A theory on flavors

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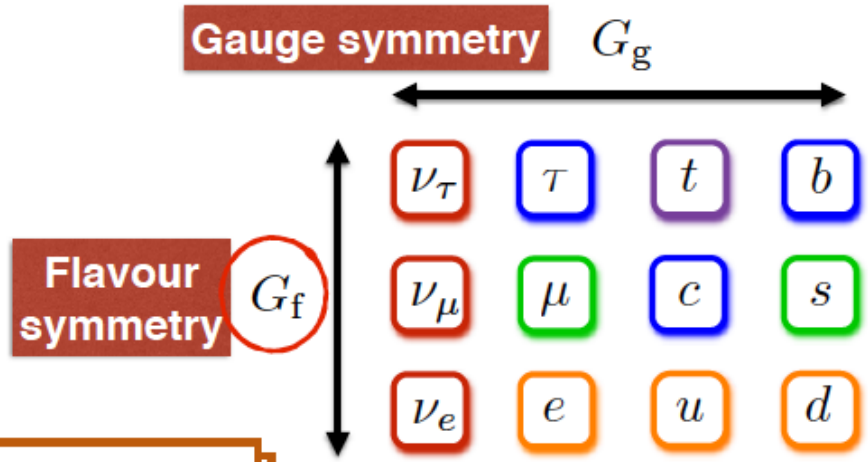
1. Introduction

Flavour symmetry



Useful for CP phase calculation.

Anarchy

	Continuous	Discrete
Abelian	U(1)	Z_n
Non-Abelian	SU(3), SO(3),…	A_4, S_4, A_5, $\Delta(48)$, …

(i) SM x (Family symmetry)

(ii) GUT x (Family symmetry)

I will focus on this

(iii) Unification of GUT families in a simple gauge group!!!

UGUTFs: SU(7) in string compactification [JEK, 1503.03104].

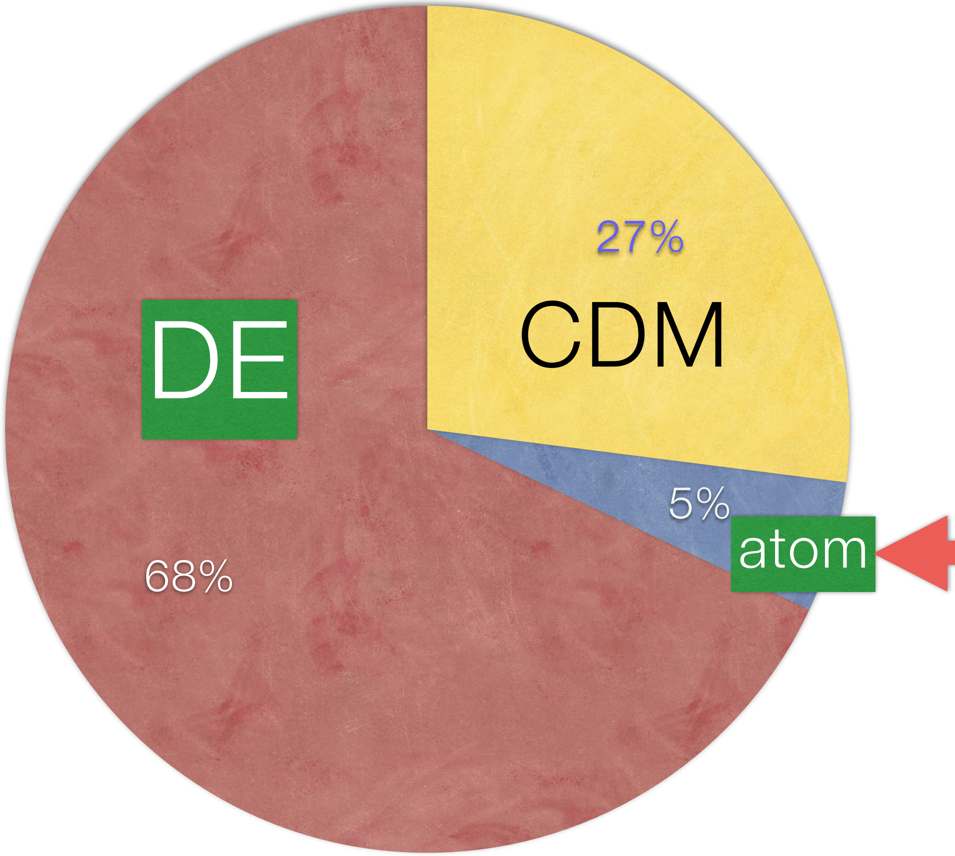
One CP phase is given in [JEK+Nam, 1506.08491].

