Predictions for $\eta_c \to \eta\pi^+\pi^-$ producing $f_0(500)$, $f_0(980)$ and $a_0(980)$

Vinícius Rodrigues Debastiani

Instituto de Física Corpuscular
Universidad de Valencia - CSIC

Excited QCD 2017
Sintra, Portugal

Outline

1. Introduction
   - Experimental measurements of $\chi_{c1} \to \eta\pi^+\pi^-$
   - Theoretical description of the experimental data
   - Predictions for the analogous reaction $\eta_c \to \eta\pi^+\pi^-$
   - Chiral unitary approach
2. The theoretical description of $\chi_{c1} \to \eta\pi^+\pi^-$
   - Introduction
   - Model - $SU(3)$ scalar
   - Model - Amplitudes
   - Model - Invariant masses
   - Results
3. Predictions for $\eta_c \to \eta\pi^+\pi^-$
   - Introduction
   - More $SU(3)$ scalars
   - 3-body kinematics
   - Results
4. Summary

Outline

1. Introduction
   - Experimental measurements of $\chi_{c1} \to \eta\pi^+\pi^-$
   - Theoretical description of the experimental data
   - Predictions for the analogous reaction $\eta_c \to \eta\pi^+\pi^-$
   - Chiral unitary approach
2. The theoretical description of $\chi_{c1} \to \eta\pi^+\pi^-$
   - Introduction
   - Model - $SU(3)$ scalar
   - Model - Amplitudes
   - Model - Invariant masses
   - Results
3. Predictions for $\eta_c \to \eta\pi^+\pi^-$
   - Introduction
   - More $SU(3)$ scalars
   - 3-body kinematics
   - Results
4. Summary

Motivation

Experimental measurements of the $\chi_{c1} \to \eta\pi^+\pi^-$ decay by BESIII Collaboration

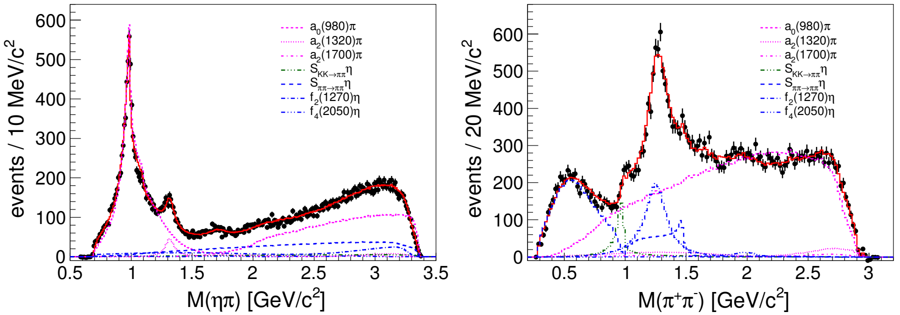


Figure: Projections in the $\eta\pi$ and $\pi^+\pi^-$ invariant mass from data, compared with the base-line fit (solid curve) and corresponding amplitudes (various dashed and dotted lines).

M. Ablikim *et al.* [BESIII Collaboration], "Amplitude analysis of the $\chi_{c1} \to \eta\pi^+\pi^-$ decays," Phys. Rev. D **95**, no. 3, 032002 (2017), arXiv:1610.02479 [hep-ex].

Motivation

Experimental measurements of the $\chi_{c1} \to \eta\pi^+\pi^-$ decay by BESIII Collaboration

- Most dominant two-body structure: $a_0(980)^\pm\pi^\mp$; $a_0(980)^\pm \to \eta\pi^\pm$.

- $f_0(500)\eta$, ($S_{\pi\pi\to\pi\pi}\eta$).
- $f_0(980)\eta$, ($S_{K\bar{K}\to\pi\pi}\eta$).

- $f_2(1270)\eta$ and $f_4(2050)\eta$.
- $a_2(1320)\pi$ and $a_2(1700)\pi$.
- Measurement of the $a_0(980)$ coupling to the $\eta'\pi$ channel.

