Constraining noncommutative space-time from GW150914

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CoEPP
ARC Centre of Excellence for Particle Physics at the Terascale

Archil Kobakhidze, CL, Adrian Manning, PRD 94 (2016) 064033

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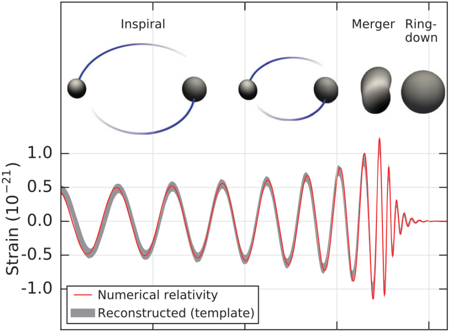
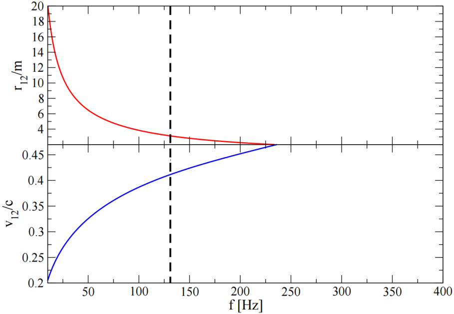
Overview

GW150914

- Inspiral, merger and ring-down of a binary black hole observed by LIGO.
- Masses of $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$.
- Frequency ranging from 35 to 250 Hz and velocity up to $\sim 0.5c$.



[LIGO/Virgo Coll., Phys. Rev. Lett. 116 (2016) 061102]



[N. Yunes, K. Yagi, F. Pretorius, arXiv:1603.08955]

An opportunity to test GR and its extensions

Einstein Field Equations (EFE) from General Relativity predicts the waveform of such GWs :

- post-Newtonian formalism provides an analytical expansion in $\frac{v}{c}$ (valid only during the inspiralling)
- numerical Relativity provides accurate simulations, including the merger and the ring-down

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[LIGO/Virgo Coll., Phys. Rev. Lett. 116 (2016) 221101]

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Our objective: constrain the scale of noncommutative space-time.

The post-Newtonian formalism

L. Blanchet, Living Rev. Rel. 17 (2014)

Definitions and notations

The full EFE in the harmonic gauge ($\partial_\mu h^{\alpha\mu} = 0$) can be written as

$$\Box h^{\alpha\beta} = \frac{16\pi G}{c^4}\tau^{\alpha\beta}$$

with the gravitational-field amplitude h and the matter-gravitational source τ:

$$h^{\alpha\beta} = \sqrt{-g}g^{\alpha\beta} - \eta^{\alpha\beta}, \qquad \tau^{\alpha\beta} = |g|T^{\alpha\beta} + \frac{c^4}{16\pi G}\Lambda^{\alpha\beta}.$$

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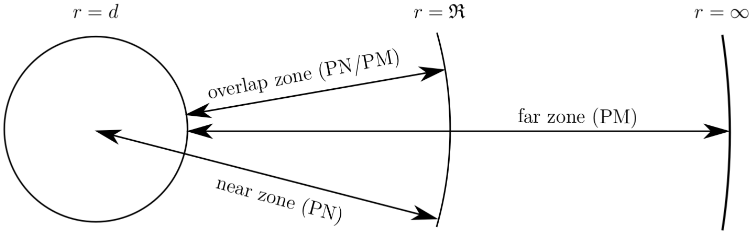
For a source term with characteristic velocity v, the post-Newtonian formalism (PN) solves the EFE as an expansion in powers of $\frac{v}{c}$. As a convention, a term of order n is called a $\frac{n}{2}$PN term and written as

$$\mathcal{O}(n) \equiv \mathcal{O}\left(\frac{v^n}{c^n}\right)$$

How to solve the full EFE?

