# Ghosts Disappearance in Lee-Wick Theories

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# Historical Background

- In 1942 Dirac proposed a method of field quantization that employes an indefinite metric in the space of State functions (Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, Vol.180, No. 980. (Mar. 18, 1942), pp. 1-40 ).
- In this method, the Hamiltonian operator is non-self adjoint in the conventional sense but of course is Hermitian in the more general sense by including the metric operator $\eta$.
- In 1943, Pauli explored the method more and put it an an elegant mathematical way and solved the problem of negative probability (Rev. Mod. Phys. 15(1943) 175).
- In 1969 Lee and Wick tried to revive the idea and built a finite QED (Nuclear Physics B9 (1969)209-243.)
- In all of these trials the Hamiltonian is non-self-adjoint in the conventional sense but through the introduction of the indefinite metric, the S-matrix is unitary and Causality isue (Macro) has been resolved.

# The Lee-Wick Standard Model

- Based on the ideas of Lee and Wick, Benjam Grinstein, Donal O'Connell, and Mark B. Wise in 2007 introduced an extension of the Standard model that solves the Hierarchy problem.
- Their idea is to add a higher derivative term to each sector in the Lagrangian.
- The Lagrangian can be reduced to the conventional second order derivative kinetic terms but duplicating the number of degrees of freedom.
- Each sector will be composed of a normal field and a Lee-Wick one.
- Quadratic divergences from normal and Lee-Wick fields cancels each other.
- In 2012, a super renormalizable higher derivative gravity theory has been introduced by *Leonardo Modesto in* Phys. Rev. D 86, 044005

# The Lee-Wick Standard Model (Cont'd)

- Here we follow the work of B. Grinstein, D. O'Connell, M. Wise, Phys.Rev.D77:025012,2008
- Consider the Lagrangian density of a toy model of the form

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2M^2}\left(\partial^2\phi\right)^2 - \frac{1}{2}m^2\phi^2 - \frac{g}{3}\phi^3 \qquad (1.1)$$

- The propagator of this theory is

$$D\left(p\right) = \frac{i}{p^2 - \frac{p^4}{M^2} - m^2} \qquad (1.2)$$

- The poles of this propagators exists at

$$p^2 = \frac{1}{2}M^2 \pm \frac{1}{2}M\sqrt{M^2 - 4m^2}. \qquad (1.3)$$

- For $M \gg m$, we get:

$$p^2 \approx M^2, \qquad p^2 \approx m^2 \qquad (1.4)$$

# The Lee-Wick Standard Model (Cont'd)

- Eq(1.1), can be written as

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \phi_2\partial^2\phi + \frac{1}{2}M^2\phi_2^2 - \frac{g}{3}\phi^3, \quad (1.5)$$

$$\frac{\partial\mathcal{L}}{\partial\left(\partial_\mu\phi_2\right)} = 0, \qquad \frac{\partial\mathcal{L}}{\partial\phi_2} = \phi_2 M^2 - \partial^2\phi. \quad (1.6)$$

Then, the auxiliary field $\phi_2$ is given by the relation $\phi_2 = \frac{1}{M^2}\partial^2\phi$. If we set

$$\phi_1 = \phi + \phi_2, \quad (1.7)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \frac{1}{2}m^2\phi_1^2 + \frac{1}{2}\left(M^2 - m^2\right)\phi_2^2$$
$$+m^2\phi_2\phi_1 - \frac{g}{3}\left(\phi_1 - \phi_2\right)^3$$

# The Lee-Wick Standard Model (Cont'd)

- To diagonalize the above Lagrangian one can introduce the transformation:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \begin{pmatrix} \phi'_1 \\ \phi'_2 \end{pmatrix}. \tag{1.8}$$

- This matrix diagonalizes the Lagrangian provided that

$$\tanh 2\theta = \frac{-2m^2/M^2}{1-2m^2/M^2} \tag{1.9}$$

which has a solution if $\underline{M > 2m}$.