Froggatt-Nielsen form,

$$\tilde{M}^{(u)} = \left(\begin{array}{c|ccc} & u_R(+5) & c_R(+4) & t_R(+2) \\ \hline \bar{q}_1(+1) & cX_{-1}^{u6} & -cX_{-1}^{u5} & \kappa_t X_{-1}^{u3} \\ \bar{q}_2(0) & -cX_{-1}^{u5} & cX_{-1}^{u4} & -\kappa_t X_{-1}^{u2} \\ \bar{q}_3(-2) & \kappa_t X_{-1}^{u3} & -\kappa_t X_{-1}^{u2} & 1 \end{array}\right) v_u, \qquad \tilde{M}^{(d)} = \left(\begin{array}{c|ccc} & d_R(-5) & s_R(0) & b_R(+2) \\ \hline \bar{q}_1(+1) & dX_{+1}^{d4} & 0 & 0 \\ \bar{q}_2(0) & 0 & sX_{+1}^{d}X_{-1}^{d} & \kappa_b X_{-1}^{d2} \\ \bar{q}_3(-2) & 0 & \kappa_b X_{+1}^{d2} & 1 \end{array}\right) v_d$$

Entries must have different phases

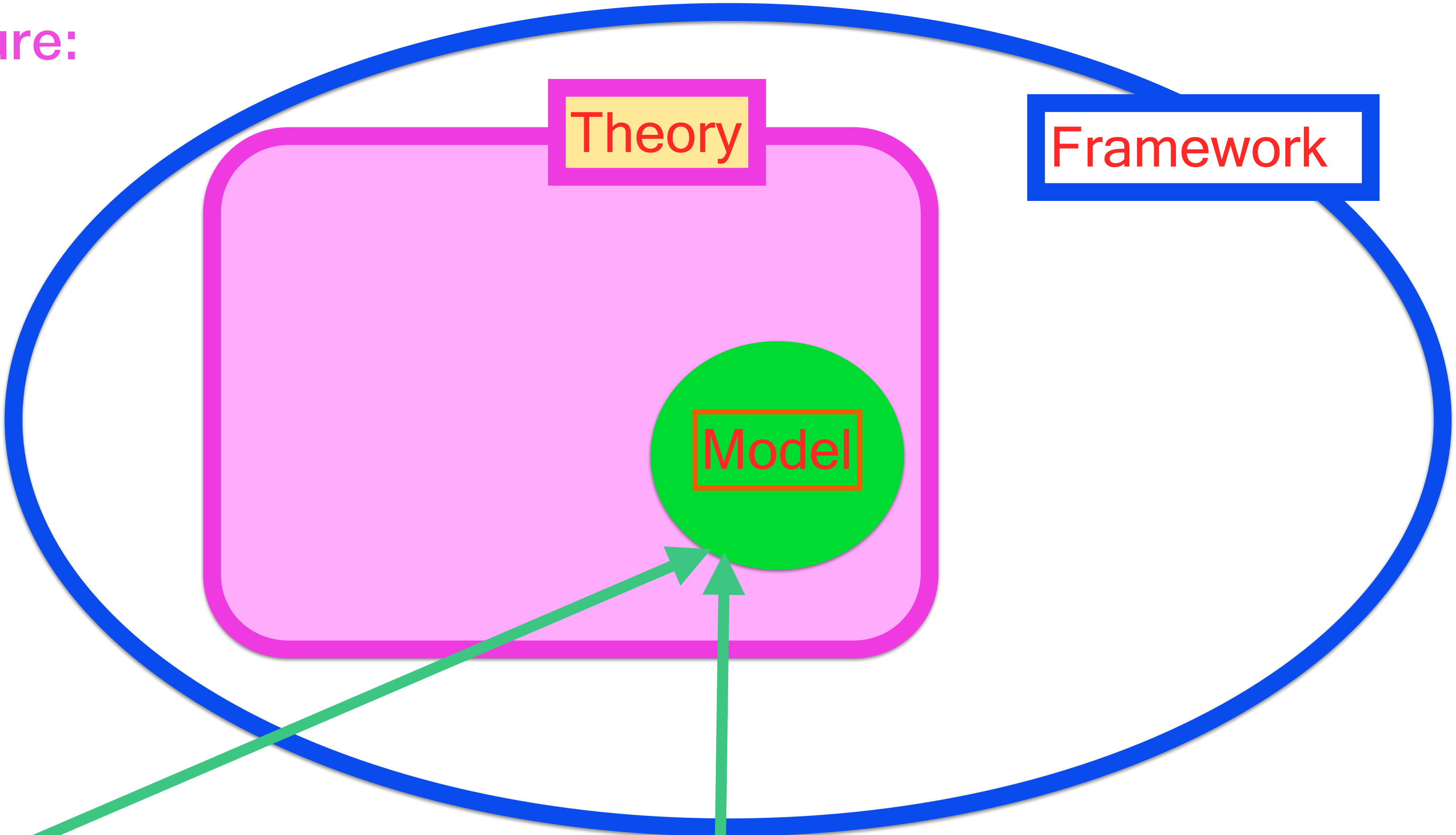
These are real, for example



Chiral fields at GUT scale
SU(5), SU(7) GUTs

UGUTF:
Kim, PRL 45, 1916 (1980);
arXiv:1503.03104.

