NNLO N-jettiness soft functions for one massive coloured particle production

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High order QCD corrections

QCD corrections

- Fixed order corrections(LO,NLO,NNLO, in expansion of strong coupling α_s)
- Analytical resummation(LL,NLL,NNLL, in expansion of $\alpha_S \times L$)
- Parton shower resummation through Monte Carlo tools (Pythia, Sherpa, Herwig $\cdot$)

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Fixed-order calculation

- General understanding of QCD radiation and one-loop amplitudes makes NLO calculation automated (MadGraph_aMC@NLO $\cdots$)
- We are now in the era of NNLO phenomenology
- There are many methods developed such as sector decomposition, transverse momentum subtraction (SCET), antenna subtraction,n-jettiness subtraction (SCET)...

What is SCET

The Soft-Collinear Effective Theory(SCET) [1] can be used to factorise multi-scale cross sections into single-scale hard, collinear, and soft parts. It is the effective theory of the soft and collinear dynamics of QCD, or more generally, any gauge theory with a weakly-coupled sector.

Q	QCD	hard	H
M	↓ SCET	collinear	J
M^2/Q	↓ SET	soft	S

Hard modes of the full theory are integrated out and represented by Wilson coefficients.

$$\mathcal{L}(\phi) = \underbrace{\mathcal{L}(\phi_C)}_{\text{collinear}} + \underbrace{\mathcal{L}(\phi_{\bar{C}})}_{\text{anti-collinear}} + \underbrace{\mathcal{L}(\phi_s)}_{\text{soft}} + \underbrace{\mathcal{L}_{c+s}(\phi_C, \phi_{\bar{C}}, \phi_s)}_{\text{disappear after field redefinition}}$$

In N-jettiness limit the cross section can be written as

$$d\sigma \propto \underbrace{B_1}_{\text{collinear}} \otimes \underbrace{B_2}_{\text{anti-collinear}} \otimes \underbrace{H}_{\text{Wilson Coefficient}} \otimes \underbrace{S}_{\text{soft}} \otimes \underbrace{\prod_i J_i}_{\text{collinear}}$$

1. see, arXiv :1410.1892 for a review.

What is N-jettiness

The N-jettiness event shape variable is

$$\mathcal{T}_N = \sum_k \min_i \{n_i \cdot q_k\}$$

Here n_i ($i = a, b, 1, ..., N$) are light-like reference vectors representing the moving directions of massless external particles. When q_k is soft or collinear with any external partons $\mathcal{T}_N \to 0$.

When $\mathcal{T}_N \to 0$, the cross section is factorised as the convolution of a hard function, two beam functions for initial states, a soft function and jet functions

$$\frac{d\sigma}{d\mathcal{T}_N} \propto \int H \otimes \underbrace{B_1 \otimes B_2}_{\mathrm{N-jettiness\ PDFs}} \otimes S \otimes \left(\prod_{n=1}^{N} J_n \right)$$

The beam functions are known up to NNLO (Gaunt et al, 2014) . The jet function has been calculated at NNLO (Becher et al, 2006, 2010). The soft function has been studied up to NNLO for massless parton production (Boughezal et al 2015). The hard function relies on two-loop virtual correction and is process dependent.

N-jettiness with massive particle

Application of factorised cross section using N-jettiness variables

- Resummed distributions (Stewart et al, 2009-2010 ; Kang et al, 2012-2015 ; Berger, 2010 ; Jouttenus, 2013 ; Alioli, 2015, $\cdots$).

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What happens for massive coloured particle production such as top quark ?

- Because there are no collinear singularities along the moving direction of the heavy particle, the definition of $\mathcal{T}_N = \sum_k \min_i \{n_i \cdot q_k\}$ can control all the infrared (both soft and collinear) singularities.

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- Because there are no collinear singularities along the moving direction of the heavy particle, the definition of $\mathcal{T}_N = \sum_k \min_i \{n_i \cdot q_k\}$ can control all the infrared (both soft and collinear) singularities.
- And the effect of including heavy external particle amounts to modifying only the soft function. The definitions of the beam functions and jet functions are the same.

