Electroweak monopoles and the electroweak phase transition[1]

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[1] S. Arunasalam and A. Kobakhidze, arXiv: 1702.04068 (2017)

Outline

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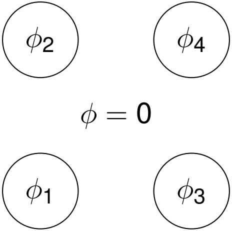
- The suppression of these processes requires a first order phase transition with $\frac{\phi_c}{T_c} \gtrsim 1$
- The EWPT is second order in the standard model
- The energy density of monopoles can contribute to the energy of the broken phase, enhancing the phase transition while satisfying nucleosynthesis constraints.

Outline

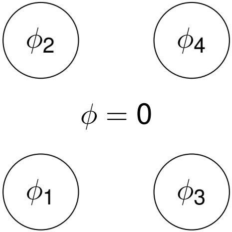
The Kibble Mechanism

- At $T = T_c$, domains of the broken phase will appear



The Kibble Mechanism

- At $T = T_c$, domains of the broken phase will appear
- The higgs field in each domain takes independent directions on the vacuum manifold



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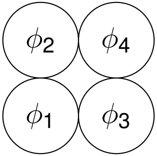
- As the Higgs field is continuous, it must be interpolated at the intersections.



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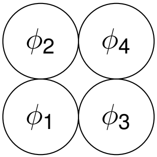
- Consider an intersection of four of these domains:



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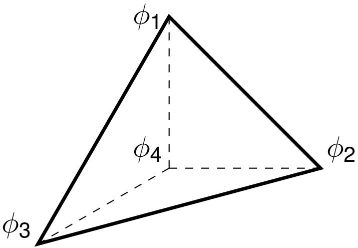

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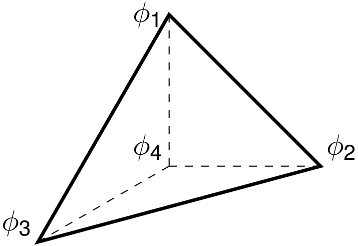

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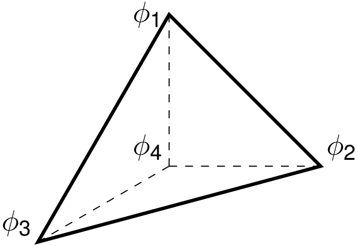

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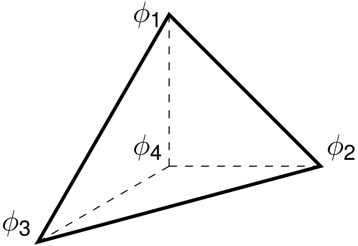

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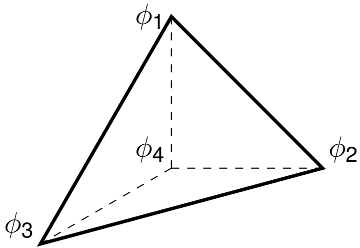
- Therefore, $\pi_2(M_{\text{vac}}) \neq 0$ implies the existence of monopoles



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- No electroweak monopoles?

Outline

The Ansatz

- Cho and Maison (1997) found electroweak monopoles through the ansatz:

$$
\begin{aligned}
\phi &= \frac{1}{\sqrt{2}}\rho\xi \\
\rho &= \rho(r) \\
\xi &= i\begin{pmatrix}\sin(\theta/2)e^{-i\varphi} \\ -\cos(\theta/2)\end{pmatrix} \\
A_\mu &= \frac{1}{g}A(r)\partial_\mu t\hat{\phi} + \frac{1}{g}(f(r)-1)\hat{\phi}\times\partial_\mu\hat{\phi} \\
B_\mu &= -\frac{1}{g'}B(r)\partial_\mu t - \frac{1}{g'}(1-\cos\theta)\partial_\mu\varphi
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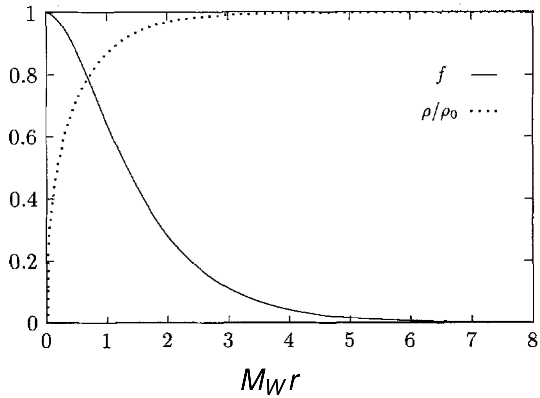
- $M_{\text{vac}} = SU(2)/U(1) \cong \mathbb{C}P^1$
- $\pi_2(M_{\text{vac}}) = \mathbb{Z}$

Solution

- Simple solution: $A = B = 0$ (Cho & Maison, 1997)



