$B \to \pi\pi$ Form Factors at Large Dipion Mass

(– in the QCD Factorization approach –)

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based on: arXiv:1608.07127 [to appear in JHEP]

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– Mini-Workshop –

"*B*-meson decays into multi-hadron final states"

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GEFÖRDERT VOM
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quark flavour physics and effective field theories

Brief Motivation

- Phenomenological determination of $|V_{ub}|$
  - one of the least precisely known CKM matrix elements
  - apparent tension between exclusive and inclusive measurements
  - $B \to \pi\pi\ell\bar{\nu}_\ell$ decays allow for independent cross-checks
    - "background" for $B \to \rho\ell\bar{\nu}_\ell$
    - full angular distribution of the 4-body final state
    - rich set of observables; different hadronic systematics [Faller et al. '14]
- Hadronic Input for other decay modes
  - e.g. for $B \to \pi\pi\pi$
- Playground for various QCD Methods:
  - QCD Sum Rules
  - Chiral Perturbation Theory
  - HQET / SCET
  - **QCD Factorization**
  - ...

Kinematics and Form-Factor Definitions

dipion momentum: $k = k_1 + k_2$

relative momentum: $\bar{k} = k_1 - k_2$

momentum transfer: $q = p - k = q_1 + q_2$

dipion helicity angle: $\cos\theta_\pi \propto (q \cdot \bar{k})$

Källén function: $\lambda = M_B^4 + q^4 + k^4 - 2\,(M_B^2 q^2 + M_B^2 k^2 + q^2 k^2)$

Hadronic matrix elements for vector and axial-vector currents

$$\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma^\mu b|\bar{B}(p)\rangle \quad \to \quad F_\perp$$

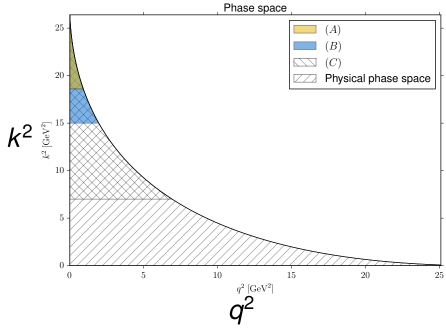
$$\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma^\mu \gamma_5 b|\bar{B}(p)\rangle \quad \to \quad F_0, F_\parallel, F_t$$

- 4 form factors $F_i \equiv F_i(q^2, k^2, \cos\theta_\pi)$
- partial wave expansion for $\cos\theta_\pi \to$ S-, P-, D-, ... waves

Phase-Space Constraints for QCDF Region

- dipion invariant mass: $k^2 = (k_1 + k_2)^2 \overset{!}{\gg} \Lambda^2_{\rm had}$
- pion energies (in B-frame): $E_{1,2} = \frac{p \cdot k_{1,2}}{M_B} \overset{!}{\gg} \Lambda_{\rm had}$

$$E_{1,2} \geq E_{\rm min}(q^2, k^2, |\cos\theta_\pi|) = \frac{M_B^2 + k^2 - q^2 - |\cos\theta_\pi|\sqrt{\lambda}}{4 M_B} \qquad (m_\pi \equiv 0)$$



Consider 3 scenarios:

A: $k^2 \geq \frac{2M_B^2}{3} \qquad \Rightarrow E_{\rm min} \simeq 1.8$ GeV

B: $k^2 \geq \frac{M_B^2}{2} \qquad \Rightarrow E_{\rm min} \simeq 1.3$ GeV

C: $k^2 \geq \frac{M_B^2}{4}$ and $|\cos\theta_\pi| \leq \frac{1}{3}$
$\Rightarrow E_{\rm min} \simeq 1.2$ GeV