Kinematics

We consider the process

$$P_1 + P_2 \to Q + X$$

where P_1 and P_2 denote incoming hadrons, Q represents the massive colored particle, and X includes any inclusive hadronic final state.
For later convenience we introduce two light-like vectors

$$n^\mu = (1,0,0,1), \qquad \bar{n}^\mu = (1,0,0,-1)$$

The momenta can be written as

$$p_1^\mu = \frac{m}{2} n^\mu \ , \quad p_2^\mu = \frac{m}{2} \bar{n}^\mu, \quad p_3^\mu = \frac{m}{2} (n^\mu + \bar{n}^\mu) \ ,$$

where m is the mass of particle Q. The 0-jettiness event shape variable in this process is defined as

$$\tau \equiv \mathcal{T}_0 = \sum_k \min\{n \cdot q_k, \bar{n} \cdot q_k\}$$

Soft function

The soft function is defined by the vacuum matrix element

$$S(\tau,\mu) = \sum_{X_s} \left\langle 0 \middle| \bar{\mathbf{T}} Y_n^\dagger Y_{\bar n} Y_v \middle| X_s \right\rangle \underbrace{\delta\left(\tau - \sum_k \min\left(n\cdot \hat{P}_k, \bar{n}\cdot \hat{P}_k\right)\right)}_{\text{measurement function}} \left\langle X_s \middle| \mathbf{T} Y_n Y_{\bar n}^\dagger Y_v^\dagger \middle| 0 \right\rangle$$

where the soft Wilson lines are defined as (Bauer et al., 2002, Chay et al., 2005, Korchemsky and Radyushkin, 1992)

$$Y_n(x) = \mathbf{P} \exp\left(i g_s \int_{-\infty}^0 ds\, n\cdot A_s^a(x+sn) \mathbf{T}^a \right),$$

$$Y_{\bar n}^\dagger(x) = \bar{\mathbf{P}} \exp\left(-i g_s \int_{-\infty}^0 ds\, \bar{n}\cdot A_s^a(x+s\bar{n}) \mathbf{T}^a \right),$$

$$Y_v^\dagger(x) = \mathbf{P} \exp\left(i g_s \int_0^{\infty} ds\, v\cdot A_s^a(x+sv) \mathbf{T}^a \right)$$

with $\mathbf{P}(\bar{\mathbf{P}})$ the (anti-)path-ordering operator acting on the colour operator $\mathbf{T}^a$.

Soft function

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Different kinds of soft function

Measurement Function	Factorised cross section	Applications
$\delta(E-\sum_i E_i)$	H $\otimes$ S	threshold resummation
$\delta(\vec{p_T}-\sum_i \vec{p^i_T})$	$B\otimes B\otimes H\otimes S$	q_T resummation
$\delta\left(\tau-\sum_k \min_i\{n_i\cdot q_k\}\right)$	$B\otimes B\otimes H\otimes S\otimes J$	n-jettiness resummation

Soft function

According to dimensional analysis, the bare soft function can be written in QCD perturbation theory as

$$S(\tau,\mu) = \delta(\tau) + \frac{1}{\tau}\sum_{n=1}^{\infty}\left(\frac{Z_{\alpha_s}\alpha_s}{4\pi}\right)^n \left(\frac{\tau}{\mu}\right)^{-2n\epsilon} s^{(n)}$$

It is convenient to discuss the renormalisation group equation by using Laplace transformed soft function, given by

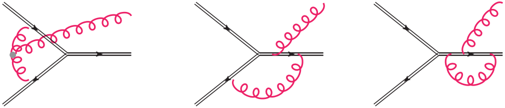
$$\begin{aligned}\tilde{S}(L,\mu) &= \int_0^\infty d\tau \exp\left(-\frac{\tau}{e^{\gamma_E}\mu e^{L/2}}\right) S(\tau) \\ &= 1 + \sum_{n=1}^{\infty}\left(\frac{Z_{\alpha_s}\alpha_s}{4\pi}\right)^n e^{n(L-2\gamma_E)\epsilon}\Gamma(-2n\epsilon)s^{(n)}\end{aligned}$$

where we use renormalised strong coupling α_s and its renormalisation factor $Z_{\alpha_s} = 1 - \beta_0\alpha_s/(4\pi\epsilon) + \mathcal{O}(\alpha_s^2)$.