Iterative expansions in the near and far zones and matching strategy in the overlap zone:



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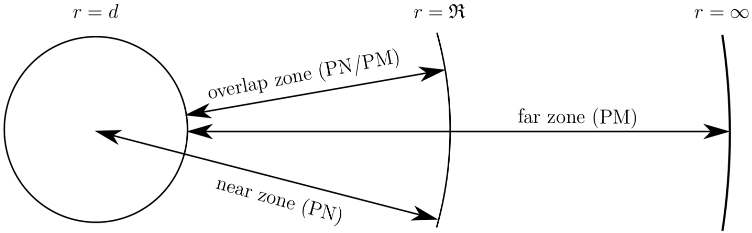

Post Minkowskian (PM) - G^n:

- $h^{\alpha\beta} = \sum_{n=1}^{\infty} G^n h_n^{\alpha\beta}$
- $\Box h^{\alpha\beta} = \Lambda^{\alpha\beta}$
- $\Box h_n^{\alpha\beta} = \Lambda_n^{\alpha\beta}[h_1, \cdots, h_{n-1}]$

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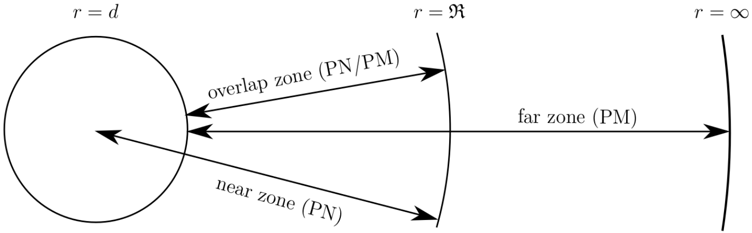

Post Newtonian (PN) - $\left(\frac{1}{c}\right)^n$:

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- $\nabla^2 h_n^{\alpha\beta} = 16\pi G\, \tau_{n-4}^{\alpha\beta} + \partial_t^2 h_{n-2}^{\alpha\beta}$

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Matter source

Consider a binary system of two black holes of masses m_1 and m_2. It is usually approximated by two point-like particles:

$$T^{\mu\nu}(\mathbf{x}, t) = \frac{m_1}{\sqrt{g g_{\rho\sigma} \frac{v_1^\rho v_1^\sigma}{c^2}}} \; v_1^\mu(t) v_1^\nu(t) \; \delta^3(\mathbf{x} - \mathbf{y}_1(t)) + 1 \leftrightarrow 2$$

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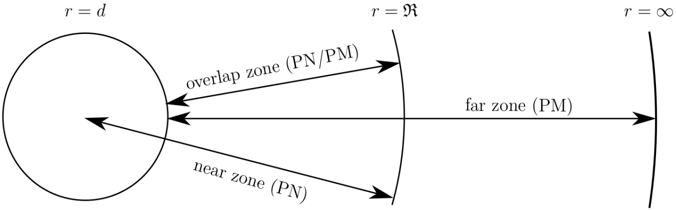
Useful parametrization:

- total mass: $M = m_1 + m_2$
- reduced mass: $\mu = \frac{m_1 m_2}{M}$
- symmetric mass ratio: $\nu = \frac{\mu}{M} = \frac{m_1 m_2}{M^2}$

The balance equation



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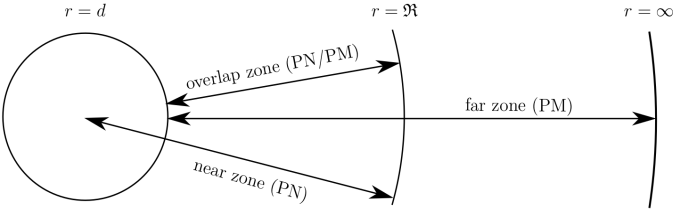

Equations of motion - energy E:

- $\nabla_\nu T^{\mu\nu} = 0$
- $\mathbf{a}_1 = -\frac{Gm_2}{r_{12}^2}\mathbf{n}_{12} + \mathcal{O}(2)$
- $E = \frac{m_1 v_1^2}{2} - \frac{Gm_1 m_2}{2r_{12}} + \mathcal{O}(2) + 1 \leftrightarrow 2$