Technical Derivation

QCD Factorization Formula

Similar structure as for non-leptonic decays [Beneke et al. '99]

$$\langle \pi^+(k_1)\pi^-(k_2)|\,\bar u\,\Gamma\,b\,|B^-(p)\rangle$$
$$\sim \xi_{\pi^-}(E_2)\int_0^1 \mathrm{d}u\,\phi_{\pi^+}(u)\,T_\Gamma^{\rm I}(u,\ldots)$$
$$+\int_0^1 \mathrm{d}u\int_0^1 \mathrm{d}v\int_0^\infty \frac{\mathrm{d}\omega}{\omega}\,\phi_{\pi^+}(u)\,\phi_{\pi^-}(v)\,\phi_B(\omega)\,T_\Gamma^{\rm II}(u,v,\omega,\ldots)$$
$$+\ \text{power corrections}$$

- universal "soft" $B\to\pi$ form factor $\xi_\pi(E_2)$ [Charles et al. '98; Beneke/TF '01]
- universal LCDAs $\phi_\pi(u)$, $\phi_B(\omega)$ for light and heavy mesons
- perturbatively calculable short-distance kernels $T_\Gamma^{\rm I,II}$ (process-dependent)

$T_\Gamma^{\rm I}\sim\mathcal{O}(\alpha_s)$ (hard gluon exchange, $\mu_{\rm hard}\sim\sqrt{k^2}$)

$T_\Gamma^{\rm II}\sim\mathcal{O}(\alpha_s^2)$ (hard and "hard-collinear" gluon exchange , $\mu_{\rm hc}\sim\sqrt{E_2\,\Lambda_{\rm had}}$)

Diagrammatic Analysis

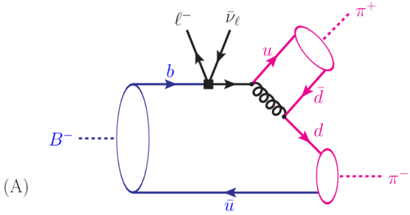
To be shown:

1. leading-power contributions involve twist-2 LCDA of π^+ only, with finite convolution integrals
2. endpoint-divergencies that would arise from spectator interactions are universal and can be absorbed into ξ_π

Caveats:

- this is not sufficient for an all order proof (→ SCET)
- factorization structure does probably not hold for power corrections; this involves twist-3 LCDAs of the pion, which can be "chirally enhanced" by a large factor $\mu_\pi = \frac{m_\pi^2}{m_u+m_d} \sim 2.5\,\text{GeV} \sim \frac{M_B}{2}$

The Kernel $T_\Gamma^{\rm I}$



- contains short-distance QCD effects that do not involve the spectator

$\rightarrow$ $\mathcal{O}(m_b^2)$ virtualities of intermediate propagators:

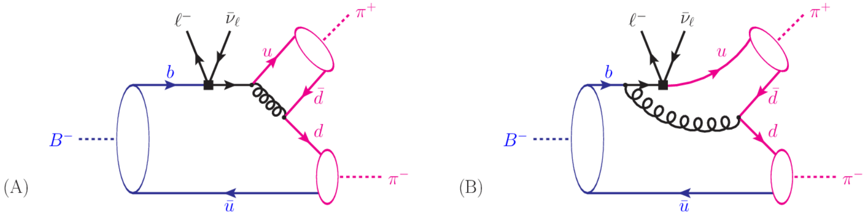
light quark (A):	$(p_b - q)^2 \simeq k^2$
heavy quark (B):	$\simeq (uk_1 + q)^2 - m_b^2 \simeq \bar u \left(k^2 - 2M_B E_1\right) - 2M_B E_2$
gluon (A,B):	$\simeq (k_2 + \bar u k_1)^2 = \bar u k^2$

(momentum fractions in π^+: $u, \bar u$)

$\rightarrow$ Dirac trace:

- propagator numerators
- projectors on leading-twist LCDA for π^+ and soft $B \rightarrow \pi$ form factor

The Kernel $T_\Gamma^{\rm I}$



- contains short-distance QCD effects that do not involve the spectator
- for instance, the result for $F_\perp$ can be written as:

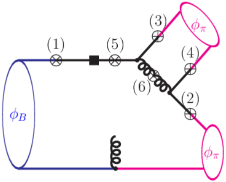
$$T_\perp^{\rm I} \propto \frac{\alpha_s C_F}{N_C}\, f_2(\bar u)\, {\rm Tr}\left[\not k_2 \not k_1 \gamma_5\, \Gamma_\perp \frac{1+\not v}{2}\right] + \mathcal{O}(\alpha_s^2)$$

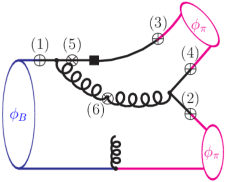
with

$$f_2(\bar u) = \frac{1}{\bar u}\, \frac{M_B^2}{2E_2 M_B + \bar u\,(2E_1 M_B - k^2)}$$

$\Rightarrow$ finite convolution integrals with $\phi_\pi(u)$ for $\bar u \to 0$ $\qquad \to$ task #1 ✓

The Kernel $T_\Gamma^{\rm II}$ (Spectator Scattering at leading power)





[hard-collinear gluon exchange between spectator quark and any of the vertices (1-6)]

Endpoint Divergencies:

- endpoint divergencies emerge in the limit $\bar{u}, \bar{v} \to 0$ and/or $\omega \to 0$

$$\text{e.g.}: \quad \int_0^1 du \, \frac{\phi_\pi(u)}{\bar{u}^2} \to \infty \,, \qquad \int_0^\infty d\omega \, \frac{\phi_B^+(\omega)}{\omega^2} \to \infty \qquad (\bar{u} = 1 - u)$$

- $\bar{u} \to 0$ divergencies do cancel, as expected from color-transparency ✓
- what about the other divergent contributions?

Cancellation of Endpoint-Divergencies (Feynman Gauge)

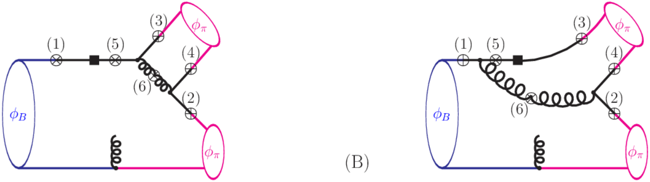
structure	A1	A2	A3 + A4	A5	A6	A1-A6
$\frac{2E_2 M_B}{\bar{u}^2 k^2} \, s_5 \, \frac{\phi_B^+(\omega)}{\omega} \, \frac{\phi_\pi(v)}{2v\bar{v}}$	0	0	$-C_{FA} \, 2v$	0	$C_A \, \frac{v-\bar{v}}{2}$	$2vC_F - \frac{C_A}{2}$
$\frac{S_A}{\bar{u}} \, \frac{\phi_B^-(\omega)}{\omega} \, \frac{\phi_\pi(v)}{\bar{v}^2}$	$C_F \, \frac{1}{v}$	$C_F \, \bar{v}$	$C_{FA} \, \frac{\bar{v}}{v}$	0	$-\frac{C_A}{2} \, \frac{\bar{v}}{v}$	$C_F \, (1+\bar{v})$
$\frac{S_A}{\bar{u}} \, \frac{\phi_B^+(\omega)}{\omega} \, \frac{\mu_\pi \phi_\sigma(v)}{6\bar{v}^3 E_2}$	C_F	0	0	0	0	C_F
$2\mu_\pi \, \frac{S_A}{\bar{u}} \, \frac{\phi_B^+(\omega)}{\omega^2} \, \frac{\phi_P(v)}{\bar{v}}$	0	C_F	0	0	0	C_F

