Center Vortices and Topological Charge

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in coop. with Derar Altarawneh, Michael Engelhardt Manfried Faber, Jeff Greensite & Urs M. Heller

Center Vortices

➜ *'t Hooft 1979, Nielsen, Ambjorn, Olesen, Cornwall, 1979*
Mack, 1980; Feynman, 1981

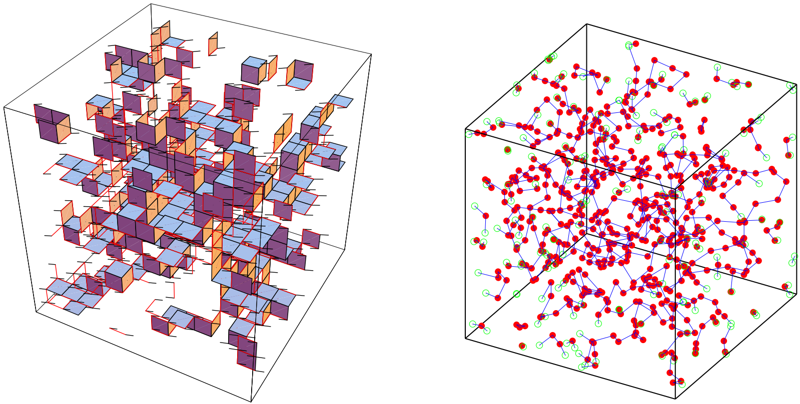
- QCD vacuum is a condensate of closed magnetic flux-lines, they have topology of tubes (3D) or surfaces (4D),
- magnetic flux corresponds to the center of the group,
- Vortex picture successfully explains ...
  - **Confinement** $\rightarrow$ piercing of Wilson loop $\equiv$ crossing of static electric flux tube and moving closed magnetic flux
  - **Topological charge**: intersections, writhings, color structure

    ➜ *Engelhardt, Reinhardt (2000), Jordan, Höllwieser, Faber, Heller (2007), Höllwieser, Engelhardt (2015)*
  - **Spontaneous chiral symmetry breaking**: vortex-only (center-projected) configurations show χSB, ILM mechanism

➜ *Forcrand, Elia (1999), Höllwieser, Faber, Greensite, Heller, Olejnik (2008), Schweigler, Höllwieser, Faber, Heller (2012,2013)*

Vortex Vacuum in SU(2)

Random Structure, Percolation Transition



3-dimensional cut through the dual of a 12^4-lattice after maximal center gauge and center projection

Wilson loops

- closed loops around rectangular $(R \times T)$, planar contour C

$$\text{Area law:} \qquad W(R,T) = \langle \prod_{x \in C} U_\mu(x) \rangle \to e^{-\sigma R T}$$

$$\text{Perimeter law:} \qquad W(R,T) = \langle \prod_{x \in C} U_\mu(x) \rangle \to e^{-\alpha(R+T)}$$

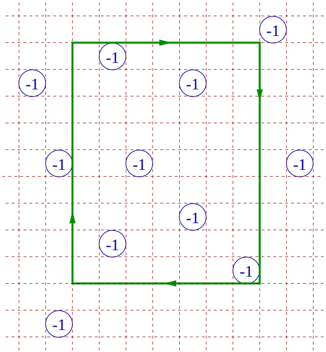
- quark-antiquark "test-pair"
- heavy quark potential in limit $T \to \infty$

$$\text{Area law:} \quad V(R) = -\lim_{T \to \infty} \frac{1}{T} \ln \langle W(R,T) \rangle \to -\sigma R$$

- Area law $\to$ Confinement
- $\sigma \ldots$ string tension $\to$ creutz ratio χ

$$\chi = \frac{W(R+1,T+1)W(R,T)}{W(R+1,T)W(R,T+1)} \to e^{-\sigma} \Rightarrow \sigma = -\ln \chi$$

