A parametrization of pion vector and scalar form factors up to $2\,\mathrm{GeV}$

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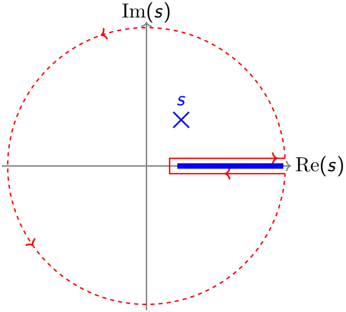
Problems of dispersion theory

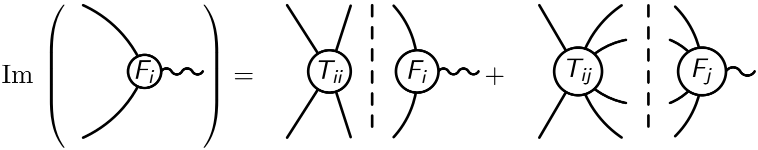
- Analyticity and Cauchy's theorem

$$F_i(s) = \frac{1}{2\pi i}\int_{s_{th}}^{\infty} \mathrm{d}z\, \frac{\mathrm{disc}\, F_i(z)}{z - s - i\epsilon}$$

$$= \frac{1}{\pi}\int_{s_{th}}^{\infty} \mathrm{d}z\, \frac{\mathrm{Im}\, F_i(z)}{z - s - i\epsilon}$$



- $\mathrm{Im}\, F_i(z)$ obtainable by the unitary equation



$$\mathrm{Im}\, F_i(s) = T_{ij}^*(s)\sigma_j(s)F_j(s) = T_{ij}^*(s)\frac{2p_j}{\sqrt{s}}F_j(s)$$

Problems of dispersion theory

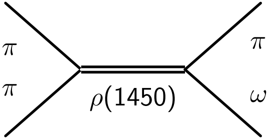
- Obtain the coupled channel integral equation

$$F_i(s) = \frac{1}{\pi} \int_{s_{th}}^{\infty} \frac{\mathrm{d}z}{z - s - i\epsilon} \, T^*_{ij}(z)\sigma_j(z) F_j(z)$$

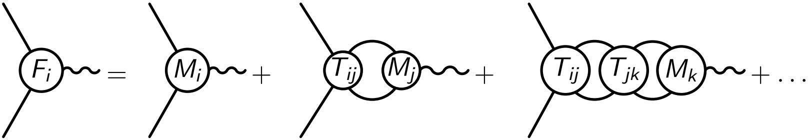
- Need $T_{ij}(s)$ for all relevant channels to arbitrary large energies (not possible/realizable)
- More complex structure of $2 \to 4$ processes (not expressible in terms of phase shifts)
- Need a parametrization that
  1. fulfills analyticity and unitarity
  2. is consistent with high precision analysis at low energies (Roy-Steiner equations) Garcia-Martin et al. [2011], Büttiker et al. [2004]
  3. describes the high energy data reasonable well
- Such a parametrization was provided by Hanhart [2012]

A new parametrization for the form factor

- At higher energies inelasticities are often accompanied by resonances
  $\Rightarrow$ Consider a resonance model that is consistent with analyticity and unitarity



- At lower energies it reduces to the Omnès solution



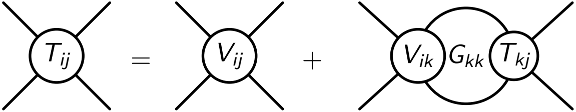
$$F_i(s) = \left(1 + a_i^1 s\right) \Omega_{ij}(s) F_j(0)$$