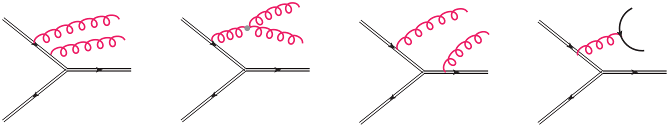
NNLO soft function

The LO and NLO soft function is easy to calculate. The NNLO contribution consists of two parts

$$S^{(2)}(\tau) = S^{(2)}_{\mathrm{VR}}(\tau) + S^{(2)}_{\mathrm{DR}}(\tau)$$

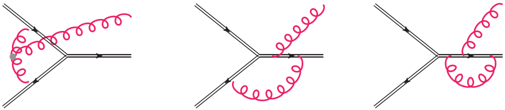


Selected diagrams for virtual real corrections



Selected diagrams double real corrections

NNLO soft function-virtual real



$$S^{(2)}_{\mathrm{VR}}(\tau) = \frac{4e^{2\gamma_E\epsilon}\mu^{4\epsilon}}{\pi^{1-\epsilon}}\mathrm{Re}\left[\int d^d q \delta(q^2) J_a^{\mu(0)\dagger} d_{\mu\nu}(q) J_a^{\nu(1)}(q) F(n,\bar{n},q)\right]$$

where

$$J_a^{\mu(1)}(q) = if_{abc}\sum_{i\neq j=1}^{3} \mathbf{T}_i^b \mathbf{T}_j^c \left(\frac{p_i^\mu}{p_i\cdot q} - \frac{p_j^\mu}{p_j\cdot q}\right) g_{ij}(\epsilon, q, p_i, p_j)\ ,$$

The measurement function is

$$F(n,\bar{n},q) = \delta(q^+ - \tau)\Theta(q^- - q^+) + \delta(q^- - \tau)\Theta(q^+ - q^-)\ ,$$

NNLO soft function-virtual real

The result has a closed-form expression, given by

$$
\begin{aligned}
s^{(2)}_{\mathrm{VR}} = & -\frac{8\mathrm{C_A}\mathrm{C_F}}{\epsilon^3} + \frac{8\mathrm{C_A^2}}{\epsilon^2} + \frac{4\mathrm{C_A}}{3\epsilon}\left((\pi^2 - 6 - 24\ln 2)\mathrm{C_A} + 3\pi^2\mathrm{C_F}\right) \\
& + \frac{4\mathrm{C_A}}{3}\left(\mathrm{C_A}(\pi^2 - 33\zeta_3 + 12(\ln^2 2 + 2\ln 2)) + 16\zeta_3\mathrm{C_F}\right) \\
& - \epsilon\frac{\mathrm{C_A}}{15}\Big[2\mathrm{C_A}\Big(\pi^4 + 30\pi^2(3 - 4\ln 2 + \ln^2 2) \\
& - 10(24 + 3\ln^4 2 + 72\mathrm{Li}_4(1/2) - 124\zeta_3 + 63\zeta_3\ln 2)\Big) + \pi^4\mathrm{C_F}\Big] \\
& + \mathcal{O}(\epsilon^2)
\end{aligned}
$$

NNLO soft function-double real

The full integrand for double-real emission can be found in literature (Catani et al, 2001 ; Czakon, 20011). The contribution from double gluon radiation is

$$S_{\rm gg}^{(2)}(\tau) = \frac{2e^{2\gamma_E \epsilon}}{\pi^{2-2\epsilon}} \int d^d q_1 d^d q_2 \delta(q_1^2)\delta(q_2^2)$$
$$\times J_{a_1 a_2}^{\mu_1 \nu_1 (0)\dagger}(q_1, q_2) d_{\mu_1 \mu_2}(q_1) d_{\nu_1 \nu_2}(q_2) J_{a_1 a_2}^{\mu_2 \nu_2 (0)}(q_1, q_2) F(n, \bar{n}, q_1, q_2)$$