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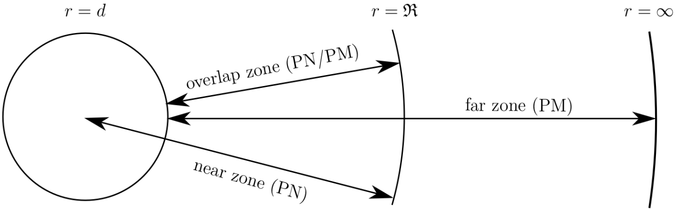

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Radiated flux $\mathcal{F}$:

- $\mathcal{F} = \frac{G}{c^5}\left(\frac{1}{5} I_{ij}^{(3)} I_{ij}^{(3)} + \mathcal{O}(2)\right)$
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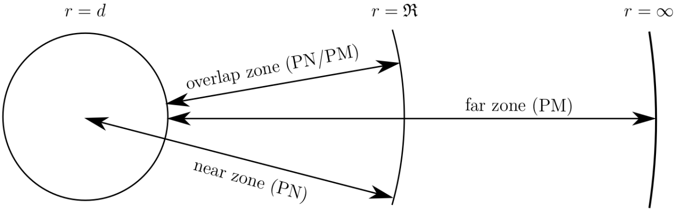

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Conservation of energy implies the balance equation and the orbital phase:

$$\frac{dE}{dt} = -\mathcal{F} \quad \Rightarrow \quad \phi = \int \Omega(t) dt$$

State-of-the-art computations

For data analysis, consider the waveform in frequency space:

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$$\psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128} \sum_{j=0}^{7} \varphi_j \left(\frac{\pi M G f}{c^3} \right)^{(j-5)/3},$$

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where the phase coefficients are

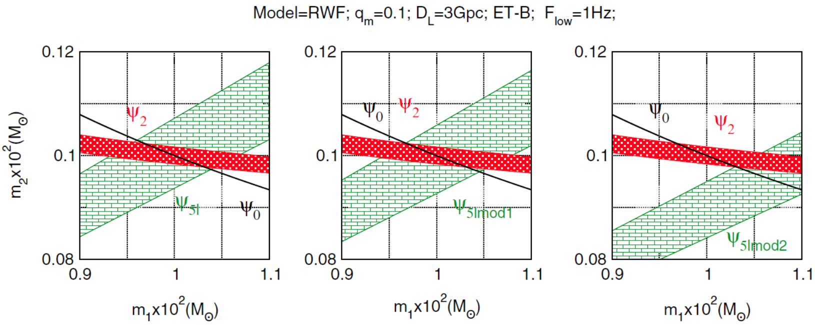
$$\begin{aligned}
\varphi_0 &= 1 \\
\varphi_1 &= 0 \\
\varphi_2 &= \frac{3715}{756} + \frac{55}{9}\nu \\
\varphi_3 &= -16\pi \\
\varphi_4 &= \frac{15293365}{508032} + \frac{27145}{504}\nu + \frac{3085}{72}\nu^2 \\
&\ldots
\end{aligned}$$

[T. Damour, B. Iyer and B. Sathyaprakash, Phys. Rev. D 63 (2001) 044023]

[G. Faye, S. Marsat, L. Blanchet, B. Iyer, Class. Quantum Grav. 29 (2012) 175004]

GR vs. GW150914

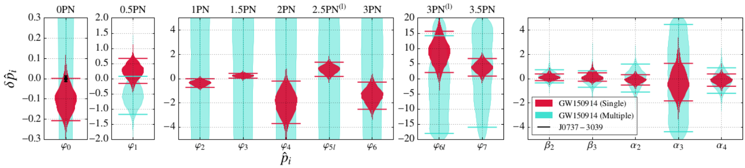
Pictorial representation on simulated data



[C. Mishra, K. Arun, B. Iyer, B. Sathyaprakash, Phys. Rev. D 82 (2010) 064010]