- Omnès matrix $\Omega_{ij}(s)$ is defined by

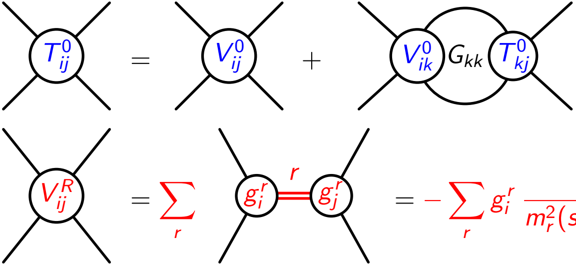
$$\Omega_{ij}(s) = \frac{1}{\pi} \int_{s_{th}}^{\infty} \mathrm{d}z \, \frac{T^*_{ik}(z)\sigma_k(z)\Omega_{kj}(z)}{z - s - i\epsilon} \qquad , \qquad \Omega_{ij}(0) = \delta_{ij}$$

A new parametrization for the form factor

- Start from the Bethe-Salpeter equation



- Separate potential $V = V^0 + V^R$ and $T = T^0 + T^R$ into two parts



$$= -\sum_r g_i^r \frac{s}{m_r^2(s - m_r^2)} g_j^r$$

A new parametrization for the form factor

- Parametrization for $T^0(s)$ given by inelasticities and scattering phases

Single channel	Two channel
$T^0(s) = \begin{pmatrix} \frac{\sin(\delta)}{\sigma_\pi} e^{i\delta} & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$	$\begin{pmatrix} \frac{\eta e^{2i\delta}-1}{2i\sigma_\pi} & g e^{i\psi} & 0 & \dots \\ g e^{i\psi} & \frac{\eta e^{2i(\psi-\delta)}-1}{2i\sigma_K} & 0 & \dots \\ 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

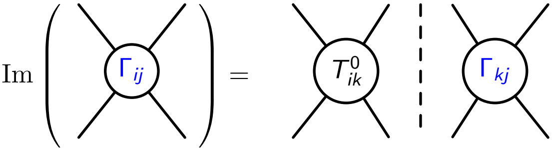
- Assumptions thus far:
  - All crossed channel interactions are contained in $T^0(s)$
  - Deviations from $T^0(s)$ come solely from s-channel resonances

A new parametrization for the form factor

- Solution for the scattering matrix $T(s)$

$$T_{ij}(s) = T^0_{ij}(s) + \Gamma_{ik}(s)\left[1 - V^R(s)\Sigma(s)\right]^{-1}_{km} V^R_{mn}(s)\Gamma_{jn}(s)$$

- Vertex factor $\Gamma(s)$



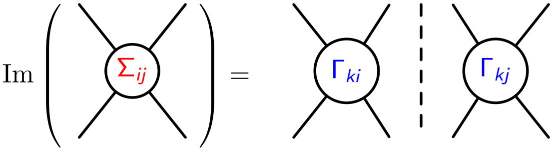
$$\Rightarrow \quad \Gamma_{ij}(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} \mathrm{d}z \, \frac{\left(T^0_{ik}(z)\right)^* \sigma_k(z)\Gamma_{kj}(z)}{z - s - i\epsilon}$$

A new parametrization for the form factor

- Solution for the scattering matrix $T(s)$

$$T_{ij}(s) = T^0_{ij}(s) + \Gamma_{ik}(s)\left[1 - V^R(s)\Sigma(s)\right]^{-1}_{km} V^R_{mn}(s)\Gamma_{jn}(s)$$

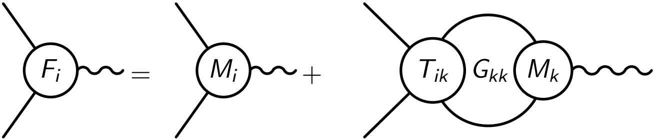
- Self energy $\Sigma(s)$



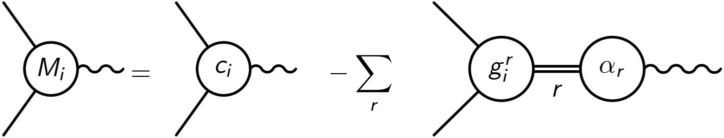
$$\Rightarrow \quad \Sigma_{ij}(s) = \frac{s}{\pi}\int_{s_{th}}^{\infty} \frac{\mathrm{d}z}{z}\,\frac{\Gamma^*_{ki}(z)\sigma_k(z)\Gamma_{kj}(z)}{z - s - i\epsilon}$$

