Romain Contant
Advisor : Markus Q. Huber

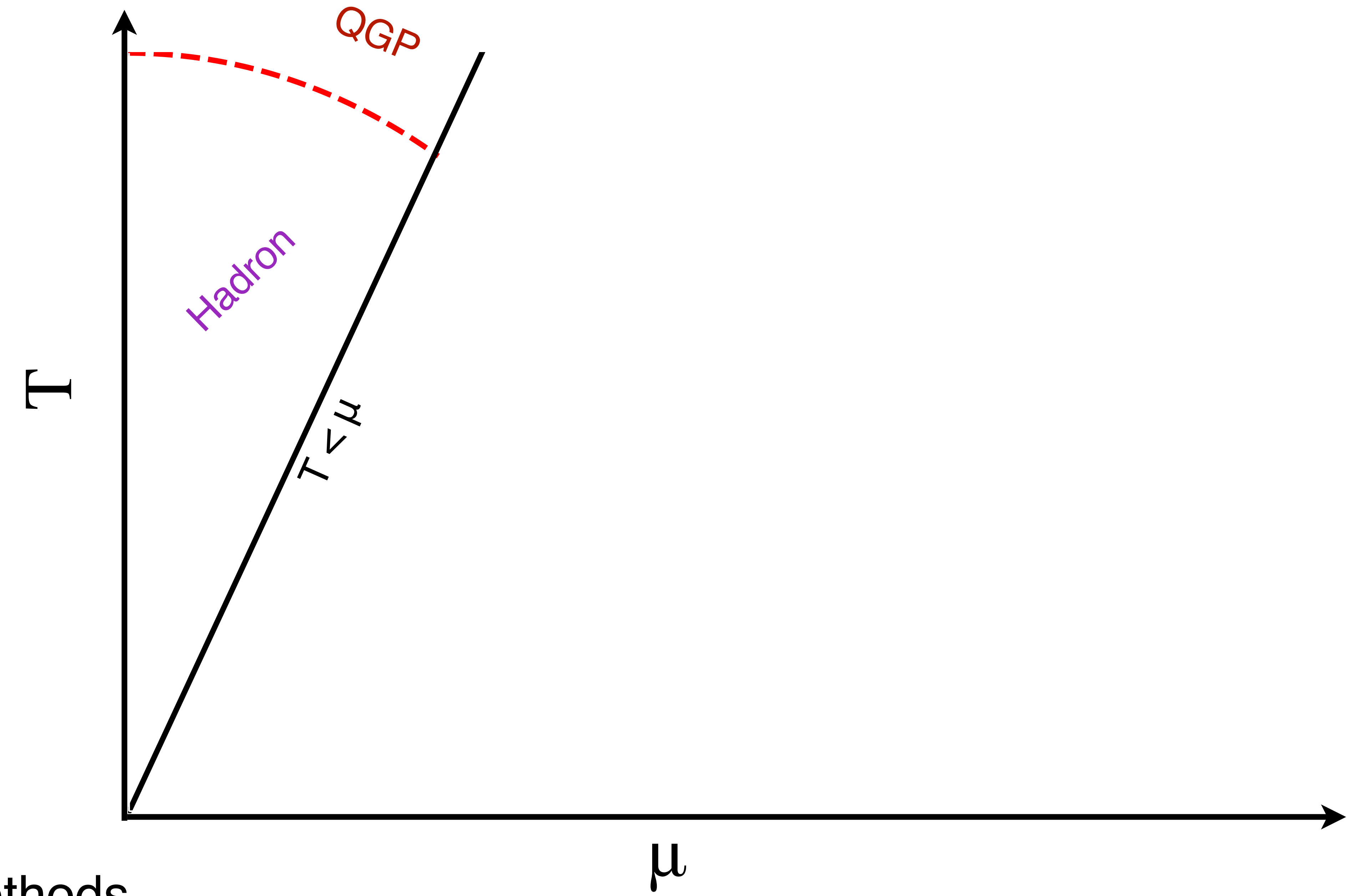
The quark propagator in QCD and QCD-like theories

arXiv:1705.xxxx

FWF Der Wissenschaftsfonds.
UNI GRAZ
NAWI Graz Natural Sciences

Excited QCD : 9 Mai 2017

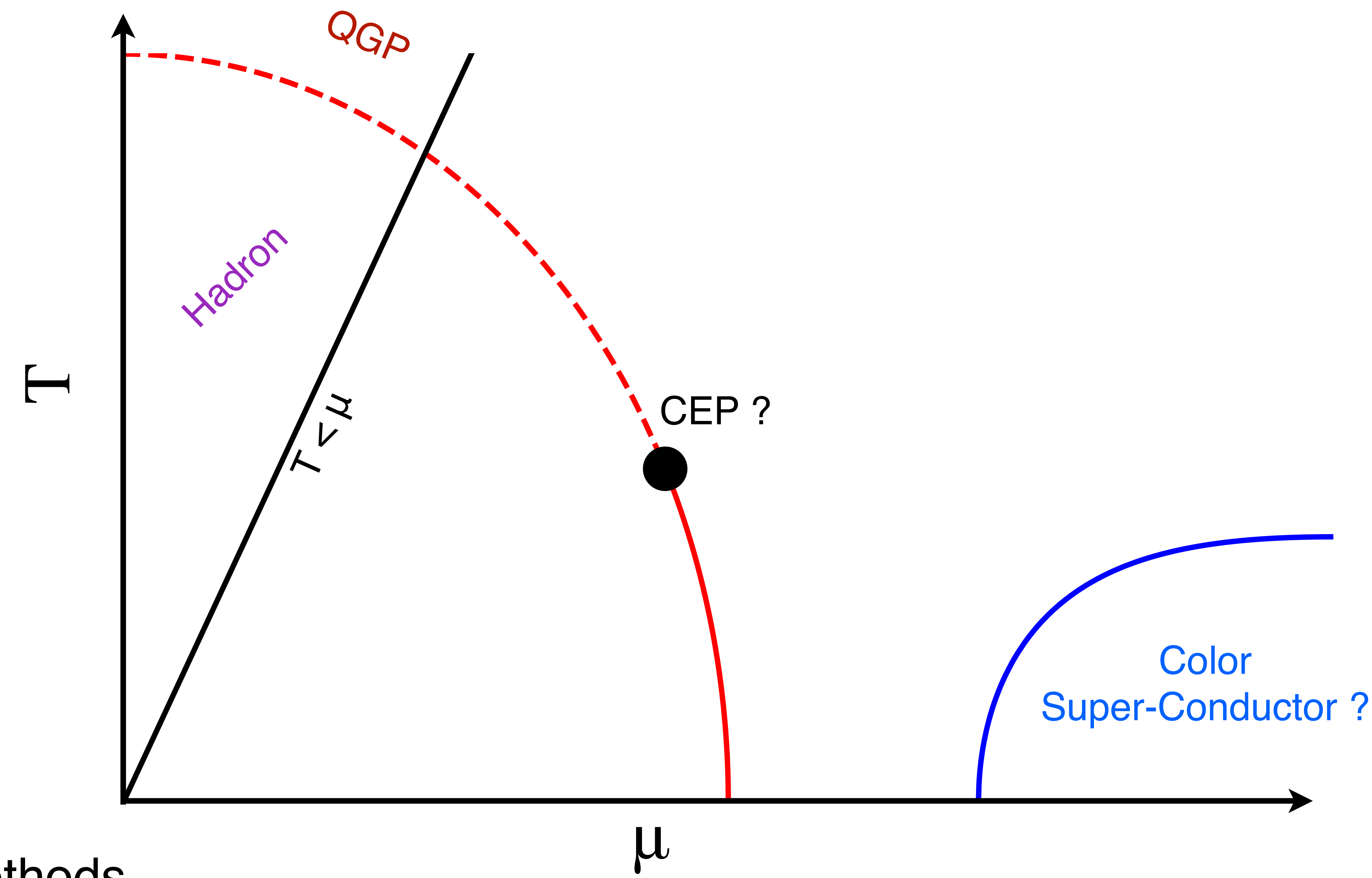
- The phase diagram of QCD



- Methods
  - Lattice QCD

→ Sign problem

- The phase diagram of QCD



- Methods
  - Lattice QCD → Sign problem
  - Effective Models → Fixing parameters
  - Functional methods → Truncation and modeling

