

# Experimental results from *b*-hadron decays to three-body final states

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Mini-workshop: multi-particle final states in B decays



#### This talk covers some recent publications from hadronic three-body decays

- Introduction to three-body decays analyses
- B decays to open charm, *i.e.* Dalitz plot analyses of B → Dhh' channels
   [Spectroscopy, CKM phase]

Status and plans for CPV measurements in charmless three-body charged decays

[Large CP violation seen in  $B^{\pm} \rightarrow h^{\pm}h^{+}h^{-}$  decays]









[Int. J. Mod. Phys. A 30 (2015) 1530022]



Technique named after Richard Dalitz (1925-2006)

Spin/parity determination of the known  $\tau/\theta$  particles in its decay products

"On the analysis of tau-meson data and the nature of the tau-meson." R. H. Dalitz, Phil. Mag. 44 (1953) 1068

#### **"I visualise geometry better than numbers"** Richard Dalitz

Scatter-plot visualisation can be interpreted as:

- Matrix element is constant, *i.e.* DP uniformly populated with events
- Non-uniform distributions gives information about the dynamics
- Interference patterns between intermediate states can be studied and parametrised



"A work of art" - gift from B. Richter, W. Panofsky, S. Drell, D. Leith, D. Aston, W. Dunwoodie and B. Ratcliff

#### Dalitz plot analysis



Intensity along bands indicates magnitude and the spin of the given resonance



Toy simulation using Laura++ package: <u>https://laura.hepforge.org</u>



Amplitude analysis can access:

- Relative phases between states
- Sensitivity to CP violating effects
- Resolve ambiguities in weak phases
- Hadron spectroscopy



A possibility is to perform an "Isobar Model", in which the total amplitude is approximated as coherent sum of quasi-two-body contributions:

$$CP \text{ violating } \begin{array}{c} Strong \ dynamics \\ CP \ conserving \end{array}$$
$$\mathcal{A}(m_{ij}^2, m_{jk}^2) = \sum_{l=1}^{N} c_l F_l(m_{ij}^2, m_{jk}^2)$$

c<sub>1</sub>: complex coefficients describing the relative magnitude and phase of the different isobars F<sub>1</sub>: dynamical amplitudes that contain the lineshape and spin-dependence of the hadronic part

$$F_l(L, m_{ij}^2, m_{jk}^2) = R_l(m_{ij}^2) \times X_L(|\vec{p}|r) \times X_L(|\vec{q}|r) \times T_l(L, \vec{p}, \vec{q})$$

Resonance mass term<br/>(e.g. Breit-Wigner)Barrier factors - p, q: momenta<br/>of bachelor and resonanceAngular probability<br/>distribution

Many observables can be accessed:  $Re(c_i)$  and  $Im(c_i)$  or  $|c_i|$  and  $arg(c_i)$ ; or derived quantities such as BF and  $A_{CP}$ 



#### B decays to open charm, *i.e.* $B \rightarrow Dhh'$ channels

Dalitz-plot analyses (e.g. spectroscopy and CKM angle measurements)

Charm and charm-strange spectroscopy

[PRL 113, 162001 (2014), PRD 90, 072003 (2014)] [PRD 91, 092002 (2015)] [PRD 92, 032002 (2015)]



 $D_s^{(**)}$  spectroscopy -  $B^0_s \rightarrow \overline{D}{}^0K^-\pi^+$ 



PRD 89, 074023 (2014)

Stephen Godfrey, Ian T. Jardine

Spectroscopy of strange-charm states has been reinvigorated due to recent observations of  $D_{s0}^{*}(2317)$  and  $D_{s1}(2460)$ 

DP analysis of  $B^{0}_{s} \rightarrow D_{s}^{**-}(\overline{D}^{0}K^{-})\pi^{+}$ 



3306 3323' 3311 3208 3218 3190 3200 3154 3193 3298 3005 3038' 3048 3186 2899 2926' 2917 3018 (MeV) 2800 2900 2673 2732 2484 2556 2592 2549 Mass 2400 2129 D<sub>s</sub> Mass Spectrum 1979 2000 1600  ${}^{1}S_{0} {}^{3}S_{1} {}^{3}P_{0} {}^{P}_{1} {}^{3}P_{2} {}^{3}D_{1} {}^{D}_{2} {}^{3}D_{3} {}^{3}F_{2} {}^{F}_{3} {}^{3}F_{4}$ 