- The Lee-Wick field here $\phi'_2$ has a negative kinetic term and is thus expected to cancel quadratic divergence from the normal field $\phi'_1$
- The higher derivative Lagrangian in the gauge sector is

$$\mathcal{L}_{\text{hd}} = -\frac{1}{2}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} + \frac{1}{M_A^2}\left(\hat{D}^{\mu}\hat{F}_{\mu\nu}\right)\left(\hat{D}^{\lambda}\hat{F}_{\lambda}{}^{\nu}\right),$$

where $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - ig[\hat{A}_\mu, \hat{A}_\nu]$ and $\hat{A}_\mu = \hat{A}^A_\mu T^A$ with $T^A$ the generators of the gauge group $G$ in the fundamental representation.

# The The Lee-Wick Standard Model (Cont'd)

- To reduce it to normal derivative theory, auxiliary massive gauge bosons $\tilde{A}$ is introduced to get:

$$\mathcal{L} = -\frac{1}{2}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - M_A^2 \tilde{A}_\mu \tilde{A}^\mu + 2\hat{F}_{\mu\nu}\hat{D}^\mu \tilde{A}^\nu,$$

where $\hat{D}_\mu \tilde{A}_\nu = \partial_\mu \tilde{A}_\nu - ig[\hat{A}_\mu, \tilde{A}_\nu]$.

- To diagonalize the kinetic terms, $\hat{A}_\mu = A_\mu + \tilde{A}_\mu$.The Lagrangian becomes

$$\mathcal{L} = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\left(D_\mu \tilde{A}_\nu - D_\nu \tilde{A}_\mu\right)\left(D^\mu \tilde{A}^\nu - D^\nu \tilde{A}^\mu\right) - ig\left(\left[\tilde{A}_\mu, \tilde{A}_\nu\right]F^{\mu\nu}\right)$$
$$- \frac{3}{2}g^2\left(\left[\tilde{A}_\mu, \tilde{A}_\nu\right]\left[\tilde{A}^\mu, \tilde{A}^\nu\right]\right) - 4ig\left(\left[\tilde{A}_\mu, \tilde{A}_\nu\right]D^\mu \tilde{A}^\nu\right) - M_A^2\left(\tilde{A}_\mu \tilde{A}^\mu\right). \quad (1.10)$$

# The The Lee-Wick Standard Model (Cont'd)

- The scalar sector now takes the form:

$$\mathcal{L}_{\rm hd} = \left(\hat{D}_\mu \hat{\phi}\right)^\dagger \left(\hat{D}^\mu \hat{\phi}\right) - \frac{1}{M_\phi^2}\left(\hat{D}_\mu \hat{D}^\mu \hat{\phi}\right)^\dagger \left(\hat{D}_\nu \hat{D}^\nu \hat{\phi}\right) - V(\hat{\phi}). \qquad (1.11)$$

- By introducing a **LW**-scalar multiplet $\tilde{\phi}$, Then the Lagrangian is given by

$$\mathcal{L} = \left(\hat{D}_\mu \hat{\phi}\right)^\dagger \left(\hat{D}^\mu \hat{\phi}\right) + M_\phi^2 \tilde{\phi}^\dagger \tilde{\phi} + \left(\hat{D}_\mu \hat{\phi}\right)^\dagger \left(\hat{D}^\mu \tilde{\phi}\right) + \left(\hat{D}^\mu \tilde{\phi}\right)^\dagger \left(\hat{D}^\mu \hat{\phi}\right) - V(\hat{\phi}), \qquad (1.12)$$

where the co-variant derivative is $\hat{D}_\mu = \partial_\mu + ig\hat{A}_\mu^A T^A$.