QCD

Lattice QCD ⟷ sign problem

Functional Methods ⟷ Truncation

QCD-like

- A theory with dynamical mass generation
- Confinement and asymptotic freedom
- A positive fermion determinant

Minimal modification of QCD

Lattice QCD ⟷ ~~sign problem~~

Functional Methods ⟷ Truncation

QCD-like

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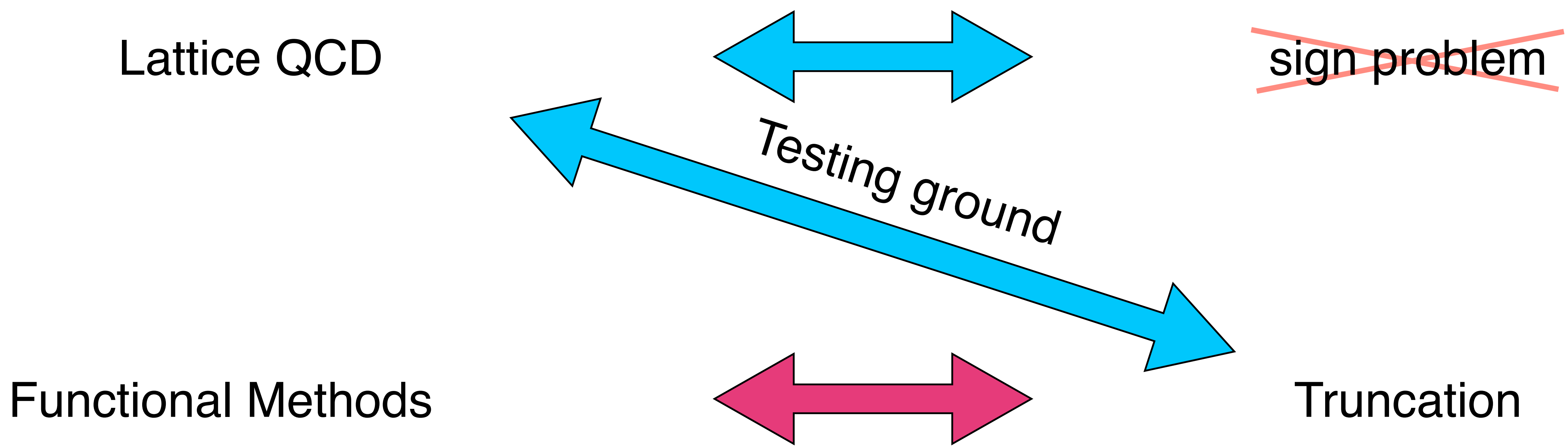
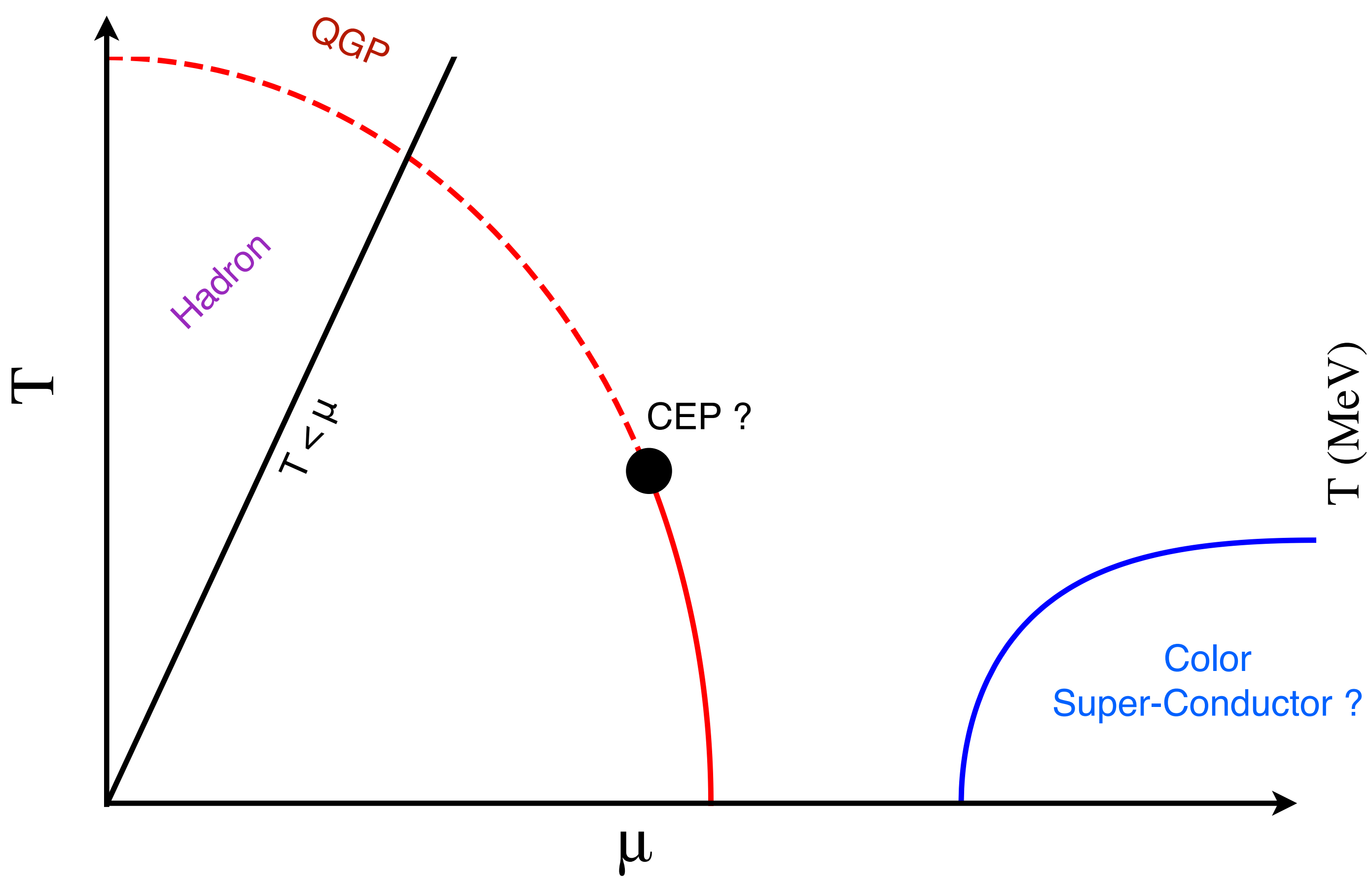
Minimal modification of QCD