- D<sub>s</sub>\* and D<sub>s0</sub>\*(2317) are too light to decay to D<sup>0</sup>K<sup>-</sup>
  Neither can states with unnatural spin-parity
- $(J^{P} = 0^{-}, 1^{+}, 2^{-}, \text{etc})$ •  $D_{s2}^{*}(2573), D_{s1}^{*}(2700)$  and  $D_{sJ}^{*}(2860)$  are possible

Analysis performed with ~11K signal events and purity of 87%

Dalitz plot analysis of  $B^{0}_{s} \rightarrow \overline{D}^{0}K^{-}\pi^{+}$ 

Backgrounds due to Combinatorial (7.3%),  $B^0 \to D^{(*)0}\pi\pi$  (2.8%) and  $\Lambda^0_b \to D^{(*)0}p\pi$  (2.3%)

## Signal region: $\pm 2.5\sigma$ around nominal mass is considered for the Dalitz plot fit

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PRD 90, 072003 (2014) PRL 113, 162001 (2014)







#### $D_{sJ}^{*}(2860)^{-}$ state

Several spin hypotheses have been investigated for the  $D_{sJ}^{*}(2860)^{-1}$ 

Two states  $[D_{s1}^*(2860)^-, D_{s3}^*(2860)^-]$ are required in the region 2.86 GeV/c<sup>2</sup> (each with a significance of 10 $\sigma$ )

1<sup>st</sup> observation of a heavy flavoured spin-3 resonance and 1<sup>st</sup> time a spin-3 state seen to be produced in B decay

Spin hypothesis	$\Delta \mathrm{NLL}$	$\sqrt{2\Delta \text{NLL}}$	Masses and widths			
1+3	0					
0	141.0	16.8	2862	57		
0+1	113.2	15.0	2446	250	2855	96
0+2	155.1	17.6	2870	61	2569	17
0+3	105.1	14.5	2415	188	2860	52
1	156.8	17.7	2866	92		
1+2	138.6	16.6	2851	99	3134	174
2	287.9	24.0	3243	81		
2	365.5	27.0	2569	17		
2+3	131.2	16.2	2878	12	2860	56
3	136.5	16.5	2860	57		





The presence of the state  $D_{s3}^{*}(2860)$  has been independently confirmed in studies of pp  $\rightarrow D^{*(+,0)}K^{0,+}X$  (LHCb) [JHEP 02 (2016) 133]





Data favours spin-1 hypothesis for the state  $D^*_J(2760)^0$  (other assignments are rejected with  $> 6\sigma$ )

Data strongly (10  $\sigma$ ) favours spin-3 assignment to the state  $D^*_{J}(2760)^-$ 

No evidence for an additional spin-1

Candidates / (40 MeV)

 $0^{2}$ 

10

2

Analogous to the B  $\rightarrow$  D $\pi$ h family, there are many interesting aspects:

- \* Spectroscopy of the Dp/p $\pi$  resonances
- $\wedge \Lambda^0_b \rightarrow D^0 p K^-$  decay should be sensitive to CKM angle  $\gamma$
- Final state also accessible to  $\Xi^{0}_{b}$ 's

First observations of  $\Lambda^{0}_{b}$  and  $\Xi^{0}_{b}$  decaying to D<sup>0</sup>ph and  $\Lambda_ch$  final states with 1 fb<sup>-1</sup> [BR and  $\Xi^{0}_{b}$  mass measurement]

[NEW] Amplitude analysis (DP + 3 angles) of  $\Lambda^{0}_{b} \rightarrow D^{0}p\pi^{-}$  decays

3.05  $D^{\circ}$  p Invariant Mass (GeV/c<sup>2</sup>)









[1] Resonance-free region: constrain reflections between Dp and  $p\pi$  channels [2] Vicinity of  $\Lambda_c(2880)$ : model-independent determination of amplitude [3]  $\Lambda_c(2880)$  down to threshold: understand near-threshold structure [4] Threshold to  $\Lambda_c(2940)$ : J<sup>P</sup>, mass and width for  $\Lambda_c(2940)$ [5]  $p\pi$  amplitude: N\* states (interplay with  $\Lambda^0_b \rightarrow J/\psi p\pi^-$ ) [Next time]

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#### The effect of $p\pi$ reflections in Dp decays can be examined in region [1]





#### Using $\Lambda^+_c(2880)$ as a reference amplitude allows us to constrain "NR" amplitude







Existence of 3/2<sup>+</sup> (1D in heavy quark-light diquark model) is suggested by many theorists (see, e.g. [arXiv:1609.07967]), mostly in the region of 2850 MeV/c<sup>2</sup>



First constraints on quantum numbers of  $\Lambda^+_c$  (2940) are obtained.

