrightarrow SU(3)_C \times SU(2)_W \times U(1)_Y$$

$$SU(3) \times SU(2) \times U(1)_Y \times U(1)'$$

$$U(1)' = \cos\theta\, U(1)_\chi + \sin\theta\, U(1)_\psi$$

E_6 inspired models

For anomaly cancelation, one can use complete E_6 matter multiplets

$$\text{New } U(1)' \text{ from } E_6$$

$$\begin{array}{l} E_6 \to SO(10) \times U(1)_\psi \\ \qquad\quad \hookrightarrow SU(5) \times U(1)_\chi \\ \qquad\qquad\quad \hookrightarrow SU(3)_C \times SU(2)_W \times U(1)_Y \end{array}$$

$$SU(3) \times SU(2) \times U(1)_Y \times U(1)'$$

$$U(1)' = \cos\theta\, U(1)_\chi + \sin\theta\, U(1)_\psi$$

- Matter from 3 complete generations of E_6

  $\Rightarrow$ automatic cancellation of gauge anomalies!

- In the E_6SSM $\tan\theta = \sqrt{15}$ $\Rightarrow$ right-handed neutrino is a gauge singlet

Exceptional Supersymmetric Standard Model (E_6SSM)

[Phys.Rev. D73 (2006) 035009 , Phys.Lett. B634 (2006) 278-284 S.F.King, S.Moretti & R. Nevzorov]

All the SM matter fields are contained in one 27-plet of E_6 per generation.

$$[27]_i \Rightarrow [10,1]_i + [5^*,2]_i + [5^*,-3]_i + [5,-2]_i + [1,5]_i + [1,0]_i$$

SU(5) reps., $U(1)_N$ charge		
$[10, 1]_i$	$Q_i,\ u_i^c,\ e_i^c$	
$[5^*, 2]_i$	$L_i,\ d_i^c$	
$[5^*, -3]_i$	$H_{1\,i} = \begin{pmatrix} H_{1\,i}^0 \\ H_{1\,i}^- \end{pmatrix},\ \bar{D}_i$	3 generations of "Higgs"; exotic quarks
$[5, -2]_i$	$H_{2\,i} = \begin{pmatrix} H_{2\,i}^+ \\ H_{2\,i}^0 \end{pmatrix},\ D_i$	3 generations of "Higgs"; exotic quarks
$[1, 5]_i$	S_i	singlets
$[1, 0]_i$	N_i^c	right handed neutrino

SU(5) reps.

$U(1)_N$ charge

E_6SSM Chiral Superfield Content

Supermultiplet	spin 0	spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_N$
$\hat{Q}_i$	$(\widetilde{u}_L \quad \widetilde{d}_L)_i$	$(u_L \quad d_L)_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	1
$\overline{u}_i$	$\widetilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	1
$\overline{d}_i$	$\widetilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	2
$\hat{L}_i$	$(\widetilde{\nu} \ \widetilde{e}_L)_i$	$(\nu \quad e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	2
$\overline{e}_i$	$\widetilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1	1
$\overline{N}_i$	$\widetilde{N}^*_{R\,i}$	$N^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	0	0
$\hat{H}_{2\,i}$	$(H^+_{2\,i} \quad H^0_{2\,i})$	$(\widetilde{H}^+_{2\,i} \quad \widetilde{H}^0_{2\,i})$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	-2
$\hat{H}_{1\,i}$	$(H^0_d \quad H^-_{1\,i})$	$(\widetilde{H}^0_{1\,i} \quad \widetilde{H}^-_d)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	-3
$\hat{S}_i$	S_i	$\widetilde{S}_i$	$\mathbf{1}$	$\mathbf{1}$	0	5
$\hat{D}_i$	$\widetilde{D}_i$	D_i	$\mathbf{3}$	$\mathbf{1}$	$-\frac{1}{3}$	-2
$\hat{\overline{D}}_i$	$\widetilde{\overline{D}}_i$	$\overline{D}_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	-3

Note: In it’s usual form there are also two extra SU(2) doublets included for single step gauge coupling unification, but these are negleected here for simplicity.

