ρ-MESON FORM FACTORS IN THE POINT FORM

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MOTIVATION

- structure of hadrons: **electromagnetic form factors** as the observables of elastic electron-hadron scattering

- simplest hadrons: $\pi^{\pm}$ and $\rho^{\pm}$ described as light $q\bar{q}$ systems

- experimental data on $\rho^{\pm}$ **form factors**:
  only little in the timelike region and **no data** in the spacelike region
  difficulty: very short life time of the ρ meson $\sim 10^{-24}s$

- knowledge of electroweak properties of the simplest hadrons important for understanding of stronly-interacting systems

THEORETICAL APPROACHES

direct QCD calculations only on lattice
various model approaches based on QFT or relativistic quantum mechanics:

- Dyson-Schwinger/Bethe-Salpeter
- quasi-potential approaches, e.g., **covariant spectator theory**
- QCD sum rules
- Nambu-Jona-Lasinio model
- hybrid model
- dynamical coupled-channel resonance models
- chiral effective field theory
- holographic approach
- light-front formalism
- constituent quark models based on instant, front, or **point form of relativistic Hamiltonian dynamics**

FORMS OF RELATIVISTIC DYNAMICS

[Dirac; Rev.Mod.Phys. 21, 1949]

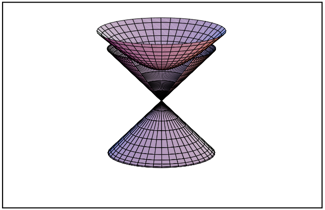
- Relativistic quantum theory for multiparticle systems:
  - Hilbert space
  - Poincaré invariance

  $\Leftrightarrow$ find representation of Poincaré generators as self-adjoint operators on Hilbert space such that the Poincaré algebra is satisfied

- free $\rightarrow$ interacting theory: include interactions in generators

- different **forms of relativistic dynamics**:
  e.g. $\left[K^j, P^k\right] = -i\delta^{jk}P^0$: if interactions in P^0 (Hamiltonian)
  $\Rightarrow$ interactions in either P^k (3-momentum operators) or/and K^j (boost generators)

- **point form**
  $\{P^0, P^k\} \equiv P^\mu$ interaction dependent
  $\{K^i, J^k\} \equiv M^{\mu\nu}$ free
  $\Rightarrow$ manifestly Lorentz covariant



BAKAMJIAN-THOMAS CONSTRUCTION

- Poincaré algebra imposes constraints on interaction terms

- systems with finite number of d.o.f: use **Bakamjian-Thomas construction** (BT)
- BT for point form: $P^\mu = P^\mu_{\text{free}} + P^\mu_{\text{int}} = (M_{\text{free}} + M_{\text{int}})V^\mu_{\text{free}} = MV^\mu_{\text{free}}$

  [Bakamjian, Thomas; PR 92, 1953]

  eigenvalue problem for BT-type **mass operator** M

  velocity-state basis: c.o.m. momenta $\mathbf{k}_i$ and overall velocity v^μ:

  $|v; \boldsymbol{k}_1, \mu_1; \ldots; \boldsymbol{k}_n, \mu_n\rangle := B_c(v)|\boldsymbol{k}_1, \mu_1; \ldots; \boldsymbol{k}_n, \mu_n\rangle$

  [Klink; PRC 58, 1998]

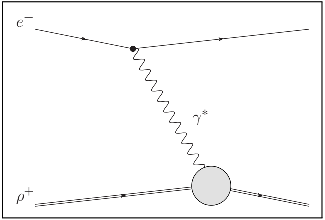
  REMARK:

- advantage: allows **instantaneous** interactions
- drawback: BT for more than 2 particles
  $\Rightarrow$ difficulties with **macroscopic locality** (cluster separability)

  [Sokolov; Theor.Math.Phys.36, 1979. Coester, Polyzou; PRD 26, 1982]

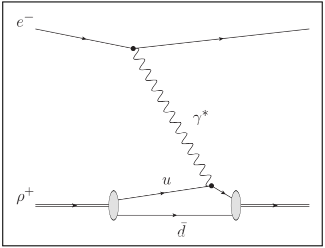
ELASTIC $e^-\rho^+$ SCATTERING

- calculate electromagnetic form factors from elastic $e^-\rho^+$-scattering in 1-γ-exchange approximation