- In terms of the shifted gauge fields the hatted co-variant derivative is $\hat{D}_\mu = D_\mu + ig\tilde{A}_\mu^A T^A$,where $D_\mu = \partial_\mu + igA_\mu^A T^A$ is the usual co-variant derivative. To diagonalize the scalar kinetic terms, we use $\hat{\phi} = \phi - \tilde{\phi}$.The scalar Lagrangian becomes

$$\begin{aligned}\mathcal{L} &= (D_\mu\phi)^\dagger D^\mu\phi - (D_\mu\tilde{\phi})^\dagger D^\mu\tilde{\phi} + M_\phi^2\tilde{\phi}^\dagger\tilde{\phi} + ig(D^\mu\phi)^\dagger\tilde{A}_\mu^A T^A\phi \\ &+g^2\phi^\dagger\tilde{A}_\mu^A T^A\tilde{A}^{B\mu}T^B\phi - ig\phi^\dagger\tilde{A}_\mu^A T^A D^\mu\phi - ig(D^\mu\tilde{\phi})^\dagger\tilde{A}_\mu^A T^A\tilde{\phi} \quad (1.13) \\ &+ig\tilde{\phi}^\dagger\tilde{A}_\mu^A T^A D^\mu\tilde{\phi} - g^2\tilde{\phi}^\dagger\tilde{A}_\mu^A T^A\tilde{A}^{B\mu}T^B\tilde{\phi}. \quad (1.14)\end{aligned}$$

# The one-particle irreducible amplitude for the normal scalar field

- At one loop, we have the diagrams:

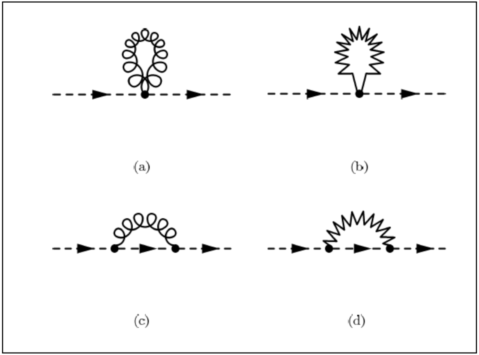


Figure: 1-One loop mass renormalization of the normal scalar field. The curly line is a gauge field while the zigzag line is the LW-gauge field. The dashed line represents the scalar field.

# The one-particle irreducible amplitude (Cont'd)

- The four diagrams give:

$$-i\Sigma_a(0) = g^2 C_2(N) \int \frac{d^n k}{(2\pi)^n} \frac{n}{k^2} \tag{1.15a}$$

$$-i\Sigma_b(0) = -g^2 C_2(N) \int \frac{d^n k}{(2\pi)^n} \left( \frac{n-1}{k^2 - M_A^2} - \frac{1}{M_A^2} \right) \tag{1.15b}$$

$$-i\Sigma_c(0) = -g^2 C_2(N) \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2} \tag{1.15c}$$

$$-i\Sigma_d(0) = -g^2 C_2(N) \int \frac{d^n k}{(2\pi)^n} \frac{1}{M_A^2}. \tag{1.15d}$$

- The sum of them cancels the quadratic divergences
- Similar calculations for the LW scalar field shows the cancellation of the quadratic divergences too.

# Negative Norm In Non-Hermitian Theories

- In the work of T. D. Lee and G. C. Wick (Phys. Rev. D 2, 1033 (1970)) a finite theory of Electrodynamics has been introduced.
- The theory suffers from negative norm states (Ghosts) and an indefinite metric was introduced for that purpose.
- In the past 20 years the non-Hermitian theories with real spectra have been stressed.
- So there is well known recipe to build up positive definite metric operator for these theories.
- As we will see this recipe can cure the ghost states in a Lee-Wick theory.

# Non-Hermitian Theories

- To show this, consider a non-Hermitian Hamiltonian $H = \frac{p^2}{2m} + gV(x)$
- $H$ is said to be pseudo Hermitian if there is an operator $\eta$ such that $\eta H = H^{\dagger}\eta$
- If there is an operator $\eta$ which is Hermitian as well as positive definite, then $H$ has a real spectrum.
- Also, one can introduce the operator $\rho = \sqrt{\eta}$, such that $\rho H \rho^{-1} = h$, where $h$ is an equivalent Hermitian Hamiltonian
- One can find the explicit form of $\eta$ by using the relation $\eta H = H^{\dagger}\eta$.