A new parametrization for the form factor

- Similar to the P-vector approach write



- Direct transition matrix element $M_i(s)$ is given by



$$M_i(s) = c_i - \sum_r g_i^r \frac{s}{s - m_r^2} \alpha_r$$

A new parametrization for the form factor

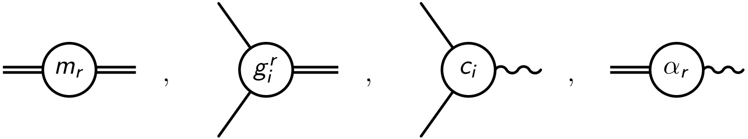
- Full parametrization for the form factor

$$F(s) = \Gamma(s)\left[1 - V^R(s)\Sigma(s)\right]^{-1} M(s)$$

- At low energies reduces to Omnès solution

$$F(s) = \Gamma(s)\left(a^0 + a^1 s\right)$$

- In principle free parameters



- For N_C physical channels and N_R resonances have $N_C + (N_C + 2)\,N_R$ free parameters

Application to the pion vector form factor

- Definition of pion vector form factor $F_V(s)$

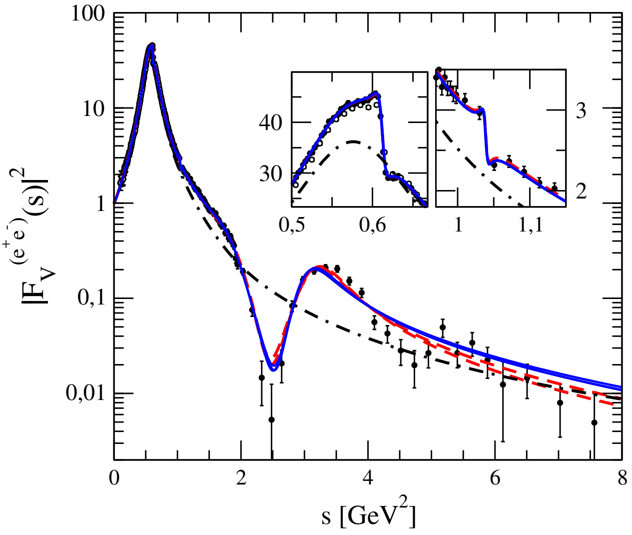
$$\langle \pi^+(q_1)\pi^-(q_2)\,|J^\mu|\,0\rangle = (q_1 - q_2)^\mu\, F_V(s)$$

- Three most relevant resonances up to $2\,\mathrm{GeV}$ are given by $\rho(770)$, $\rho(1450)$ and $\rho(1700)$
- Relevant channels (1+2)
  1. $\pi\pi\,(\sqrt{s_{th}} \approx 0.279\,\mathrm{GeV})$: Elastic channel treatment works up to $1\,\mathrm{GeV}$
  2. $4\pi\,(\sqrt{s_{th}} \approx 0.558\,\mathrm{GeV})$: Heavily phase space suppressed at low energies
  3. $\pi\omega\,(\sqrt{s_{th}} \approx 0.922\,\mathrm{GeV})$: Could play a strong role in the $\pi\pi$ inelasticity
- Elastic scattering matrix

$$T^0(s) = \begin{pmatrix} \frac{\sin(\delta(s))}{\sigma_\pi(s)}\, e^{i\delta(s)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