structure	B1	B2	B3+B5	B4	B6	B1-B6
$\frac{2E_2 M_B}{\bar{u}^2 k^2} \, s_5 \, \frac{\phi_B^+(\omega)}{\omega} \, \frac{\phi_\pi(v)}{2v\bar{v}}$	0	0	0	$C_{FA} \, 2v$	$C_A \, \frac{\bar{v}-v}{2}$	$\frac{C_A}{2} - 2vC_F$
$\frac{S_B^{(i)}}{\bar{u}} \, \frac{\phi_B^+(\omega)}{\omega} \, \frac{\phi_\pi(v)}{\bar{v}^2}$	0	0	$-C_{FA} \, v_\perp^2$	$C_{FA} \, v_\perp^2$	0	0
$\frac{S_B^{(i)}+S_B^{(ii)}}{\bar{u}} \, \frac{\phi_B^-(\omega)}{\omega} \, \frac{\phi_\pi(v)}{\bar{v}^2}$	$C_F \, \frac{1}{v}$	$C_F \, \bar{v}$	$C_{FA} \, \frac{1}{v}$	$-C_{FA}$	$-\frac{C_A}{2} \, \frac{\bar{v}}{v}$	$C_F \, (1+\bar{v})$
$\frac{S_B^{(i)}+S_B^{(ii)}}{\bar{u}} \, \frac{\phi_B^+(\omega)}{\omega} \, \frac{\mu_\pi \phi_\sigma(v)}{6\bar{v}^3 E_2}$	C_F	0	$-C_{FA} \, v_\perp^2$	$C_{FA} \, v_\perp^2$	0	C_F
$\frac{S_B^{(i)}}{\bar{u}} \, \frac{\phi_B^-(\omega)}{\omega} \, \frac{\mu_\pi \phi_\sigma(v)}{6\bar{v}^3 E_2}$	0	0	$C_{FA} \, v_\perp^2$	$-C_{FA} \, v_\perp^2$	0	0
$2\mu_\pi \, \frac{S_B^{(i)}+S_B^{(ii)}}{\bar{u}} \, \frac{\phi_B^+(\omega)}{\omega^2} \, \frac{\phi_P(v)}{\bar{v}}$	0	C_F	0	0	0	C_F

NLO Spectator Scattering - The Kernel $T_\Gamma^{\rm II}$

(A)

(B)



Cancellation of endpoint divergencies:

- sum of all diagrams reproduces the very same structure of endpoint divergencies as in the $B \to \pi$ case: [Beneke, Feldmann '01]
  - endpoint divergencies are universal and can be absorbed in ξ_π

$$\langle \pi\pi | \bar u \Gamma b | B \rangle = \frac{2\pi f_\pi \, \xi_\pi(E_2)}{k^2} \int_0^1 du\, \phi_\pi(u) T_\Gamma^{\rm I}(u,\ldots) \quad + \quad \text{finite terms}$$

- remaining factorizable (i.e. endpoint-finite) contributions determine $T_\Gamma^{\rm II}$

$\to$ task #2 ✓

Twist-3 Contributions to $T_\Gamma^{\rm I}$

- $T_\Gamma^{\rm II}$ corrections conceptually important, but clearly suppressed by α_s
  $\rightarrow$ can be neglected to first approximation
- Contributions from twist-3 LCDAs in π^+ "chirally enhanced" (s.a.)
  $\rightarrow$ to be included in numerical estimate

- Neglect 3-particle contributions $\rightarrow$ Wandzura-Wilczek relations
- Corresponding contributions to $B \rightarrow \pi\pi$ matrix elements can now be written as

$$\langle \pi^+\pi^- | \bar\psi_u \, \Gamma \, \psi_b | B^- \rangle \Big|_{\text{twist-3}} \propto \xi_\pi(E_2) \int_0^1 du \, \Big(\phi_P(u) \, T_\Gamma^{(\rm I,P)}(u,\ldots) + \phi_\sigma(u) \, T_\Gamma^{(\rm I,\sigma)}(u,\ldots) \Big)$$

- For instance,

$$T_\Gamma^{(\rm I,P)} = i \, \frac{\alpha_s C_F}{N_C} \, \frac{2 M_B \mu_\pi}{k^2} \, \frac{{\rm Tr}\left[\not{k}_2 \not{k}_1 \gamma_5 \, \Gamma_\perp \, \frac{1+\not{v}}{2} \right]}{\bar u} + \mathcal{O}(\alpha_s^2)$$

- Endpoint divergence from $\phi_P(u)/\bar u$ cancels with a corresponding term in $T_\Gamma^{(\rm I,\sigma)}$.