- Investigation of the production of mesons with exotic quantum numbers $J^{PC} = 1^{-+}$. Upper limits for the branching fractions of $\chi_{c1} \to \pi_1(1400/1600/2015)^\pm\pi^\mp$; $\pi_1(X)^\pm \to \eta\pi^\mp$.

Outline

1. Introduction
   - Experimental measurements of $\chi_{c1} \to \eta\pi^+\pi^-$
   - Theoretical description of the experimental data
   - Predictions for the analogous reaction $\eta_c \to \eta\pi^+\pi^-$
   - Chiral unitary approach
2. The theoretical description of $\chi_{c1} \to \eta\pi^+\pi^-$
   - Introduction
   - Model - $SU(3)$ scalar
   - Model - Amplitudes
   - Model - Invariant masses
   - Results
3. Predictions for $\eta_c \to \eta\pi^+\pi^-$
   - Introduction
   - More $SU(3)$ scalars
   - 3-body kinematics
   - Results
4. Summary

Motivation

Theoretical description of the $f_0(500)$, $f_0(980)$, and $a_0(980)$ production in $\chi_{c1} \to \eta\pi^+\pi^-$ decay

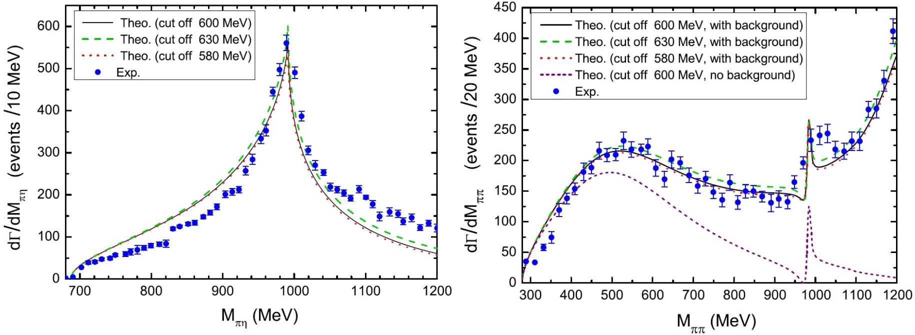


Figure: Results for the $\pi\eta$ (left) and $\pi\pi$ (right) mass distribution in the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction, producing $a_0(980)$ (left), $f_0(500)$ and $f_0(980)$ (right).

W. H. Liang, J. J. Xie and E. Oset, "$f_0(500)$, $f_0(980)$, and $a_0(980)$ production in the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction," Eur. Phys. J. C **76**, no. 12, 700 (2016), arXiv:1609.03864 [hep-ph].

Motivation

Theoretical description of the $f_0(500)$, $f_0(980)$, and $a_0(980)$ production in $\chi_{c1} \to \eta\pi^+\pi^-$ decay

- $SU(3)$ symmetry to see the weight of different trios of pseudoscalars produced in this decay.
- The scalar mesons $f_0(500)$, $f_0(980)$, and $a_0(980)$ are described as dynamically generated meson-meson molecules.
- Final state interaction of pseudoscalar mesons using chiral unitary approach in coupled channels.
- Good agreement with the experimental data.

Outline

1. Introduction
   - Experimental measurements of $\chi_{c1} \to \eta\pi^+\pi^-$
   - Theoretical description of the experimental data
   - Predictions for the analogous reaction $\eta_c \to \eta\pi^+\pi^-$
   - Chiral unitary approach
2. The theoretical description of $\chi_{c1} \to \eta\pi^+\pi^-$
   - Introduction
   - Model - $SU(3)$ scalar
   - Model - Amplitudes
   - Model - Invariant masses
   - Results
3. Predictions for $\eta_c \to \eta\pi^+\pi^-$
   - Introduction
   - More $SU(3)$ scalars
   - 3-body kinematics
   - Results
4. Summary

Predictions for the analogous reaction $\eta_c \to \eta\pi^+\pi^-$

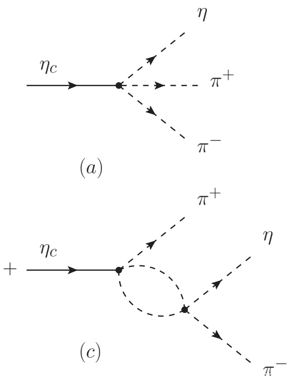


Figure: Diagrams involved in the $\eta_c \to \eta\pi^+\pi^-$ reaction including final state interaction of pairs of mesons.

