Lattice Landau gauge gluon propagator at finite temperature

non-zero Matsubara frequencies and spectral densities

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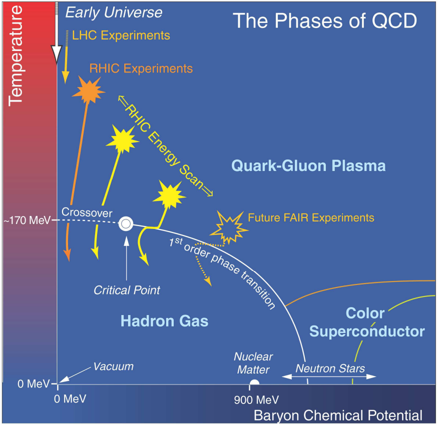
Outline

1. Introduction and Motivation
2. Gluon propagator @ finite T
   - How-to
   - Results
3. Spectral densities
   - Definitions and methods
   - Spectral densities @ finite T
4. Conclusions and outlook

QCD Phase Diagram

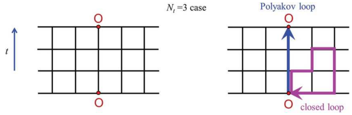


QCD Phase Diagram

- study of the phase diagram of QCD relevant e.g. for heavy ion experiments
- QCD has phase transition where quarks and gluons become deconfined for sufficiently high T

- Polyakov loop
  - order parameter
  - $L = \langle L(\vec{x}) \rangle \propto e^{-F_q/T}$
  - On the lattice:

$$L(\vec{x}) = \mathrm{Tr} \prod_{t=0}^{N_t-1} \mathcal{U}_4(\vec{x}, t)$$



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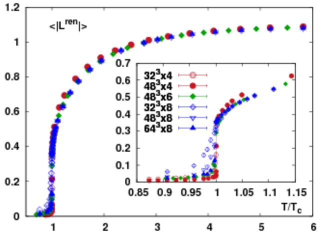

- $T < T_c$: $L = 0$ (center symmetry)
- $T > T_c$: $L \neq 0$ (spontaneous breaking)

Lattice QCD at finite temperature

- expectation values in a heat bath

$$\langle A \rangle = \frac{1}{Z} \mathrm{Tr} \left[e^{-\beta H} A \right]$$

- thermal Green's function for bosons:

$$\begin{aligned} G(x,y;\tau,0) &= Z^{-1}\mathrm{Tr}\left[e^{-\beta H}\hat{\phi}(x,\tau)\hat{\phi}(y,0)\right] \\ &= Z^{-1}\mathrm{Tr}\left[\hat{\phi}(y,0)e^{-\beta H}\hat{\phi}(x,\tau)\right] \\ &= Z^{-1}\mathrm{Tr}\left[e^{-\beta H}e^{\beta H}\hat{\phi}(y,0)e^{-\beta H}\hat{\phi}(x,\tau)\right] \\ &= Z^{-1}\mathrm{Tr}\left[e^{-\beta H}\mathcal{T}_\tau\left(\hat{\phi}(y,\beta)\hat{\phi}(x,\tau)\right)\right] \\ &= G(x,y;\tau,\beta) \end{aligned}$$

- temperature plays the role of imaginary time $T = \frac{1}{aL_t}$
- $\phi(y,0) = \phi(y,\beta) \Rightarrow$ Matsubara frequencies $\omega_n = 2\pi n T$

QCD Green's functions

- In a Quantum Field Theory, knowledge of all Green's functions allows a complete description of the theory
- In QCD, propagators of fundamental fields (e.g. quark, gluon and ghost propagators) encode information about non-perturbative phenomena
  - In particular, gluon propagator encodes information about confinement/deconfinement
- Since the gluon propagator is a gauge dependent quantity, we need to choose a gauge
  - in our works: Landau gauge $\partial_\mu A_\mu = 0$