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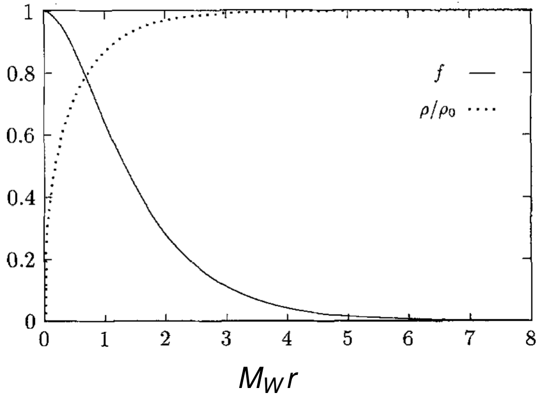
- $h = \frac{4\pi}{e}$



The energy

$$E = E_0 + E_1$$

$$E_0 = 4\pi \int_0^\infty \frac{dr}{2r^2} \left\{ \frac{1}{g'^2} + \frac{1}{g^2}(f^2 - 1)^2 \right\}$$

$$E_1 = 4\pi \int_0^\infty dr \left\{ \frac{1}{2}(r\dot{\rho})^2 + \frac{1}{g^2}\left(\dot{f}^2 + \frac{1}{2}(r\dot{A})^2 + f^2 A^2 \right) \right.$$

$$+ \frac{1}{2g'^2}(r\dot{B})^2 + \frac{\lambda r^2}{8}(\rho^2 - \rho_0^2)^2$$

$$\left. + \frac{1}{4} f^2 \rho^2 + \frac{r^2}{8}(B - A)^2 \rho^2 \right\}$$

- The first term of E_0 is divergent at the origin.

Regularisation

- Cho, Kim and Yoon(2015) proposed a regularisation of the form:

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- However, g' becomes non-peturbative as $\phi \to 0$.
- This is undesirable in an EFT framework.
- We instead propose a Born-Infeld modification for the $U(1)_Y$ kinetic term.

Born-Infeld modification

- We regularise the $U(1)_Y$ kinetic term by replacing it with:

$$\beta^2\left[1-\sqrt{-\det\left(\eta_{\mu\nu}+\frac{1}{\beta}B_{\mu\nu}\right)}\right]$$

$$=\beta^2\left[1-\sqrt{1+\frac{1}{2\beta^2}B_{\mu\nu}B^{\mu\nu}-\frac{1}{16\beta^4}(B_{\mu\nu}\tilde{B}^{\mu\nu})^2}\right]$$

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- The corresponding energy is

$$\int_0^\infty dr\beta^2 \left[\sqrt{(4\pi r^2)^2 + \left(\frac{4\pi}{g'\beta}\right)^2} - 4\pi r^2\right]$$

$$= \frac{4\pi^{5/2}}{3\Gamma\left(\frac{3}{4}\right)^2}\sqrt{\frac{\beta}{g'^3}}$$

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- Hence, β acts as a mass parameter for the monopoles.

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- The Gibbs free energy:

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- Assuming $T << M$,the monopoles are decoupled and $E_{monopoles} = M \times n_M$

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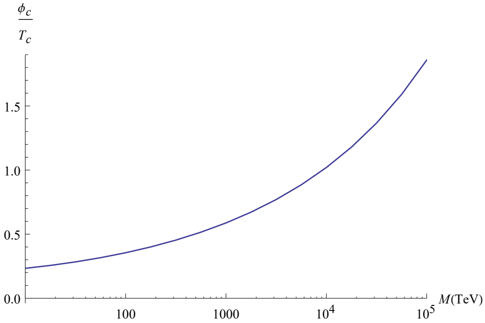
- Hence, $n_M \approx \left(\frac{4\pi}{h^2}\right)^3 T^3$

Results



- Sphaleron processes are suppressed for $M > 0.9 \cdot 10^4$ TeV.

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- $\left.\frac{n}{T^3}\right|_{T=1\text{MeV}} < \frac{1\text{MeV}}{M}$
- Hence, the evolution of the number density over time must be considered.

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- $B = \frac{3}{4\pi^2} \zeta(3) \sum_i (hq_i/4\pi)^2$

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- For $T < T_f$, the monopole density simply dilutes as $n \propto T^3$.

Nucleosynthesis constraint

- Solving the Boltzmann equation, one obtains (Preskill, 1979)

$$\frac{n}{T^3} = \frac{1}{Bh^2}\left(\frac{4\pi}{h^2}\right)^2 \frac{M}{CM_{pl}}, \ (T > T_f)$$

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- This constrains the mass of the monopole to $M \lesssim 2.3 \cdot 10^4$ TeV.

Summary and future work

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- This occurs for monopoles with a mass of $(0.9 - 2.3) \cdot 10^4$TeV.
- This could possibly be extended to a new mechanism for electroweak baryogenesis