V. R. Debastiani, W. H. Liang, J. J. Xie and E. Oset, "Predictions for $\eta_c \to \eta\pi^+\pi^-$ producing $f_0(500)$, $f_0(980)$ and $a_0(980)$," Phys. Lett. B **766**, 59 (2017), arXiv:1609.09201 [hep-ph].

Predictions for the analogous reaction $\eta_c \to \eta\pi^+\pi^-$

- S-wave *vs.* P-wave $\to$ 3-body kinematics $\to$ double differential mass distribution $\to$ background and interference.
- $SU(3)$ symmetry $\to$ more possibilities of $SU(3)$ scalars.
- Predictions $\to$ further test for the theory and the nature of the low mass scalar mesons.

Outline

Chiral unitary approach

We use an effective chiral Lagrangian where the pseudoscalar mesons are the degrees of freedom:

$$\mathcal{L}_2 = \frac{1}{12\, f_\pi^2} \mathrm{Trace}[\, (\partial_\mu \Phi\, \Phi - \Phi\, \partial_\mu \Phi)^2 + M \Phi^4 \,]\,, \tag{1}$$

$$\phi \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}, \tag{2}$$

$$M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2\, m_K^2 - m_\pi^2 \end{pmatrix}, \tag{3}$$

and we neglect the term η' which play only a marginal role in the building of the $f_0(500)$, $f_0(980)$, $a_0(980)$ resonances, because of its large mass and small couplings.

Chiral unitary approach

From this Lagrangian we extract the kernel of each channel which are then inserted into the Bethe-Salpeter equation, summing the contribution of every meson-meson loop.

$$t = (1 - v\, G)^{-1}\, v\,, \tag{4}$$

where G is the loop-function, which we regularize with a cut-off. In these works we use $q_{max} \sim 600$ MeV. After the integration in q^0 and $\cos\theta$ we have:

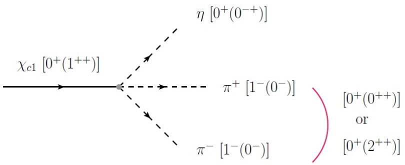
$$G = \int_0^{q_{max}} \frac{q^2 dq}{(2\pi)^2} \frac{\omega_1 + \omega_2}{\omega_1 \omega_2 [(P^0)^2 - (\omega_1 + \omega_2) + i\epsilon]}\,, \tag{5}$$

$$\omega_i = \sqrt{q^2 + m_i^2}, \quad P^0 = s \tag{6}$$

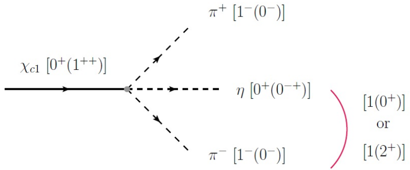
Each contribution is projected in S-wave and a normalization factor is included when identical particles are present.

The matrix T gives us the scattering amplitude and transitions between each channel, which in charge basis are: 1) $\pi^+\pi^-$, 2) $\pi^0\pi^0$, 3) K^+K^-, 4) $K^0\bar{K}^0$, 5) $\eta\eta$ and 6) $\pi\eta$.

Outline



(a) The $\pi^+\pi^-$ pair combines to $f_0(500)$, $f_0(980)$ and $f_2(1270)$.



(b) The $\eta\pi^-$ pair combines to $a_0(980)$ and $a_2(1320)$.

Figure: $\chi_{c1} \to \eta\pi^+\pi^-$ process with the quantum numbers of the mesons produced.