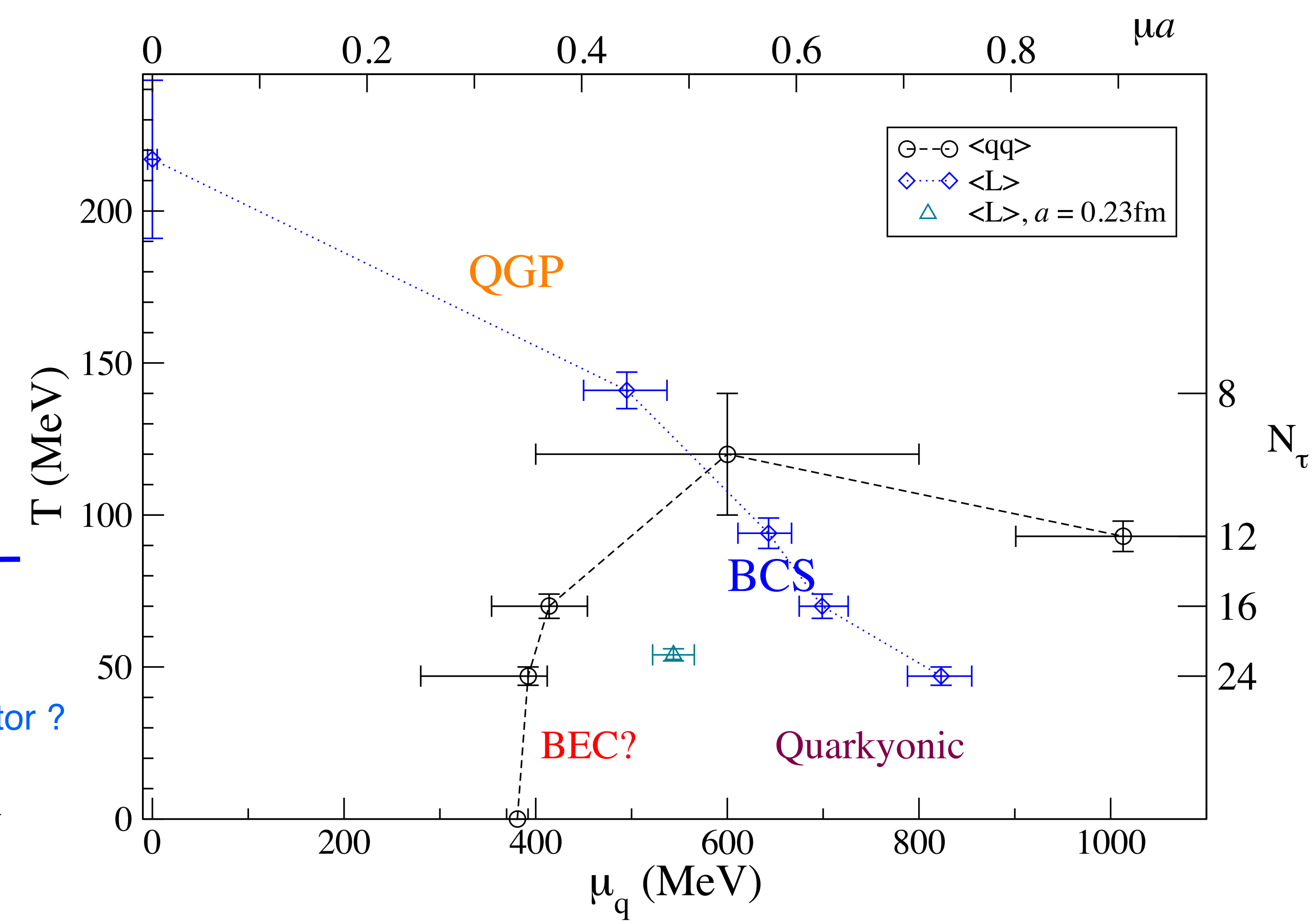


→ Apply the truncation to physical QCD

1 Introduction
QCD-like phase diagram

- Phase diagrams of QCD-like theories
  - QCD sketch
  - SU(2)





Is the same truncation sufficient to encode different gauge groups ?

Diquark condensate : *[S. Cotter et al. Phys.rev., vol. D87, pp. 034507, (2013)]*

Polyakov loop : *[S. Hands et al. Eur.phys.j., vol. C48, pp. 193, (2006)]*

- Phase diagrams of QCD-like theories
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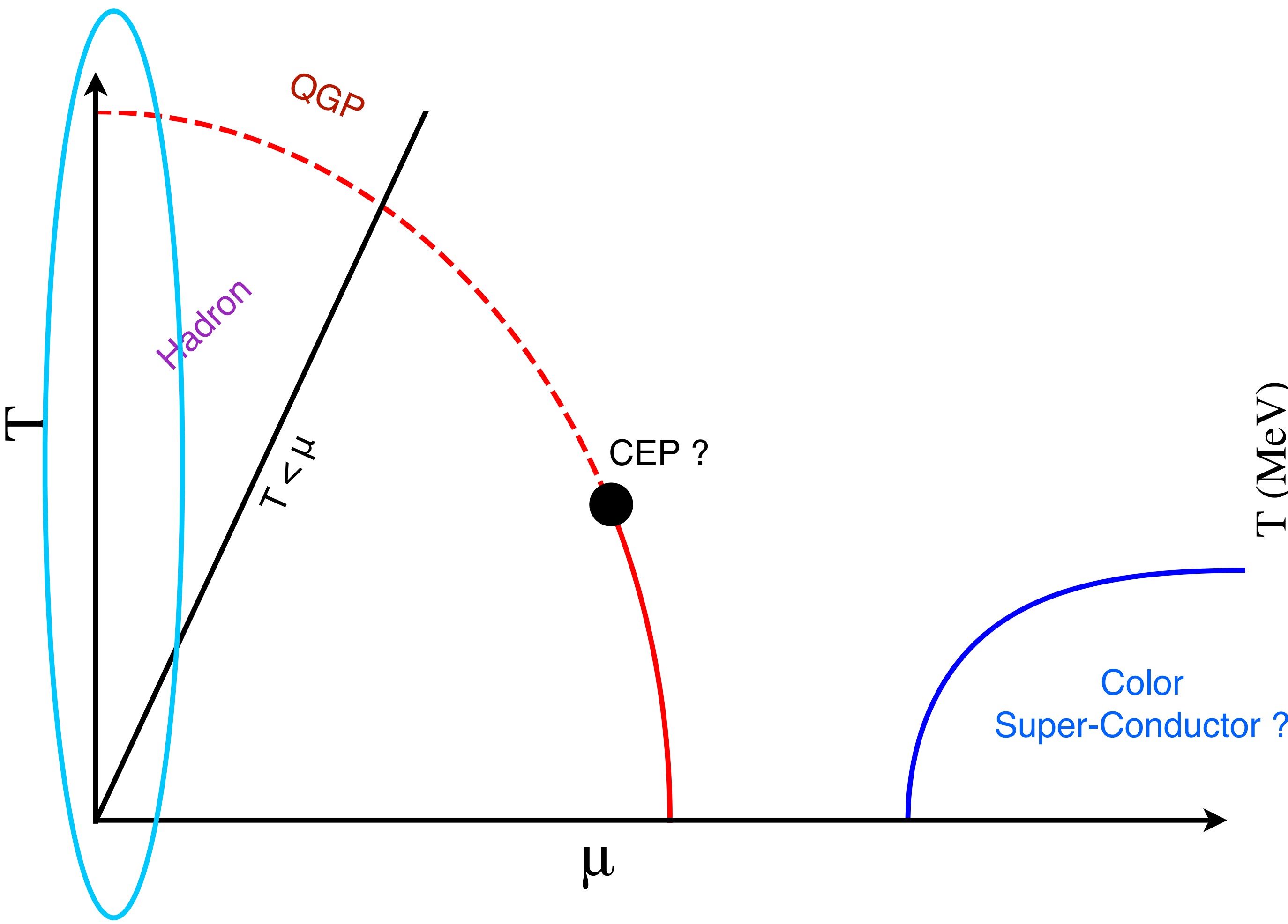