Gross’s picture:



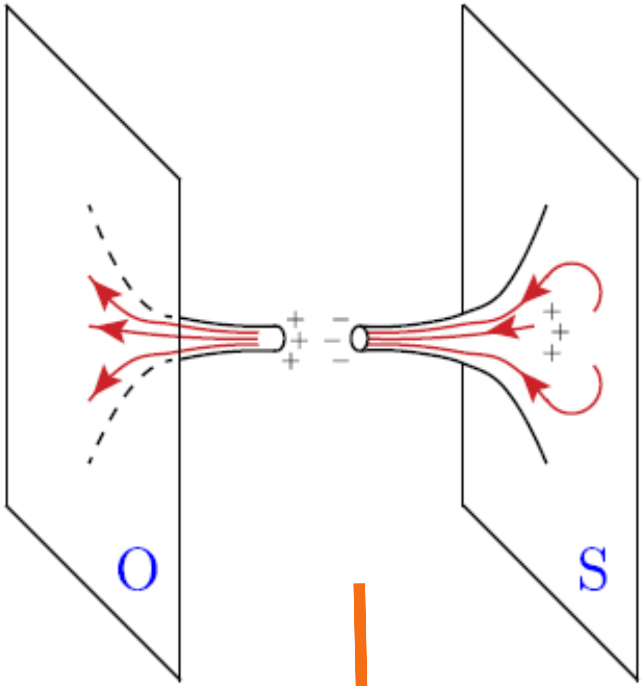
“Model” is a working example. Even though the design is fantastic, without a model example some will say that it is a religion.

Efforts to find a working model is our job toward THEORY/FRAMEWORK.

Let us work in string compactification.

The dominant contribution is
QCD anomaly term

How about using $U(1)_{PQ}$ as a family symmetry?



Wormholes:
Gidding-Strominger,
Coleman, Cline

For PQ,
Barr-Seckel,
Kamionkowski-MarchRussel,
Holdom et al.
Exclude terms up to dim 8.

The example of acc symm.

Discrete gauge symmetry:
Krauss-Wilczek

2. ’t Hooft mechanism

't Hooft mechanism:

If a gauge symmetry and a global symmetry are broken by one complex scalar by the BEHGHK mechanism, then the gauge symmetry is broken and a global symmetry remains unbroken.

Q_{gauge}	Q_{global}
1	1

Unbroken $X=Q_{global}-Q_{gauge}$

$$\phi \to e^{i\alpha(x)Q_{\mathrm{gauge}}} e^{i\beta Q_{\mathrm{global}}} \phi$$

the α direction becomes the longitudinal mode of heavy gauge boson. The above transformation can be rewritten as

$$\phi \to e^{i(\alpha(x)+\beta)Q_{\mathrm{gauge}}} e^{i\beta(Q_{\mathrm{global}}-Q_{\mathrm{gauge}})} \phi$$

Redefining the local direction as $\alpha'(x) = \alpha(x) + \beta$, we obtain the transformation

$$\phi \to e^{i\alpha'(x)Q_{\mathrm{gauge}}} e^{i\beta(Q_{\mathrm{global}}-Q_{\mathrm{gauge}})} \phi.$$

$$|D_\mu \phi|^2 = |(\partial_\mu - igQ_a A_\mu)\phi|^2_{\rho=0} = \frac{1}{2}(\partial_\mu a_\phi)^2 - gQ_a A_\mu \partial^\mu a_\phi + \frac{g^2}{2} Q_a^2 v^2 A_\mu^2$$
$$= \frac{g^2}{2} Q_a^2 v^2 (A_\mu - \frac{1}{gQ_a v}\partial^\mu a_\phi)^2$$

So, the gauge boson becomes heavy and there remains the x-independent transformation parameter beta. The corresponding charge is a combination:

$X = Q_{\mathrm{global}} - Q_{\mathrm{gauge}}$

3. Model-independent axion in string theory

Green-Schwarz mechanism:

The gravity anomaly in 10D requires 496 spin-1/2 fields. Possible non-Abelian gauge groups are rank 16 groups SO(32) and E8xE8'. The anti-symmetric field B_{MN} has field strength (in diff notation), $H= dB+w_{3Y}{}^0-w_{3L}{}^0$:SO(32). Three indices matched.

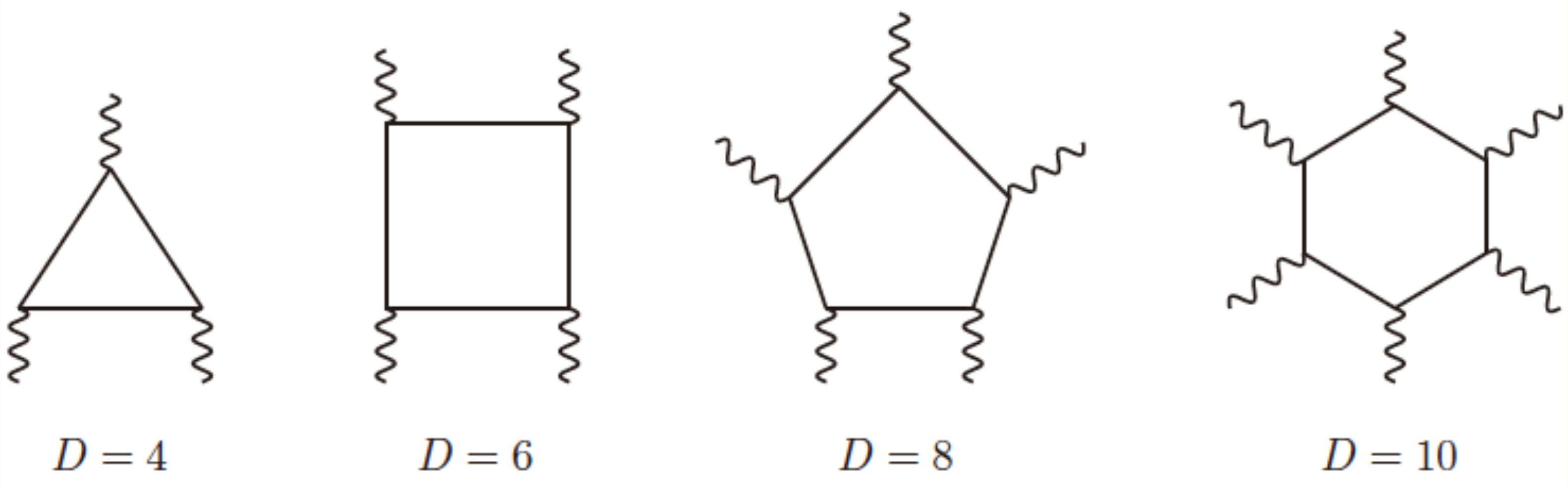
$$-\frac{3\kappa^2}{2g^4\,\varphi^2}H_{MNP}H^{MNP},\ \text{with}\ M,N,P=\{1,2,\cdots,10\}$$