- (a) η leaves in P-wave while $\pi^+\pi^-$ go through final state interaction with $I = 0$ to form the $f_0(500)$, $f_0(980)$ in S-wave and $f_2(1270)$ in D-wave.
- (b) π^+ (or π^-) leaves in P-wave while $\eta\pi^+$ (or $\eta\pi^-$) go through final state interaction with $I = 1$ to form the $a_0(980)$ in S-wave and $a_2(1320)$ in D-wave.

Outline

We consider that the charmonium state $c\bar{c}$ behaves as a $SU(3)$ scalar, and use the ϕ matrix to get the weight of every trio of pseudoscalar meson:

$$\phi \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}, \quad (7)$$

$$SU(3)[\text{scalar}] \equiv \text{Trace}(\phi\phi\phi). \quad (8)$$

Selecting only the ones that can go to $\eta\pi^+\pi^-$ in the final state we isolate one of them and let the other pair interact:

$$C_1: \quad \eta\left(\frac{6}{\sqrt{3}}\pi^+\pi^- + \frac{3}{\sqrt{3}}\pi^0\pi^0 + \frac{1}{3\sqrt{3}}\eta\eta\right), \quad (9)$$

$$C_2: \quad \pi^+\left(\frac{6}{\sqrt{3}}\pi^-\eta + 3K^0K^-\right), \quad (10)$$

$$C_3: \quad \pi^-\left(\frac{6}{\sqrt{3}}\pi^+\eta + 3K^+\bar{K}^0\right). \quad (11)$$

Outline

The sum of each primary amplitude where one meson of the trio leaves in P-wave and the other pair interact in S-wave is

$$t = A\ \vec{\epsilon}_{\chi_{c1}} \cdot \vec{p}_\eta + B\ \vec{\epsilon}_{\chi_{c1}} \cdot \vec{p}_{\pi^+} + C\ \vec{\epsilon}_{\chi_{c1}} \cdot \vec{p}_{\pi^-}\,, \tag{12}$$

But the crossed terms in $|t|^2$ after averaging over the polarization of the χ_{c1} state go as

$$\begin{aligned}&\overline{\sum} 2\mathrm{Re}(AB^*)\vec{\epsilon}_{\chi_{c1}} \cdot \vec{p}_\eta\ \vec{\epsilon}_{\chi_{c1}} \cdot \vec{p}_{\pi^+} \\ =&2\mathrm{Re}(AB^*)\frac{1}{3}\delta_{ij}p_{\eta i}p_{\pi^+ j} = \frac{2}{3}\mathrm{Re}(AB^*)\vec{p}_\eta \cdot \vec{p}_{\pi^+}\,,\end{aligned} \tag{13}$$

which will vanish upon integration over angles in phase space.

Theoretical description of $\chi_{c1} \to \eta\pi^+\pi^-$

Model - Amplitudes

For instance, in the case where η leaves in P-wave and $\pi^+\pi^-$ interact, we shall consider the following diagrams:

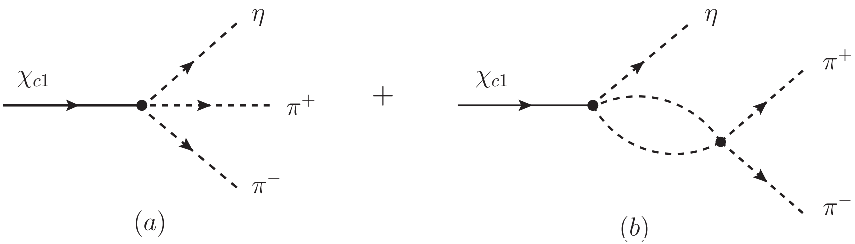


Figure: Production of $\eta\pi^+\pi^-$ through tree level (a) or rescattering (b) of $\pi^+\pi^-$ pair.

$$t_\eta = V_p \sum_i h_i S_i G_i(M_{\rm inv}(\pi^+\pi^-)) t_{i,\pi^+\pi^-}(M_{\rm inv}(\pi^+\pi^-)), \quad (14)$$

The factor V_P is the only unknown quantity in our approach, which provides a global normalization, and it is fitted to the data.