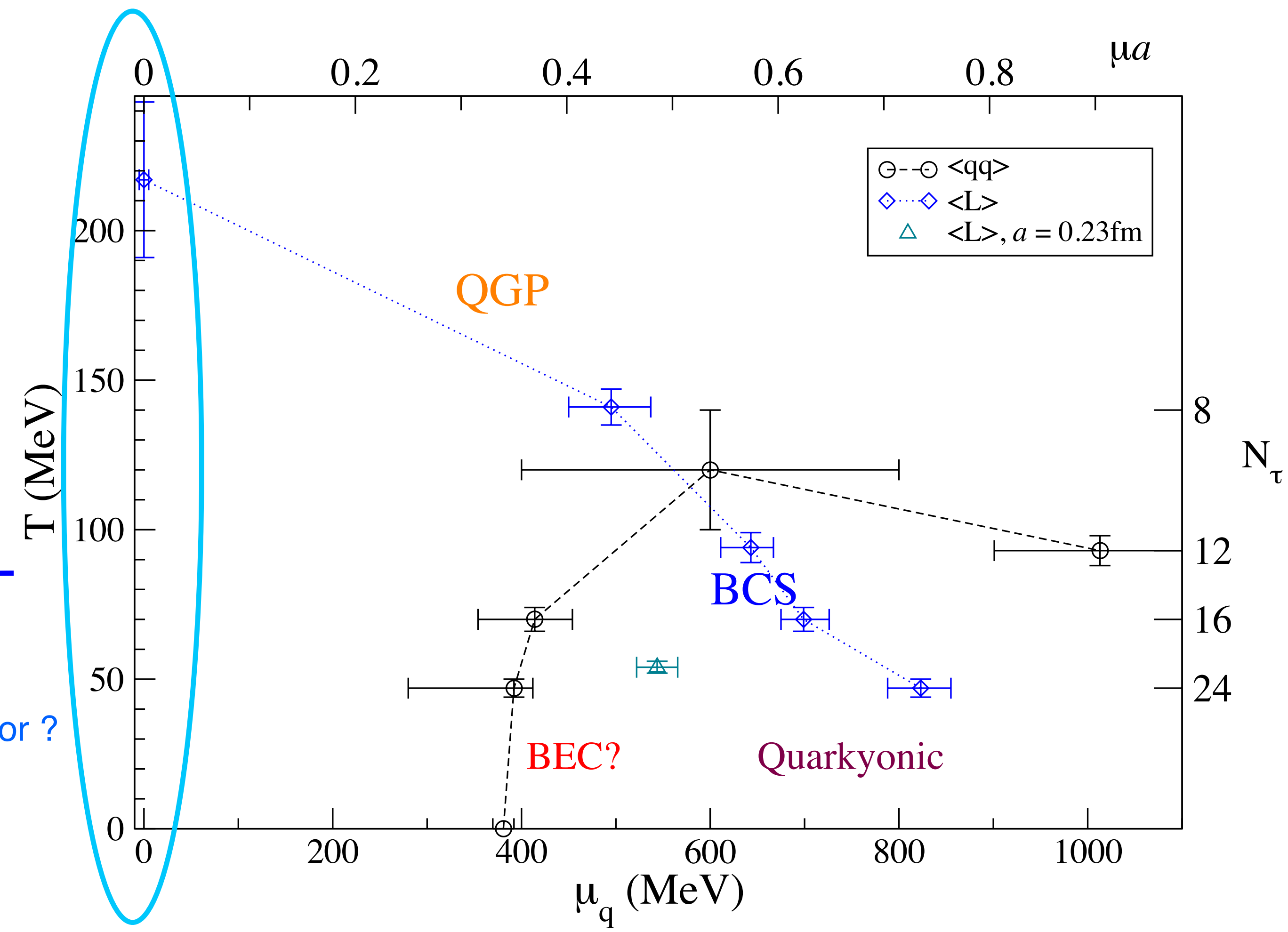

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Study the quark propagator at finite T for QCD and QCD-like theories

Synopsis

Dyson-Schwinger Equation

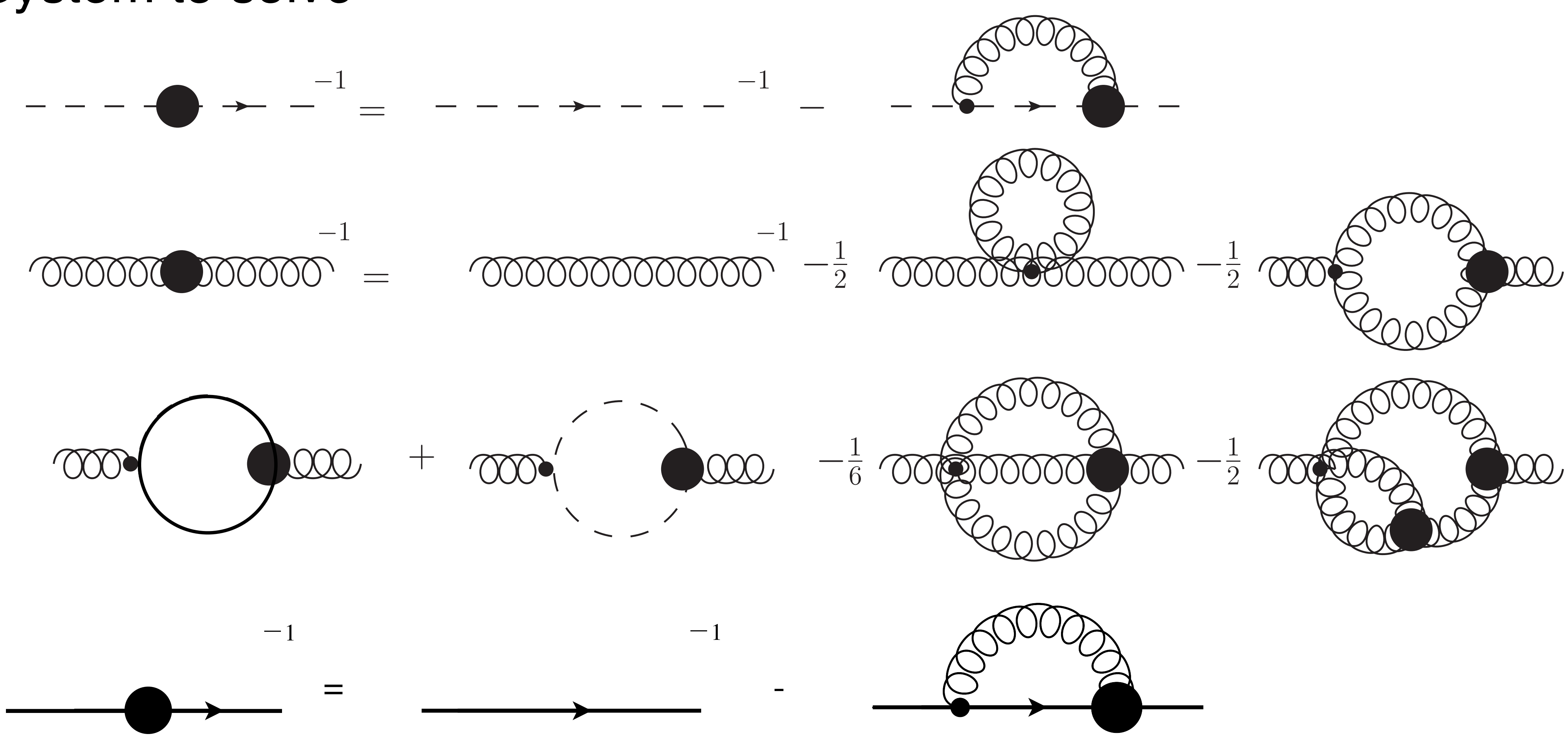
$$Z = \int d[\Phi]\, \mathrm{Exp}\left[-\int S[\Phi] + \Phi J\right] \longrightarrow W[\Phi] = \mathrm{Log}[Z] \longrightarrow \Gamma[J]$$