Bayesian analysis from GW150914

waveform regime	parameter	f−dependence	median single	median multiple	GR quantile single	GR quantile multiple	$\log_{10} B^{GR}_{model}$ single	$\log_{10} B^{GR}_{model}$ multiple
early-inspiral regime	$\delta\hat{\varphi}_0$	$f^{-5/3}$	$-0.1^{+0.1}_{-0.1}$	$1.3^{+3.0}_{-3.2}$	0.94	0.30	1.9 ± 0.2	3.7 ± 0.6
	$\delta\hat{\varphi}_1$	$f^{-4/3}$	$0.3^{+0.4}_{-0.4}$	$-0.5^{+0.6}_{-0.6}$	0.16	0.93	1.6 ± 0.2	
	$\delta\hat{\varphi}_2$	f^{-1}	$-0.4^{+0.3}_{-0.4}$	$-1.6^{+18.8}_{-16.6}$	0.96	0.56	1.2 ± 0.2	
	$\delta\hat{\varphi}_3$	$f^{-2/3}$	$0.2^{+0.2}_{-0.2}$	$2.0^{+13.4}_{-13.9}$	0.02	0.42	1.2 ± 0.2	
	$\delta\hat{\varphi}_4$	$f^{-1/3}$	$-1.9^{+1.6}_{-1.7}$	$-1.9^{+19.3}_{-16.4}$	0.98	0.56	0.3 ± 0.2	
	$\delta\hat{\varphi}_{5l}$	$\log(f)$	$0.8^{+0.5}_{-0.6}$	$-1.4^{+18.6}_{-16.9}$	0.01	0.55	0.7 ± 0.4	
	$\delta\hat{\varphi}_6$	$f^{1/3}$	$-1.4^{+1.1}_{-1.1}$	$1.2^{+16.8}_{-18.9}$	0.99	0.47	0.4 ± 0.2	
	$\delta\hat{\varphi}_{6l}$	$f^{1/3}\log(f)$	$8.9^{+6.8}_{-6.8}$	$-1.9^{+19.1}_{-16.1}$	0.02	0.57	-0.3 ± 0.2	
	$\delta\hat{\varphi}_7$	$f^{2/3}$	$3.8^{+2.9}_{-2.9}$	$3.2^{+15.1}_{-19.2}$	0.02	0.41	-0.0 ± 0.2	
intermediate regime	$\delta\hat{\beta}_2$	$\log f$	$0.1^{+0.4}_{-0.3}$	$0.2^{+0.6}_{-0.5}$	0.24	0.28	1.4 ± 0.2	2.3 ± 0.2
	$\delta\hat{\beta}_3$	f^{-3}	$0.1^{+0.6}_{-0.3}$	$-0.0^{+0.8}_{-0.7}$	0.31	0.56	1.2 ± 0.4	
merger-ringdown regime	$\delta\hat{\alpha}_2$	f^{-1}	$-0.1^{+0.4}_{-0.4}$	$0.0^{+1.0}_{-1.2}$	0.68	0.50	1.2 ± 0.2	2.1 ± 0.4
	$\delta\hat{\alpha}_3$	$f^{3/4}$	$-0.3^{+1.9}_{-1.5}$	$0.0^{+4.4}_{-4.4}$	0.60	0.51	0.7 ± 0.2	
	$\delta\hat{\alpha}_4$	$\tan^{-1}(af+b)$	$-0.1^{+0.5}_{-0.5}$	$-0.1^{+1.1}_{-1.0}$	0.68	0.62	1.1 ± 0.2	



[LIGO/Virgo Coll., Phys. Rev. Lett. 116 (2016) 221101]

Noncommutative corrections to the waveform

A. Kobakhidze, CL, A. Manning, PRD 94 (2016) 064033

Noncommutative space-time

NC space-time arises in a number of contexts:

- Originally proposed by Heisenberg as an effective UV cutoff.
- Snyder formalized the idea [Phys. Rev. 71 (1947) 38].
- Noncommutative geometry [A. Connes, Inst. Hautes Etudes Sci. Publ. Math. 62 (1985) 257].
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Previous constraints on noncommutative scale at inverse $\sim$ TeV.