We will have

$$t_\eta = (\vec{\epsilon}_{\chi_{c1}} \cdot \vec{p}_\eta)\, \tilde{t}_\eta, \tag{15}$$

with

$$\tilde{t}_\eta = V_P \left(h_{\pi^+\pi^-} + \sum_i h_i S_i G_i(M_{\rm inv}) t_{i,\pi^+\pi^-} \right), \tag{16}$$

where

$$h_{\pi^+\pi^-} = \frac{6}{\sqrt{3}}, \qquad h_{\pi^0\pi^0} = \frac{3}{\sqrt{3}}, \qquad h_{\eta\eta} = \frac{1}{3\sqrt{3}} \tag{17}$$

are the weights of Eq. (9) and S_i are symmetry and combination factors for the identical particles,

$$S_{\pi^0\pi^0} = 2 \times \frac{1}{2} \;\; (\text{for two } \pi^0); \qquad S_{\eta\eta} = 3!\frac{1}{2} \;\; (\text{ for three } \eta). \tag{18}$$

Outline

With all these ingredients we can write the differential mass distribution for $\pi^+\pi^-$ as

$$\frac{d\Gamma}{dM_{\rm inv}(\pi\pi)} = \frac{1}{(2\pi)^3}\frac{1}{4M^2_{\chi_{c1}}}\frac{1}{3}p^2_\eta p_\eta \tilde{p}_\pi \left|\tilde{t}_\eta\right|^2 , \tag{19}$$

where p_η is the η momentum in the χ_{c1} rest frame

$$p_\eta = \frac{\lambda^{1/2}(M^2_{\chi_{c1}}, m^2_\eta, M^2_{\rm inv}(\pi\pi))}{2M_{\chi_{c1}}}, \tag{20}$$

and $\tilde{p}_\pi$ is the pion momentum in the $\pi^+\pi^-$ rest frame

$$\tilde{p}_\pi = \frac{\lambda^{1/2}(M^2_{\rm inv}(\pi\pi), m^2_\pi, m^2_\pi)}{2M_{\rm inv}(\pi\pi)}. \tag{21}$$

Outline

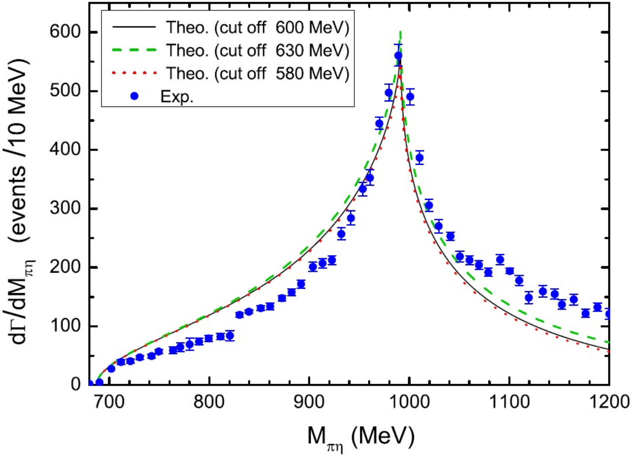


Figure: Results for the $\pi\eta$ mass distribution in the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction, producing $a_0(980)$.

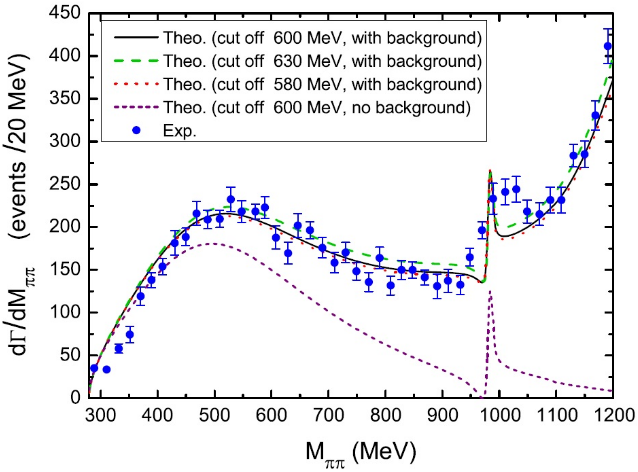


Figure: Results for the $\pi\pi$ mass distribution in the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction, producing $f_0(500)$ and $f_0(980)$. A linear background is fitted to the data.