# Non-Hermitian Theories (Cont'd)

- Let the non-Hermitian Hamiltonian be in the form ($H_0$ is the free Hamiltonian)

$$H = H_0 + gH_I \tag{2.1}$$

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- The metric operator can be defined in terms of another operator $Q$ such that $\eta = \exp(-Q)$,
$$Q = Q_0 + gQ_1 + g^2 Q_2 + +g^3 Q_3 + ..$$

# Non-Hermitian Theories (Cont'd)

- Let the non-Hermitian Hamiltonian be in the form ($H_0$ is the free Hamiltonian)

$$H = H_0 + gH_I \tag{2.1}$$

- The metric operator can be defined in terms of another operator $Q$ such that $\eta = \exp(-Q)$,

$$Q = Q_0 + gQ_1 + g^2 Q_2 + +g^3 Q_3 + ..$$

- Since $H^\dagger = \eta H \eta^{-1}$, we get

$$-2gH_I = \left[-\left(Q_0 + gQ_1 + g^2 Q_2 + +g^3 Q_3 + ...\right), H\right] + \frac{\begin{array}{c}[-(Q_0 + gQ_1 + g^2 Q_2 + +g^3 Q_3 + ...), \\ [-(Q_0 + gQ_1 + g^2 Q_2 + +g^3 Q_3 + ...), H]]\end{array}}{2!} + \frac{\left[\begin{array}{c}[-(Q_0 + gQ_1 + g^2 Q_2 + +g^3 Q_3 + ...), \\ [-(Q_0 + gQ_1 + g^2 Q_2 + +g^3 Q_3 + ...), \\ [-(Q_0 + gQ_1 + g^2 Q_2 + +g^3 Q_3 + ...), H]]\end{array}\right]}{3!}$$

- So we get;

$$g^1 : -2H_I = [-Q_1, H_0],$$

$$g^3 : 0 = [-Q_3, H_0] + \frac{[-Q_1, [-Q_1, [-Q_1, H_0]]]}{3!} + \frac{[-Q_1, [-Q_1, H_I]]}{2!}$$

$$g^5 : 0 = [-Q_5, H_0] + \frac{[-Q_1, [-Q_1, [-Q_1, [-Q_1, [-Q_1, H_0]]]]]}{5!}$$
$$+ \frac{[-Q_3, [-Q_1, [-Q_1, H_0]]]}{3!}$$
$$+ \frac{[-Q_1, [-Q_3, [-Q_1, H_0]]]}{3!} + \frac{[-Q_1, [-Q_1, [-Q_3, H_0]]]}{3!}$$
$$+ \frac{[-Q_1, [-Q_1, [-Q_1, [-Q_1, H_I]]]]}{4!} + \frac{[-Q_1, [-Q_3, H_I]]}{2!}$$
$$+ \frac{[-Q_3, [-Q_1, H_I]]}{2!},$$

- Solving these operator equations one can get the operator $\eta$ perturbatively.
- In some cases, one can get a closed form for $\eta$.

# Ghost States in the Lee-Wick Standard Model

- Consider the Higher derivative Scalar model

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2M^2}\left(\partial^2\phi\right)^2 - \frac{1}{2}m^2\phi^2. \tag{3.1}$$

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- It can be reduced to

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \frac{1}{2}m^2\left(\phi_1 - \phi_2\right)^2 + \frac{1}{2}M^2\phi_2^2. \qquad (3.2)$$