Z : Partition function → *W :* Connected Diagrams → *Γ :* irreducible Diagrams

$$\left\langle \frac{\delta\Gamma}{\delta\Phi} - J \right\rangle = 0$$

Equations of motion of a QFT

- System to solve

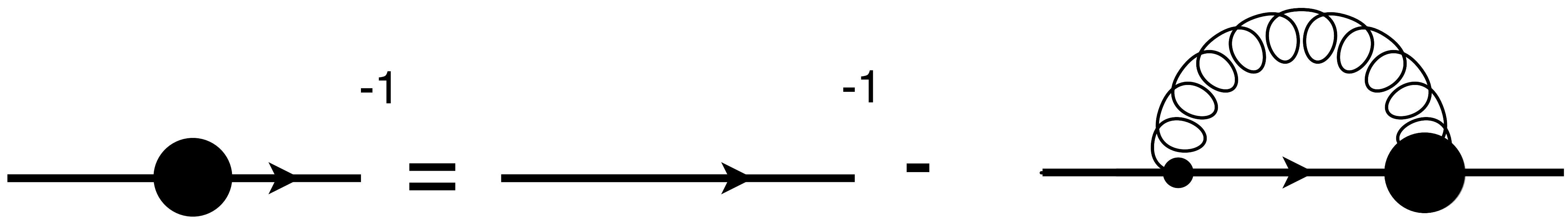


+ higher equations

- System to solve
    - Truncations are mandatory

→ We want a realistic temperature dependence of the gluon

$$S^{-1}(\vec{p},\omega_n) = A(\vec{p},\omega_n)\,\vec{\gamma}\,\vec{p} + C(\vec{p},\omega_n)\,\omega_n\gamma_4 + B(\vec{p},\omega_n) + \omega_n\gamma_4\,\vec{p}\,\vec{\gamma}\,D(\vec{p},\omega_n)$$



Setup
The truncation and modeling

- System to solve
  - Truncations are mandatory

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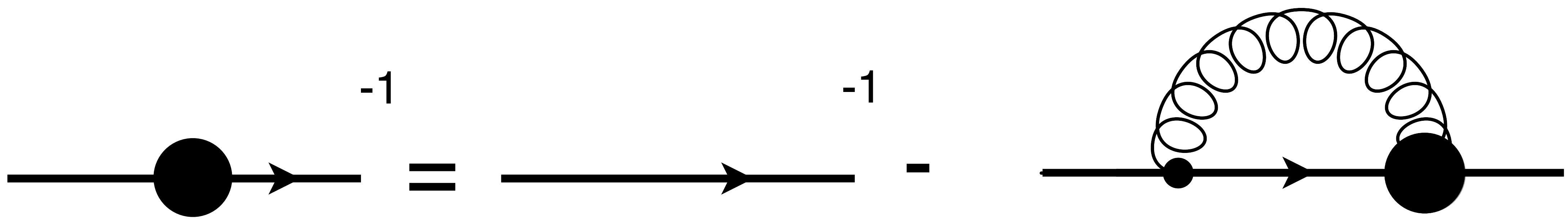
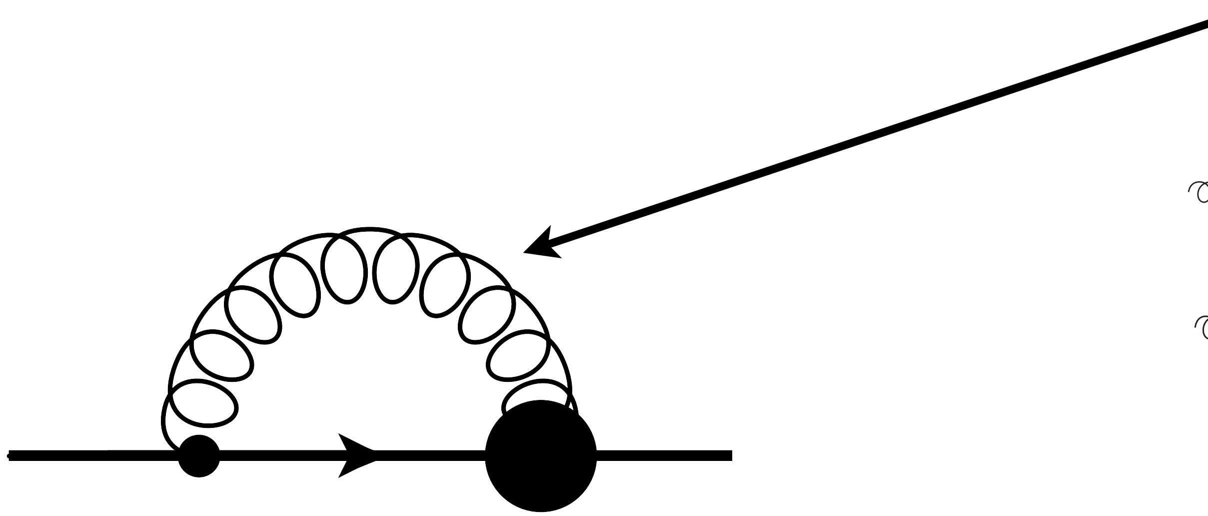
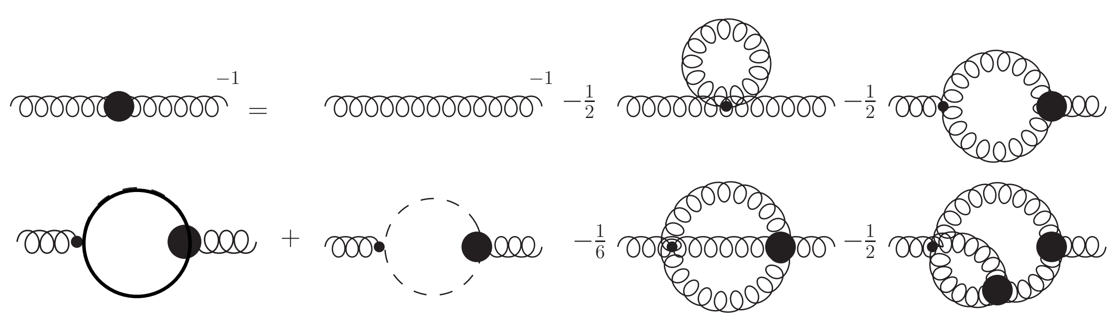
$$S^{-1}(\vec{p}, \omega_n) = A(\vec{p}, \omega_n)\, \vec{\gamma}\, \vec{p} + C(\vec{p}, \omega_n)\, \omega_n \gamma_4 + B(\vec{p}, \omega_n) + \omega_n \gamma_4\, \vec{p}\, \vec{\gamma}\, D(\vec{p}, \omega_n)$$



$$D_{\mu\nu}(p) = \frac{1}{p^2}\left(Z_T(p) P^T_{\mu\nu} + Z_L(p) P^L_{\mu\nu}\right)$$