Area law for center projected loops



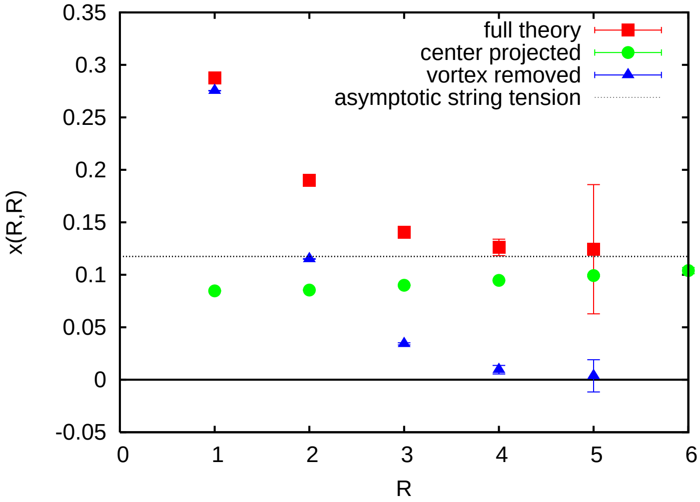
denote f the probability that a plaquette has the value -1

$$\langle W(A)\rangle = [f\cdot(-1)+(1-f)\cdot 1]^A = \exp[\underbrace{\ln(1-2f)}_{-\sigma}A], =$$

$$= \exp[-\sigma R\times T], \qquad \sigma \equiv -\ln(1-2f)\approx 2f$$

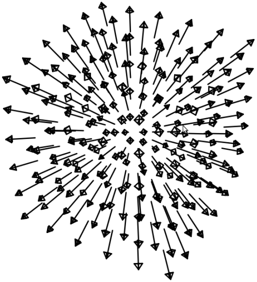
Center Dominance and Precocious Linearity

Creutz ratios: $\chi(I,J) = \frac{W(I,J)\,W(I-1,J-1)}{W(I-1,J)\,W(I,J-1)} \to \sigma$



Topological charge Q

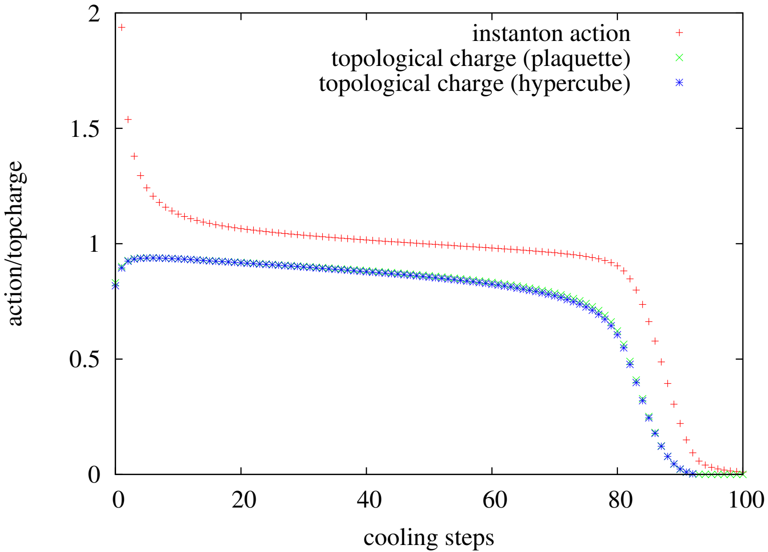
- QCD-vacua characterised by winding number n_w
- scalar (gauge) function: $g(x) = \mathrm{e}^{-\mathrm{i}\vec{\alpha}(x)\vec{\sigma}} \in$ SU(2)$\simeq$ S^3
- R^3 $\to$ S^3: $n_w = -\frac{1}{4\pi^2}\int_{\mathrm{S}^3} \mathrm{d}^3x \mathrm{Tr}(\partial_i g g^\dagger \partial_j g g^\dagger \partial_k g g^\dagger)$
  - hedgehog $g(x_\mu) = \frac{x_0 + \vec{x}\vec{\sigma}}{\sqrt{x_0^2 + \vec{x}^2}}$
  - $\mathrm{i}\partial_\mu g g^\dagger = \mathcal{A}_\mu = \frac{\vec{\sigma}}{2}\vec{A}_\mu$
  - $F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu - \varepsilon_{ijk} A^j_\mu A^k_\nu = 0$
  - apply profile or set boundary links trivial



➜ *Creutz (2010)*

- Topological charge $Q = \frac{1}{32\pi^2}\int \mathrm{d}^4x\, F^i_{\mu\nu}\tilde{F}^i_{\mu\nu} = \frac{1}{4\pi^2}\vec{E}\vec{B}$
- Lattice: $F_{\mu\nu} = \frac{1}{2\mathrm{i}}(U_{\mu\nu} - U^\dagger_{\mu\nu})$