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- By the canonical transformation $\phi_2 \longrightarrow i\phi_2, \pi_2 \longrightarrow -i\pi_2$, we get the Hamiltonian corresponding to the above Lagrangian as

$$H = \frac{\pi_1^2}{2} + \frac{1}{2}\left(\nabla\phi_1\right)^2 + \frac{1}{2}m^2\phi_1^2 + \frac{\pi_2^2}{2} + \frac{1}{2}\left(\nabla\phi_2\right)^2 \quad (3.3)$$

$$+\frac{1}{2}\left(M^2 - m^2\right)\phi_2^2 - im^2\phi_1\phi_2 \quad (3.4)$$

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- By the canonical transformation $\phi_2 \longrightarrow i\phi_2, \pi_2 \longrightarrow -i\pi_2$, we get the Hamiltonian corresponding to the above Lagrangian as

$$H = \frac{\pi_1^2}{2} + \frac{1}{2}\left(\nabla\phi_1\right)^2 + \frac{1}{2}m^2\phi_1^2 + \frac{\pi_2^2}{2} + \frac{1}{2}\left(\nabla\phi_2\right)^2 \tag{3.3}$$

$$+\frac{1}{2}\left(M^2 - m^2\right)\phi_2^2 - im^2\phi_1\phi_2 \tag{3.4}$$

- Consider first $0+1$ dimensions (Quantum mechanics), and let

$$\eta = \exp\left(2\left(\omega_1\pi_1\phi_2 + \omega_2\pi_2\phi_1\right)\right)$$

$\omega_1$ and $\omega_2$ are two real parameters to be obtained later.

# Ghost States in the Lee-Wick Standard Model in 0+1 dimensions

- Now,

$$\eta H \eta^{-1} = H^{\dagger}. \tag{3.5}$$

Also, $\rho = \sqrt{\eta}$ has the property

$$\rho H \rho^{-1} = h, \tag{3.6}$$

$$\begin{aligned}
h = &\frac{1}{2}\pi_1^2 + i\pi_1\omega_2\pi_2 - \frac{1}{2}\omega_2^2\pi_2^2 \\
&+ \frac{1}{2}m^2\phi_1^2 - \frac{1}{2}m^2\omega_1^2\phi_2^2 \\
&+ \frac{1}{2}\pi_2^2 + i\pi_2\omega_1\pi_1 - \frac{1}{2}\omega_1^2\pi_1^2 \\
&\frac{1}{2}\left(M^2 - m^2\right)\left(\phi_2^2 - \omega_2^2\phi_1^2\right) - m^2\omega_2\phi_1^2 - m^2\omega_1\phi_2^2 \\
&- im^2\phi_1\phi_2 + im^2\omega_1\phi_2\omega_2\phi_1 - i\phi_2\omega_2\phi_1 M^2
\end{aligned}$$

# Ghost States in the Lee-Wick Standard Model (Cont'd)

- For $h$ to be Hermitian, one has to put the constraints

$$i\omega_2 + i\omega_1 = 0,$$
$$\left(-m^2 + m^2\omega_1\omega_2 - \omega_2 M^2 + \omega_2 m^2 - m^2\omega_1\right) = 0,$$

on the introduced parameters $\omega_1$ and $\omega_2$. Equivalently, we have the relations

$$\omega_1 = -\omega_2, \quad (3.7)$$
$$-m^2 - m^2\omega_1^2 + \omega_1 M^2 - 2m^2\omega_1 = 0. \quad (3.8)$$

In terms of the mass parameters, $\omega_1$ can be obtained as

$$\omega_1 = \frac{1}{2m^2}\left(M^2 - 2m^2 \pm \sqrt{M^4 - 4M^2 m^2}\right). \quad (3.9)$$

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In terms of the mass parameters, $\omega_1$ can be obtained as

$$\omega_1 = \frac{1}{2m^2}\left(M^2 - 2m^2 \pm \sqrt{M^4 - 4M^2m^2}\right). \tag{3.9}$$

- For $\eta$ to be Hermitian, $\omega_1$and $\omega_2$ should be real and we have the condition $M^2 \geqslant 4m^2$,which agrees with the result in Ref. B. Grinstein, D. O'Connell, M. Wise, Phys.Rev.D77:025012,2008