Outline

Predictions for $\eta_c \to \eta\pi^+\pi^-$

In the analogous reaction $\eta_c \to \eta\pi^+\pi^-$ the dominant structure will be the one where every final state meson goes out in S-wave. Therefore one must consider the interference between each term in the amplitude

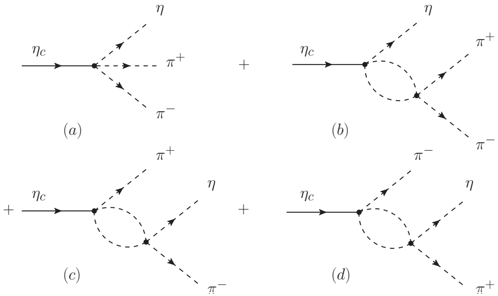


Figure: Diagrams involved in the $\eta_c \to \eta\pi^+\pi^-$ reaction including final state interaction of pairs of mesons.

$$t = t_{tree} + t_\eta + t_{\pi^+} + t_{\pi^-}, \qquad t_{tree} = V_p h_{\eta\pi^+\pi^-}. \tag{22}$$

Outline

There are three independent $SU(3)$ scalars that we can build with the ϕ matrix:

$$\mathrm{Trace}(\phi\phi\phi) = 2\sqrt{3}\eta\pi^+\pi^- + \sqrt{3}\eta\pi^0\pi^0 + \frac{\sqrt{3}}{9}\eta\eta\eta \\ + 3\pi^+K^0K^- + 3\pi^-K^+\bar{K}^0, \tag{23}$$

$$\mathrm{Trace}(\phi)\mathrm{Trace}(\phi\phi) = \frac{\sqrt{3}}{3}\,\eta\,(2\pi^+\pi^- + \pi^0\pi^0 + 2K^+K^- \\ + 2K^0\bar{K}^0 + \eta\eta), \tag{24}$$

$$[\mathrm{Trace}(\phi)]^3 = \frac{\sqrt{3}}{9}\,\eta\eta\eta. \tag{25}$$

Predictions for $\eta_c \to \eta\pi^+\pi^-$

More $SU(3)$ scalars

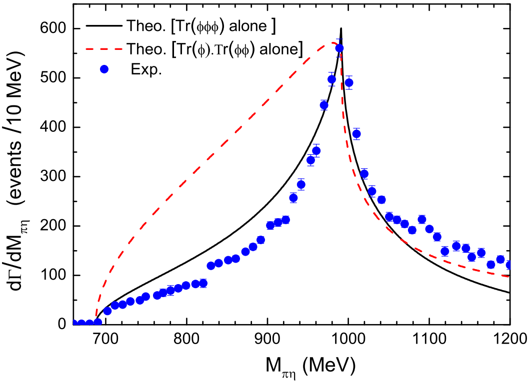


Figure: Results for the $\pi\eta$ mass distribution in the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction. Solid curve: results using $\mathrm{Trace}(\phi\phi\phi)$. Dashed line: results using $\mathrm{Trace}(\phi)\mathrm{Trace}(\phi\phi)$ normalized to the peak of the distribution.

Predictions for $\eta_c \to \eta\pi^+\pi^-$

More $SU(3)$ scalars

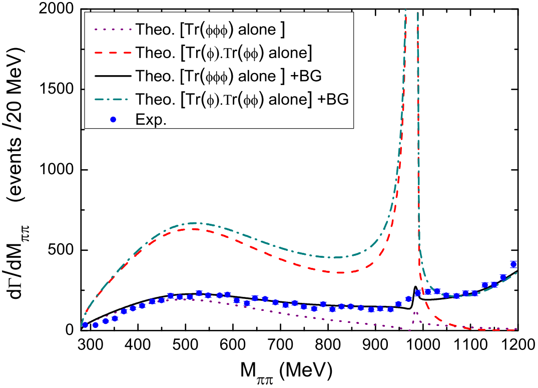


Figure: Results for the $\pi^+\pi^-$ mass distribution in the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction. Dotted and solid lines: results using $\mathrm{Trace}(\phi\phi\phi)$, with and without background contribution. Dash-dotted and dashed lines: results using $\mathrm{Trace}(\phi)\mathrm{Trace}(\phi\phi)$, with and without background.