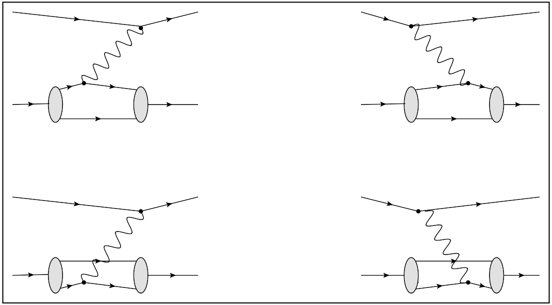
- constituent quark model: ρ^+ meson as bound state of a u and a $\bar{d}$ quarks

- extract the **microscopic ρ-meson current** J^μ from 1-γ-exchange optical potential



1-γ-EXCHANGE OPTICAL POTENTIAL

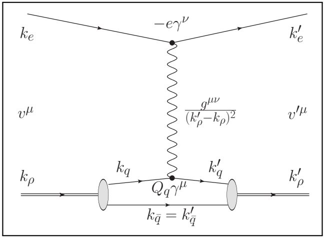
- treat $e^-\rho^+$ scattering as coupled-channel problem ($eq\bar{q}$ and $eq\bar{q}\gamma$) for BT-type mass operator:
$$\begin{pmatrix} M_{eq\bar{q}} + M_{\text{conf}} & K^\dagger \\ K & M_{eq\bar{q}\gamma} + M_{\text{conf}} \end{pmatrix} \begin{pmatrix} |\Psi_{e\rho}\rangle \\ |\Psi_{e\rho\gamma}\rangle \end{pmatrix} = m \begin{pmatrix} |\Psi_{e\rho}\rangle \\ |\Psi_{e\rho\gamma}\rangle \end{pmatrix}$$
confining interaction M_{conf} between the quarks
- Feshbach reduction to $eq\bar{q}$-channel
$\Rightarrow$ **optical potential** $V_{\text{opt}}(m) = K^\dagger(m - M_{eq\bar{q}\gamma} - M_{\text{conf}})^{-1}K$



ρ-MESON CURRENT

[EB, Fuchsberger, Klink, Schweiger; PRC 79, 2009; EB, Schweiger; PRC 89, 2014]

- on-shell matrix element of optical potential in velocity-state basis
- assumption: **total velocity conservation** at the interaction vertex:



$$\langle v'; \boldsymbol{k}'_e, \mu'_e; \boldsymbol{k}'_q, \mu'_q; \mu'_{\bar q}; \boldsymbol{k}'_\gamma, \mu'_\gamma | K | v; \boldsymbol{k}_e, \mu_e; \boldsymbol{k}_q, \mu_q; \mu_{\bar q}\rangle$$
$$\propto v^0 \delta^3(\boldsymbol{v} - \boldsymbol{v}') \langle \boldsymbol{k}'_e, \mu'_e; \boldsymbol{k}'_q, \mu'_q; \mu'_{\bar q}; \boldsymbol{k}'_\gamma, \mu'_\gamma | [\mathcal{L}^{q\gamma}(0) + \mathcal{L}^{e\gamma}(0)] | \boldsymbol{k}_e, \mu_e; \boldsymbol{k}_q, \mu_q; \mu_{\bar q}\rangle$$

[Klink; NPA 716, 2003]

- current-current interaction: $\left\langle v'; \boldsymbol{k}'_e, \mu'_e; n, \mu'_j, j \left| V_{\text{opt}}(m) \right| v; \boldsymbol{k}_e, \mu_e; n, \mu_j, j \right\rangle$
  $\propto v^0 \delta^3(\boldsymbol{v} - \boldsymbol{v}') J^\mu \left(\boldsymbol{k}'_\rho, \mu'_j; \boldsymbol{k}_\rho, \mu_j; k_e + k'_e\right) \frac{g_{\mu\nu}}{(k'_\rho - k_\rho)^2} |e| \bar{u}_{\mu'_e}(\boldsymbol{k}'_e) \gamma^\nu u_{\mu_e}(\boldsymbol{k}_e)$

