$B \to K^*$ form factors on the lattice

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AA, V. Bernard, U.-G. Meißner, A. Rusetsky, Nucl. Phys. B 910 (2016)

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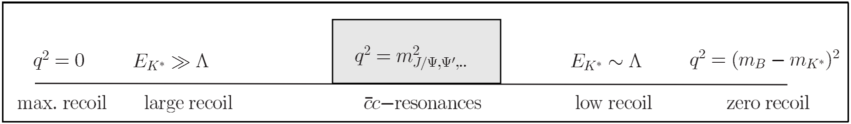
$B \to K^*(892)l^+l^-$



Source: C. Hambrock et al., Phys. Rev. D 89 (2014), 074014

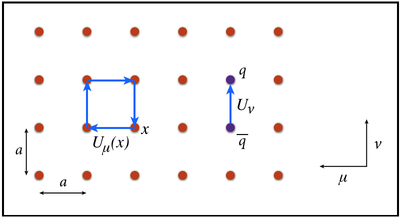
- Pattern of deviations observed by LHCb and Belle Collaboration

BSM or QCD?

- Hadronic uncertainties: form factors, long-distance effects
- In the low recoil region, lattice QCD simulations are reliable

Lattice QCD

- Path integral formulation in Euclidean space-time



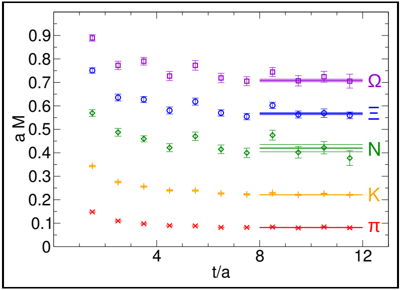
- Space-time is discretized and finite ⇒ natural UV cut-off $\sim 1/a$

  K. G. Wilson, Phys. Rev. D **10** (1974) 2445

- Integration → Monte Carlo methods
- Correlation functions → energy levels, current matrix elements

FLAG: S. Aoki *et al.*, arXiv:1607.00299 (2016)

Stable hadrons



Source: BMW Collaboration, Science **322**, 1224 (2008)

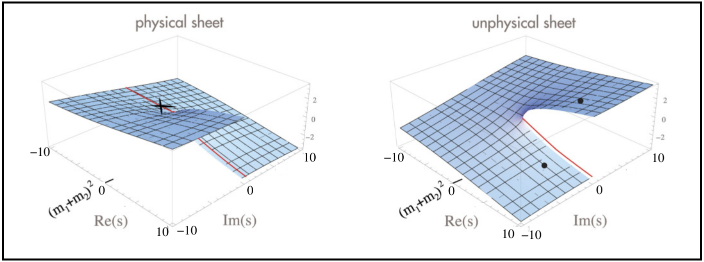
- Two-point correlation function:

$$C(t) = \sum_{\mathbf{x}} \langle 0|O(\mathbf{x},t)O^\dagger(\mathbf{0},0)|0\rangle = |Z_O|^2 e^{-mt}\left[1 + \sum_n |Z_n|^2 e^{-\Delta E_n t}\right]$$

▷ $O(\mathbf{x},t)$ - field operator, Z_O, Z_n - overlap factors, $\Delta E_n > 0$

- Effective mass: $M(t) = \frac{1}{a}\log\frac{C(t)}{C(t+a)}$, $\quad M(t) \to m$

Resonances



Riemann sheets on the complex s-plane

- Energy levels → do not correspond to any resonance
- Resonances → complex poles of the scattering amplitude:

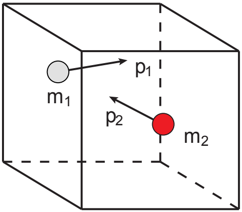
$$E_R = m_R - i\frac{\Gamma_R}{2}, \quad s_R = E_R^2$$

▷ m_R - mass, Γ_R - width

Lüscher method

- A standard tool to study resonances on the lattice
- scattering phase shift $\leftrightarrow$ finite volume energy spectrum

M. Lüscher, Nucl. Phys. B **354** (1991) 531.



$$\cot \delta_0(p) = -\cot \phi(q)$$

−Lüscher equation, no partial-wave mixing

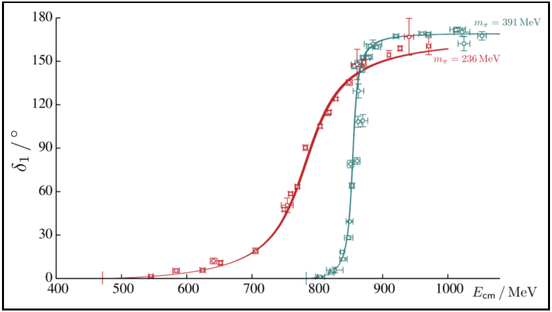
$$p^2 = \lambda(s, m_1^2, m_2^2)/4s, \quad q = \frac{pL}{2\pi}$$

$$\cot \phi(q) = -\frac{1}{2\pi^2 q^2} \sum_{\mathbf{n} \in \mathbb{Z}^3} \frac{1}{\mathbf{n}^2 - q^2} \quad (l = 0,\ m = 0)$$