Counter term is introduced to cancel the anomalies: $E_8 \times E_8'$

$$S_1' = \frac{c}{108\,000} \int \{30B\,[(\mathrm{tr}_1 F^2)^2 + (\mathrm{tr}_2 F^2)^2 - \mathrm{tr}_1 F^2\, \mathrm{tr}_2 F^2] + \cdots\}$$

One needs a term (GS-term) to cancel the gauge and gravitational anomalies.

Anomalies: even dimensions



In 10D, the hexagon anomaly. It is cancelled by the previous GS term.

One may look this in the following way.

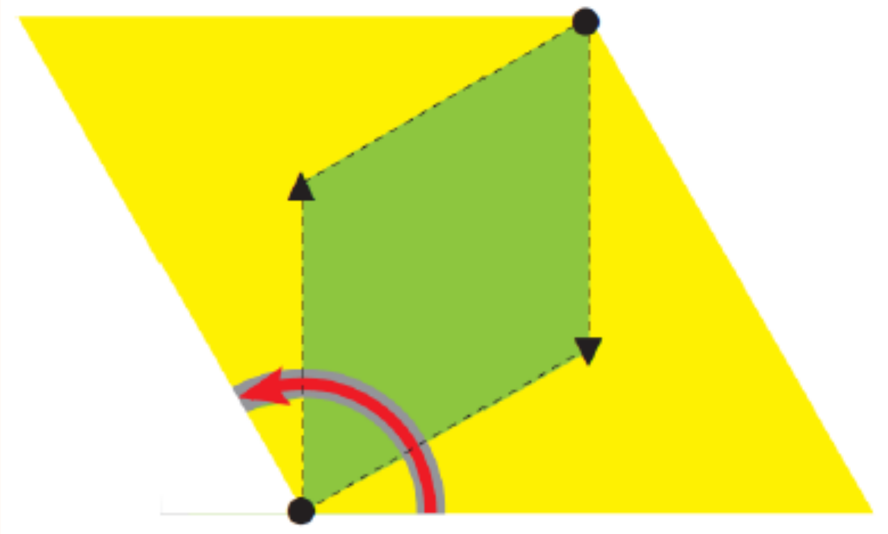
The10 supergravity quantum field theory with SO(32) and E8xE8' gauge groups has gauge and gravity anomalies. Let us believe that string theory is consistent, effectively removing all divergences, i.e. removing all anomalies. The point particle limit of 10D string theory should not allow any anomalies. There must be some term in the string theory removing all these anomalies. It is the Green-Schwarz term. One may remember the Wess-Zumino term removing anomalies by some term involving pseudoscalar fields.

For the GS term, already there is the field B_{MN} needed for the anomaly cancellation.

In the orbifold compactification, e.g. at a Z_3 torus, there are 3 fixed points. Here, we interpret that the flux is located at the fixed points. We take the limit of string loop almost sitting at the fixed points.

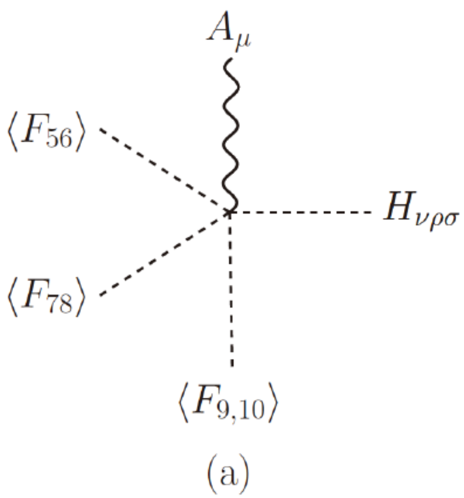
It involves 2nd rank antisymmetric field B_{MN}.