Predictions for $\eta_c \to \eta\pi^+\pi^-$

More $SU(3)$ scalars

- The results have been normalized in both cases to the peak of the $\pi\eta$ invariant mass distribution.
- The shape for the case of $\mathrm{Trace}(\phi)\mathrm{Trace}(\phi\phi)$ is completely off from experiment.
- The strength of the $\pi^+\pi^-$ distribution is also much bigger than experiment and it produces a huge $f_0(980)$ peak.
- We have also tried different linear combinations of $\mathrm{Trace}(\phi\phi\phi)$, $\mathrm{Trace}(\phi)\mathrm{Trace}(\phi\phi)$ and $[\mathrm{Trace}(\phi)]^3$.
- It is clear that the most symmetrical term $\mathrm{Trace}(\phi\phi\phi)$ yields results in much better agreement with the data of the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction.
- In view of these results, we make predictions for the $\eta_c \to \eta\pi^+\pi^-$ with only the term $\mathrm{Trace}(\phi\phi\phi)$.

Outline

Each of the amplitude terms are functions of an invariant mass. We selected $M_{\rm inv}(\pi^+\pi^-)$ and $M_{\rm inv}(\eta\pi^+)$ as variables and the third one is determined by the relation:

$$M_{13}^2 = M_{\eta_c}^2 + 2m_\pi^2 + m_\eta^2 - M_{12}^2 - M_{23}^2. \tag{26}$$

It is also necessary to consider the double differential mass distribution

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{dM_{inv}}(\pi^+\pi^-)\mathrm{dM_{inv}}(\pi^+\eta)} = \frac{1}{(2\pi)^3}\frac{1}{8M_{\eta_c}^3} M_{\rm inv}(\pi^+\pi^-) M_{\rm inv}(\pi^+\eta)|t|^2. \tag{27}$$

where one should integrate in one of the invariant masses to get the other one. This way the background of $\pi^+\eta$ appears naturally in the $\pi^+\pi^-$ mass distribution and vice-versa.

For instance, to get $\frac{\mathrm{d}\Gamma}{\mathrm{dM_{inv}}(\pi^+\eta)}$ the integration limits are:

$$\begin{aligned}(M_{23}^2)_{max} &= (E_2^* + E_3^*)^2 \\ &\quad - \left(\sqrt{{E_2^*}^2 - m_2^2} - \sqrt{{E_3^*}^2 - m_3^2}\ \right)^2, \\ (M_{23}^2)_{min} &= (E_2^* + E_3^*)^2 \\ &\quad - \left(\sqrt{{E_2^*}^2 - m_2^2} + \sqrt{{E_3^*}^2 - m_3^2}\ \right)^2,\end{aligned} \tag{28}$$

where we have labeled 1, 2, 3 to $\eta\pi^+\pi^-$, respectively. The energies E_2^* and E_3^* are:

$$\begin{aligned}E_2^* &= (M_{12}^2 - m_1^2 + m_2^2)/2M_{12}, \\ E_3^* &= (M_{\eta_c}^2 - M_{12}^2 - m_3^2)/2M_{12}.\end{aligned} \tag{29}$$

Predictions for $\eta_c \to \eta\pi^+\pi^-$

3-body kinematics

The effective model we use is not valid in energies higher than ~ 1.2 GeV, so we need to perform a smooth cut in the amplitudes to perform the integration into the double differential mass distributions:

We evaluate $Gt(M_{\rm inv})$ combinations up to $M_{\rm inv} = M_{\rm cut}$. From there on, we multiply Gt by a smooth factor to make it gradually decrease at large $M_{\rm inv}$.

$$Gt(M_{\rm inv}) = Gt(M_{\rm cut}) \mathrm{e}^{-\alpha(M_{\rm inv} - M_{\rm cut})}, \quad \text{for} \quad M_{\rm inv} > M_{\rm cut}. \tag{30}$$

We take the value $M_{\rm cut} = 1100$ MeV, with $\alpha = 0.0037$ MeV^{-1}, 0.0054 MeV^{-1} and 0.0077 MeV^{-1}, which reduce Gt by about a factor 3, 5 and 10, respectively, at $M_{\rm cut} + 300$ MeV.