▷ energy levels $\rightarrow$ Lüscher equation $\rightarrow$ scattering phase

▷ effective-range expansion (ERE) $\rightarrow$ resonance pole position E_R

An example: ρ resonance



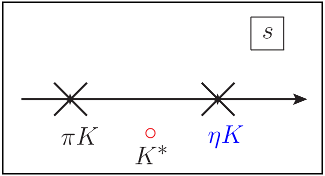
Source: Hadron Spectrum Collaboration, Phys. Rev. D **92** (2015) 094502

m_π [MeV]	m_R [MeV]	Γ_R [MeV]
236	783(2)	85(2)
391	853(2)	12.4(6)

▷ I=1, P-wave $\pi\pi$ scattering phase shift

▷ rapid change of phase shift → resonance

Theoretical issues

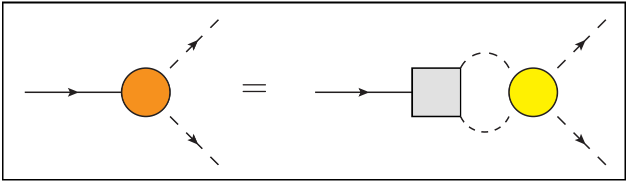


- $K^*(892)$ is a resonance $\rightarrow$ Lüscher method
- Lattice data might be affected by another threshold (ηK)
- Resonance matrix elements should be properly defined
- Form factors: real axis vs. complex resonance pole

Electroweak processes

- Seminal work on $K \to \pi\pi$ by

  L. Lellouch and M. Lüscher, Commun.Math.Phys. **219**, 31 (2001)



Lellouch-Lüscher formula

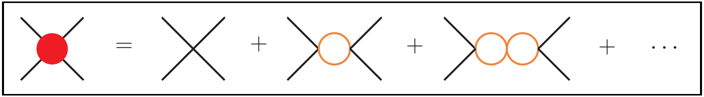
$$|A(K \to \pi\pi)| \propto |\langle \pi\pi | H_{\mathrm{weak}} | K \rangle| \times \left(\frac{p^2}{\frac{d\delta(p)}{dp} + \frac{d\phi(q)}{dp}} \right)^{-1/2}$$

- The range of applicability:

  large volumes ($m_\pi L \gtrsim 4$), below 3(4)-particle threshold

The framework: non-relativistic EFT

- Ideally suited for the problem we study



Bubble-chain diagrams

$$T = V + VGV + VGVGV + \ldots = \frac{1}{V^{-1} - G}$$

J. Gasser, B. Kubis and A. Rusetsky, Nucl. Phys. B **850** (2011) 96

- In a finite volume: $G \to G_L$ (the loop function)

- A bridge: finite volume spectrum $\leftrightarrow$ scattering sector

Form factors

- on the real energy axis → model- and process-dependent

$$|\mathrm{Im}\,\mathcal{A}(E_{BW},|\mathbf{q}|)| = \sqrt{\frac{8\pi}{p_{BW}\Gamma}}|F_A(E_{BW},|\mathbf{q}|)|$$

▷ $F_A(E_{BW},|\mathbf{q}|)$ → current matrix elements, Γ – resonance width

- at the resonance pole → process-independent ⇒ favourable!

$$\langle P,\,\text{resonance}|J(0)|Q,\,\text{stable}\rangle = \lim_{P^2\to s_R,\,Q^2\to M^2} Z_R^{-1/2} Z^{-1/2}(s_R-P^2)(M^2-Q^2)F(P,Q)$$

▷ $F(P,Q)$ – 3-point Green's function

▷ Z, Z_R – wave-function renormalization constants

generalization of: S. Mandelstam, Proc. Roy. Soc. Lond. A **233** (1955) 248

$B \to K^*(892) l^+ l^-$: low recoil

- For $q^2 \gg \Lambda_{QCD}$, the decay amplitude A is approximately

$$A \approx \sum_{M=1}^{7} c_M(C_7, ...) f^M(q^2)$$

$\hookrightarrow$ see, however, J. Lyon and R. Zwicky, arXiv:1406.0566 [hep-ph]

- Seven semileptonic form factors $f^M(q^2), M = 1, ...7$,

$$\langle K^*(k,\lambda)|\bar{s}\gamma^\mu b|B(p)\rangle = \frac{2iV(q^2)}{m_B + m_V}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu k_\rho p_\sigma, \quad \text{etc.}$$