Supersymmetric Models

- Minimal Supersymmetric Standard Model (MSSM)

$$\mathcal{W}_{MSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \mu H_u H_d$$

- Next to Minimal Supersymmetric Standard Model (NMSSM)

[Dine, Fischler and Srednicki]
[Ellis, Gunion, Haber, Roszkowski, Zwirner]

$$\mathcal{W}_{NMSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \lambda S H_u H_d + \frac{1}{3}\kappa S^3$$

$$\mu_{eff} = \lambda \langle S \rangle$$

- U(1) extended Supersymmetric Standard Model (USSM)

- Exceptional Supersymmetric Standard Model (E_6SSM)

[S.F. King, S. Moretti, R. Nevzrov, Phys.Rev. D73 (2006) 035009]

Back up slides

Beyond the CMSSM
(Relaxing high scale constraints)

For universal gauginos we have a (one loop) relation:

$$\frac{d}{dt}g_i = \frac{b_i}{(4\pi)^2}g_i^3 \qquad \frac{d}{dt}M_i = 2\frac{b_i}{(4\pi)^2}g_i^2 M_i$$

$$\Rightarrow \frac{d}{dt}g_i^{-2} = -2\frac{b_i}{(4\pi)^2} \qquad \frac{d}{dt}\left(\frac{M_i}{g_i^2}\right) = 0 \qquad \Rightarrow M_i(t) = \frac{g_i^2}{g_0^2}M_{1/2}$$

$$\Rightarrow \text{the ratio } M_1 : M_2 : M_3 \text{ is fixed to } 0.15 : 0.25 : 0.7$$

Testable predictions for gaugino universality!

<u>Non-universal Gaugino masses</u>

$$\boxed{\begin{array}{rcl} g_i(M_X) & = & g_0 \;\; m_i^2(M_X) = m_0 \;\; M_1(M_X) \neq M_2(M_X) \neq M_3(M_X) \\ A_i(M_X) & = & A_0 \;\; B\mu(M_X) = B\mu \end{array}}$$

Breaks ratio $\longrightarrow$ get different gaugino mass patterns: $M_i(t) = \frac{g_i^2}{g_0^2}M_i(M_X)$

One can also ignore the universality more parameters to consider the model with less prejudice, e.g. <u>pMSSM</u>

$$\boxed{\begin{array}{l} m_{Q_3}, m_{Q_1}, m_{L_3}, m_{L_1}, m_{u_3}, m_{u_1}, m_{d_3}, m_{d_1}, m_{e_3}, m_{e_1} \\ \qquad\qquad M_1, M_2, M_3, A_t, A_b, A_\tau, \mu, M_A, \tan\beta \end{array}}$$

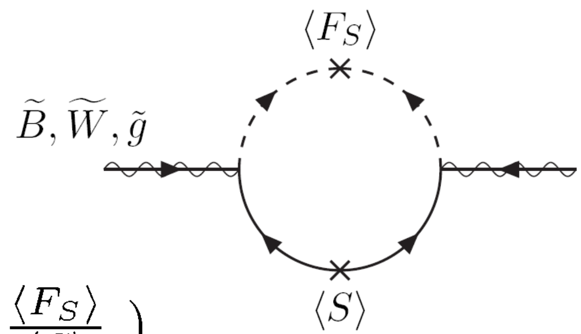
Gauge Mediation

In gauge mediated symmetry breaking the SUSY breaking is transmitted from the hidden sector via SM gauge interactions of heavy messenger fields.

Chiral Messenger fields couple to Hidden sector
→ SUSY breaking in messenger spectrum

$$\mathcal{W}_{mess} = \sum_i \lambda_i S \Phi_i \overline{\Phi}_i$$

SM Gauge interactions couple them to visible sector
Loops from gauge interactions with virtual messengers
→ flavour diagonal soft masses.



Loop diagram: → $M_a = \frac{\alpha_a}{4\Pi}\Lambda \ \ (\Lambda = \frac{\langle F_S \rangle}{\langle S \rangle})$

Two loop diagrams: $m^2_{\phi_i}(M_m) = 2\Lambda^2 N_5 \sum_{a=1}^{3} C_a(i) \left(\frac{g_a^2}{16\pi^2}\right)^2$ $\qquad A_i \approx 0$

Light gravitino $\quad m_{3/2} = \frac{1}{\sqrt{3}M_{Pl}} \left(\sum_i |\langle F_i \rangle|^2\right)^{\frac{1}{2}}$

→ Soft mass relations imposed at messenger scale

→ Non-universal soft gaugino masses since they depend on gauge interactions!

Minimal Gauge Mediated SUSY Breaking (mGMSB)

- Messenger fields form Complete SU(5) representations
- Assumptions ($\frac{\langle F_S \rangle}{\lambda_i \langle S \rangle^2}$ small) $\Rightarrow$ messenger couplings λ_i don't affect spectrum.