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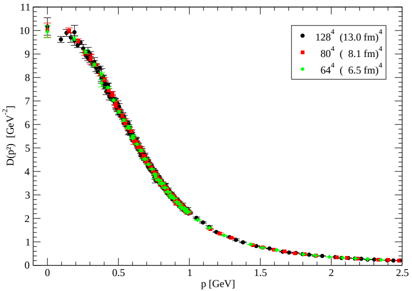
Gluon propagator at zero temperature

- Definition

$$D^{ab}_{\mu\nu}(\hat{q}) = \frac{1}{V}\langle A^a_\mu(\hat{q})A^b_\nu(-\hat{q})\rangle$$

- Tensor structure

$$D^{ab}_{\mu\nu}(q) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)D(q^2)$$



A. G. Duarte, O. Oliveira, PJS, Phys.Rev. D94 (2016) 014502

Gluon propagator at finite temperature

- Two components:
  - transverse D_T
  - longitudinal D_L

$$D^{ab}_{\mu\nu}(\hat{q}) = \delta^{ab}\left(P^T_{\mu\nu}D_T(q_4,\vec{q}) + P^L_{\mu\nu}D_L(q_4,\vec{q})\right)$$

Projectors are defined as

$$P^T_{\mu\nu} = (1-\delta_{\mu 4})(1-\delta_{\nu 4})\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{\vec{q}^2}\right)$$

$$P^L_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) - P^T_{\mu\nu}$$

Gluon propagator

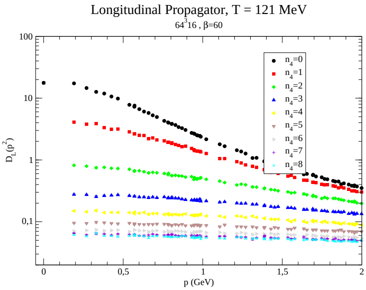
D_T and D_L extracted from the two equations below:

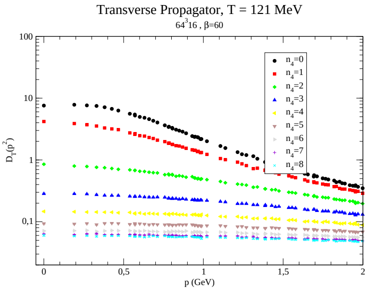
$$D^{aa}_{ii}(q) = \frac{2}{V}\left\langle \mathrm{Tr}\left[A_i(\hat{q})A^{\dagger}_i(\hat{q})\right]\right\rangle = \delta^{aa}\left(P^T_{ii}D^T + P^L_{ii}D^L\right)$$

$$D^{aa}_{44}(q) = \frac{2}{V}\left\langle \mathrm{Tr}\left[A_4(\hat{q})A^{\dagger}_4(\hat{q})\right]\right\rangle = \delta^{aa}\left(P^T_{44}D^T + P^L_{44}D^L\right)$$

Outline

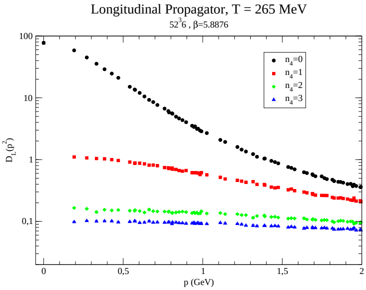
Gluon propagator at finite temperature

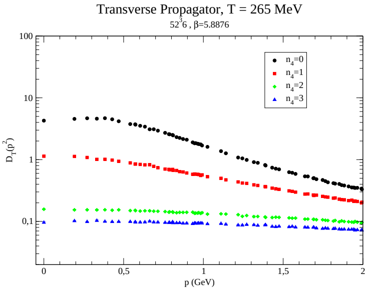




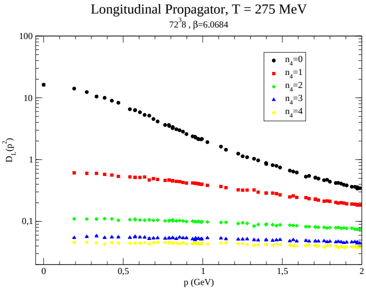
smaller D in the infrared $\rightarrow$ larger mass scales