  ρ-**meson electromagnetic current** J^μ: overlap integral of wave functions with quark currents, Wigner rotation and kinematical factors

  $J^\mu \left(\boldsymbol{k}'_\rho, \mu'_j; \boldsymbol{k}_\rho, \mu_j; k_e + k'_e\right)$
  $\propto \sum \int \mathrm{d}^3 \tilde{k}'_q \cdots \sqrt{\frac{m_{q\bar q}}{m'_{q\bar q}}} \Psi^*_{nj\mu'_j}\left(\tilde{\boldsymbol{k}}'_q\right) \mathcal{S}^{\frac{1}{2}}_{\mu_q \mu'_q} \bar{u}_{\mu'_q}\left(\boldsymbol{k}'_q\right) \gamma^\mu u_{\mu_q}\left(\boldsymbol{k}_q\right) \Psi_{nj\mu_j}\left(\tilde{\boldsymbol{k}}_q\right)$

CURRENT PROPERTIES

[EB, Schweiger; PRC 89, 2014]

- hermiticity: $J^\mu\left(\boldsymbol{k}'_\rho, \mu'_j; \boldsymbol{k}_\rho, \mu_j; K_e\right) = J^{*\mu}\left(\boldsymbol{k}_\rho, \mu_j; \boldsymbol{k}'_\rho, \mu'_j; k_e + k'_e\right)$

- covariance: $J^\mu\left(\boldsymbol{k}'_\rho, \mu'_j; \boldsymbol{k}_\rho, \mu_j; k_e + k'_e\right)$ (with e-ρ center-of-mass momenta) **does not** transform like a 4-vector under Lorentz transformations!
  $\rightarrow$ **physical** momenta $p_i^{(\prime)} = B_c(v)k_i^{(\prime)}$ and spins $\sigma_j^{(\prime)}$:

  $$I^\mu\left(\boldsymbol{p}'_\rho, \sigma'_j; \boldsymbol{p}_\rho, \sigma_j; p_e + p'_e\right) := B_c(v)^\mu_{\ \nu} J^\nu\left(\boldsymbol{k}'_\rho, \mu'_j; \boldsymbol{k}_\rho, \mu_j; k_e + k'_e\right) \times D^1_{\mu_j\sigma_j}(R_W^{-1}(B_c(v), k_\rho)) D^{1*}_{\mu'_j\sigma'_j}(R_W^{-1}(B_c(v), k'_\rho))$$

  transforms like a 4-vector

- current conservation conservation **cannot** be shown:
  $(k'_\rho - k_\rho)_\mu J^\mu\left(\boldsymbol{k}'_\rho, \mu'_j; \boldsymbol{k}_\rho, \mu_j; k_e + k'_e\right) \neq 0$

CLUSTER PROPERTIES

[EB, Schweiger; PRC 89, 2014]

- BT framework $\Rightarrow$ bound-state current J^μ does **not** satisfy cluster separability $\Rightarrow$ depends also on electron momenta k_e, k_e'
  1. current J^μ parametrized by 3 physical from factors f_1, f_2, g_M and additional **spurious** form factors $\{b_j\}$
  2. all form factors depend on the 2 Mandelstam variables of the process: $t \equiv q^2 = (k_\rho' - k_\rho)^2 = -Q^2$ and $s = (p_\rho + p_e)^2 \equiv (\sqrt{m_\rho^2 + \boldsymbol{k}^2} + \sqrt{m_e^2 + \boldsymbol{k}^2})^2$ does not spoil Poincaré invariance!