Cooling



instanton action
topological charge (plaquette)
topological charge (hypercube)
action/topcharge
cooling steps
2
1.5
1
0.5
0
0
20
40
60
80
100

Atiyah-Singer index theorem

- zero-modes of fermionic matrix: $D[A]\psi(x) = 0$
- ψ has definite chirality:

$$\psi_{\substack{R\\L}} = \frac{1}{2}(1 \pm \gamma_5)\psi, \quad \Rightarrow \quad \gamma_5\psi_{\substack{R\\L}} = \pm\psi_{\substack{R\\L}}$$

- Index theorem (Wilson, overlap fermions):

n_-, n_+: number of left-/right-handed zeromodes

$$\mathrm{ind} D[A] = n_- - n_+ = Q[A]$$

- (Asqtad) staggered fermions:

$$\mathrm{ind}\ D[A] = 2Q[A] \text{ (SU(2), double degeneracy)}$$

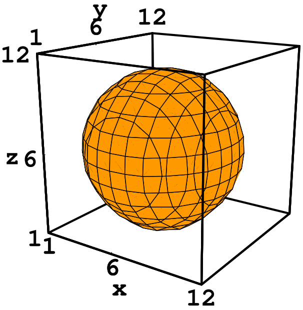
- Adjoint overlap fermions:

$$\mathrm{ind}\ D[A] = 2NQ[A] = 4Q[A] \text{ (real representation)}$$

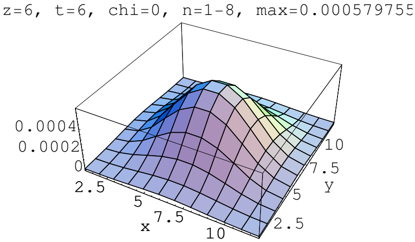
➜ *Neuberger, Fukaya (1999)*

Dirac zeromodes

topological charge distribution



scalar eigenmode density



The instanton attracts Dirac zeromodes.

Vortex topological charge Q

P-Vortices: closed surfaces of quantised flux

$$d^2\sigma_{\mu\nu} = \epsilon_{ab}\frac{\partial \bar{x}_\mu}{\partial \sigma_a}\frac{\partial \bar{x}_\nu}{\partial \sigma_b} d^2\sigma$$

$Q =$ Topological winding number
$Q =$ Self intersection number

➜ *Engelhardt, Reinhardt (2000)*

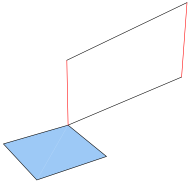
$$Q = -\frac{1}{16}\epsilon_{\mu\nu\alpha\beta}\int_S d^2\sigma_{\alpha\beta}\int_S d^2\sigma'_{\mu\nu}\delta^4(\bar{x}(\sigma) - \bar{x}(\sigma'))$$

one intersection contributes $\pm\frac{1}{2}$
Specify surface orientation !