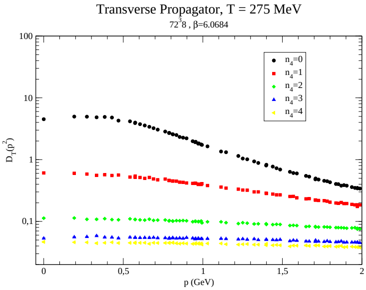
Gluon propagator at finite temperature



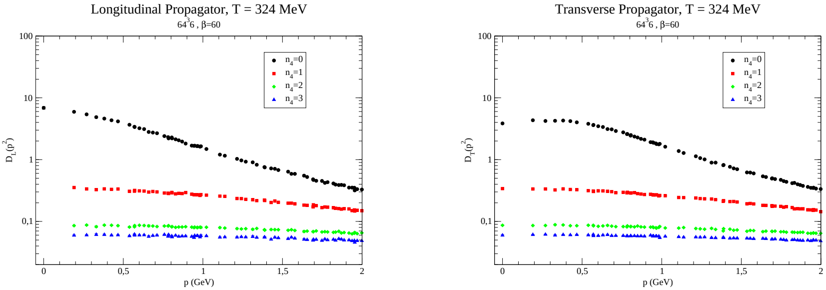


Gluon propagator at finite temperature

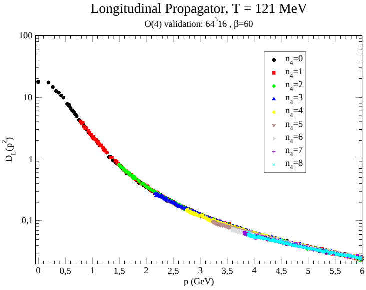


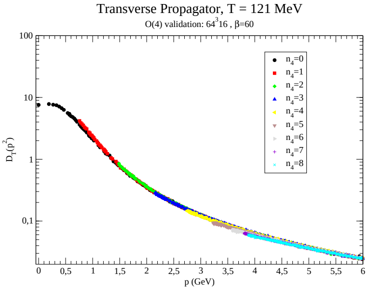


Gluon propagator at finite temperature



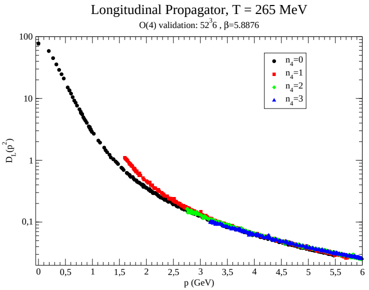
O(4) scaling

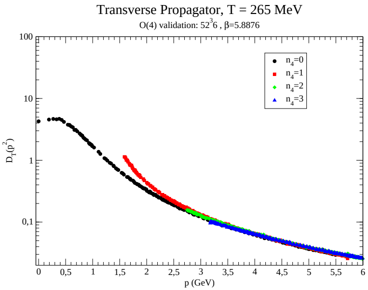




$$D(q_4, \vec{q}) = D(0, \sqrt{(}q_4^2 + \vec{q}^2))$$

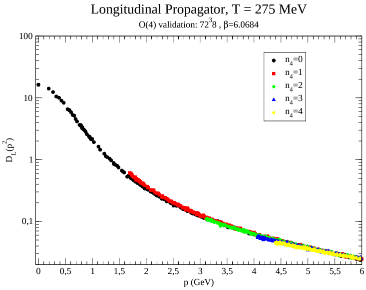
O(4) scaling

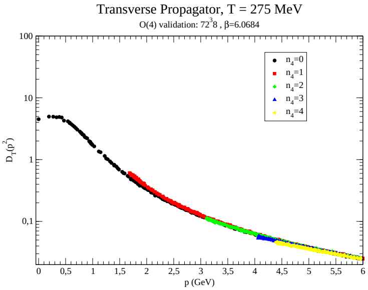




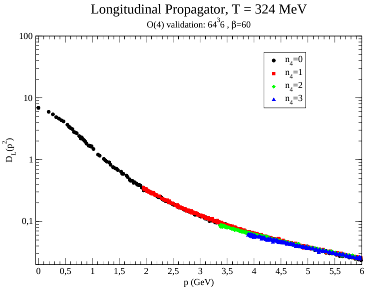
small violation for a few temperatures below T_c

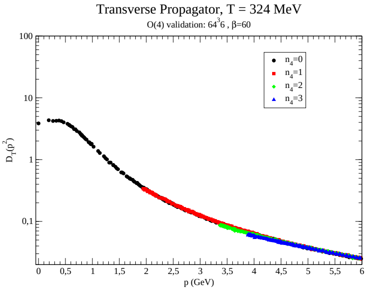
Gluon propagator at finite temperature



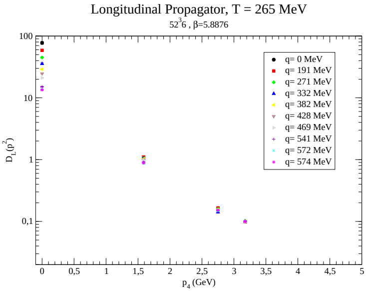


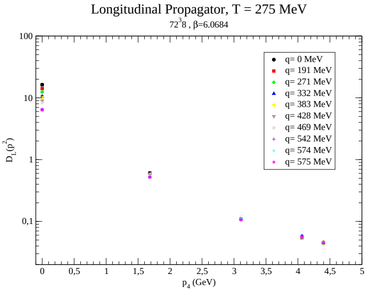
Gluon propagator at finite temperature





Dependence on q_4





Outline

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Spectral density

- Euclidean momentum-space propagator of a (scalar) physical degree of freedom

$$\mathcal{G}(p^2) \equiv \langle \mathcal{O}(p)\mathcal{O}(-p)\rangle$$

- Källén-Lehmann spectral representation

$$\mathcal{G}(p^2) = \int_0^{\infty} \mathrm{d}\mu \frac{\rho(\mu)}{p^2+\mu}\,, \qquad \text{with } \rho(\mu) \geq 0 \text{ for } \mu \geq 0\,.$$

- spectral density contains information on the masses of physical states described by the operator $\mathcal{O}$