- identify **spurious (unphysical)** contribution as a structure proportional to $K_e \equiv k_e + k_e'$

COVARIANT STRUCTURE

[EB, Schweiger; PRC 89, 2014]

$J^\mu\left(\boldsymbol{k}'_\rho,\mu'_j;\boldsymbol{k}_\rho,\mu_j;K_e\right)$ has 11 independent matrix elements
$\Rightarrow$ usual 3 physical + **8 spurious** form factors:

$$\frac{1}{|e|}J^\mu\left(\boldsymbol{k}'_\rho,\mu'_j;\boldsymbol{k}_\rho,\mu_j;K_e\right)=\left[f_1\epsilon'^*\cdot\epsilon+f_2\frac{(\epsilon'^*\cdot q)(\epsilon\cdot q)}{2m_\rho^2}\right]K_\rho^\mu+g_M\left[\epsilon'^{*\mu}(\epsilon\cdot q)-\epsilon^\mu(\epsilon'^*\cdot q)\right]$$
$$+\frac{m_\rho^2}{2K_e\cdot k_\rho}\left[b_1(\epsilon'^*\cdot\epsilon)+b_2\frac{(q\cdot\epsilon'^*)(q\cdot\epsilon)}{m_\rho^2}+b_3m_\rho^2\frac{(K_e\cdot\epsilon'^*)(K_e\cdot\epsilon)}{(K_e\cdot k_\rho)^2}+b_4\frac{[(q\cdot\epsilon'^*)(K_e\cdot\epsilon)-(q\cdot\epsilon)(K_e\cdot\epsilon'^*)]}{2(K_e\cdot k_\rho)}\right]K_e{}^\mu$$
$$+\left[b_5m_\rho^2\frac{(K_e\cdot\epsilon'^*)(K_e\cdot\epsilon)}{(K_e\cdot k_\rho)^2}+b_6\frac{[(q\cdot\epsilon'^*)(K_e\cdot\epsilon)-(q\cdot\epsilon)(K_e\cdot\epsilon'^*)]}{2K_e\cdot k_\rho}\right]K_\rho^\mu$$
$$+b_7m_\rho^2\frac{[\epsilon'^{*\mu}(\epsilon\cdot K_e)+\epsilon^\mu(\epsilon'^*\cdot K_e)]}{K_e\cdot k_\rho}+b_8q^\mu\frac{[(q\cdot\epsilon'^*)(K_e\cdot\epsilon)+(q\cdot\epsilon)(K_e\cdot\epsilon'^*)]}{2K_e\cdot k_\rho}$$
$$K_\rho=k_\rho+k'_\rho,\ \epsilon^{(\prime)}\equiv\epsilon_{\mu_j^{(\prime)}}(\boldsymbol{k}_\rho^{(\prime)})$$

REMARKS:

1. only this decomposition gives **correct normalization** of charge form factor!
2. b_7 and b_8 contributions **violate** current conservation
3. K_e-dependence resembles ω-dependence in **covariant light-front dynamics**

   [Karmanov, Smirnov; NPA 575, 1994. Carbonell, Desplanques, Karmanov, Mathiot; Phys.Rept.300, 1998]

DEPENDENCE ON MANDELSTAM s

[EB, Schweiger; PRC 89, 2014]

- spurious s-dependence in $f_i(Q^2, s)$, $g_M(Q^2, s)$, and $b_i(Q^2, s)$ vanishes quickly with increasing s
  $\Rightarrow$ take limit $s \to \infty$: $F_i(Q^2) \equiv \lim_{s\to\infty} f_i(Q^2, s)$ and $B_i(Q^2) \equiv \lim_{s\to\infty} b_i(Q^2, s)$
- corresponds to $|\boldsymbol{k}_\rho| \to \infty$; means that subprocess $\gamma^* \rho^\pm \to \rho^\pm$ is considered in **infinite momentum frame**
- removes 4 spurious form factors: $B_1(Q^2) = B_2(Q^2) = B_3(Q^2) = B_4(Q^2) = 0$
- does **not** remove $B_5(Q^2), B_6(Q^2), B_7(Q^2)$ and $B_8(Q^2)$
  1. B_7, B_8: **violation** of current conservation
  2. B_6 **spoils** extraction of G_M
  3. $(B_5 + B_7)$: **violation** of angular condition for current matrix elements $J^0_{\mu_j'\mu_j}$:
     $(1+2\eta)J^0_{11} + J^0_{1-1} - 2\sqrt{2\eta}J^0_{10} - J^0_{00} = -(B_5 + B_7) \neq 0$
     $(\sqrt{\eta} = \frac{Q}{2m_\rho}$, kinematics: $q = (0, Q, 0, 0))$