- The first unquenched lattice calculation (with a stable K^*) :

R. R. Horgan et al., Phys. Rev. D **89**, 094501 (2014)

$\triangleright$ consistent with LCSR, A. Khodjamirian et al., JHEP **1009**, 089 (2010)

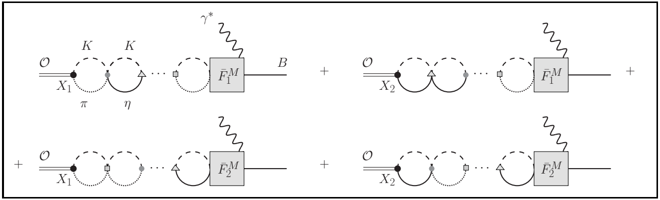
Kinematics

Little group	Irrep	Form factor
C_{4v}	$\mathbb{E}$	V, A_1, T_1, T_2
	$\mathbb{A}_1$	A_0, A_{12}, T_{23}

Table: The irreps without $S - P$ partial-wave mixing

- broken rotational symmetry $\rightarrow$ cubic group: choose some irrep(s)
- K^* is at rest $\mathbf{k} = 0$ and the B momentum is $\mathbf{p} = \mathbf{q} = \frac{2\pi}{L}(0, 0, n)$

Lellouch-Lüscher formula for $B \to K^*(892) l^+ l^-$



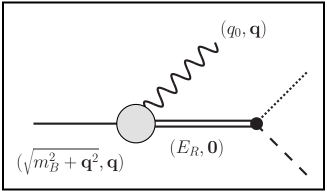
$$|F^M(E_n, |\mathbf{q}|)| = \frac{\mathcal{V}^{-1}}{8\pi E} \left| v_1\, \bar{F}_1^M + v_2\, \bar{F}_2^M \right| \Big|_{E=E_n}$$

- $\mathcal{V}$ – (asymmetric) volume
- $v_1 = v_1(E, L)$, $v_2 = v_2(E, L)$ – known functions (gener. of LL factor)
- $F^M(E_n, |\mathbf{q}|) \to$ current matrix elements, measured on the lattice

- Two-particle irreducible vertices $\bar{F}_1^M(E_n, |\mathbf{q}|)$, $\bar{F}_2^M(E_n, |\mathbf{q}|) \to$

$\to$ decay amplitudes $\mathcal{A}^M(B \to \pi K l^+ l^-)$ | Watson's theorem, agrees with S. Sharpe and M. Hansen, Phys. Rev. D **86** (2012) 016007

Form factors at the K^* pole



- Suppose, the K^* pole is on the Riemann sheet *II*
- Analytic continuation: vary E with $|\mathbf{q}|$ fixed (analog of the ERE)

$$F_R^M(E_R, |\mathbf{q}|) = -\frac{i}{8\pi E}\left(w_1 \bar{F}_1^M - w_2 \bar{F}_2^M\right)\Big|_{E=E_R}$$

▷ $w_1 = w_1(E)$, $w_2 = w_2(E)$ – volume-independent quantities

- Form factors at the pole → complex

Infinitely narrow width

- Results are simplified in the limit $\Gamma \to 0$ (K^* is above ηK)
- Assume the Breit-Wigner form in the vicinity of $E = E_{BW}$
- Real axis:

$$\big|F^M(E_n, |\mathbf{q}|)\big| = \frac{\mathcal{V}^{-1}}{\sqrt{2E_n}}\big|F_A^M(E_n, |\mathbf{q}|)\big| + O(\Gamma^{1/2}), \quad E_n = E_{BW} + O(\Gamma)$$

- Complex plane ($E_R \to E_{BW}$):

$$F_R^M(E_R, |\mathbf{q}|)\big|_{\Gamma\to 0} = F_A^M(E_{BW}, |\mathbf{q}|) + O(\Gamma^{1/2})$$

$\hookrightarrow$ similar to the one-channel case of the $\Delta N \gamma^*$ transition:

AA, V. Bernard, U.-G. Meißner and A. Rusetsky, Nucl. Phys. B **886** (2014) 1199

Summary

- Extraction of the $B \to K^*$ form factors on the lattice is studied
- Possible admixture of the ηK to πK channels is taken into account
- Equation for the $B \to K^* l^+ l^-$ amplitude at low recoil is derived
- Form factors at the K^* pole are determined
- Infinitely-narrow width approximation of the results is considered

- An open issue: long-distance effects $\Leftrightarrow$ non-local matrix elements

$$T^\mu \propto i \int d^4x e^{iqx} \langle K^* | T j^\mu_{\rm em}(x) H_{4q}(0) | B \rangle$$

$\hookrightarrow$ simpler cases: $K \to \pi \nu \bar{\nu}$, $K \to \pi l^+ l^-$