Outline

1. Introduction
   - Experimental measurements of $\chi_{c1} \to \eta\pi^+\pi^-$
   - Theoretical description of the experimental data
   - Predictions for the analogous reaction $\eta_c \to \eta\pi^+\pi^-$
   - Chiral unitary approach
2. The theoretical description of $\chi_{c1} \to \eta\pi^+\pi^-$
   - Introduction
   - Model - $SU(3)$ scalar
   - Model - Amplitudes
   - Model - Invariant masses
   - Results
3. Predictions for $\eta_c \to \eta\pi^+\pi^-$
   - Introduction
   - More $SU(3)$ scalars
   - 3-body kinematics
   - Results
4. Summary

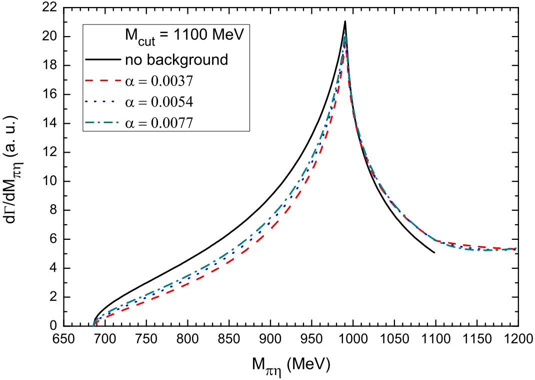


Figure: $\frac{\mathrm{d}\Gamma}{\mathrm{dM}_{\pi\eta}}$ as a function of $M_{\pi\eta}$ for $M_{\mathrm{cut}} = 1100$ MeV and three different values of α.

We also show the results using the single mass distribution (solid curve) denoted by "no background", used in the $\chi_{c1} \to \eta\pi^+\pi^-$ theoretical work.

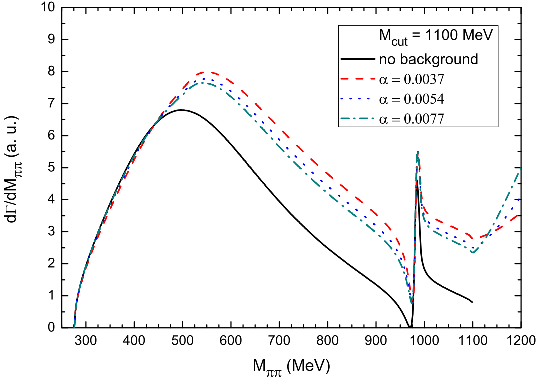


Figure: $\frac{\mathrm{d}\Gamma}{\mathrm{dM}_{\pi\pi}}$ as a function of $M_{\pi\pi}$ and for $M_{\mathrm{cut}} = 1100$ MeV and three different values of α.

One can see that in both mass distribution the background introduced goes in the direction where there was a small discrepancy with the experimental data.

Summary

- The experimental measurements of the $\chi_{c1} \to \eta\pi^+\pi^-$ decay provides data where the $f_0(500)$, $f_0(980)$ and specially the $a_0(980)$ can be seen clearly.
- The theoretical description of the these scalar mesons shows a successful and simple explanation of the experimental data up to 1.2 GeV.
- This fact reinforces the valid of the model and the picture of these states as dynamically generate meson-meson molecules in coupled channels.
- The predictions for the $\eta_c \to \eta\pi^+\pi^-$ decay provides further tests for the theory and the picture of these states. The implementation of the experiment and comparison with the theoretical work would be useful to the discussion of the nature of these scalar mesons.