PHYSICAL FORM FACTORS AND MODEL

[EB, Schweiger; PRC 89, 2014]

- **unambiguous prescription** to separate physical from spurious form factors:

  1. $F_1(Q^2) = -J^0_{11}(Q^2) - J^0_{1-1}(Q^2)$
  2. $F_2(Q^2) - \frac{1}{\eta} J^0_{1-1}(Q^2)$

  with overlap integrals $J^0_{\mu_j'\mu_j}(Q^2) = \frac{1}{4\pi}\int \mathrm{d}^3\tilde{k}_q' \sqrt{\frac{m_{q\bar{q}}}{m'_{q\bar{q}}}}\, u^*(\tilde{k}_q')u(\tilde{k}_q)\mathcal{S}^1_{\mu_j'\mu_j}$

  3. $G_M(Q^2) = -\frac{i}{Q} J^2_{11}(Q^2)$ with

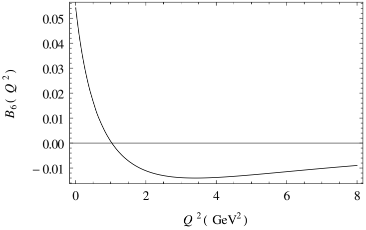
  $J^2_{\mu_j'\mu_j}(Q^2) = \frac{1}{4\pi}\int \mathrm{d}^3\tilde{k}_q' \sqrt{\frac{m_{q\bar{q}}}{m'_{q\bar{q}}}}\, u^*(\tilde{k}_q')u(\tilde{k}_q)\frac{m'_{q\bar{q}}}{(m'_{q\bar{q}}+2\tilde{k}_q'^3)}(\tilde{k}_q'^2\mathcal{S}^1_{\mu_j'\mu_j} + \frac{iQ}{2}\mathcal{S}^3_{\mu_j'\mu_j})$

- simple model for ρ-meson: confinement described by harmonic oscillator potential $\Rightarrow$ 2 parameters: constituent quark mass $m_q = 340$ MeV and oscillator length $a = 312$ MeV fixed to reproduce mass splitting between ground ($m_\rho = 770$ MeV) and 1st excited state

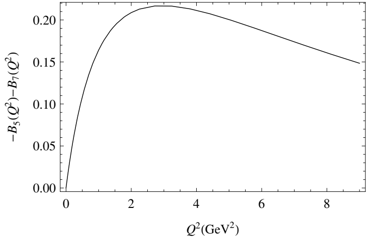
RESULTS: SPURIOUS CONTRIBUTIONS

[EB, Schweiger; PRC 89, 2014]

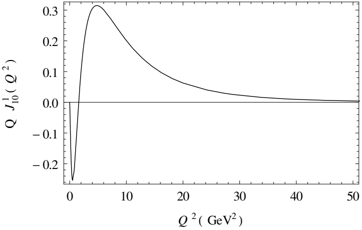
B_6 spoils G_M



violation of angular condition



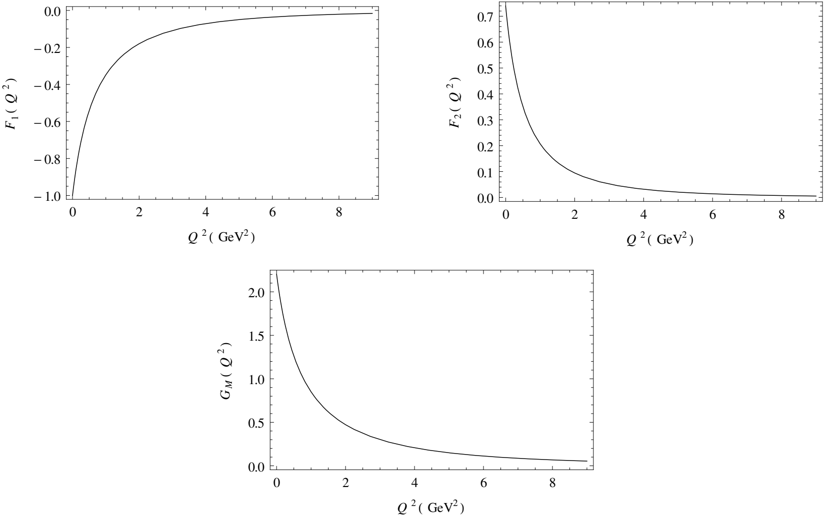
violation of current conservation



cannot be neglected!

