Gravitational Instabilities of the Cosmic Neutrino Background with Non-zero Lepton Number

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Outline

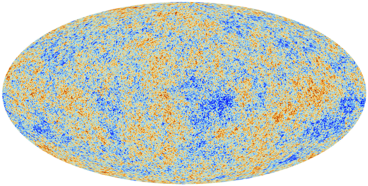
Cosmic Neutrino Background

- Decoupling of neutrinos results in the Cosmic Neutrino Background (CνB), $T \sim 1$ MeV

- The CνB temperature is related to that of the CMB:

$$\frac{T_\nu}{T_0} = \left(\frac{4}{11}\right)^{\frac{1}{3}}$$

where $T_0 = 2.725$ K is the temperature of the CMB today.



- Weakly interacting nature and low temperature $\rightarrow$ not yet observed

Neutrino Masses and Relic Asymmetry

- A very large lepton asymmetry can be stored in the CνB.

- Flavour dependent bound on the asymmetries is

$$L_\alpha^{C\nu B} = \frac{n_{\nu_\alpha} - \bar{n}_{\nu_\alpha}}{n_\gamma} = \frac{\pi^2}{12\zeta(3)} \left(\xi_\alpha + \frac{\xi_\alpha^3}{\pi^2} \right)$$

  where the flavour independent bounds on ξ_α are $-0.07 < \xi < 0.22$.

- Majorana particles $\rightarrow$ the CνB will be parity violating.

- Possible indirect evidence of the CνB via induced parity violating radiative corrections to the graviton propagator.

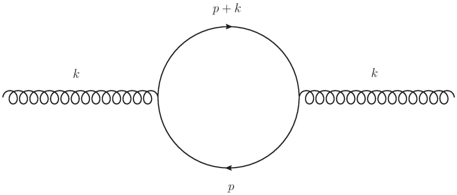
- Homogeneous neutrino gas with a chemical potential μ.

Addition to the Graviton Propagator in CνB

- $\mathcal{L}_\mu = \mu \bar{\nu} \gamma_0 \gamma_5 \nu$, which alters the neutino propagator

- First order μ parity violating contribution to the fermion propagator:

$$S(p) = S_0(p) + \mu \gamma_0 \gamma^5 \frac{(\not{p} + m)^2}{(p^2 - m^2)^2} + ...$$



- Parity violating polarisation tensor contribution:

$$\Pi_{\mu\nu\rho\sigma} = -\frac{1}{2} \int \frac{d^4p}{(2\pi)^4} (2p+k)_\nu (2p+k)_\sigma \left[Tr(\gamma_\mu S_0(p+k) \gamma_\rho S_1(p)) + Tr(\gamma_\rho S_0(p) \gamma_\mu S_1(p+k)) \right]$$

Graviton Polarisation Tensor

- We find a divergent quantity:

$$\Pi^{(div)}_{\mu\nu\rho\sigma} = -\frac{1}{\epsilon}\frac{\mu}{2\pi^2}k^\alpha \varepsilon_{\mu\rho\alpha 0} m^2 \eta_{\nu\sigma}$$

  - This term violates gauge invariance ($h_{\mu\nu} \to h_{\mu\nu} + k_\mu \lambda_\nu + k_\nu \lambda_\mu$).
  - Transversality requires: $k^\mu \Pi_{\mu\nu\rho\sigma} = 0$ and $k^\nu \Pi_{\mu\nu\rho\sigma} = 0$.

- We obtain the following simple form for the polarisation tensor,

$$\Pi_{\mu\nu\rho\sigma} = \mu \varepsilon_{\mu\rho\alpha 0} k^\alpha [k_\nu k_\sigma - k^2 \eta_{\nu\sigma}] C(k^2)$$

where

$$C(k^2) = \begin{cases} -\frac{1}{1920\pi^2}\frac{k^2}{m^2}, & \text{if } k^2/m^2 \ll 1 \\ \frac{1}{192\pi^2}, & \text{if } k^2/m^2 \gg 1 \end{cases}$$

Effective Graviton Action

- In the limit $k^2/m_\nu^2 \gg 1$,

$$S_{eff} = -\frac{\mu}{192\pi^2}\int d^4x \varepsilon_{\mu\rho\alpha 0} h^{\mu\nu}\partial^\alpha(\Box h^{\rho\sigma}\eta_{\nu\sigma} - \partial_\nu\partial_\sigma h^{\rho\sigma})$$
$$= \frac{\mu}{48\pi^2}\int d^4x \; K^0$$