$$S_1' \propto -\frac{c}{10800}\left\{H_{\mu\nu\rho}A_\sigma\,\epsilon^{\mu\nu\rho\sigma}\epsilon^{ijklmn}\langle F_{ij}\rangle\langle F_{kl}\rangle\langle F_{mn}\rangle+\cdots\right\}\to\frac{1}{3!}\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}A^\sigma$$



$$S_1' \propto -\frac{c}{10800}\left\{H_{\mu\nu\rho}A_\sigma\,\epsilon^{\mu\nu\rho\sigma}\epsilon^{ijklmn}\langle F_{ij}\rangle\langle F_{kl}\rangle\langle F_{mn}\rangle+\cdots\right\} \to \frac{1}{3!}\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}A^\sigma$$

$$\frac{1}{2\cdot 3! M_{MI}^2}H_{\mu\nu\rho}H^{\mu\nu\rho},\ \text{with } \mu,\nu,\rho=\{1,2,3,4\}.$$



$$M_{MI} A_\mu \partial^\mu a_{MI}$$

$$\frac{1}{2} M_{MI}^2 A_\mu A^\mu$$

$$\frac{1}{2} M_{MI}^2 (A_\mu + \frac{1}{M_{MI}} \partial_\mu a_{MI})^2$$

This gives with $$H_{\mu\nu\rho} = M_{MI}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma a_{MI}.$$

$$\frac{1}{2}\partial^\mu a_{MI}\partial_\mu a_{MI} + M_{MI}A_\mu\partial^\mu a_{MI}.$$

This is the Higgs mechanism, i.e. a_{MI} becomes the longitudinal mode of the gauge boson. The previous two terms from the GS counter term gives

$$\frac{1}{2}(\partial_\mu a_{MI})^2 + M_{MI}A_\mu\partial^\mu a_{MI} + \frac{1}{2\cdot 3!}A_\mu A^\mu \to \frac{1}{2}M_{MI}^2(A_\mu + \frac{1}{M_{MI}}\partial_\mu a_{MI})^2.$$

It is the 't Hooft mechanism working in the string theory. So, the continuous direction a_{MI} –> a_{MI} + (constant) survives as a global symmetry at low energy. "Invisible" axion!!!!!

$$|D_\mu\phi|^2 = |(\partial_\mu - igQ_aA_\mu)\phi|^2_{\rho=0} = \frac{1}{2}(\partial_\mu a_\phi)^2 - gQ_aA_\mu\partial^\mu a_\phi + \frac{g^2}{2}Q_a^2v^2A_\mu^2$$

$$= \frac{g^2}{2}Q_a^2v^2(A_\mu - \frac{1}{gQ_av}\partial^\mu a_\phi)^2$$

$$\boxed{\frac{1}{2}\left(M_{MI}^2 + g^2Q_a^2v^2\right)(A_\mu)^2 + A_\mu(M_{MI}\partial^\mu a_{MI} - gQ_av\partial^\mu a_\phi) + \frac{1}{2}\left[(\partial_\mu a_{MI})^2 + (\partial^\mu a_\phi)^2\right]}$$

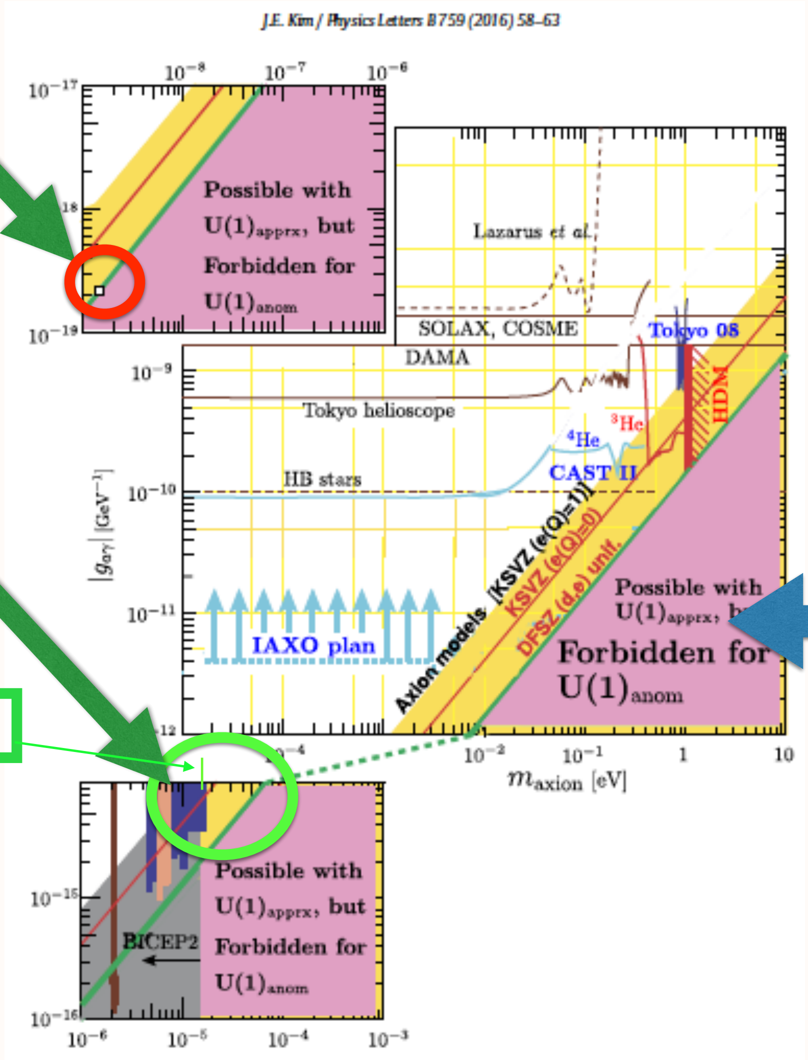
$$a = \cos\theta\, a_\phi + \sin\theta\, a_{MI}$$

$$\sin\theta = \frac{gQ_av}{\sqrt{M_{MI}^2 + g^2Q_a^2v^2}}.$$

MI axion

A small allowed region by $U(1)_{anom}$

Yale (2017)