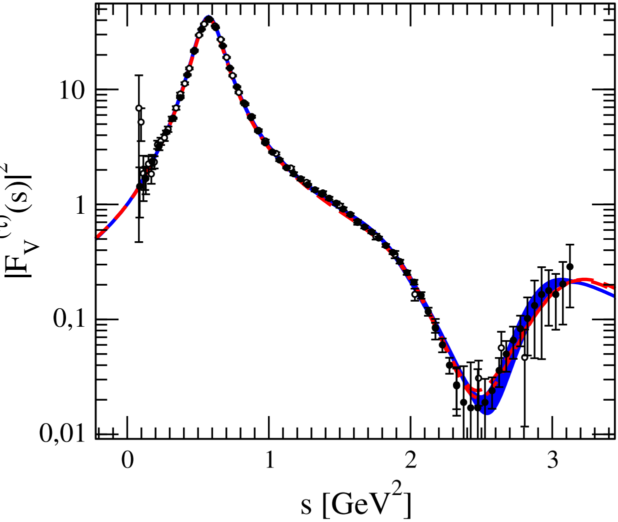
- The $\pi\pi$ phase shift $\delta(s)$ from Garcia-Martin et al. [2011]

Application to the pion vector form factor



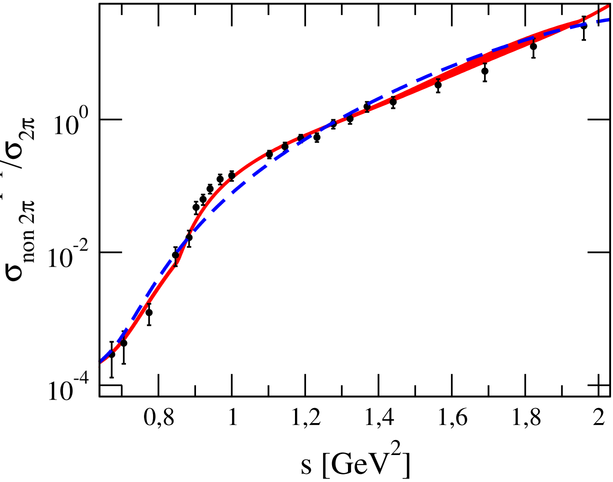
- Blue solid band: Only $\pi\pi$ and 4π $\left(\frac{\chi^2}{\mathrm{d.o.f.}} = 1.2\right)$
- Red dashed lines: $\pi\pi$, 4π and $\pi\omega$ $\left(\frac{\chi^2}{\mathrm{d.o.f.}} = 1.4\right)$
- Data: BaBar [2009] and KLOE [2011]

Application to the pion vector form factor



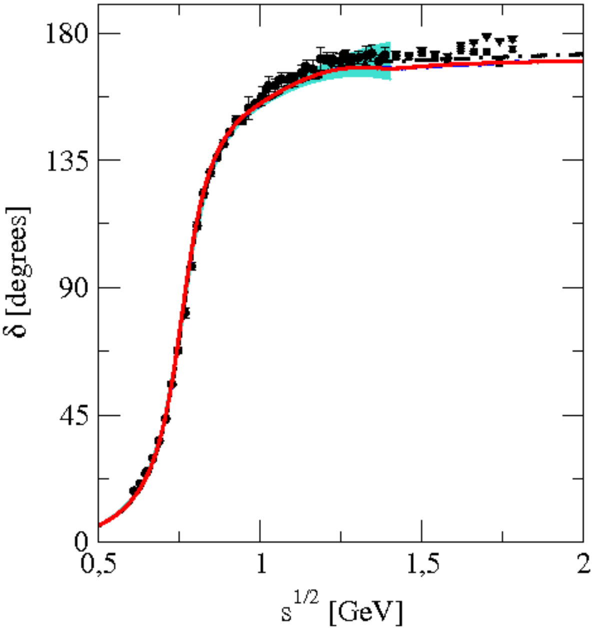
- Blue solid band: Only $\pi\pi$ and 4π (Prediction)
- Red dashed lines: $\pi\pi$, 4π and $\pi\omega$ (Prediction)
- Data: Belle [2008] and CLEO [2000]