$$\rho(\mu) = \sum_{\ell} \delta(\mu - m_\ell^2)\, |\langle 0|\mathcal{O}|\ell_0\rangle|^2\,,$$

Spectral density: motivation

- Main goal: compute the spectral density of gluons and other (un)physical degrees of freedom
  - important for e.g. DSE/BSE spectrum studies (Minkowski space)
  - spectral density is not strictly positive
  - traditional Maximum Entropy Method does not allow negative spectral densities

D. Dudal, O. Oliveira, PJS, PRD 89 (2014) 014010

Spectral density

- $\mathcal{G} = \mathcal{L}^2\hat{\rho} = \mathcal{L}\mathcal{L}^*\hat{\rho}$ where $(\mathcal{L}f)(t) \equiv \int_0^\infty \mathrm{d}s e^{-st} f(s)$ is a Laplace transform
- inversion of Laplace transform: ill-posed problem
- Way out: Tikhonov regularization
  - ill-posed problem $y = \mathcal{K}x$
  - minimize $||\mathcal{K}x - y|| + \lambda||x||^2$
    - $\lambda > 0$ is a regularization parameter
  - x^λ is the unique solution of the normal equation

$$\mathcal{K}^*\mathcal{K}x^\lambda + \lambda x^\lambda = \mathcal{K}^*y$$

  the operator $\mathcal{K}^*\mathcal{K} + \lambda$ is strictly positive, hence invertible
- Morozov discrepancy principle: choose $\overline{\lambda}$ s.t. $||\mathcal{K}x^{\overline{\lambda}} - y^\delta|| = \delta$
  - δ: "noise of input data"
  - A unique solution $x^{\overline{\lambda},\delta}$ exists