Twist-3 Contributions to $T_\Gamma^{\rm I}$

- $T_\Gamma^{\rm II}$ corrections conceptually important, but clearly suppressed by α_s
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  $\rightarrow$ to be included in numerical estimate

For illustration:

- use $q^2 \sim \lambda \ll M_B^2$
- perform Gegenbauer expansion of the twist-2 pion LCDA
- use asymptotic form for twist-3 LCDAs

This leads to relations like:

$$F_0^{(S)} \approx \frac{\sqrt{\lambda}}{2M_B\sqrt{q^2}}\, F_t^{(S)} \approx \frac{i\alpha_s C_F}{N_C}\,\frac{2\pi f_\pi}{M_B}\,\frac{2\sqrt{\lambda}}{M_B\sqrt{q^2}}\left(1+\frac{3a_2^\pi}{4}+\frac{\mu_\pi}{M_B}\right)\xi_\pi(\frac{M_B}{2})\,,$$

and similar for

$$F_0^{(P)} \simeq \frac{1}{\sqrt{2}}\, F_\parallel^{(P)}\,, \quad \text{and} \quad F_0^{(D)} \simeq \sqrt{\frac{2}{3}}\, F_\parallel^{(D)}$$

[some relations are a simple consequence of Lorentz invariance]

Phenomenological Implications

Partially integrated branching ratio

- branching ratios $\mathcal{B}(B^- \to \pi^+\pi^-\mu^-\bar{\nu}_\mu)$
- integrated over benchmark phase-space regions A,B,C (see above)

phase space region	central	$\delta_{\rm param}$	δ_{f_+}	unit
A	2.93	$^{+0.87}_{-0.40}$	$^{+0.49}_{-0.35}$	$10^{-8}\,\lvert V_{ub}\rvert^2$
B	9.60	$^{+2.80}_{-1.30}$	$^{+1.89}_{-0.79}$	$10^{-7}\,\lvert V_{ub}\rvert^2$
C	3.18	$^{+0.63}_{-0.63}$	$^{+0.48}_{-0.33}$	$10^{-5}\,\lvert V_{ub}\rvert^2$

The total uncertainty is $\delta^2_{\rm tot} = \delta^2_{\rm param} + \delta^2_{f_+}$.

$\to$ Even for Scenario C, the BR is too small
to be measured with reasonable accuracy (!)

(reason: perturbative suppression by $\mathcal{O}(\alpha_s^2)$)

Forward-backward asymmetry

$$A^{\pi}_{\rm FB}(k^2,q^2) \equiv \frac{\int_{-1}^{+1} d\cos\theta_\pi \;\; {\rm sign}(\cos\theta_\pi)\, \mathcal{B}(k^2,q^2,\cos\theta_\pi)}{\int_{-1}^{+1} d\cos\theta_\pi \; \mathcal{B}(k^2,q^2,\cos\theta_\pi)} \, .$$

phase space region	central	$\delta_{\rm param}$	δ_{f_+}	unit
A	-1.96	$^{+0.15}_{-0.19}$	$^{+0.04}_{-0.07}$	10^{-1}
B	-0.32	$^{+0.19}_{-0.21}$	$^{+0.07}_{-0.11}$	10^{-1}
C	$+1.25$	$^{+0.07}_{-0.07}$	$^{+0.03}_{-0.08}$	10^{-1}

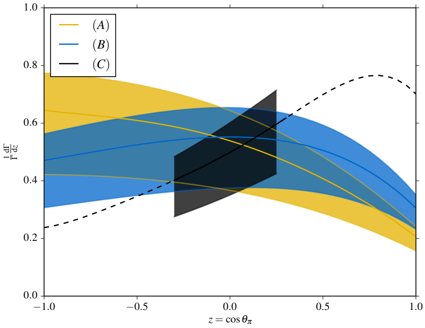
The total uncertainty is $\delta^2_{\rm tot} = \delta^2_{\rm param} + \delta^2_{f_+}$.