Fits favour JP = 3/2, but other solutions cannot yet be excluded, depending on the non-resonant model

Mass and width of the  $\Lambda^+_c$  (2940) are consistent with and have comparable precision to the current world average



 $\Lambda^+_c$  (2940)+ has different explanations depending on J<sup>P</sup>, 3/2 is a typical molecular assignment (D\*N) (e.g. [arXiv:1212.5325])



Spectroscopy studies have received a great attention from the community with numerous recent results

- Additional insights can be obtained in near future through from  $B \rightarrow D^*hh^2$  channels, where unnatural spin-parity states can appear
- Similar modes, e.g. B → D<sup>(\*)</sup>D<sup>(\*)</sup>h are of great interest (e.g. for leptonic decays)
   recently performed by BaBar [Phys. Rev. D 91, 052002 (2015)]

Measurements of CKM weak phases (*i.e.*  $\gamma$ ,  $\beta$  and  $\beta_s$ ) are being gradually performed

- LHCb performed a simultaneous analysis of  $B^0 \rightarrow \overline{D}{}^0K^+\pi^-$ , with  $\overline{D}{}^0 \rightarrow K^+\pi^-$  and of  $B^0 \rightarrow D_{CP}K^+\pi^-$  (+cc) with  $D_{CP} \rightarrow \pi^+\pi^-$  or  $K^+K^-$  [Phys.Rev. D93 (2016) 112018]
- Similar approach can be applied to other final states. Moreover, time-dependent amplitude analysis can provide clean measurements of mixing phases



Dalitz-plot analysis of  $B^{\pm} \rightarrow h^{\pm}h^{\pm}h^{\prime} \pm decays$ , where  $h^{(\prime)} \in \{\pi^{\pm}, K^{\pm}\}$ 

LHCb results :  $\mathcal{L} = 3 \, \text{fb}^{-1} - 2011 + 2012 \, \text{dataset}$ 

Large local phase-space asymmetries observed in charmless charged B decays [PRD 90, 112004 (2014), PRL 112 (2014) 011801, PRL 111 (2013) 101801]





The presence of multiple amplitudes leads can be modelled as

$$A(B \to f) = \sum_{i} |A_i| e^{i(\delta_i + \phi_i)} \qquad \bar{A}(\bar{B} \to \bar{f}) = \sum_{i} |A_i| e^{i(\delta_i - \phi_i)}$$

Strong phase ( $\delta$ ) invariant under *CP*, while weak phase ( $\varphi$ ) changes sign under *CP* 

$$\mathcal{A}_{CP}(B \to f) \equiv \frac{|\bar{A}|^2 - |A|^2}{|\bar{A}|^2 + |A|^2} \propto \sum_{i,j} |A_i| |A_j| \sin(\delta_i - \delta_j) \sin(\phi_i - \phi_j)$$

Conditions for CP violation in decay

- At least two amplitudes
- Non-zero strong phase difference
- Non-zero weak phase difference

Source of weak phase differences come from different CKM phases of each amplitude

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Each source of strong phase leaves a unique signature in the Dalitz plot

**[Illustrative example]** Consider  $B^{\pm} \rightarrow K^{\pm}\pi^{+}\pi^{-}$  with only two isobars, *i.e.*  $B^{\pm} \rightarrow \rho^{0}K^{\pm}$  and a flat non-resonant (NR) component