Application to the pion vector form factor



- Blue dashed lines: Only $\pi\pi$ and 4π
- Red solid band: $\pi\pi$, 4π and $\pi\omega$
- Data: Eidelman and Lukaszuk [2004]

Application to the pion vector form factor

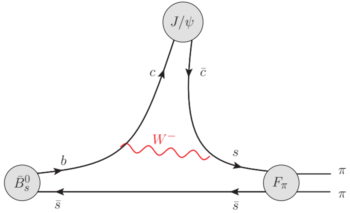


- Blue dashed lines: Only $\pi\pi$ and 4π
- Red solid band: $\pi\pi$, 4π and $\pi\omega$
- Cyan solid band: Garcia-Martin et al. [2011]
- Data: Hyams et al. [1973], Hyams et al. [1975], W. Ochs and Protopopescu et al. [1973]

Application to the scalar pion form factor

- Definition of the strange scalar pion form factor

$$\langle \pi^+(p_1)\pi^-(p_2)\,|\bar{s}s|\,0\rangle = \frac{2M_K^2 - M_\pi^2}{2m_s} F_S^\pi(s)$$



- Decay $\bar{B}_s^0 \to J/\psi\pi^+\pi^-$ measured by LHCb [2014]
- Experimental Dalitz plot shows that left-hand cuts from $J/\psi\pi$ interaction are negligible
- Isoscalar source $s\bar{s} \Rightarrow \pi\pi$ system can be in an even partial wave
- Dispersive analysis up to $1\,\mathrm{GeV}$ done in Daub et al. [2016] using

$$\begin{pmatrix} F_S^\pi(s) \\ \frac{2}{\sqrt{3}} F_s^K(s) \end{pmatrix} = \mathrm{const} \times \begin{pmatrix} \Omega_{11}(s) & \Omega_{12}(s) \\ \Omega_{21}(s) & \Omega_{22}(s) \end{pmatrix} \times \begin{pmatrix} F_S^\pi(0) \\ \frac{2}{\sqrt{3}} F_S^K(0) \end{pmatrix}$$

- Test the new parametrization on the isoscalar S-wave

Application to the scalar pion form factor

- Relevant observables are the angular momentum averages

$$\left\langle Y_L^0 \right\rangle (s) = \int_{-1}^{1} \mathrm{d}\cos\Theta_\pi \, \frac{\mathrm{d}^2\Gamma}{\mathrm{d}\sqrt{s}\,\mathrm{d}\cos\Theta_\pi} \, Y_L^0(\cos\Theta_\pi)$$

- Expressing them by a normalization $\mathcal{N}$, the scalar pion form factor F_S^π and the D-wave amplitude F_T^τ

$$\sqrt{4\pi}\left\langle Y_0^0 \right\rangle = X\sigma_\pi\sqrt{s}\left\{X^2\mathcal{N}^2|F_S^\pi|^2 + \sum_{\tau=0,\parallel,\perp}|F_T^\tau|^2\right\}$$

$$\sqrt{4\pi}\left\langle Y_2^0 \right\rangle = X\sigma_\pi\sqrt{s}\left\{2X\mathcal{N}\mathrm{Re}\left(F_S^\pi\left(F_T^0\right)^*\right) + \frac{\sqrt{5}}{7}\left(2\left|F_T^0\right|^2 + \sum_{\tau=\perp,\parallel}|F_T^\tau|^2\right)\right\}$$

- D-wave amplitudes F_T^τ are modeled for simplicity with Breit-Wigner functions of $f_2(1270)$ and $f_2'(1525)$