- The 4 dimensional Chern-Simons topological current:

$$K^\beta = \varepsilon^{\beta\alpha\mu\nu}(\Gamma^\sigma_{\alpha\rho}\partial_\mu\Gamma^\rho_{\nu\sigma} - \frac{2}{3}\Gamma^\sigma_{\alpha\rho}\Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma}).$$

- Replicating Chern-Simons modified gravity.

$$S_{CS} = \int d^4x \; (\partial_\mu\theta)K^\mu = \int d^4x \; \theta({}^*RR)$$

Birefringent Gravitational Wave Propagation

Graviton Propagation Effects

- Chern-Simons modification induces a birefringence effect.
- eLISA can measure sources $z \lesssim 30$, and differentiates polarisations.
- Consider propagation in a FRW universe.

Consider $h_{ij} = \frac{\mathcal{A}_{ij}}{a(\eta)} \exp[-i(\phi(\eta) - \kappa n_k x^k)]$

- Decomposing into the two circularly polarised states: e^R_{ij} and e^L_{ij}
- Take the action $S = S_{EH} + S_{eff}$
- Find the accumulated phase over propagation.

Graviton Propagation Effects

- From the equations of motion:

$$(i\phi^{R,L}_{,\eta\eta} + (\phi^{R,L}_{,\eta})^2 + \mathcal{H}_{,\eta} + \mathcal{H}^2 - \kappa^2)\left(1 - \frac{\lambda_{R,L}\kappa\theta_{,\eta}}{a^2}\right) = \frac{i\lambda_{R,L}\kappa}{a^2}(\theta_{,\eta\eta} - 2\mathcal{H}\theta_{,\eta})(\phi^{R,L}_{,\eta} - i\mathcal{H})$$

- Solve in the matter dominated epoch, $a(\eta) = a_0\eta^2 = \frac{a_0}{1+z}$.

- Accumulated phase to first order in θ,

$$\Delta\phi^{R,L} = i\lambda_{R,L}kH_0\int_{\eta}^{1}\left[\frac{1}{4}\theta_{,\eta\eta} - \frac{1}{\eta}\theta_{,\eta}\right]\frac{d\eta}{\eta^4}$$

- Temperature dependence of $\mu \Rightarrow \theta_{,\eta} = \left(\frac{a(\eta_0)}{a(\eta)}\right)^2\frac{\mu_0}{48\pi^2 M_p^2}$

Birefringence of Gravitational Waves

- For the CνB,

$$\Delta\phi^{R,L} = -i\frac{\lambda_{R,L}\mu H_0}{288\pi^2 M_p^2}\left(\frac{k}{1\ \mathrm{GeV}}\right)(1+z)^4$$

- Ratio of the polarisations:

$$\frac{h_R}{h_L} \propto e^{-2|\Delta\phi|}$$

- From the current bounds on the CνB, $|i\Delta\phi^{R,L}| \lesssim 10^{-87}\left(\frac{k}{1\ \mathrm{GeV}}\right)$, for $z \sim 30$

- Now consider the propagation of GWs from early sources, larger μ_ν and longer accumulated propagation time. Conceivably, any source could provide constraints.

Birefringence of Gravitational Waves

- Consider GWs produced in the radiation dominated epoch:

$$\Delta\phi^{R,L}_{rad} = i\lambda_{R,L}\frac{|\mathbf{k}|}{\Omega_{\mathrm{r},0}H_0^2}\int_{\eta}^{1}\left[\frac{1}{2}\theta_{,\eta\eta} - \frac{1}{\eta}\theta_{,\eta}\right]\frac{d\eta}{\eta^2}$$
$$\simeq -i\lambda_{R,L}\xi_{\nu}\left(\frac{|\mathbf{k}|}{1\ \mathrm{GeV}}\right)\left(\frac{T_s}{1\ \mathrm{TeV}}\right)^4$$

where $\Omega_{\mathrm{r},0} \sim 9.2\cdot 10^{-5}$ is the radiation density parameter today.

- Redshift defined in terms of the source/asymmetry temperature T_s (whichever is lowest).

- It is possible to get significant birefringent behaviour in the propagation of GWs from primordial sources.