$$\begin{aligned} A_{+} &= |a_{+}^{\rho}|e^{i\delta_{+}^{\rho}}F_{\rho}^{\mathrm{BW}}\cos\theta + |a_{+}^{\mathrm{NR}}|e^{i\delta_{+}^{\mathrm{NR}}} \\ A_{-} &= |a_{-}^{\rho}|e^{i\delta_{-}^{\rho}}F_{\rho}^{\mathrm{BW}}\cos\theta + |a_{-}^{\mathrm{NR}}|e^{i\delta_{-}^{\mathrm{NR}}} \\ \mathcal{A}_{CP} &\propto |A_{-}|^{2} - |A_{+}|^{2} \\ &\propto (|a_{-}^{\rho}|^{2} - |a_{+}^{\rho}|^{2})|F_{\rho}^{\mathrm{BW}}|^{2}\cos^{2}\theta \\ &-2(m_{\rho}^{2} - s)|F_{\rho}^{\mathrm{BW}}|^{2}\cos\theta \dots \\ &+2m_{\rho}\Gamma_{\rho}|F_{\rho}^{\mathrm{BW}}|^{2}\cos\theta \dots \end{aligned}$$

$$ToyMC$$

### Dalitz plot strong phase manifestation









Dalitz plot strong phase manifestation



 $K^+$ 

 $\pi^+$ 

 $\pi^+$ 

 $\pi^+$ 

20.

 $\pi^+$ 

 $\pi K^+$ 

 $\rho^0$ 

 $P^{-}$ 

θ

 $\rho^0$ 

ĥ

ľ



Interference term from imaginary part of Breit-Wigner, maximum at ρ pole, linear in helicity

$$\begin{aligned} \mathcal{A}_{CP} \propto & (|a_{-}^{\rho}|^{2} - |a_{+}^{\rho}|^{2})|F_{\rho}^{\mathrm{BW}}|^{2}\cos^{2}\theta \dots \\ & -2(m_{\rho}^{2} - s)|F_{\rho}^{\mathrm{BW}}|^{2}\cos\theta \dots \\ & +2m_{\rho}\Gamma_{\rho}|F_{\rho}^{\mathrm{BW}}|^{2}\cos\theta\dots \end{aligned}$$



[4] Final state re-scattering contributions (e.g KK  $\leftrightarrow \pi\pi$ )

Can occur between decay channels with the same flavour quantum numbers:

*e.g.*  $B^{\pm} \rightarrow K^{\pm}K^{+}K^{-}$  and  $B^{\pm} \rightarrow K^{\pm}\pi^{+}\pi^{-}$ 

CPT conservation constrains hadron re-scattering:

- For given quantum numbers, sum of partial widths equal for charge-conjugate decays
- KK  $\leftrightarrow \pi\pi$  re-scattering generates a strong phase
- If re-scattering phase in one decay channel generates direct *CP* violation in this region
- Re-scattering phase should generate opposite sign direct *CP* violation in partner decay channel

CP violation LHCb inclusive results



PRD 90, 112004 (2014)