Application to the scalar pion form factor

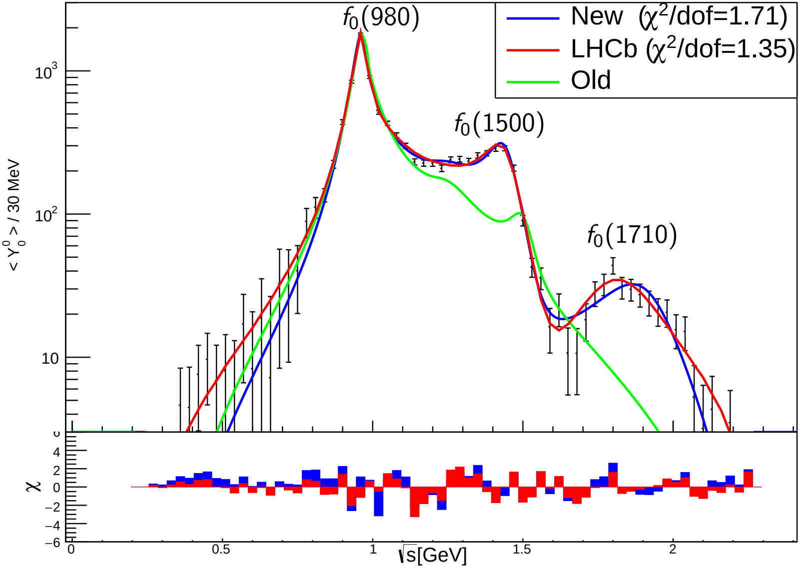
- Relevant channels for the process (2+1)
  1. $\pi\pi$ ($\sqrt{s_{th}} \approx 0.279\,\mathrm{GeV}$): Strongly coupled to $K\bar{K}$ via $f_0(980)$ resonance
  2. $K\bar{K}$ ($\sqrt{s_{th}} \approx 0.987\,\mathrm{GeV}$): Strongly coupled to $\pi\pi$ via $f_0(980)$ resonance
  3. $4\pi(\rho\rho)$ ($\sqrt{s_{th}} \approx 1.55\,\mathrm{GeV}$): Strongly phase space suppressed at low energies
- The relevant resonances at higher energies measured by LHCb [2014] are $f_0(1500)$ and $f_0(1710)$
- Elastic scattering matrix

$$T^0(s) = \begin{pmatrix} \frac{\eta(s)\exp(2i\delta(s))-1}{2i\sigma_\pi(s)} & g(s)\exp\left(i\psi(s)\right) & 0 \\ g(s)\exp\left(i\psi(s)\right) & \frac{\eta(s)\exp(2i(\psi(s)-\delta(s)))}{2i\sigma_K(s)} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

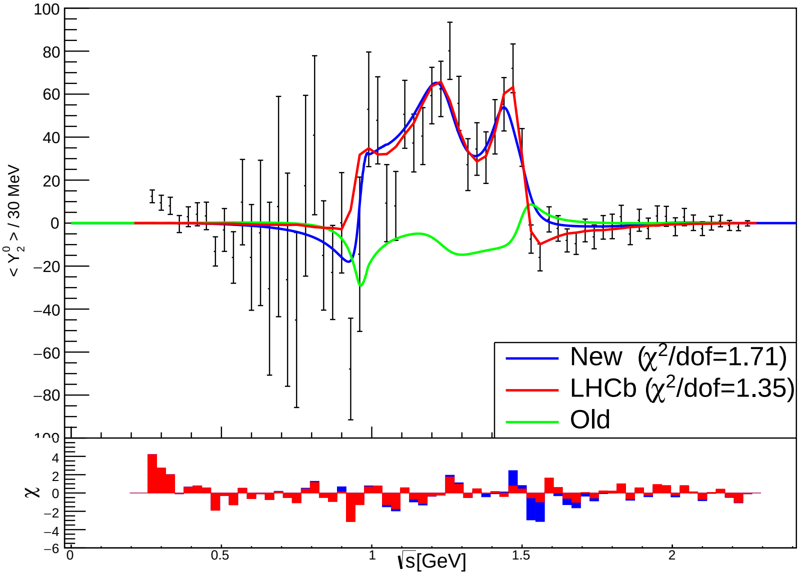
$$\eta(s) = \sqrt{1 - 4g^2(s)\sigma_\pi(s)\sigma_K(s)\Theta\left(s - 4M_K^2\right)}$$