# Ghost States in the Lee-Wick Standard Model (Cont'd)

- Then the Hermitian Hamiltonian $h$ has the form;

$$
\begin{aligned}
h &= \frac{1}{2}\pi_1^2\left(1-\omega_1^2\right)+\frac{1}{2}m^2\phi_1^2+\frac{1}{2}\left(1-\omega_1^2\right)\pi_2^2-\frac{1}{2}m^2\omega_1^2\phi_2^2 \\
&+\frac{1}{2}\left(M^2-m^2\right)\left(\phi_2^2-\omega_1^2\phi_1^2\right)+m^2\omega_1\phi_1^2-m^2\omega_1\phi_2^2, \\
&= \frac{1}{2}\pi_1^2\left(1-\omega_1^2\right)+\frac{1}{2}m^2\phi_1^2+\frac{1}{2}\left(1-\omega_1^2\right)\pi_2^2 \\
&+\left(\frac{1}{2}m^2\omega_1^2+m^2\omega_1-\frac{1}{2}M^2\omega_1^2\right)\phi_1^2 \\
&+\left(-\frac{1}{2}m^2\omega_1^2+\frac{1}{2}M^2-m^2\omega_1-\frac{1}{2}m^2\right)\phi_2^2.
\end{aligned}
$$

# Ghost States in the Lee-Wick Standard Model (Cont'd)

- Then the Hermitian Hamiltonian $h$ has the form;

$$
\begin{aligned}
h &= \frac{1}{2}\pi_1^2\left(1-\omega_1^2\right)+\frac{1}{2}m^2\phi_1^2+\frac{1}{2}\left(1-\omega_1^2\right)\pi_2^2-\frac{1}{2}m^2\omega_1^2\phi_2^2\\
&+\frac{1}{2}\left(M^2-m^2\right)\left(\phi_2^2-\omega_1^2\phi_1^2\right)+m^2\omega_1\phi_1^2-m^2\omega_1\phi_2^2,\\
&=\frac{1}{2}\pi_1^2\left(1-\omega_1^2\right)+\frac{1}{2}m^2\phi_1^2+\frac{1}{2}\left(1-\omega_1^2\right)\pi_2^2\\
&+\left(\frac{1}{2}m^2\omega_1^2+m^2\omega_1-\frac{1}{2}M^2\omega_1^2\right)\phi_1^2\\
&+\left(-\frac{1}{2}m^2\omega_1^2+\frac{1}{2}M^2-m^2\omega_1-\frac{1}{2}m^2\right)\phi_2^2.
\end{aligned}
$$

- Note that the mass spectra here in this formula agrees with those obtained in the paper introduced the Lee-Wick Standard Model: Phys.Rev.D77:025012,2008

# Ghost States in the Lee-Wick Standard Model in 3+1 dimensions

- In higher dimensions (Quantum field theory), the metric takes the form:

$$\eta = \int dz \exp\left(2\left(\omega_1 \pi_1(z)\,\phi_2(z) + \omega_2 \pi_2(z)\,\phi_1(z)\right)\right).$$

Accordingly, we have the relations

$$\rho\phi_1(x)\,\rho^{-1} = \phi_1(x) - i\omega_1 \int dz\phi_2(z)\,\delta^3(x-z),$$

$$\rho\pi_1\rho^{-1} = \pi_1 + i\omega_2 \int dz\pi_2(z)\,\delta^3(x-z),$$

$$\rho\phi_2^{-1}\rho^{-1} = \phi_2 - i\omega_2 \int dz\phi_1(z)\,\delta^3(x-z),$$