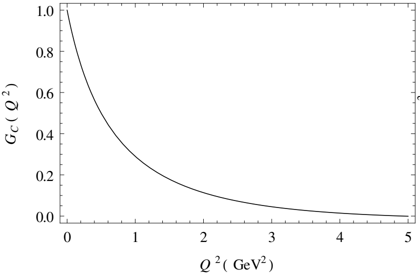
RESULTS FOR F_1, F_2, G_M

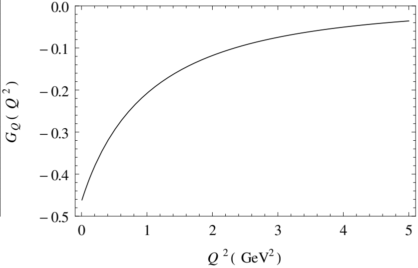
[EB, Schweiger; PRC 89, 2014]



RESULTS FOR G_C, G_M, G_Q AND COMPARISONS

[EB, Schweiger; PRC 89, 2014]





Ref.	m_q (MeV)	a (MeV)	μ_ρ ($\|e\|/2m_\rho$)	Q_ρ ($\|e\|/m_\rho^2$)
this work	340	312	2.20	-0.47
Choi et al. [CJ]	220	365.9	1.92	-0.43
this work	220	365.9	2.33	-0.33
Jaus [J]	250	280	1.83	-0.33
this work	250	280	2.25	-0.33
Carbonell et al. [CD]	250	262	**2.23**	**-0.005**
this work	250	262	**2.231**	**-0.0058**
Chung et al. [CC]	250	316	**2.23**	**-0.19**
this work	250	316	**2.27344**	**-0.253915**

References: CJ: Choi, Ji; PRD 70, 2004 J: Jaus; PRD 67, 2003 CD: Carbonell et al.; Phys.Rep.300, 1998
CC: Chung et al.; PRC 37, 1988

COMPARISONS OF MOMENTS WITH OTHER APPROACHES

[EB, Schweiger; PRC 89, 2014]

Ref.	μ_ρ	Q_ρ
this work	2.20	-0.47
Bagdasaryan et al. [BE]	2.30	-0.45
Samsonov [S]	2.00±0.3	-
Aliev et al. [AS]	2.30	-
Cardarelli et al. [C]	2.23	-0.61
Bhagwat et al. [BM]	2.01	-0.41
Hawes et al. [HP]	2.69	-0.84
De Melo et al. [MF]	2.14	-0.79

References:
BE: Bagdasaryan, Esaybegyan; Yad.Fiz.42, 1985
MF: de Melo, Frederico; PRC 55, 1997
C: Cardarelli et al.; PLB 349, 1995
S: Samsonov; JHEP 12, 2003
AS: Aliev, Savci; PRD 70, 2004
BO: Braguta, Onishchenko; PRD 70, 2004
BM: Bhagwat, Maris; PRC 77, 2008
HP: Hawes, Pichowski; PRC 59, 1999

taken from Carrillo-Serrano, Bentz, Cloet, Thomas, PRC 92, (2015):