Getting gluon spectral density

$D = \mathcal{L}^2 \rho$

- setting $D_i \equiv D(p_i^2)$; N data points
- minimization of

$$\mathcal{J}_\lambda = \sum_{i=1}^{N} \left[\int_{\mu_0}^{+\infty} \mathrm{d}\mu \frac{\rho(\mu)}{p_i^2 + \mu} - D_i \right]^2 + \lambda \int_{\mu_0}^{+\infty} \mathrm{d}\mu \; \rho^2(\mu)$$

- linear perturbation of ρ: vanishing of

$$\sum_{i=1}^{N} \underbrace{\left[\int_{\mu_0}^{+\infty} \mathrm{d}\nu \frac{\rho(\nu)}{p_i^2 + \nu} - D_i \right]}_{\equiv c_i} \frac{1}{p_i^2 + \mu} + \lambda \rho(\mu) = 0 \;\; (\mu \geq \mu_0)$$

D. Dudal, O. Oliveira, PJS, PRD **89** (2014) 014010

Getting gluon spectral density

- Källén-Lehmann inverse given by

$$\rho_\lambda(\mu) = -\frac{1}{\lambda}\sum_{i=1}^{N}\frac{c_i}{p_i^2+\mu}\theta(\mu-\mu_0)\,,$$

- linear system for coefficients c_i: $\lambda^{-1}\mathcal{M}c + c = -D$

$$\mathcal{M}_{ij} = \int_{\mu_0}^{+\infty} \mathrm{d}\nu \frac{1}{p_i^2+\nu}\frac{1}{p_j^2+\nu} = \frac{\ln\frac{p_j^2+\mu_0}{p_i^2+\mu_0}}{p_j^2-p_i^2}\,.$$

Getting gluon spectral density

- Reconstructed propagator:

$$D^{reconstructed}(p^2) = \int_{\mu_0}^{+\infty} \mathrm{d}\mu \frac{\rho_\lambda(\mu)}{p^2+\mu} = -\frac{1}{\lambda}\sum_{i=1}^{N} \frac{c_i \ln \frac{p^2+\mu_0}{p_i^2+\mu_0}}{p^2-p_i^2} .$$

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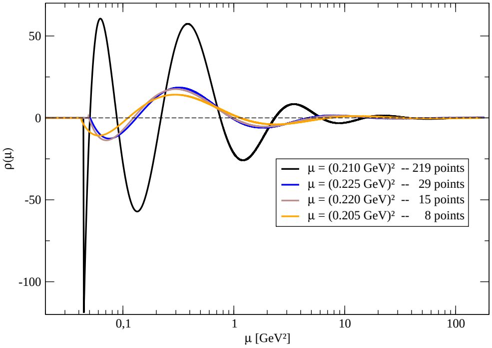
Spectral density at finite temperature

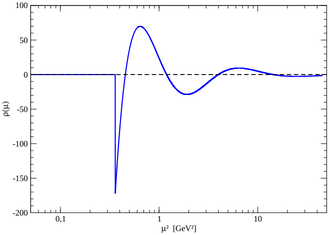
$$\mathcal{D}(q_4, \vec{q}) = \int_0^\infty \mathrm{d}\mu \frac{\rho(\mu, \vec{q})}{q_4^2 + \mu}$$

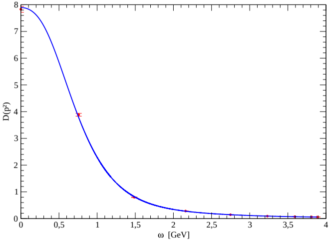
- Problem: small number of Matsubara frequencies
- How does the inversion look like when we consider just a few data points?
- Preliminary results just for $\vec{p} = (1, 0, 0)$