→ FB asymmetries of the order 10%

→ sign of FB asymmetry changes for Scenario C

Forward-backward asymmetry

- normalized partial decay rate $\frac{1}{\Gamma}\,\frac{d\Gamma}{d\cos\theta_\pi}$ as a function of $\cos\theta_\pi$



(integrated over k^2, q^2 – shaded areas = 68% intervals from variation of all input parameters)

→ FB asymmetries of the order 10%

→ sign of FB asymmetry changes for Scenario C

Summary and Outlook

- Factorization formula for $B \to \pi\pi$ form factors works as expected
  - leading-power contributions emerge from twist-2 LCDAs
  - non-trivial cancellation of endpoint-divergencies
- Caveats:
  - very small decay rate in the QCDF region
  - chiral enhancement of twist-3 contributions

Outlook

- set constraints for extrapolation from other phase-space regions
  (using other approaches: [Meißner, Wang '14], [Kang et al. '14], [Hambrock, Khodjamirian '15])
- apply to other decay topologies, e.g. in (off-resonant) $B \to K\pi\ell\ell$ decays

Backup Slides

Input Parameters

parameter	value/interval	unit	prior
QCD input parameter			
$\alpha_s(m_Z)$	0.1184 ± 0.0007	—	gaussian @ 68%
μ	$M_B/2 \pm M_B/4$	GeV	gaussian† @ 68%
$\overline{m}_{u+d}(2\,\text{GeV})$	7.8 ± 0.9	MeV	uniform @ 100%
hadron masses			
m_B	5279.58	MeV	—
m_π	139.57	MeV	—
parameters of the pion DAs			
f_π	130.4	MeV	—
$a_2^\pi(1\,\text{GeV})$	[0.09, 0.25]	—	uniform @ 100%
$\mu_\pi(2\,\text{GeV})$	2.5 ± 0.3	GeV	—

Prior distribution are expressed as a product of individual priors that are either uniform or gaussian.
The prior for the parameters describing the $B \to \pi$ form factor f_+ taken from [Imsong et al. 14].

Minimal pion energy: $E_{\min} = \min\left[\frac{k^2 + M_B^2 - q^2 - |\cos\theta| \sqrt{\lambda(k^2,q^2)}}{4M_B}\right]$

- Decreasing function of $|\cos\theta| \Rightarrow$ minimal value at: $|\cos\theta|_{\max}$
- Decreasing function of $k^2 \Rightarrow$ minimal value at $k^2_{\min}$
- q^2-dependence (for fixed $k^2_{\min}$ and $|\cos\theta|_{\max} \equiv 1/a$): minimum at

$$q_\star^2 = M_B^2 + k^2_{\min} - \frac{2aM_B\sqrt{k^2_{\min}}}{\sqrt{a^2-1}} \leq q^2_{\max} = (M_B - \sqrt{k^2_{\min}})^2 .$$

Two cases:

- $q_\star^2 \geq 0$ with $E_{\min} < \frac{a-1}{a}\frac{M_B}{2}$ and

$$E_{\min}(k^2_{\min}, q_\star^2, 1/a) = \frac{\sqrt{a^2-1}}{2a}\sqrt{k^2_{\min}}$$

- $q_\star^2 < 0$, with $E_{\min} > \frac{a-1}{a}\frac{M_B}{2}$ and

$$E_{\min}(k^2_{\min}, 0, 1/a) = \frac{(a+1)\,k^2 + (a-1)\,M_B^2}{4aM_B}$$