→ Spuriously divergent terms

→ Accessible on lattice

2 Setup

The truncation and modeling

- System to solve
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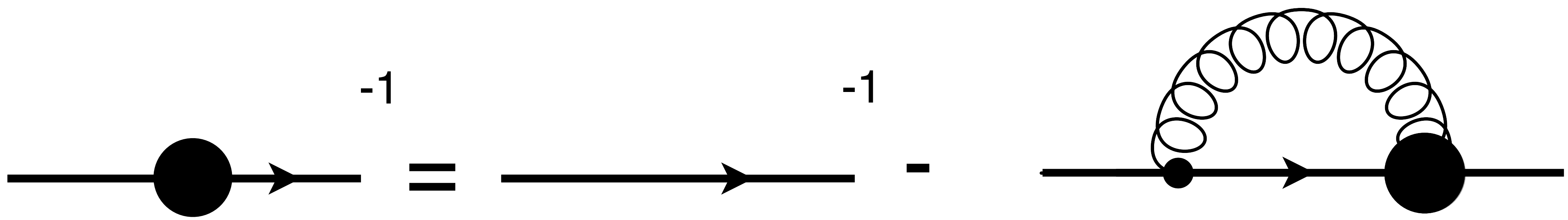
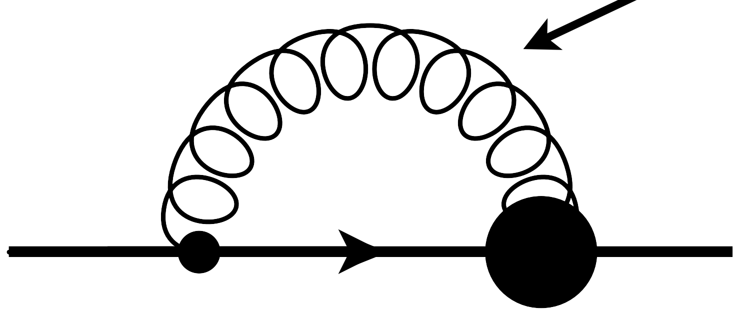


$$Z_{T,L}(x) = \frac{x}{(x+1)^2}\left(\left(\frac{c}{x+a_{T,L}(T)}\right)^{b_{T,L}(T)} + x\left(\frac{\beta_0\alpha(\mu)}{4\pi}\ln(x+1)\right)^{\gamma}\right)$$



Coefficients are fitted to reproduce lattice data

[A. Maas, J.M Pawlowski, L. von Smekal, D. Spielmann (2012)]
[C.S. Fischer, A. Maas, J.A. Muller (2010)]

The truncation and modeling

- System to solve
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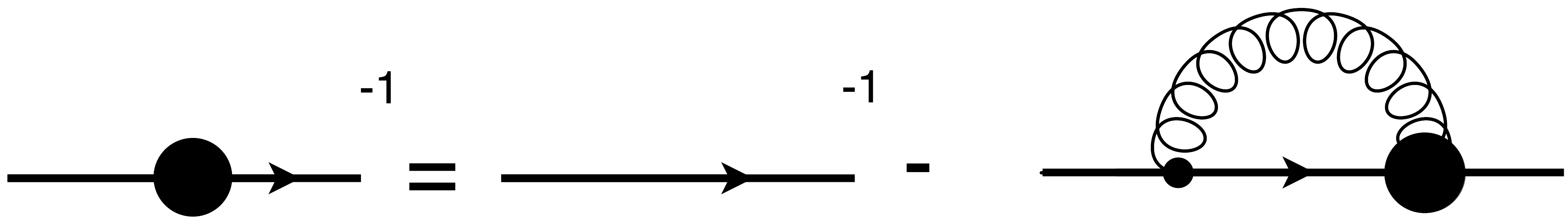
$$S^{-1}(\vec{p}, \omega_n) = A(\vec{p}, \omega_n)\, \vec{\gamma}\, \vec{p} + C(\vec{p}, \omega_n)\, \omega_n \gamma_4 + B(\vec{p}, \omega_n) + \omega_n \gamma_4\, \vec{p}\, \vec{\gamma}\, D(\vec{p}, \omega_n)$$

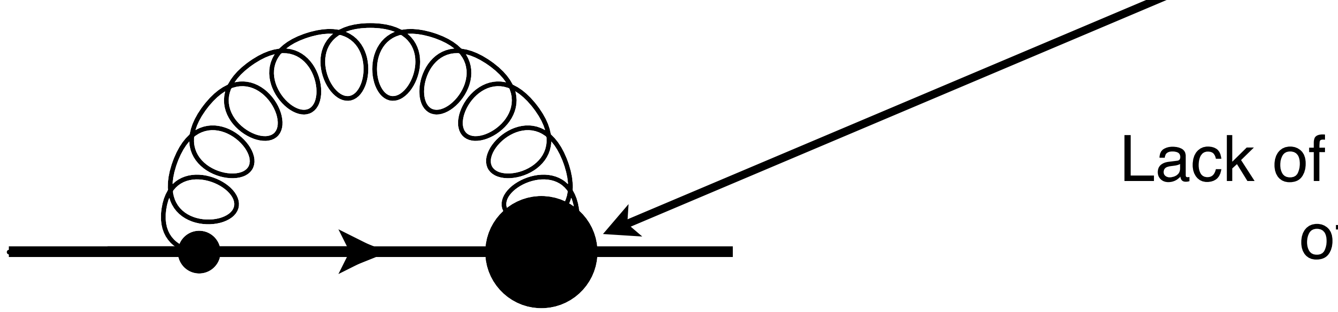


$\Gamma_{q\text{-}gl}(p,q,l)$

24 tensors parts

Difficult to obtain from lattice

Lack of information of the temperature dependence of this quantity from continuum studies



- System to solve
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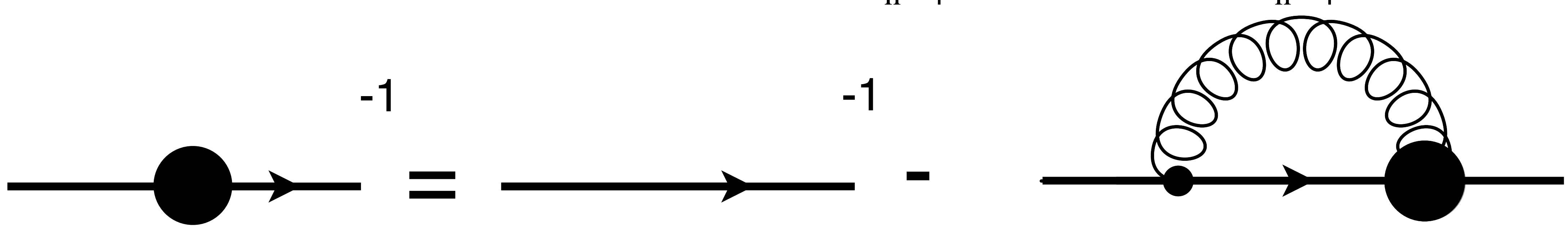
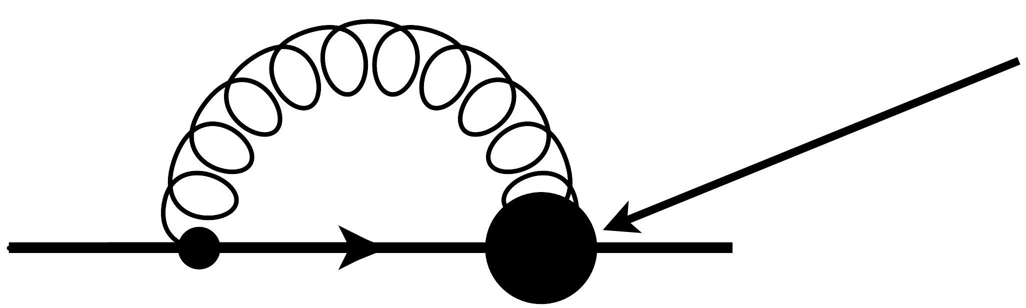
$$S^{-1}(\vec{p},\omega_n) = A(\vec{p},\omega_n)\,\vec{\gamma}\,\vec{p} + C(\vec{p},\omega_n)\,\omega_n\gamma_4 + B(\vec{p},\omega_n) + \omega_n\gamma_4\,\vec{p}\,\vec{\gamma}\,D(\vec{p},\omega_n)$$