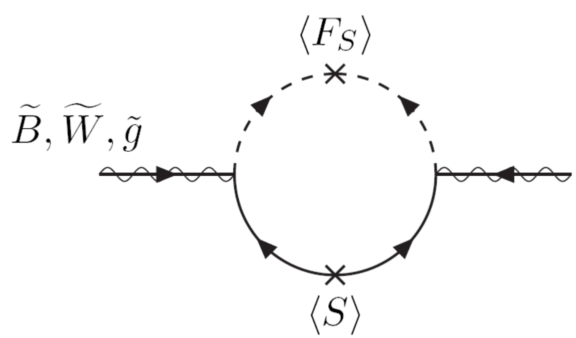
$$\Lambda = \frac{\langle F_S \rangle}{\langle S \rangle}$$

Number of SU(5) multiplets

$$\mathcal{W}_{mess} = \sum_i \lambda_i S \Phi_i \overline{\Phi}_i$$

$$\Lambda, \quad M_m, \quad N_5, \quad \underbrace{\tan\beta, \quad \mathrm{sgn}(\mu)}_{\text{From EWSB as in CMSSM}}$$

Messenger scale



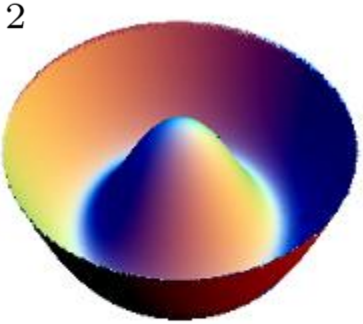
$$M_a(M_m) = \frac{g_a^2}{(4\pi)^2} \Lambda N_5$$

$$m_{\phi_i}^2(M_m) = 2\Lambda^2 N_5 \sum_{a=1}^{3} C_a(i) \left(\frac{g_a^2}{16\pi^2} \right)^2 \qquad A_i \approx 0$$

What about the Higgs boson
and electroweak symmetry breaking?

Electroweak Symmetry Breaking (EWSB)

Recall in the SM the Higgs potential is: $V(\phi) = \mu^2 \phi^\dagger \phi + |\lambda| \left(\phi^\dagger \phi\right)^2$



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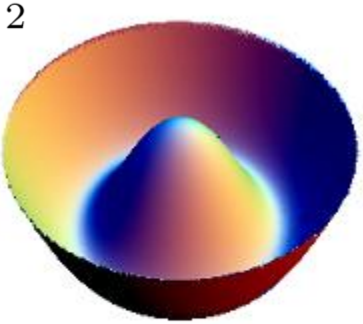

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$\longrightarrow$ Vacuum Expectation Value (VEV)

ϕ is an $SU(2)$ doublet with two complex components $\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$

Underlying SU(2) invariance $\Rightarrow$ the direction of the VEV in SU(2) space is arbitrary.

Any choice breaks $SU(2) \times U(1)_Y$ in the vacuum, choosing $\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}$

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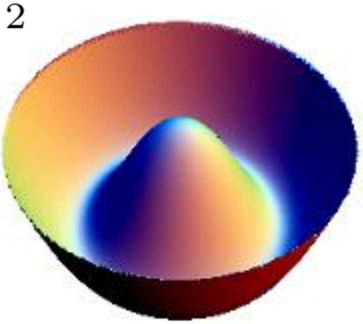

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$\longrightarrow$ All $SU(2) \times U(1)_Y$ generators broken: $\sigma^i \langle \phi \rangle_0 \neq 0, \qquad Y \langle \phi \rangle_0 \neq 0.$

But for this choice $Q \langle \phi \rangle_0 = \frac{1}{2}(\sigma^3 + Y) \langle \phi \rangle_0 = 0.$

$\longrightarrow$ $\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$ can be written as $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

Showing the components' charge under unbroken generator Q

EWSB

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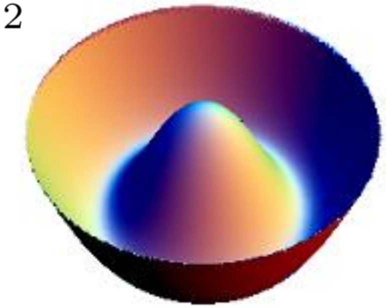

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The MSSM Higgs Potential

$$\boxed{\begin{aligned} V_{\mathrm{H}} \;=\; & (m_{H_d}^2 + |\mu|^2)(|H_d^0|^2 + |H_d^-|^2) + (m_{H_u}^2 + |\mu|^2)(|H_u^+|^2 + |H_u^0|^2) \\ & + B\mu(H_u^+ H_d^- - H_u^0 H_d^0 + \mathrm{h.c.}) + \frac{1}{8}(g^2 + g'^2)\left(|H_d^0|^2 + |H_d^-|^2 - |H_u^+|^2 - |H_u^0|^2\right)^2 \\ & + \frac{1}{2} g^2 (H_u^{+*} H_d^0 + H_u^{0*} H_d^-)(H_u^+ H_d^{0*} + H_u^0 H_d^{-*}) \end{aligned}}$$

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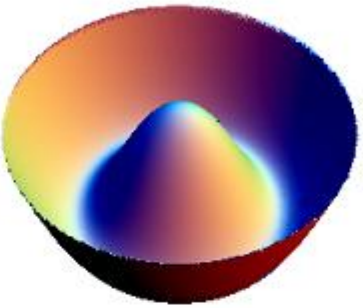