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Noncommutative QFT - fields product replaced by Moyal product:

$$f(x) \star g(x) = f(x)g(x) + \sum_{n=1}^{+\infty} \left(\frac{i}{2}\right)^n \frac{1}{n!} \theta^{\alpha_1 \beta 1} \cdots \theta^{\alpha_n \beta_n} \partial_{\alpha_1} \cdots \partial_{\alpha_n} f(x) \, \partial_{\beta_1} \cdots \partial_{\beta_n} g(x)$$

Noncommutative effects on GWs

Expect both modifications on the matter source and on the EFE.

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- Consider a Schwarzschild black hole described by a massive scalar field in noncommutative QFT:

$$T^{\mu\nu}_{NC}(x) = \frac{1}{2}\left(\partial^\mu\phi \star \partial^\nu\phi + \partial^\nu\phi \star \partial^\mu\phi\right) - \frac{1}{2}\eta^{\mu\nu}\left(\partial_\rho\phi \star \partial^\rho\phi - m^2\phi \star \phi\right)$$

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- Neglect corrections to the laws of GR, since noncommutative gravity appears at $\mathcal{O}(|\theta|^2)$ and is model-dependent.

[X. Calmet, A. Kobakhidze, Phys. Rev. D74 (2006) 047702] [P. Mukherjee, A. Saha, Phys. Rev. D74 (2006) 027702]

Energy-momentum tensor in noncommutative space-time

After quantising and keeping leading-order corrections of the Moyal product:

$$T^{\mu\nu}_{NC}(\mathbf{x},t) \approx T^{\mu\nu}_{GR}(\mathbf{x},t) + \frac{m^3G^2}{8c^4} v^\mu v^\nu \Theta^{kl} \partial_k \partial_l \, \delta^3(\mathbf{x} - \mathbf{y}(t))$$

with

$$\Theta^{kl} = \frac{\theta^{0k}\theta^{0l}}{l_P^2 t_P^2} + 2\frac{v_p}{c}\frac{\theta^{0k}\theta^{pl}}{l_P^3 t_P} + \frac{v_p v_q}{c^2}\frac{\theta^{kp}\theta^{lq}}{l_P^4} = \frac{\theta^{0k}\theta^{0l}}{l_P^2 t_P^2} + \mathcal{O}(1)$$

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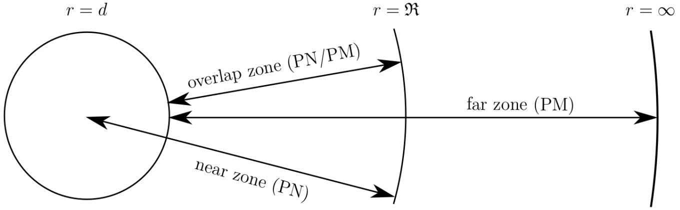
Binary black hole EMT with 2PN noncommutative corrections:

$$T^{\mu\nu}(\mathbf{x},t) = m_1 \gamma_1 v_1^\mu v_1^\nu \delta^3(\mathbf{x} - \mathbf{y}_1(t)) + \frac{m_1^3 G^2 \Lambda^2}{8c^4} v_1^\mu v_1^\nu \theta^k \theta^l \partial_k \partial_l \, \delta^3(\mathbf{x} - \mathbf{y}_1(t)) + 1 \leftrightarrow 2$$

where

$$\Lambda \theta^i = \frac{\theta^{0i}}{l_P t_P}.$$

The modified balance equation



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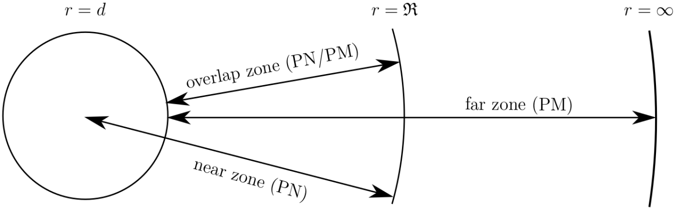