Reference	$\langle r_C^2 \rangle$ (fm^2)	μ_ρ (μ_N)	Q_ρ (fm^2)
This work	0.67	3.14	−0.070
Garcia Gudiño [6]		2.6(6)	
Cardarelli [8]	0.35	2.76	−0.024
De Melo [9]	0.37	2.61	−0.052
Melikhov [11]	0.33	2.87	−0.031
Jaus [12]		2.23	−0.022
Choi [13]		2.34	−0.028
Biernat [14]		2.68	−0.027
Samsonov [16]		2.4(4)	
Aliev [18]		2.8(6)	
Hawes [19]	0.37	3.28	−0.055
Bhagwat [20]	0.54	2.54	−0.026
Roberts [21]	0.31	2.14	−0.037
Pitschmann [22]		2.13	
Owen [25]	0.670(68)	2.613(97)	−0.0452(61)
Shultz [26]	0.30(6)	2.00(9)	−0.020(4)

COMPARISONS OF G_C, G_M, G_Q

[EB, Schweiger; PRC 89, 2014]

same model wave function:

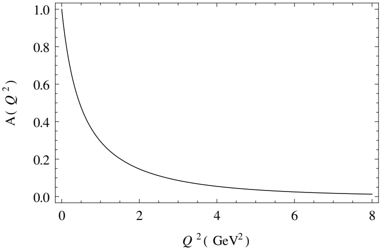
$Q^2(\mathrm{GeV}^2)$		Choi et al. [CJ]	this work
$Q^2 = 1$	G_C	0.38	0.29
	G_M	0.93	0.93
	G_Q	-0.23	-0.21
$Q^2 = 2$	G_C	0.18	0.12
	G_M	0.59	0.58
	G_Q	-0.15	-0.14
$Q^2 = 3$	G_C	0.08	0.05
	G_M	0.41	0.41
	G_Q	-0.10	-0.10

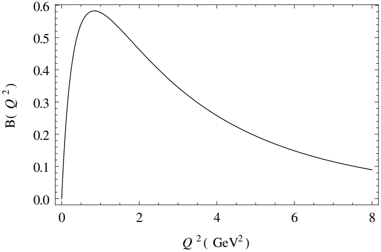
different wave functions:

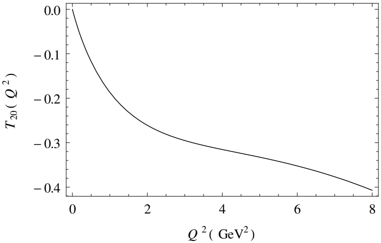
$Q^2(\mathrm{GeV}^2)$		this work	[BM]	[HP]	[AS]	[BO]
$Q^2 = 0$	G_C	1	1	1	1	-
	G_M	2.20	2.01	2.69	2.30	-
	G_Q	-0.47	-0.41	-0.84	-	-
$Q^2 = 1$	G_C	0.29	0.22	0.17	0.25	0.10
	G_M	0.85	0.57	0.85	0.58	0.46
	G_Q	-0.21	-0.11	-0.51	-0.49	-0.16
$Q^2 = 2$	G_C	0.11	0.08	0.04	0.13	0.16
	G_M	0.47	0.27	0.45	0.28	0.27
	G_Q	-0.12	-0.05	-0.32	-0.24	-0.11
$Q^2 = 3$	G_C	0.05		0.11	0.08	-0.03
	G_M	0.30		0.25	0.17	0.18
	G_Q	-0.07		-0.23	-0.15	-0.10

ELASTIC SCATTERING OBSERVABLES

[EB, Schweiger; PRC 89, 2014]







SUMMARY

- Point-form relativistic quantum mechanics: coupled-channel formalism to calculate electromagnetic form factors from physical process of electron-ρ-meson scattering

- derive ρ-meson current within Bakamjian-Thomas framework with

  1. correct covariance properties
  2. wrong cluster properties $\Rightarrow$ spurious contributions that cannot be neglected

- unambiguous procedure to separate physical from spurious contributions: infinite invariant mass limit
  $\Rightarrow$ extract 3 physical form factors
  $\Rightarrow$ physical ρ-meson current with all required properties

- other applications (Schweiger, Jung, Gomez-Rocha, Kupelwieser, Senekowitsch, EB)

  1. deuteron, pion, heavy-light meson form factors
  2. meson transition from factors
  3. nucleon form factors, pion-cloud effects
  4. baryon transition from factors, e.g. N-Δ transition form factors
  5. meson-baryon vertices

THANK YOU!