Induced Vaccum Decay

Vaccum Decay from Induced Ghost-like Modes

- Interaction describing $0 \to g\gamma\gamma$, $(h^{can}_{\mu\nu} = m_{can} h^{can}_{\mu\nu})$:

$$S_{\rm int} \sim \frac{1}{m_{can}} \int d^4x \frac{1}{2} h^{can} F_{\mu\nu} F^{\mu\nu} - h^{can}_{\mu\nu} F^{\mu\alpha} F^{\nu}_{\alpha}$$

where

$$m_{can} = M_p \sqrt{1 + \lambda_{R,L} \frac{|\mathbf{k}|}{a m_{CS}}}, \quad \text{where } m_{CS}(t) = \frac{M_p^2}{\mu_\nu} = \frac{a(t) M_p^2}{\mu_0}$$

- Consider when we have ghost modes. Require a phase space cutoff, or will decay to infinity. Take comoving momenta cutoff: $|\mathbf{k}| \sim \Lambda \to m_{can}(t) \simeq \sqrt{\lambda \frac{\mu\Lambda}{a}}$

- Total decay width is approximately:

$$\Gamma \sim \frac{\Lambda^6}{m_{can}^2} = \frac{a(t)\Lambda^6}{|\mathbf{k}|\mu_\nu} = \frac{a(t)^2 \Lambda^5}{\mu_0} \, .$$

Temperature-Asymmetry relation

- After reheating the universe is radiation dominated:

$$a(t) = \sqrt{2\Omega_{r,0}^{1/2} H_0 t}$$

where $\Omega_{r,0} \sim 9.2 \cdot 10^{-5}$.

- The ghost term is no longer present when $T = T_*$:

$$1 = \frac{\Lambda}{a(t_*) m_{CS}(t_*)} \quad \Rightarrow \quad a(t_*) = \frac{\xi T_* \Lambda}{M_p^2}$$

- Given the known temperature dependence of the scale factor:

$$T_* \simeq \frac{440}{\sqrt{\xi}} \text{ GeV} \sqrt{\frac{M_p}{\Lambda}}$$

Generated Photon Spectrum and Energy Density

- Calculate the spectrum and the energy density of photons generated by the induced vacuum decay:

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n(k,t)) = \Gamma\delta\left(\frac{|\mathbf{k}|}{\Lambda}-1\right) \quad \text{and} \quad \frac{dE}{d^3x d\ln|\mathbf{k}|} \sim |\mathbf{k}| n_0(|\mathbf{k}|)$$

- Integrating this effect between T_a and T_*:

$$\frac{dE}{d^3x d\ln k} \sim \frac{\xi^4 T_*^5}{10 T_a^2}\sqrt{\frac{M_p^3}{H_0}}\left(\frac{\Lambda}{M_p}\right)^{11}$$

- Conservative bound: the universe is not radiation dominated today,

$$\frac{dE}{d^3x d\ln k} \lesssim M_p^2 H_0^2$$

Photon Energy Density Produced

- Assuming $T_a > T_*$:

$$\eta_\nu \lesssim 10^{-41}\left(\frac{T_a}{10^{15}\ \mathrm{GeV}}\right)^{4/3}\left(\frac{M_p}{\Lambda}\right)^{17/3}$$

or

$$T_* \gtrsim 10^{23}\ \mathrm{GeV}\left(\frac{T_a}{10^{15}\ \mathrm{GeV}}\right)^{-2/3}\left(\frac{\Lambda}{M_p}\right)^{17/6}$$

- Conflicts with observation unless $\Lambda \ll M_p$.

- If instead $T_a \lesssim T_*$, i.e. no vacuum decay:

$$\eta_\nu \lesssim 0.033\left(\frac{2000\ \mathrm{GeV}}{T_a}\right)^{2}\frac{\Lambda}{M_p}$$

where $\eta_\nu \lesssim 0.033$ is from BBN constraints.

Conclusion and Future Work

- Parity asymmetric CνB $\rightarrow$ Chern-Simons gravity term induced.
- Potentially observable birefringent effect for large z sources.
- Bounds from vacuum decay on the temperature a given neutrino asymmetry can be generated.

$$\eta_\nu \lesssim 0.033 \left(\frac{2000 \text{ GeV}}{T_a}\right)^2 \frac{\Lambda}{M_p}$$

Future work

- Further exploration of the mechanism, and GW propagation.
- Early universe implications: Baryogenesis, dark matter, parity asymmetries.