- $\pi\pi$ scattering phase δ from Caprini et al. [2012]
- $\pi\pi \to K\bar{K}$ scattering amplitude g and phase ψ from Büttiker et al. [2004]

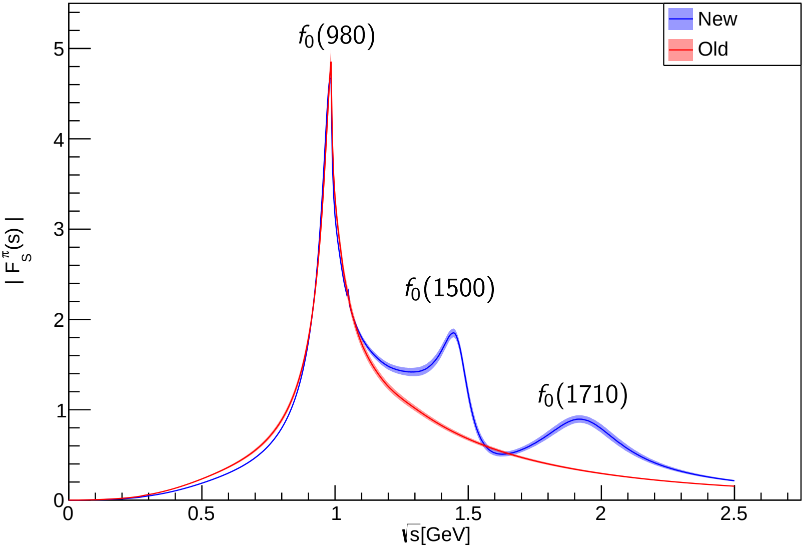
Application to the scalar pion form factor



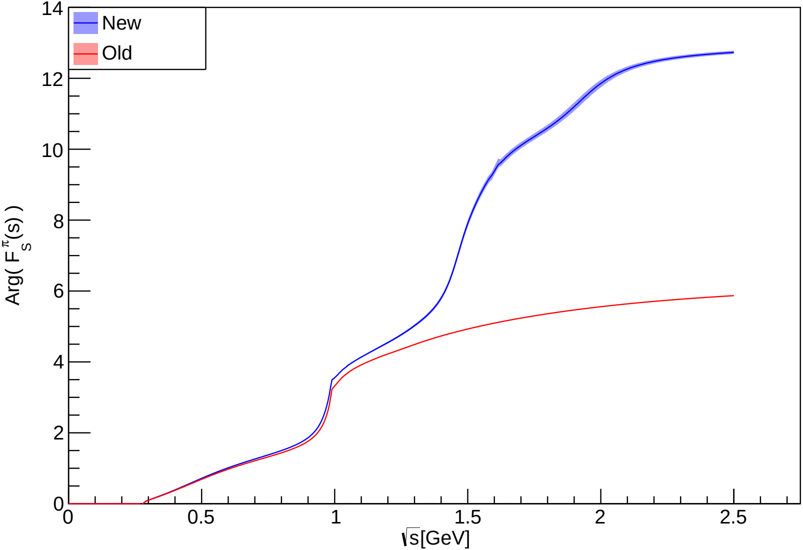
Application to the scalar pion form factor



Application to the scalar pion form factor



Application to the scalar pion form factor



Summary and outlook

Summary

- Were able to write down a new parametrization of the form factor with improved analyticity and unitarity properties
- Tested on the pion vector and scalar form factor
- Both give reasonable fit results
- Form factor phase at low energies is reproduced quite well
- Parametrization introduces various unknown parameters which have to be fitted
- Fitted parameters are process dependent

Outlook

- Estimate systematic error of this parametrization
- Infer information from the coupled channel system
- Extend the scattering matrix into the complex plane to search for resonance poles

Thank you for your attention!

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