Spectral density — test T=0

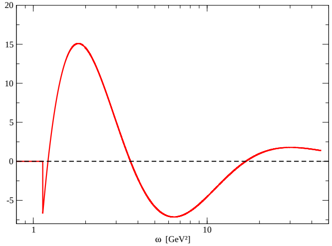


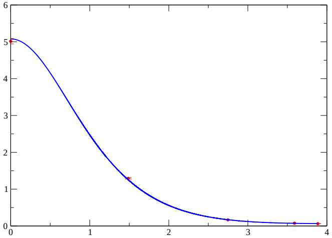
Tranverse component, T=121 MeV



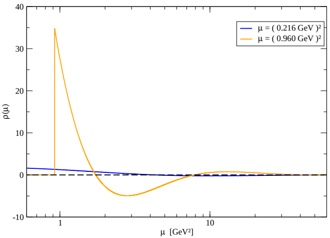


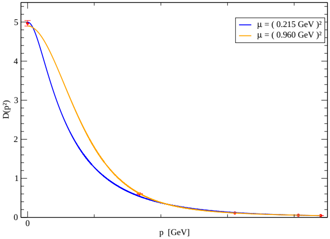
Transverse component, T=243 MeV



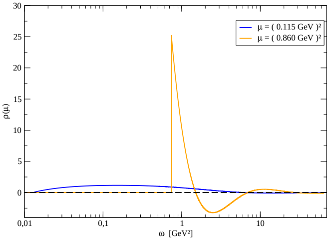


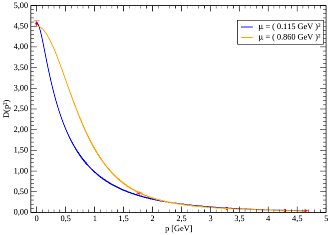
Transverse component, T=275 MeV



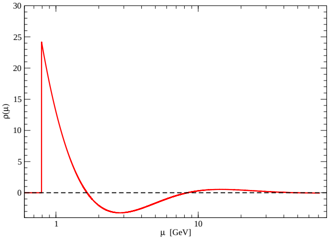


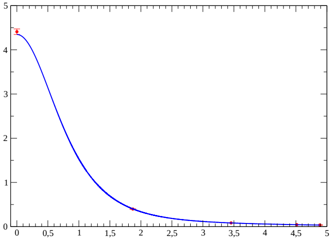
Transverse component, T=290 MeV



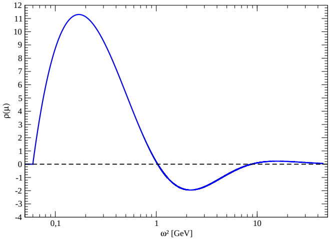


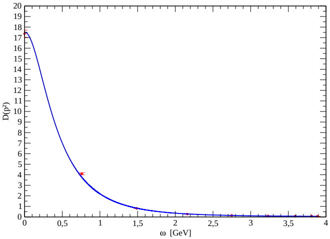
Transverse component, T=305 MeV



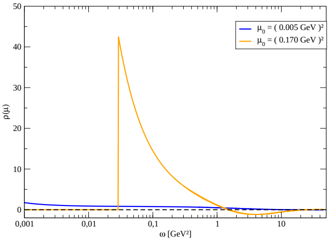


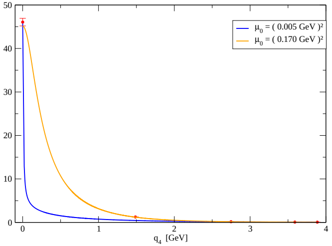
Longitudinal component, T=121 MeV



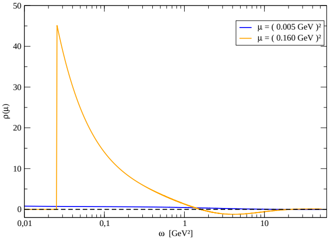


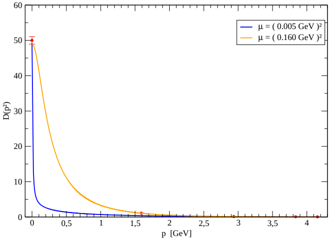
Longitudinal component, T=243 MeV



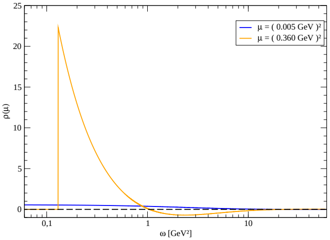


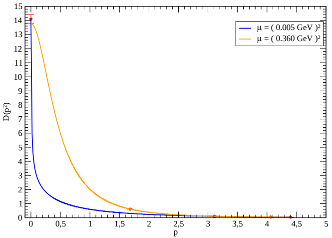
Longitudinal component, T=260 MeV



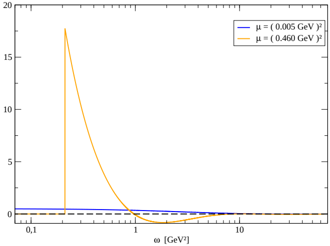


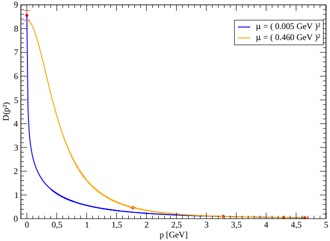
Longitudinal component, T=275 MeV



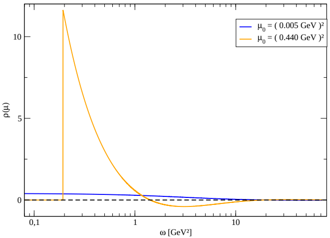