$$\Gamma_{q\text{-}gl}(p,q,l) = \left(\frac{A(p)+A(q)}{2}\vec{\gamma},\frac{C(p)+C(q)}{2}\gamma_4\right)W(p,q,l)$$

$$W(p,q,l) = \frac{d_1}{d_2+l^2} + \frac{l^2}{1+l^2}\left(\frac{\beta_0\,\alpha(\mu)}{4\pi}\ln(l^2+1)\right)^{2\delta}$$

[C.S. Fischer (2009)]



Contant Romain

- System to solve
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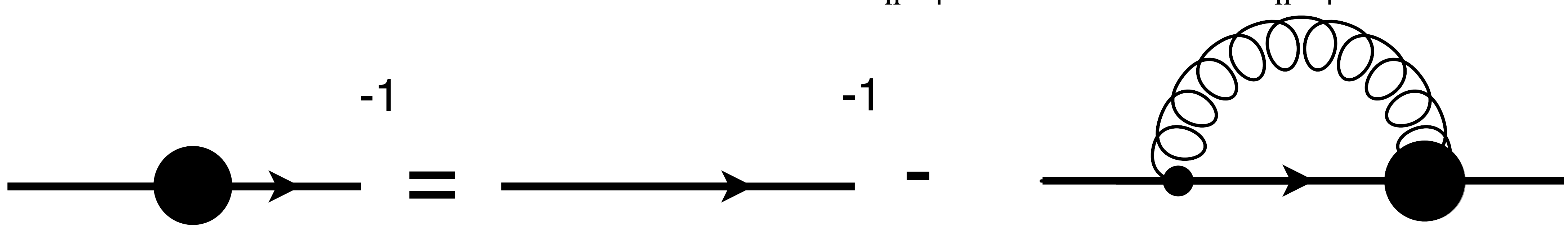
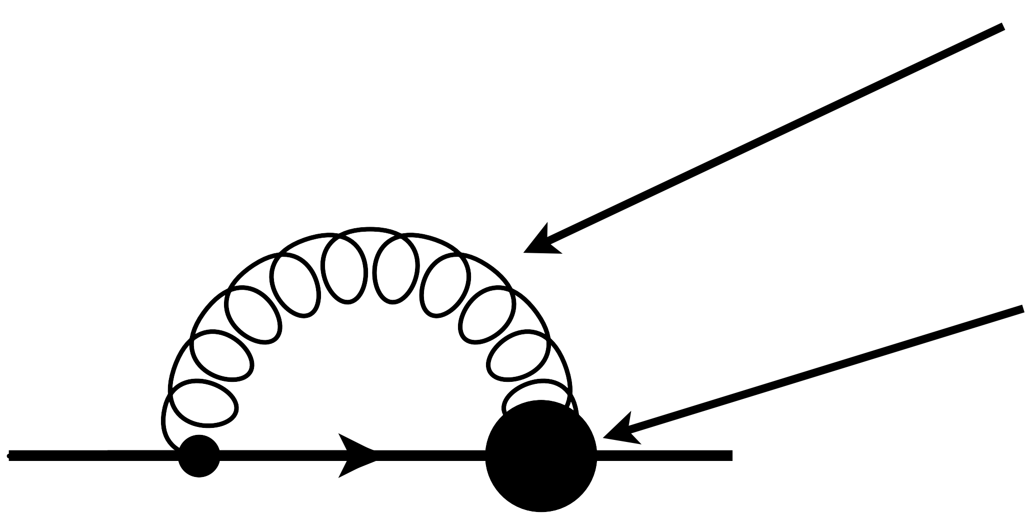


$$Z_{T,L}(p^2) = \frac{p^2}{(1+p^2)^2}\left(\left(\frac{c}{p^2+a_{T,L}(T)}\right)^{b_{T,L}(T)} + p^2\left(\frac{\beta_0\,\alpha(\mu)}{4\pi}\ln(p^2+1)\right)^{\gamma}\right)$$



$$\Gamma_{q\text{-}gl}(p,q,l) = \left(\frac{A(p)+A(q)}{2}\vec{\gamma},\frac{C(p)+C(q)}{2}\gamma_4\right)W(p,q,l)$$

→ The system can be solved

Chiral condensate

- Chiral Symmetry Breaking

$$S^{-1}(\vec{p},\omega_n) = A(\vec{p},\omega_n)\,\vec{\gamma}\,\vec{p} + C(\vec{p},\omega_n)\,\omega_n\gamma_4 + B(\vec{p},\omega_n) + \omega_n\gamma_4\,\vec{p}\,\vec{\gamma}\,D(\vec{p},\omega_n)$$

$$\Delta_\pi(T) = Z_2(Z_m)N_c\,T\sum_{\omega_n}\int_{\vec{p}}\frac{d^3\vec{p}}{(2\pi)^3}\mathrm{Tr}[S(\vec{p},\omega_n)]$$