$$\rho\pi_2^{-1}\rho^{-1} = \pi_2 + i\omega_1 \int dz\pi_1(z)\,\delta^3(x-z).$$

# Ghost States in the Lee-Wick Standard Model in 3+1 dimensions

- And thus

$$\begin{aligned} \rho\phi_1^{-1}\rho^{-1} &= \phi_1 - i\omega_1\phi_2, \\ \rho\pi_1^{-1}\rho^{-1} &= \pi_1 + i\omega_2\pi_2, \\ \rho\phi_2^{-1}\rho^{-1} &= \phi_2 - i\omega_2\phi_1, \\ \rho\pi_2^{-1}\rho^{-1} &= \pi_2 + i\omega_1\pi_1. \end{aligned} \tag{3.10}$$

$$\begin{aligned} \rho\frac{1}{2}\left(\nabla\phi_1(x)\right)^2\rho^{-1} &= \frac{1}{2}\left(\nabla\phi_1(x)\right)^2 - i\omega_1\nabla_x\phi_1(x)\nabla_x\phi_2(x) \\ &- \frac{\omega_1^2}{2}\left(\nabla_x\phi_2(x)\right)^2, \\ \rho\frac{1}{2}\left(\nabla\phi_2(x)\right)^2\rho^{-1} &= \frac{1}{2}\left(\nabla\phi_2(x)\right)^2 - i\omega_2\nabla_x\phi_1(x)\nabla_x\phi_2(x) \\ &- \frac{\omega_2^2}{2}\left(\nabla_x\phi_1(x)\right)^2. \end{aligned} \tag{3.11}$$

## Ghost States in the Lee-Wick Standard Model in 3+1 dimensions

- Again, with the choice $\omega_1 = -\omega_2 = \omega$, one get

$$\rho \left( \frac{1}{2} \left( \nabla \phi_1(x) \right)^2 + \frac{1}{2} \left( \nabla \phi_2(x) \right)^2 \right) \rho^{-1} = \frac{1}{2} \left( 1 - \omega^2 \right) \left( \nabla \phi_1(x) \right)^2 + \left( \nabla \phi_2(x) \right. \tag{3.12}$$

and the quantum field Hermitian Hamiltonian takes the form;

$$\begin{aligned} h = & \frac{1}{2} \pi_1^2 \left( 1 - \omega^2 \right) + \left( 1 - \omega^2 \right) \left( \nabla \phi_1(x) \right)^2 + \frac{1}{2} m^2 \phi_1^2 \\ & + \frac{1}{2} \left( 1 - \omega^2 \right) \pi_2^2 + \left( 1 - \omega^2 \right) \left( \nabla \phi_2(x) \right)^2 + \frac{1}{2} M^2 \phi_2^2 \\ & + \left( \frac{1}{2} m^2 \omega^2 + m^2 \omega - \frac{1}{2} M^2 \omega_1^2 \right) \phi_1^2 \\ & + \left( -\frac{1}{2} m^2 \omega_1^2 - m^2 \omega_1 - \frac{1}{2} m^2 \right) \phi_2^2 . \end{aligned} \tag{3.13}$$

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- To compare the mass spectra with those obtained before: we apply the Canonical transformations of the form; $\psi_1 = \frac{1}{\sqrt{1-\omega_1^2}}\,\phi_1$, $\Pi_1 = \sqrt{(1-\omega_1^2)}\pi_1$, $\psi_2 = \frac{1}{\sqrt{1-\omega_1^2}}\,\phi_2$, $\Pi_2 = \sqrt{(1-\omega_1^2)}\pi_2$

$$\begin{aligned} h &= \frac{1}{2}\Pi_1^2 + \frac{1}{2}\Pi_2^2 + \frac{1}{2}\frac{M^2}{2}\left(1-\sqrt{1-\frac{4m^2}{M^2}}\right)\psi_1^2 \\ &+\frac{1}{2}\left(\frac{M^2 - m^2\left(1+\omega_1\right)^2}{1-\omega_1^2}\right)\psi_2^2. \end{aligned} \tag{3.14}$$

$$m_{\psi_1}^2 = \frac{m^2}{2}\left(1-\sqrt{1-\frac{4m^2}{M^2}}\right), \tag{3.15}$$

- Which is the same result obtained by Wise et.al but now the theory has a positive norm.