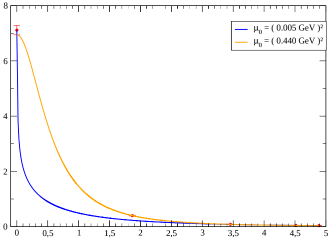
Longitudinal component, T=290 MeV



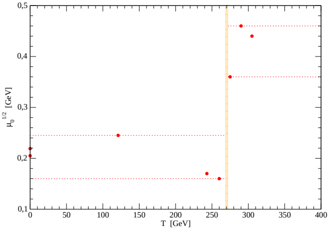


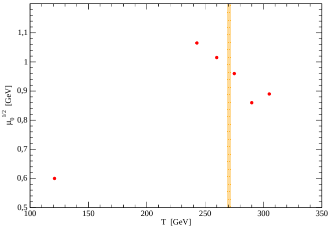
Longitudinal component, T=305 MeV





Infrared cut-offs





Interpretation as a mass scale?

Conclusions

- Very preliminary results for the spectral densities of the gluon propagator considering the non-zero Matsubara frequencies
- Results for the gluon propagator similar to recent results

Ilgenfritz, Pawlowski, Rothkopf, Trumm, arXiv:1701.08610

- Longitudinal spectral densities:
  - similar behaviour for all T — different from $T = 0$ case
  - positivity violation scale $\sim$ 1 GeV
- Transverse spectral densities:
  - $T < T_c$ — similar to $T = 0$ — pos. vio. scale $\sim$ 1.9 GeV
  - $T > T_c$ — similar to ρ_L — pos. vio. scale $\sim$ 1.4 GeV

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Outlook

- Other $\vec{p}$
- Other temperatures
- improve the inversion procedure?

Computing time provided by the Laboratory for Advanced Computing at the University of Coimbra. Supported by CFisUC. Work of P. J. Silva supported by FCT under Contracts No. SFRH/BPD/40998/2007 and SFRH/BPD/109971/2015.