$g_{a\gamma}(= 1.57 \times 10^{-10}\, c_{a\gamma\gamma})$ vs. m_a plot

Kim-Semertzidis-Tsujikawa, Front. Phys. 2 (2014) 60

Kim-Nam, 1603.02145[hep-ph]

$U(1)_{anom}$ forbidden

4. $U(1)_{\rm anom}$ as a family symmetry

JEK, Kyae, Nam, 1703.1703.05345 [Eur. Phys. J. C77 (2017) 847]


$U(1)_{global}$
Z_2


TOE

arXiv:1703.05345 [hep-ph]

Z(12-I) orbifold compactification:

a flipped SU(5) model x SU(5)' x SU(2)' x U(1)s [Huh-Kim-Kyae:0904.1108]

7 U(1)s: $U(1)_Y$, $U(1)_1$, $U(1)_2$, $U(1)_3$, $U(1)_4$, $U(1)_5$, $U(1)_6$.

$$
\begin{aligned}
Q_1 &= (0^5; 12, 0, 0)(0^8)', \\
Q_2 &= (0^5; 0, 12, 0)(0^8)', \\
Q_3 &= (0^5; 0, 0, 12)(0^8)', \\
Q_4 &= (0^8)(0^4, 0; 12, -12, 0)', \\
Q_5 &= (0^8)(0^4, 0; -6, -6, 12)', \\
Q_6 &= (0^8)(-6, -6, -6, -6, 18; 0, 0, 6)'.
\end{aligned}
$$

Flipped SU(5) is the best GUT from heterotic string compactification: Adjoint representation is not needed to break the GUT.

$$\hat{Q}' = Q_{\rm anom} + x_1 Q_a + x_2 Q_b$$

$$
\begin{aligned}
X &= (-2, -2, -2, -2, -2\,;\ 0^3)(0^8)', \\
Q_{\rm anom} &= 84Q_1 + 147Q_2 - 42Q_3 - 63Q_5 - 9Q_6,
\end{aligned}
$$

$$\hat{Q}' = 63(0^5; 16, 28, -8)(0^5; 6, 6, -12)'.$$

$U(1)_{\rm anom}$, U(1)' give the same anomaly.

$$\mathrm{Tr}\, \hat{Q}' Q_p Q_q = Q_{\rm anom} Q_p Q_q \text{ where } \{p, q\} = \{a, b\}$$

These are family q.n.

Table 1 The $SU(5)\times U(1)_X$ states. Here, + represents helicity $+\frac{1}{2}$ and − represents helicity $-\frac{1}{2}$. Sum of $Q_{\rm anom}$ is multiplied by the index of the fundamental representation of $SU(3)_c$, $\frac{1}{2}$. The PQ symmetry, being chiral, counts quark and antiquark in the same way. The right-handed states in T_3 and T_5 are converted to the left handed ones of T_9 and T_7, respectively. The bold entries are $Q_{\rm anom}/126$

Sect.	Colored states	$SU(5)_X$	Mult.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	Label	$Q_a^{\gamma\gamma}$
U	$(+++--;--+)(0^8)'$	$\overline{\mathbf{10}}_{-1}$		-6	-6	$+6$	0	0	0	$-1638(\mathbf{-13})$	C_2	-3276
U	$(+----;+--)(0^8)'$	$\mathbf{5}_{+3}$		$+6$	-6	-6	0	0	0	$-126(\mathbf{-1})$	C_1	-294
T_4^0	$(+----;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^8)'$	$\mathbf{5}_{+3}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_3$	-882
T_4^0	$(+++--;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^8)'$	$\overline{\mathbf{10}}_{-1}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_4$	-756
T_4^0	$(10000;\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$	$\mathbf{5}_{-2}$	2	$+4$	$+4$	$+4$	0	0	0	$+756(\mathbf{+6})$	$2C_5$	$+1008$
T_4^0	$(-10000;\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$	$\overline{\mathbf{5}}_{+2}$	2	$+4$	$+4$	$+4$	0	0	0	$+756(\mathbf{+6})$	$2C_6$	$+1008$
T_6^0	$(10000;000)(0^5;\frac{-1}{2}\,\frac{+1}{2}\,0)'$	$\mathbf{5}_{-2}$	3	0	0	0	-12	0	0	0	$3C_7$	0
T_6^0	$(-10000;000)(0^5;\frac{+1}{2}\,\frac{-1}{2}\,0)'$	$\overline{\mathbf{5}}_{+2}$	3	0	0	0	$+12$	0	0	0	$3C_8$	0
T_7^0	$(-10000;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\overline{\mathbf{5}}_{+2}$	1	-2	-2	-2	0	$+9$	$+3$	$-972(-\frac{54}{7})$	C_9	-1296
T_7^0	$(+10000;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\mathbf{5}_{-2}$	1	-2	-2	-2	0	$+9$	$+3$	$-972(-\frac{54}{7})$	C_{10}	-1296
T_3^0	$(+++--;000)(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\overline{\mathbf{10}}_{-1}$	1	0	0	0	0	$+9$	$+3$	$-594(-\frac{33}{7})$	C_{11}	-1188
T_9^0	$(++---;000)(0^5;\frac{+1}{4}\,\frac{+1}{4}\,\frac{-2}{4})'$	$\mathbf{10}_{+1}$	1	0	0	0	0	-9	-3	$+594(+\frac{33}{7})$	C_{12}	$+1188$
				-16	-28	$+8$	0	$+18$	$+6$	-3492		-5406

Two families from T4 and one family from U

$$c_{a\gamma\gamma} \simeq \frac{-9312}{-3492} - 2 = \frac{2}{3}$$

Adding contributions from other tables, -9312

The unification value

Even if this example is not a final theory, some interesting features are demonstrated.