where $F(n, \bar{n}, q_1, q_2)$ is the measurement function

$$\begin{aligned} F(n, \bar{n}, q_1, q_2) = & \ \delta(q_1^+ + q_2^+ - \tau)\Theta(q_1^- - q_1^+)\Theta(q_2^- - q_2^+) \\ & + \delta(q_1^+ + q_2^- - \tau)\Theta(q_1^- - q_1^+)\Theta(q_2^+ - q_2^-) \\ & + \delta(q_1^- + q_2^+ - \tau)\Theta(q_1^+ - q_1^-)\Theta(q_2^- - q_2^+) \\ & + \delta(q_1^- + q_2^- - \tau)\Theta(q_1^+ - q_1^-)\Theta(q_2^+ - q_2^-) \end{aligned}$$

The whole phase space is partitioned into four pieces.

We have adopted two different methods to deal with phase space integration so that they can provide a cross-check.

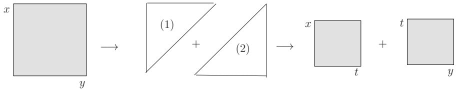
- Mellin-Barnes representation

$$\frac{1}{(X+Y)^\lambda} = \frac{1}{\Gamma(\lambda)} \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} dz \frac{Y^z}{X^{\lambda+z}} \Gamma(\lambda+z)\Gamma(-z)$$

  The integration over z was done numerically by MB-tools [2]

- Sector decomposition method : the basic idea is to factorise the overlapping singularities

$$\int_0^1 dx \int_0^1 dy \, x^{-1-a\epsilon} \, y^{-b\epsilon} \left(x + (1-x)\, y\right)^{-1}$$



$$\int_0^1 dx \, x^{-1-(a+b)\epsilon} \int_0^1 dt \, t^{-b\epsilon} \left(1 + (1-x)\, t\right)^{-1} + \int_0^1 dy \, y^{-1-(a+b)\epsilon} \int_0^1 dt \, t^{-1-a\epsilon} \left(1 + (1-y)\, t\right)^{-1}$$

2. https ://mbtools.hepforge.org/

There are in total over two hundreds of integrals in our calculation because of the different kinematic structure. For illustration, one example of such integrals is

$$I = \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dt \frac{x^{1+2\epsilon} y^{-1+2\epsilon} (1-z)^{-2\epsilon} z^{-1-2\epsilon} (1-t)^{-\frac{1}{2}-\epsilon} t^{-\frac{1}{2}-\epsilon}}{\left(x^2 + z - zx^2\right)\left(1 - 2xy + x^2 y^2 + 4txy\right)} .$$

$$I|_{\mathrm{MB}} = \frac{0.392699}{\epsilon^3} + \frac{1.08879}{\epsilon^2} + \frac{4.09324}{\epsilon} + 15.1676 + 50.7918\epsilon \, ,$$

$$I|_{\mathrm{SecDec}} = \frac{0.392695}{\epsilon^3} + \frac{1.08876}{\epsilon^2} + \frac{4.09301}{\epsilon} + 15.1668 + 50.7878\epsilon \, .$$

We find that the results from both methods agree well with each other.

Cross-Check with RG equations

Because of the independence of the cross section on the renormalisation scale μ, in Laplace space, RG equation for soft function is

$$\frac{d\ln\tilde{s}}{d\ln\mu} = \gamma_s = -\frac{d\ln H}{d\ln\mu} - \frac{d\ln\tilde{B}_1}{d\ln\mu} - \frac{d\ln\tilde{B}_2}{d\ln\mu}$$

The expression for renormalisation factor Z_s is

$$\ln Z_s = \frac{\alpha_s}{4\pi}\left(\frac{\gamma_s^{(0)\prime}}{4\epsilon^2} + \frac{\gamma_s^{(0)}}{2\epsilon}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left(-\frac{3\beta_0\gamma_s^{(0)\prime}}{16\epsilon^2} + \frac{\gamma_s^{(1)\prime} - 4\beta_0\gamma_s^{(0)}}{16\epsilon^2} + \frac{\gamma_s^{(1)}}{4\epsilon}\right)$$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}
SCET prediction	-24.8889	96.8889	158.568	354.032
Numerical calculation	-24.8889	96.8888	158.577	353.820
Difference	0	-1 $\times 10^{-4}$	9$\times 10^{-3}$	-0.212

Comparison of the coefficients of ϵ^{-i}, $i = 1, 2, 3, 4$ in double real contribution.