Contributions to topological charge Q

1 contribution

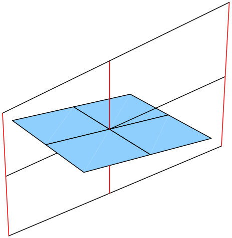
$Q = \pm\frac{1}{32}$



vortex intersection

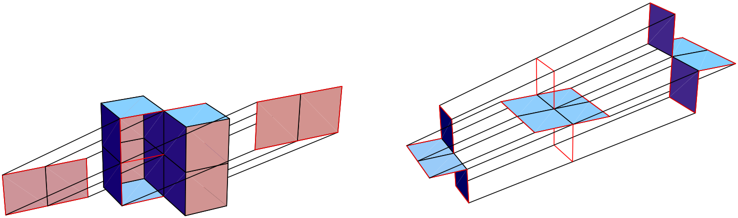
$4 \cdot 4 = 16$ contributions

$Q = \pm\frac{1}{2}$



➜ *Engelhardt, Reinhardt (2000)*

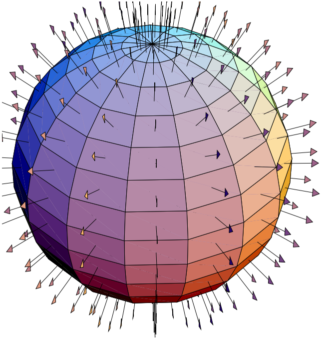
Writhing points

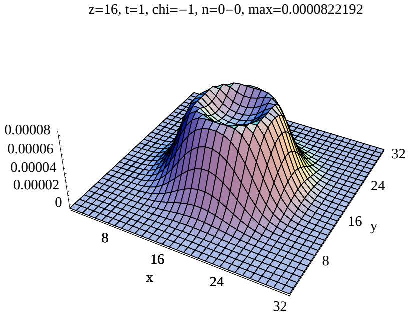


contributions to topological charge

- intersections
- writhing points
- color structure

Thick Spherical SU(2)-Vortices



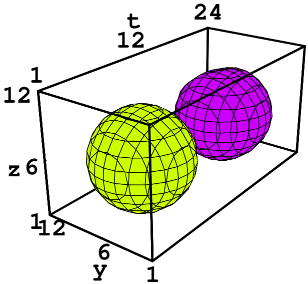


$$U_\mu(\vec{r}) = \begin{cases} \exp\{\mathrm{i}\alpha(r)\frac{\vec{r}}{r}\vec{\sigma}\}, & t=1, \mu=4 \\ 1 & \text{else} \end{cases}$$

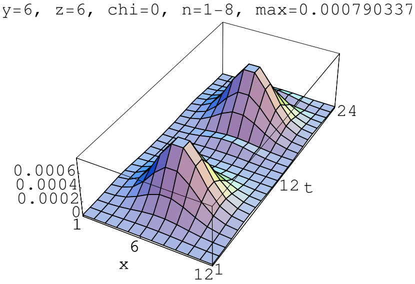
Spherical vortices attract zero modes just like instantons

Instanton/vortex ensembles and near-zeromodes

instanton antiinstanton pair
vortex antivortex pair



lowest eigenmodes density



Near-zeromodes produce a finite chiral condensate $\Rightarrow$ SCSB.

Chiral Symmetry Breaking

- parity acting on a Dirac fermion is called **chiral symmetry**
- two chiralities of quarkfield ψ

$$\psi_{\substack{L\\R}} = \frac{1}{2}(1 \pm \gamma_5)\psi, \quad \bar{\psi}_{\substack{L\\R}} = \psi^{\dagger}_{\substack{L\\R}}\gamma_0 = \bar{\psi}\frac{1}{2}(1 \mp \gamma_5)$$

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3, \quad \gamma_5^2 = 1, \quad \gamma_5^{\dagger} = \gamma_5, \quad \{\gamma_\mu, \gamma_5\} = 0$$

- chiral projection $P_{\substack{L\\R}} = \frac{1}{2}(1 \mp \gamma_5)$ on QCD-Lagrangian:
  - kinetic term $\bar{\psi}\gamma^\mu D_\mu \psi$ gives

$$\bar{\psi}\gamma_\mu\psi = \bar{\psi}_R\gamma_\mu\psi_R + \bar{\psi}_L\gamma_\mu\psi_L$$

  - whereas mass term $m\bar{\psi}\psi$ gives interaction

$$\bar{\psi}\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R$$

  and breaks chiral symmetry explicitly

Banks-Casher relation

Chiral symmetry breaking $\Longrightarrow$

$\Longrightarrow$ Low-lying eigenmodes of Dirac operator

$$\bar{\psi}\psi = -\lim_{m\to 0}\lim_{V\to\infty}\left\langle \frac{1}{V}\sum_n \frac{1}{\mathrm{i}\lambda_n + m}\right\rangle$$

Non-zero eigenvalues appear in pairs $\pm\mathrm{i}\lambda_n$

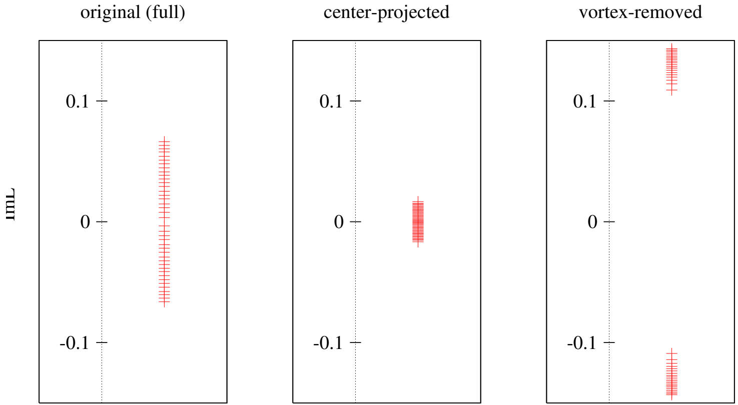
$$\lim_{m\to 0}\frac{2m}{\lambda_n^2 + m^2} \longrightarrow \pi\delta(0)$$