 $\mathcal{A}_{CP} = -0.036 \pm 0.004 \pm 0.002 \pm 0.007$   $\times 10^{3}$  $\mathcal{A}_{CP} = +0.025 \pm 0.004 \pm 0.004 \pm 0.007$ <u>×</u>10<sup>3</sup> Candidates /  $(0.01 \text{ GeV}/c^2)$ Candidates /  $(0.01 \text{ GeV}/c^2)$ - Model LHCb - Model LHCb  $B^{\pm} \rightarrow K^{\pm} K^{+} K^{-}$  $B^{\pm} \rightarrow K^{\pm} \pi^{+} \pi^{-}$ 16 10 ····Combinatorial Combinatorial 14E **–** B→4-body B→4-body 12 8 •••  $B^{\pm} \rightarrow \pi^{\pm} K^{+} K^{-}$ •••  $B^{\pm} \rightarrow \eta'(\rho^0 \gamma) K^{\pm}$ 10 6  $-B^{\pm}\rightarrow K^{\pm}\pi^{+}\pi^{-}$ − B<sup>±</sup>→π<sup>±</sup>π<sup>+</sup>π<sup>-</sup> 6 0**6** 5.1 5.3 5.4 5.5 5.3 5.2 5.1 5.2 5.4 5.1 5.5 5.4 5.2 5.3 5.5 5.1 5.2 5.3 5.4 5.5  $m(K^{-}K^{+}K^{-})$  [GeV/ $c^2$ ]  $m(K^{+}K^{+}K^{-})$  [GeV/ $c^{2}$ ]  $m(K^{-}\pi^{+}\pi^{-})$  [GeV/*c*<sup>2</sup>]  $m(K^{+}\pi^{+}\pi^{-})$  [GeV/*c*<sup>2</sup>]  $\mathcal{A}_{CP} = +0.058 \pm 0.008 \pm 0.009 \pm 0.007$  $\mathcal{A}_{CP} = -0.123 \pm 0.017 \pm 0.012 \pm 0.007$ ×10<sup>3</sup> 3F<sup>1</sup> <u>×10<sup>3</sup></u> Candidates /  $(0.01 \text{ GeV}/c^2)$ Candidates /  $(0.01 \text{ GeV}/c^2)$ - Model LHCb - Model LHCb  $B^{\pm} \rightarrow \pi^{\pm} K^{+} K^{-}$ 2.5 <sup>....</sup> B<sup>±</sup>→π<sup>±</sup>π<sup>+</sup>π<sup>−</sup> ...Combinatorial B<sub>s</sub>→4-body "Combinatorial 0.6 B→4-body B→4-body **١.5**⊦  $B^{\pm} \rightarrow K^{\pm} K^{+} K^{-}$ •••  $B^{\pm} \rightarrow K^{\pm} \pi^{+} \pi^{-}$  $-B^{\pm} \rightarrow K^{\pm} \pi^{+} \pi^{-}$ 0.4 0.2 0.5 5.2 5.3 5.4 5.5 5.1 5.2 5.4 5.5 5.3 5.4 5.5 5.1 5.2 5.3 5.4 5.3 5.2 5.5 5.1 5.1  $m(\pi^{-}\pi^{+}\pi^{-})$  [GeV/*c*<sup>2</sup>]  $m(\pi^{+}\pi^{+}\pi^{-})$  [GeV/ $c^{2}$ ]  $m(\pi^{-}K^{+}K^{-})$  [GeV/*c*<sup>2</sup>]  $m(\pi^{+}K^{+}K^{-})$  [GeV/*c*<sup>2</sup>]



PRD 90, 112004 (2014)









PRD 90, 112004 (2014)





Full amplitude analysis is clearly the next step, in particular to understand the origin of the strong phase difference

#### Such analyses are currently ongoing at LHCb!

However, building models for these decays is challenging:

- Unprecedented statistics (e.g. 180K events for  $B^{\pm} \rightarrow K^{\pm}\pi^{\mp}\pi^{\pm}$ ): simplified theoretical descriptions **are not sufficient** to accommodate the data
- How to model the large non-resonant components?
- How to describe re-scattering effects? Connect two (or all) different final states?
- How to include thee-body final state interaction (FSI)?

"Guinea pig":  $B^{\pm} \rightarrow \pi^{\pm}\pi^{-}$  decays with Run-I





Main A<sub>CP</sub> features of LHCb data Run-I are present in a simple simulation of the previous **BaBar** model

More work clearly required to fully reproduce all features seen in data

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Two alternative fits are also investigated

K-matrix approach

Resonances don't necessarily manifest as Breit-Wigner structures



• Quasi-model independent method  $(d_{\underline{a}}, d_{\underline{a}}, d_{\underline{a}}, d_{\underline{b}}, d_{\underline{b}},$ 





#### General conclusions

- Enormous wealth of physics to be found in three-body hadronic decays of b-hadrons (*e.g.* CKM phase measurements, CP violation)
- Some very interesting and intriguing results obtained recently
  - Latest results in multi-body charmless hadronic decays are using increasingly sophisticated amplitude analysis techniques
- Still many interesting results are foreseen with LHCb Run-I dataset (*e.g.* charmless DP analyses, *b*-baryon and  $B^+_c$  decays)
  - ✤ Potential for improving the spectroscopy and CKM measurements from B → Dhh using Run 1 + Run 2.
  - Larger datasets from the LHCb upgrade and Belle II will provide in the future the possibility to fully explore the potential of the field

#### Dalitz plot fit results



PRL 113, 162001 (2014) PRD 90, 072003 (2014)

Contributions to the amplitude fit model (resonances labelled with subscript v are virtual)