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- In Physics Letters A 373 (2009) 3304–3308 and Int J Theor Phys (2011) 50: 1071–1080, **H.F. Jones** argued that Path integral calculations already implements the metric.
- So, one can obtain the amplitudes directly without resorting to the calculations of the complicated metric operator

# Closely Related theories that solve the Hierarchy problem too

- The $\mathcal{PT}-$symmetric $\left(-\phi^4\right)$ field theory has the Hamiltonian of the form,

$$H = \frac{1}{2}\left((\nabla\phi)^2 + \pi^2 + m^2\phi^2\right) - \frac{g}{4!}\phi^4 \qquad (3.16)$$

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$$\begin{aligned} e = &-\left(\frac{1}{2}G\,(t-1)\,(\gamma + \ln t - \ln 4\pi - 1) - 1\right) b^2 \\ &- \frac{G}{12}b^4 + \left(-\frac{1}{4}\,(t^2-1)\,(2\gamma + 2\ln t - 2\ln 4\pi - 1)\right) \\ &- \frac{G\,((t-1)\,(\gamma - 1 - \ln 4\pi + \ln t))^2}{4} \\ &+ ((t-1)\,(\gamma - 1 - \ln 4\pi + \ln t))\,, \quad e = \frac{32\pi^2}{m^4}E \end{aligned} \tag{3.17}$$

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- In the conventional Hermitian $\phi^4$ theory used in the Standard model to break the symmetry, the Higgs mass blow up at UV scales causing the Hierarchy problem.

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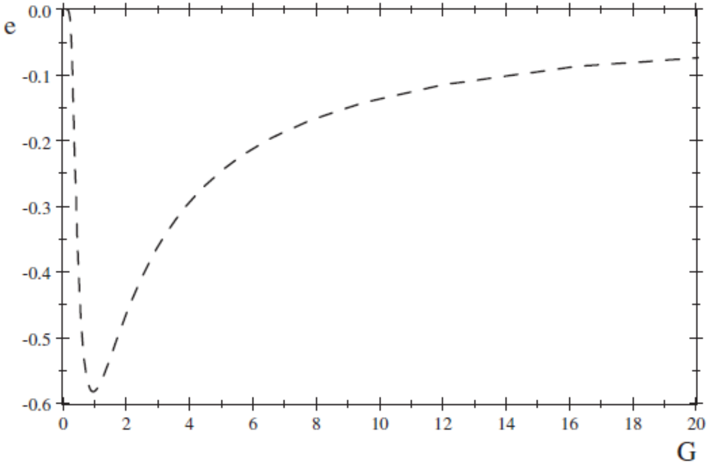


Figure: 2: The vacuum energy is finite for the whole domain of the renormalized coupling

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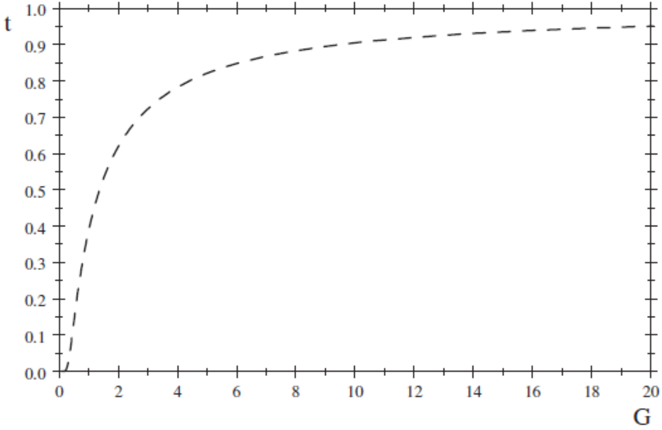


Figure: 3: The Higgs mass is finite for all scales

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- This shows that critical phenomena in magnetic systems can be used to test the validity of $\mathcal{PT}-$symmetric theories.

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