Normalised soft function

In the end, we obtain the following renormalised soft function in Laplace space for a colour octet production

$$\tilde{s}^{(1)} = 2\mathrm{C_A}(L - 2\ln 2) - \mathrm{C_F}(2L^2 + \pi^2),$$

$$\tilde{s}^{(2)} = \mathrm{C_A^2} K_{AA} + \mathrm{C_A C_F} K_{AF} + \mathrm{C_F^2} K_{FF} + \mathrm{C_A} n_f K_{Af} + \mathrm{C_F} n_f K_{Ff} - \frac{A_1}{4}$$

with

$$K_{AA} = \frac{17L^2}{3} + \Big(4\zeta_3 + \frac{98}{9} - \frac{2\pi^2}{3} - \frac{68}{3}\ln 2\Big)L + \frac{17\pi^2}{6}\,,$$

$$K_{AF} = -\frac{58L^3}{9} + \Big(-\frac{134}{9} + \frac{2\pi^2}{3} + 8\ln 2\Big)L^2 + \Big(28\zeta_3 - \frac{808}{27} - \frac{40\pi^2}{9}\Big)L - \frac{568\zeta_3}{3} + \frac{4\pi^4}{9} - \frac{268\pi^2}{27} + \frac{52\pi^2}{3}\ln 2\,,$$

$$K_{FF} = 2L^4 + 2\pi^2 L^2 + \frac{247\pi^4}{90}\,,$$

$$K_{Af} = -\frac{2}{3}L^2 + \Big(\frac{8}{3}\ln 2 - \frac{20}{9}\Big)L - \frac{\pi^2}{3}\,,$$

$$K_{Ff} = \frac{4}{9}L^3 + \frac{20}{9}L^2 + \Big(\frac{112}{27} + \frac{4\pi^2}{9}\Big)L + 8\zeta_3 + \frac{40}{27}\pi^2$$

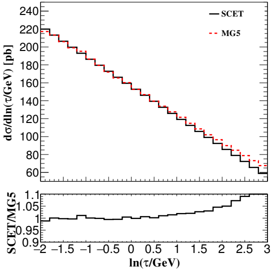
Application

In the limit $\tau \ll M$, the cross section can be written as

$$\frac{d\sigma}{dYd\tau} = \sigma_0 H(\mu^2) \int dt_a dt_b d\tau_s B_1(t_a, x_a, \mu) B_2(t_b, x_b, \mu)$$
$$\times S(\tau_s, \mu) \delta\left(\tau - \tau_s - \frac{t_a + t_b}{m}\right)\left(1 + \mathcal{O}\left(\frac{\Lambda^2}{m^2}, \frac{\tau}{m}\right)\right)$$

The NLO and NNLO τ distribution is

$$d\sigma|_{NLO} = \underbrace{d\sigma(\tau < \tau^{\rm cut})}_{SCET} + \underbrace{d\sigma(\tau \geq \tau^{\rm cut})}_{LO}; \qquad d\sigma|_{NNLO} = \underbrace{d\sigma(\tau < \tau^{\rm cut})}_{SCET} + \underbrace{d\sigma(\tau \geq \tau^{\rm cut})}_{NLO}$$



Conclusion and Outlook

- Discussed the N-jettiness method for massive particle production
- Present the N-jettiness NNLO soft function for one massive particle production
- This soft function can be used in NNLO calculation with N-jettiness subtraction method
- This soft function is one boundary condition for moving coloured particle production such as single top production
- This soft function can be extended to the case of single top production or top pair production at the LHC

Thank you for your attention !