Resonance	$\operatorname{Spin}$	Dalitz plot axis	Model	Parameters $(MeV/c^2)$	
$\overline{K}^{*}(892)^{0}$	1	$m^2(K^-\pi^+)$	RBW	$m_0 = 895.81 \pm 0.19,  \Gamma_0 = 47.4 \pm 0.6$	
$\overline{K}^{*}(1410)^{0}$	1	$m^2(K^-\pi^+)$	$\operatorname{RBW}$	$m_0 = 1414 \pm 15,  \Gamma_0 = 232 \pm 21$	
$\overline{K}_{0}^{*}(1430)^{0}$	0	$m^2(K^-\pi^+)$	LASS	Floating parameters	
$\overline{K}_{2}^{*}(1430)^{0}$	2	$m^2(K^-\pi^+)$	RBW	$m_0 = 1432.4 \pm 1.3, \ \Gamma_0 = 109 \pm 5$	
$\overline{K}^{*}(1680)^{0}$	1	$m^2(K^-\pi^+)$	$\operatorname{RBW}$	$m_0 = 1717 \pm 27,  \Gamma_0 = 322 \pm 110$	
$\overline{K}_{0}^{*}(1950)^{0}$	0	$m^2(K^-\pi^+)$	RBW	$m_0 = 1945 \pm 22,  \Gamma_0 = 201 \pm 90$	
$D_{s2}^{*}(2573)^{-}$	2	$m^2(\overline{D}{}^0K^-)$	$\operatorname{RBW}$	Floating parameters	
$D_{s1}^{*}(2700)^{-}$	1	$m^2(\overline{D}{}^0K^-)$	$\operatorname{RBW}$	$m_0 = 2709 \pm 4,  \Gamma_0 = 117 \pm 13$	
$D_{sJ}^{*}(2860)^{-}$	1	$m^2(\overline{D}{}^0K^-)$	RBW	Floating parameters + Multiple spin hypotheses	
$D_{sJ}^{*}(2860)^{-}$	3	$m^2(\overline{D}{}^0K^-)$	$\operatorname{RBW}$		
Nonresonant		$m^2(\overline{D}{}^0K^-)$	EFF	Floating parameters	
$D_{sv}^{*-}$	1	$m^2(\overline{D}{}^0K^-)$	RBW	$m_0 = 2112.3 \pm 0.5,  \Gamma_0 = 1.9$	
$D^*_{s0v}(2317)^-$	0	$m^2(\overline{D}{}^0K^-)$	$\operatorname{RBW}$	$m_0 = 2317.8 \pm 0.6,  \Gamma_0 = 3.8$	
$B_v^{*+}$	1	$m^2(\overline{D}{}^0\pi^+)$	RBW	$m_0 = 5325.2 \pm 0.4,  \Gamma_0 = 0$	

RBW = Relativistic Breit-Wigner, LASS = K S-wave parameter from LASS experiment and EFF = exponential form factor

DP analysis of  $B^0 \rightarrow \overline{D}{}^0\pi^+\pi^-$ 



PRD 92, 032002 (2015)



#### Charm spectroscopy at LHCb



Recent measurements of e<sup>+</sup>e<sup>-</sup>/pp indicated the presence of higher excited states (both BaBar and LHCb)



PRD 92, 032002 (2015) PRD 91, 092002 (2015)

 $B^- \to D^+ K^- \pi^-$ 

Initial investigation of angular moments to guide the modelling



Two different DP fit framework: Isobar model and K-matrix parametrisation of the S-wave

Large A<sub>CP</sub> in charmless B<sup>+</sup> decays





#### Baryonic final states







#### Adaptive binning algorithm





Adaptive binning algorithm





Adaptive binning algorithm



Example: for the biggest bin in figure we calculate the  $acp_i$ :

$$acp_i = \frac{79 - 85}{164} = -0.037$$

Further details on the re-scattering approach



Phys. Rev. D 92, 054010 (2015)  $B --> \pi \pi \pi$  $B \rightarrow \pi \pi \pi$ 200 200  $S = \begin{bmatrix} \eta e^{2i\delta_{\pi\pi}} & i\sqrt{1-\eta^2}e^{i(\delta_{\pi\pi}+\delta_{KK})} \\ i\sqrt{1-\eta^2}e^{i(\delta_{\pi\pi}+\delta_{KK})} & \eta e^{2i\delta_{KK}} \end{bmatrix}$  $\cos\theta < 0$  $\cos\theta > 0$ 100 100 yields yields  $\mathbf{B}^+$ Only off-diagonal elements are m 



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