Correction to E:

- $\nabla_\nu T^{\mu\nu} = 0$
- $E_{NC} = \frac{G^3 M^3 \mu (1-2\nu)\Lambda^2}{8c^4 r^3} + \mathcal{O}(5)$

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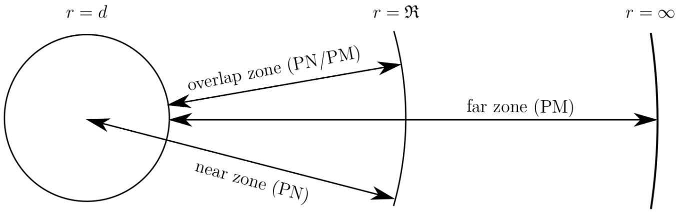

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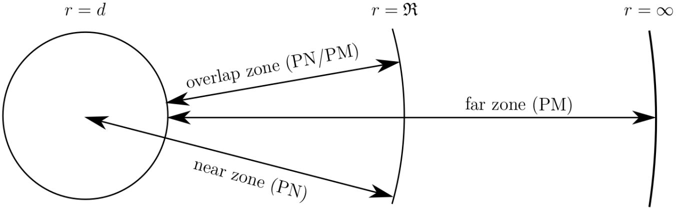

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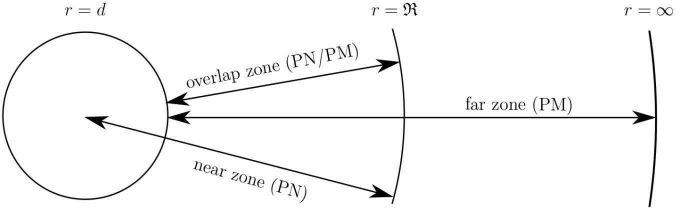
$$\frac{d(E + E_{NC})}{dt} = -\mathcal{F} - \mathcal{F}_{NC}$$

The modified balance equation



Correction to E:

- $\nabla_\nu T^{\mu\nu} = 0$
- $E_{NC} = \frac{G^3 M^3 \mu (1-2\nu)\Lambda^2}{8c^4 r^3} + \mathcal{O}(5)$

Correction to $\mathcal{F}$:

- $\mathcal{F} = \frac{G}{c^5}\left(\frac{1}{5} I_{ij}^{(3)} I_{ij}^{(3)} + \mathcal{O}(2)\right)$
- $\mathcal{F}_{NC} = \frac{G}{c^5}\left(-\frac{36}{5}\frac{G^5 M^7}{c^4 r^7}\nu^2(1-2\nu)\Lambda^2 + \mathcal{O}(5)\right)$

$$\frac{d(E + E_{NC})}{dt} = -\mathcal{F} - \mathcal{F}_{NC}$$

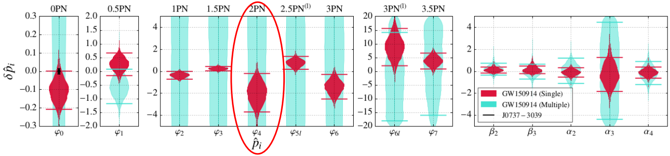
Lowest-order correction to the orbital phase:

$$\varphi_4 = \frac{15293365}{508032} + \frac{27145}{504}\nu + \frac{3085}{72}\nu^2 + \frac{5}{4}(1-2\nu)\Lambda^2$$