Chiral condensate $\Longrightarrow$ Density of Near-Zero-modes.

$$\bar{\psi}\psi = \frac{\pi\rho(0)}{V}$$

➜ *Banks, Casher, 1980*

Vortices and Staggered Fermions



Vortex-only configurations produce finite chiral condensate.
Vortex removal destroys topological charge, confinement & SCSB.

Chiral Symmetry Breaking

- Instanton Liquid Model:
  - action minima localized in space-time carry $Q = \pm 1$
  - attract zero modes according to Atiyah-Singer index theorem
  - overlapping would-be zero modes lead to near-zero modes
  - chiral symmetry breaking via Banks-Casher relation

→ *Diakonov, Petrov (1984)*

- spherical vortices behave like instantons

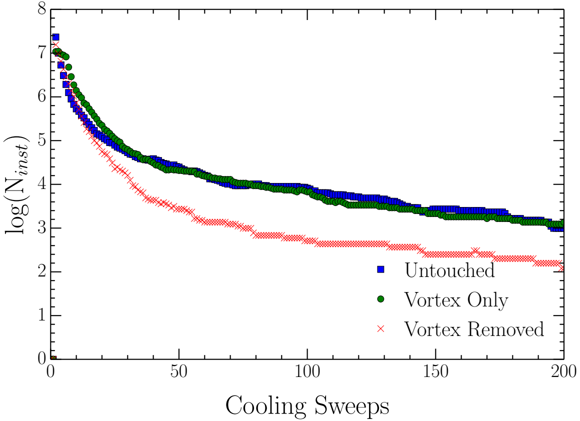
→ *Schweigler, R.H., Faber, Heller (2012)*

- also intersection points give chiral condensate

→ *R.H., Faber, Heller, Schweigler (2013)*

- not an exclusive picture of chiral symmetry breaking
- any source of topological charge can contribute (monopoles, instantons, merons, bions, calorons,...)

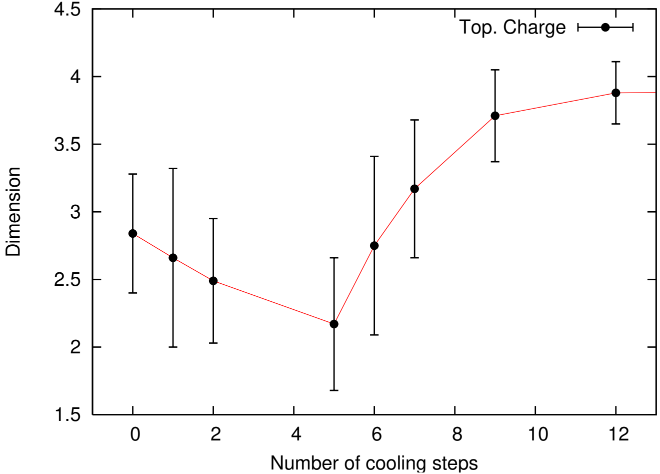
Vortices stabilize instantons



A log plot of the number of instanton-like objects per configuration as a function of $\mathcal{O}(a^4)$-improved cooling sweeps.

➜ *Trewartha, Kamleh, Leinweber, 2015*

Fractal Dimensionality



Fractal dimension of topologiclal structures during cooling.

➜ *Buividovich, Kalaydzhyan, Polikarpov, 2011*

Thank You &

Derar Altarawneh, Falk Bruckmann, Matthias Burkardt, Michael Engelhardt, Roman Höllwieser, Martin Gal, Jeff Greensite, James Hetrick, Urs M. Heller, Andrei Ivanov, Thomas Layer, Štefan Olejník, Mario Pitschmann, Hugo Reinhardt, Jesus Saenz, Thomas Schweigler, Lorenz von Smekal, Wolfgang Söldner, Mithat Unsal, Markus Wellenzohn