$T_4\, T_4\, T_4$ coupling is the lowest term.

$$M^{2\times 2}_{\mathrm{demo}} = \begin{pmatrix} M & M \\ M & M \end{pmatrix}$$

$T_i\, F_j\, H_u$ coupling is not distinguished by changing i and j. So, it is completely democratic. We use this kind for the multiplicities of representations in the tables. We will use the bases where one mass is zero explicitly.

$$M_u \propto \begin{array}{c c} & \begin{array}{ccc} \mathbf{5}_3^U & \mathbf{5}_3^{T_4^A} & \mathbf{5}_3^{T_4^S} \end{array} \\ \begin{array}{c} \overline{\mathbf{10}}_{-1}^U \\ \overline{\mathbf{10}}_{-1}^{T_4^A} \\ \overline{\mathbf{10}}_{-1}^{T_4^S} \end{array} & \begin{pmatrix} \frac{\sigma_2\sigma_4}{M_{\rm int}^2}, & 0, & \frac{\alpha_1\sigma_4}{M_{\rm int}} \\ 0, & 0, & 1 \\ \frac{\alpha_2\sigma_2}{M_{\rm int}}, & 1, & 0 \end{pmatrix} \end{array} v_u \ , \quad \text{with } \langle H_u \rangle = \frac{v_u}{\sqrt{2}};$$

$$M_\nu \propto \begin{array}{c c} & \begin{array}{ccc} \mathbf{5}_{+3}^U & \mathbf{5}_{+3}^{T_4^A} & \mathbf{5}_{+3}^{T_4^S} \end{array} \\ \begin{array}{c} \mathbf{5}_{+3}^U \\ \mathbf{5}_{+3}^{T_4^A} \\ \mathbf{5}_{+3}^{T_4^S} \end{array} & \begin{pmatrix} \frac{\sigma_4\sigma_6^2\sigma_{13}\sigma_{16}}{M_{\rm int}^5}, & 0, & \frac{\{\sigma_2^2\sigma_4, \sigma_2\sigma_5\sigma_9\}}{M_{\rm int}^3} \\ 0, & 0, & \frac{\sigma_1^3\sigma_4^2}{M_{\rm int}^5} \\ \frac{\{\sigma_2^2\sigma_4, \sigma_2\sigma_5\sigma_9\}}{M_{\rm int}^3}, & \frac{\sigma_1^3\sigma_4^2}{M_{\rm int}^5}, & 0 \end{pmatrix} \end{array} \frac{v_u^2}{M_{\rm int}}$$

$$M_d \propto \begin{array}{cc} & \begin{array}{ccc} \overline{\mathbf{10}}^{U}_{-1} & \overline{\mathbf{10}}^{T_4^A}_{-1} & \overline{\mathbf{10}}^{T_4^S}_{-1} \end{array} \\ \begin{array}{c} \overline{\mathbf{10}}^{U}_{-1} \\ \overline{\mathbf{10}}^{T_4^A}_{-1} \\ \overline{\mathbf{10}}^{T_4^S}_{-1} \end{array} & \begin{pmatrix} \frac{\sigma_4^2}{M_{\text{int}}^2}, & 0\,, & \frac{\beta_1\sigma_4}{M_{\text{int}}} \\ 0\;\;, & 0\,, & 1 \\ \frac{\beta_1\sigma_4}{M_{\text{int}}}, & 1\,, & 0 \end{pmatrix} \end{array} v_d\;, \quad \text{with } \langle H_d \rangle = \frac{v_d}{\sqrt{2}}$$

$$M_e \propto \begin{array}{cc} & \begin{array}{ccc} \mathbf{5}^{U}_{3} & \mathbf{5}^{T_4^A}_{3} & \mathbf{5}^{T_4^S}_{3} \end{array} \\ \begin{array}{c} \mathbf{1}^{U}_{-5} \\ \mathbf{1}^{T_4^A}_{-5} \\ \mathbf{1}^{T_4^S}_{-5} \end{array} & \begin{pmatrix} \frac{\sigma_2\sigma_3}{M_{\text{int}}^2}, & 0, & \frac{\gamma_1\sigma_3}{M_{\text{int}}} \\ 0, & 0, & 1 \\ \frac{\gamma_2\sigma_2}{M_{\text{int}}}, & 1, & 0 \end{pmatrix} \end{array} v_d$$

Scales:

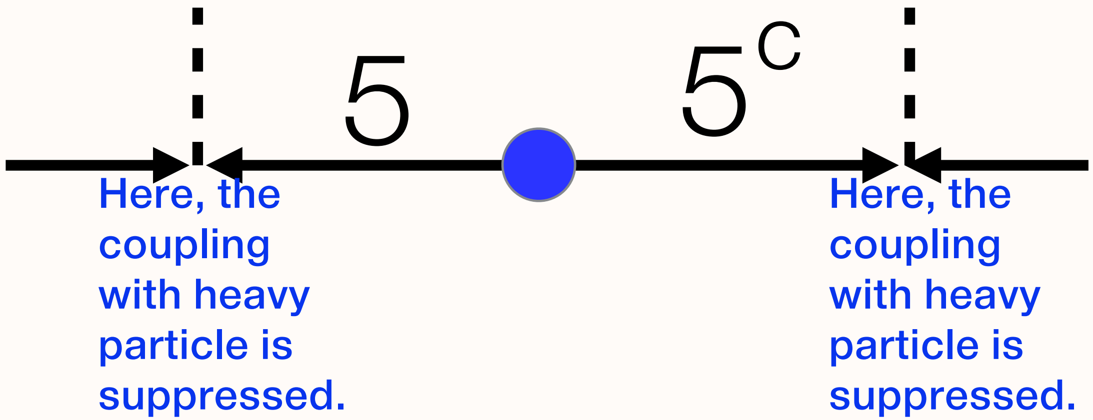
$$E_8 \times E_8' \underset{M_{\rm vec}}{\longrightarrow} \text{GUT} \underset{M_{\rm GUT}}{\longrightarrow} \text{SM and "invisible" axion} \underset{M_{\rm int}}{}$$

$$\underset{v_{\rm ew}}{\longrightarrow} SU(3)_c \times U(1)_{\rm em}$$

The doublet-triplet splitting is obtained.

Non-renormailzation theorem used.

For example,

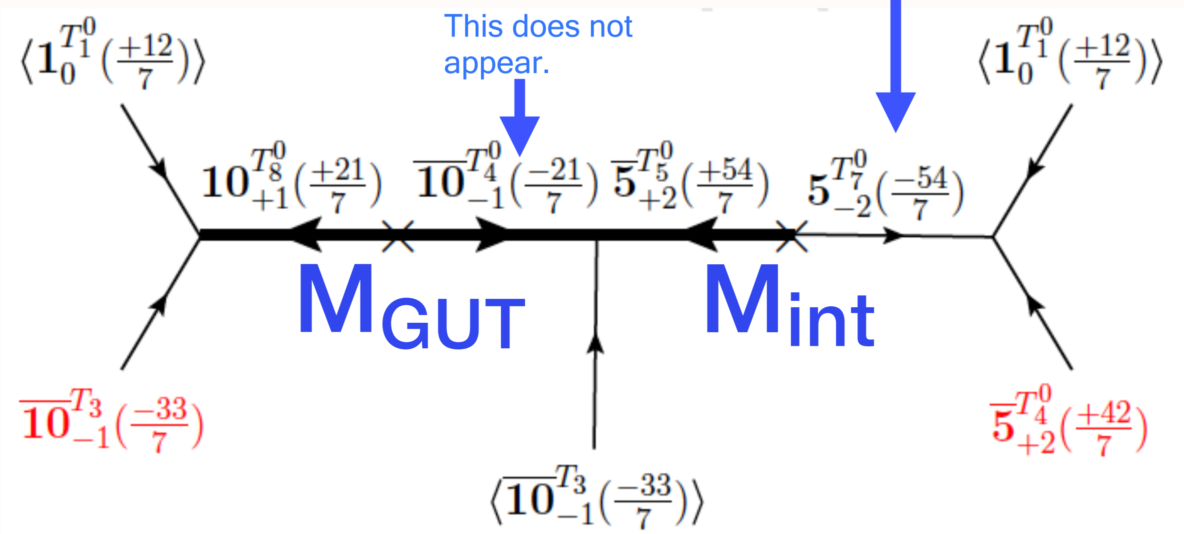


Diagrams with GUT mass scales must appear in loops, which is forbidden by the non-renormalization theorem.

If high dimensional terms of light fields appear, the mass suppression factor is not M_{GUT} but M_{int} .

Doublet-triplet splitting.

This appears in Table 1



Determination of the CP phase.

$$W = m\sigma_6\sigma_8 + \frac{1}{M^2}\sigma_6\sigma_7\sigma_2\sigma_4^2 \tag{32}$$

where m and M are real parameters, and all fields develop nonvanishing VEVs. Then

$$\sigma_4 = \pm i \left| mM^2 \frac{\sigma_8}{\sigma_2\sigma_7} \right|^{1/2} e^{i(\delta_8-\delta_2-\delta_7)/2} \tag{33}$$

where δ_i are the phases of σ_i. If $\delta_8 = \delta_2 = \delta_7 = 0$, the CKM and PMNS phases are determined as $\pm\frac{\pi}{2}$.

sigmas are neutral singlets.

6. Conclusion

1. Introduction
2. ’t Hooft mechanism
3. Model-independent axion from string theory

   In the compactification, if an anomalous gauge U(1) is created, then the ’t Hooft mechanism works and a global PQ symmetry comes down to the intermediate energy scale.

4. Flipped SU(5) GUT and $U(1)_{\rm anom}$ as a family symmetry
5. Doublet-triplet splitting and CKM, PMNS matrices