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As in the SM, underlying $SU(2)_W$ invariance means we can choose one component of one doublet to have no VEV:

Choose: $\langle H_u^+ \rangle_0 = 0$

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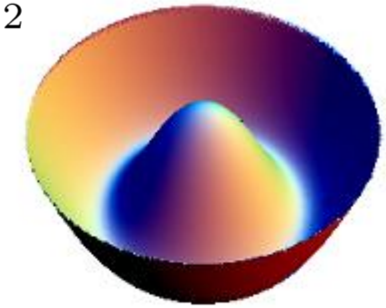

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As in the SM, underlying $SU(2)_W$ invariance means we can choose one component of one doublet to have no VEV:

Choose: $\langle H_u^+ \rangle_0 = 0$

$$\Rightarrow B\mu + \frac{1}{2} g^2 H_u^{0\,*} H_d^{0\,*} = 0 \text{ OR } H_d^- = 0 \qquad \Rightarrow \langle H_d^- \rangle_0 = 0$$

bad for stable EWSB

EWSB

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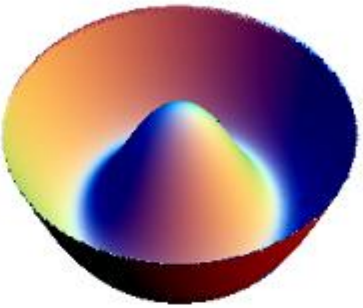

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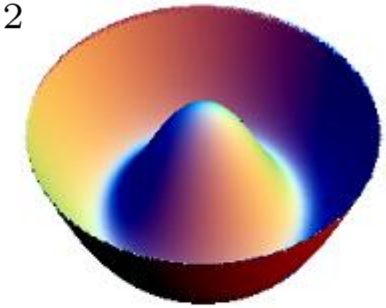

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For successful EWSB:

$$\begin{aligned} (m_{H_d}^2 + m_{H_u}^2 + 2|\mu|^2) &\geq 2B\mu \\ (m_{H_d}^2 + |\mu|^2)(m_{H_u}^2 + |\mu|^2) &\leq (B\mu)^2 \end{aligned}$$

EWSB conditions

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$$m_1^2 v_d = B\mu v_u + \frac{1}{8}(g^2 + g'^2)(v_u^2 - v_d^2)v_d \qquad \langle H_u \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_u \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v\sin\beta \end{pmatrix}$$

$$m_2^2 v_u = B\mu v_d - \frac{1}{8}(g^2 + g'^2)(v_u^2 - v_d^2)v_u \qquad \langle H_d \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_d \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} v\cos\beta \\ 0 \end{pmatrix}$$

With: $\tan\beta = \frac{v_u}{v_d}$

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With: $\tan\beta = \frac{v_u}{v_d}$ $\quad M_Z^2 = \frac{g^2+g'^2}{4}(v_u^2 + v_d^2) = \frac{g^2+g'^2}{4}(v^2)$

$$\begin{aligned} \sin(2\beta) &= \frac{2B\mu}{m_{H_u}^2 + m_{H_u}^2 + 2|\mu^2|} \\ M_Z^2 &= \frac{|m_{H_d}^2 - m_{H_u}^2|}{\sqrt{1 - \sin^2(2\beta)}} - m_{H_u}^2 - m_{H_d}^2 - 2|\mu|^2 \end{aligned}$$

Constranied MSSM (cMSSM)

From SUperGRAvity (SUGRA)

Take minimal set of couplings:

Universal soft scalar mass: $m_0^2 = \kappa \frac{|\langle F \rangle|^2}{M_{Pl}^2}$

Universal soft gaugino mass: $M_{1/2} = f \frac{\langle F \rangle}{m_{Pl}}$

Universal soft trilinear mass: $A = \frac{\alpha \langle F \rangle}{M_{Pl}}$

Universal soft bilinear mass: $B = \frac{\beta \langle F \rangle}{M_{Pl}}$

Gauge coupling Unification Scale

$$\underbrace{g_i(M_X) = g_0 \quad y_i(M_X) = y_i}_{\text{Fixed}}$$

$$m_i^2(M_X) = m_0 \quad M_i(M_X) = M_{1/2} \quad A_i(M_X) = A_0$$

$$\underbrace{\mu(M_X) = \mu \quad B\mu(M_X) = B\mu}_{\text{Traded for } M_Z \text{ and } \tan\beta \text{ (via EWSB conditions)}}$$

$$\tan\beta = \text{ratio of vevs} = \frac{v_u}{v_d} \qquad \langle H_u \rangle = v_u \quad \langle H_d \rangle = v_d$$

Free parameters: $\{m_0, M_{1/2}, A, \tan\beta, \mathrm{sgn}(\mu)\}$