Constraint on the scale of noncommutativity

Noncommutativity vs. GW150914

waveform regime	parameter	f−dependence	median single	median multiple	GR quantile single	GR quantile multiple	$\log_{10} B^{GR}_{model}$ single	$\log_{10} B^{GR}_{model}$ multiple
early-inspiral regime	$\delta\hat{\varphi}_0$	$f^{-5/3}$	$-0.1^{+0.1}_{-0.1}$	$1.3^{+3.0}_{-3.2}$	0.94	0.30	1.9 ± 0.2	3.7 ± 0.6
	$\delta\hat{\varphi}_1$	$f^{-4/3}$	$0.3^{+0.4}_{-0.4}$	$-0.5^{+0.6}_{-0.6}$	0.16	0.93	1.6 ± 0.2	
	$\delta\hat{\varphi}_2$	f^{-1}	$-0.4^{+0.3}_{-0.4}$	$-1.6^{+18.8}_{-16.6}$	0.96	0.56	1.2 ± 0.2	
	$\delta\hat{\varphi}_3$	$f^{-2/3}$	$0.2^{+0.2}_{-0.2}$	$2.0^{+13.4}_{-13.9}$	0.02	0.42	1.2 ± 0.2	
	$\delta\hat{\varphi}_4$	$f^{-1/3}$	$-1.9^{+1.6}_{-1.7}$	$-1.9^{+19.3}_{-16.4}$	0.98	0.56	0.3 ± 0.2	
	$\delta\hat{\varphi}_{5l}$	$\log(f)$	$0.8^{+0.5}_{-0.6}$	$-1.4^{+18.6}_{-16.9}$	0.01	0.55	0.7 ± 0.4	
	$\delta\hat{\varphi}_6$	$f^{1/3}$	$-1.4^{+1.1}_{-1.1}$	$1.2^{+16.8}_{-18.9}$	0.99	0.47	0.4 ± 0.2	
	$\delta\hat{\varphi}_{6l}$	$f^{1/3}\log(f)$	$8.9^{+6.8}_{-6.8}$	$-1.9^{+19.1}_{-16.1}$	0.02	0.57	-0.3 ± 0.2	
	$\delta\hat{\varphi}_7$	$f^{2/3}$	$3.8^{+2.9}_{-2.9}$	$3.2^{+15.1}_{-19.2}$	0.02	0.41	-0.0 ± 0.2	
intermediate regime	$\delta\hat{\beta}_2$	$\log f$	$0.1^{+0.4}_{-0.3}$	$0.2^{+0.6}_{-0.5}$	0.24	0.28	1.4 ± 0.2	2.3 ± 0.2
	$\delta\hat{\beta}_3$	f^{-3}	$0.1^{+0.6}_{-0.3}$	$-0.0^{+0.8}_{-0.7}$	0.31	0.56	1.2 ± 0.4	
merger-ringdown regime	$\delta\hat{\alpha}_2$	f^{-1}	$-0.1^{+0.4}_{-0.4}$	$0.0^{+1.0}_{-1.2}$	0.68	0.50	1.2 ± 0.2	2.1 ± 0.4
	$\delta\hat{\alpha}_3$	$f^{3/4}$	$-0.3^{+1.9}_{-1.5}$	$0.0^{+4.4}_{-4.4}$	0.60	0.51	0.7 ± 0.2	
	$\delta\hat{\alpha}_4$	$\tan^{-1}(af+b)$	$-0.1^{+0.5}_{-0.5}$	$-0.1^{+1.1}_{-1.0}$	0.68	0.62	1.1 ± 0.2	



$$\delta\varphi_4^{NC} = \frac{\varphi_4^{NC}}{\varphi_4^{GR}} = \frac{1270080\,(1-2\nu)}{4353552\,\nu^2 + 5472432\,\nu + 3058673}\Lambda^2$$

$$|\delta\varphi_4^{NC}| \lesssim 20 \Rightarrow \sqrt{\Lambda} \lesssim 3.5$$

Conclusion

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- Derivation of the lowest-order (2PN) noncommutative correction to the GW waveform.
- Constraint on the scale of noncommutativity to around the Planck scale:

$$|\theta^{0i}| \lesssim 12 \cdot l_P t_P$$

$\sim$ 15 orders of magnitude improvement