- chiral limit $m_{bare} = 0$

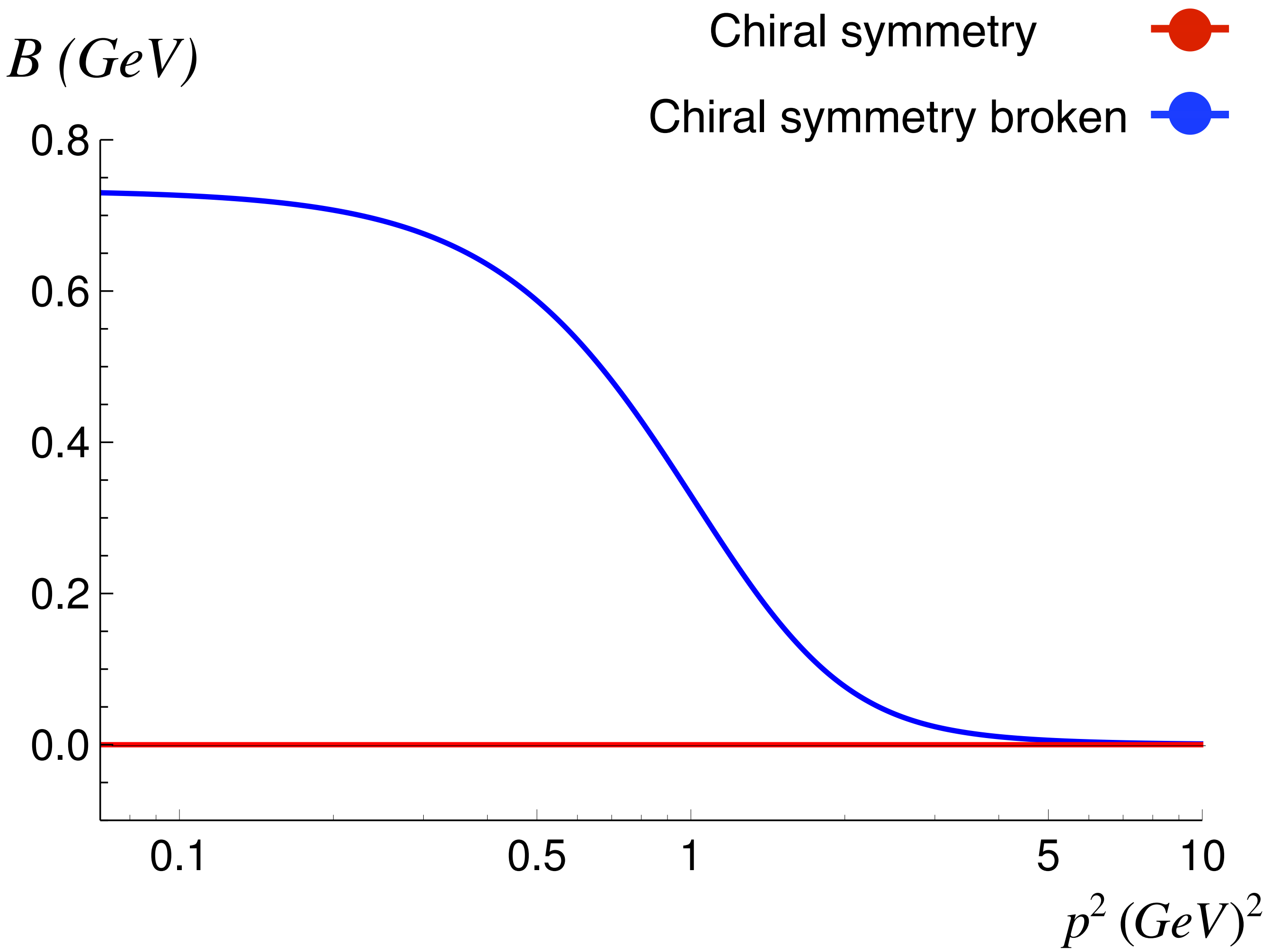
→ $B \neq 0$, formation of a chiral condensate

Chiral symmetry broken

→ $B = 0$, no chiral condensate

Chiral symmetry

<u>order parameter</u>



(Pseudo)-order parameter

- Chiral Symmetry Breaking

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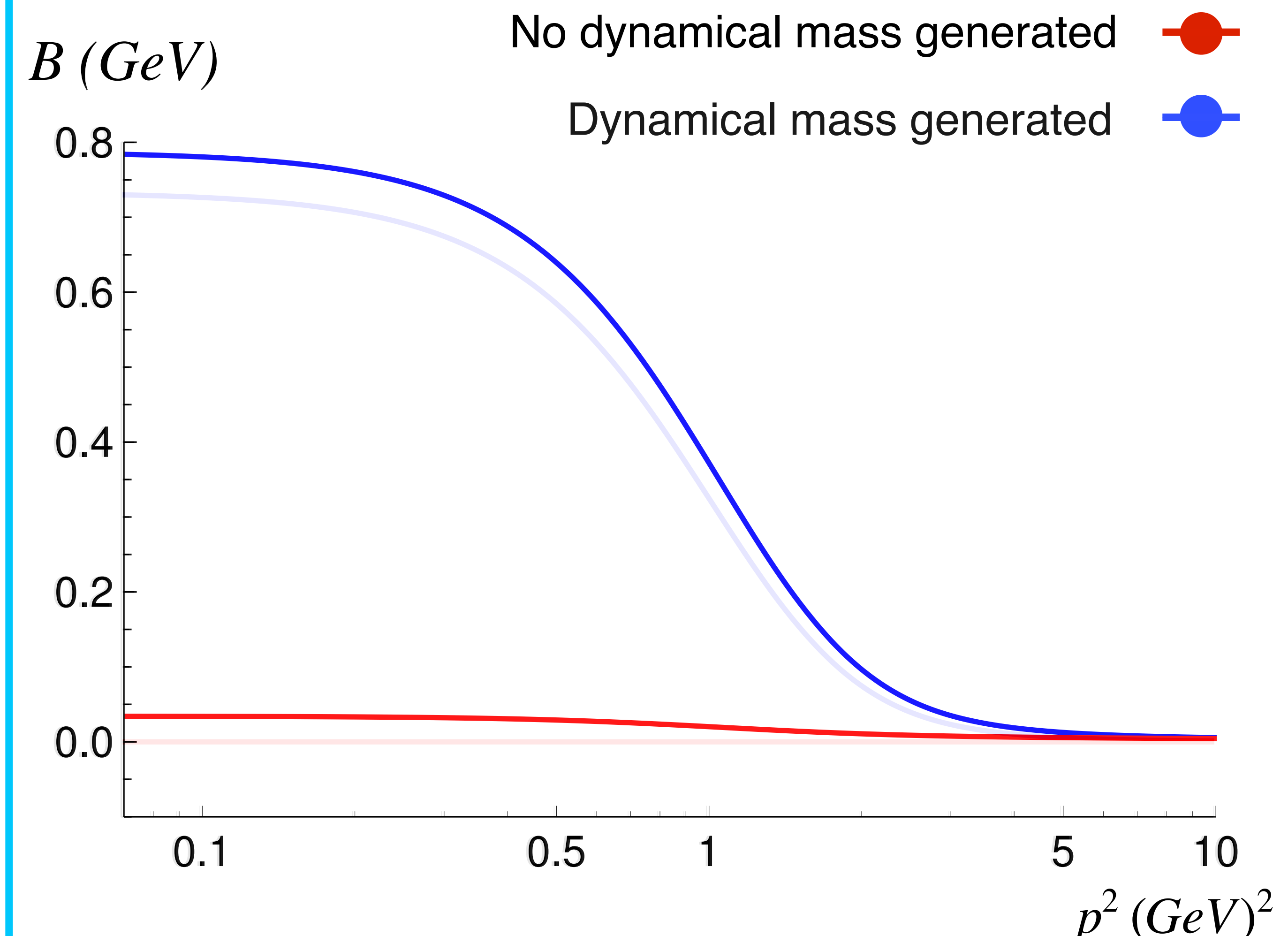
- $m_{bare} > 0$

→ $B \neq 0$, formation of a chiral condensate

Chiral symmetry broken

Pseudo-order parameter



D function

- Chiral Symmetry Breaking

$$S^{-1}(\vec{p},\omega_n) = A(\vec{p},\omega_n)\,\vec{\gamma}\,\vec{p} + C(\vec{p},\omega_n)\,\omega_n\gamma_4 + B(\vec{p},\omega_n) + \omega_n\gamma_4\,\vec{p}\,\vec{\gamma}\,D(\vec{p},\omega_n)$$

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→ $[\vec{p}\vec{\gamma}\,\omega_n\,\gamma_4\,,\gamma_5] = 0$ $\quad D \neq 0$

Chiral symmetry broken

→ $D = 0$ at $T = 0$

→ D power-law supressed in UV

→ $D = 0$ in Rainbow ladder

→ D small after the chiral restoration

We expect a small contribution of *D*

- Quark confinement

$$S^{-1}(\vec{p},\omega_n) = A(\vec{p},\omega_n)\,\vec{\gamma}\,\vec{p} + C(\vec{p},\omega_n)\,\omega_n\gamma_4 + B(\vec{p},\omega_n) + \omega_n\gamma_4\,\vec{p}\,\vec{\gamma}\,D(\vec{p},\omega_n)$$

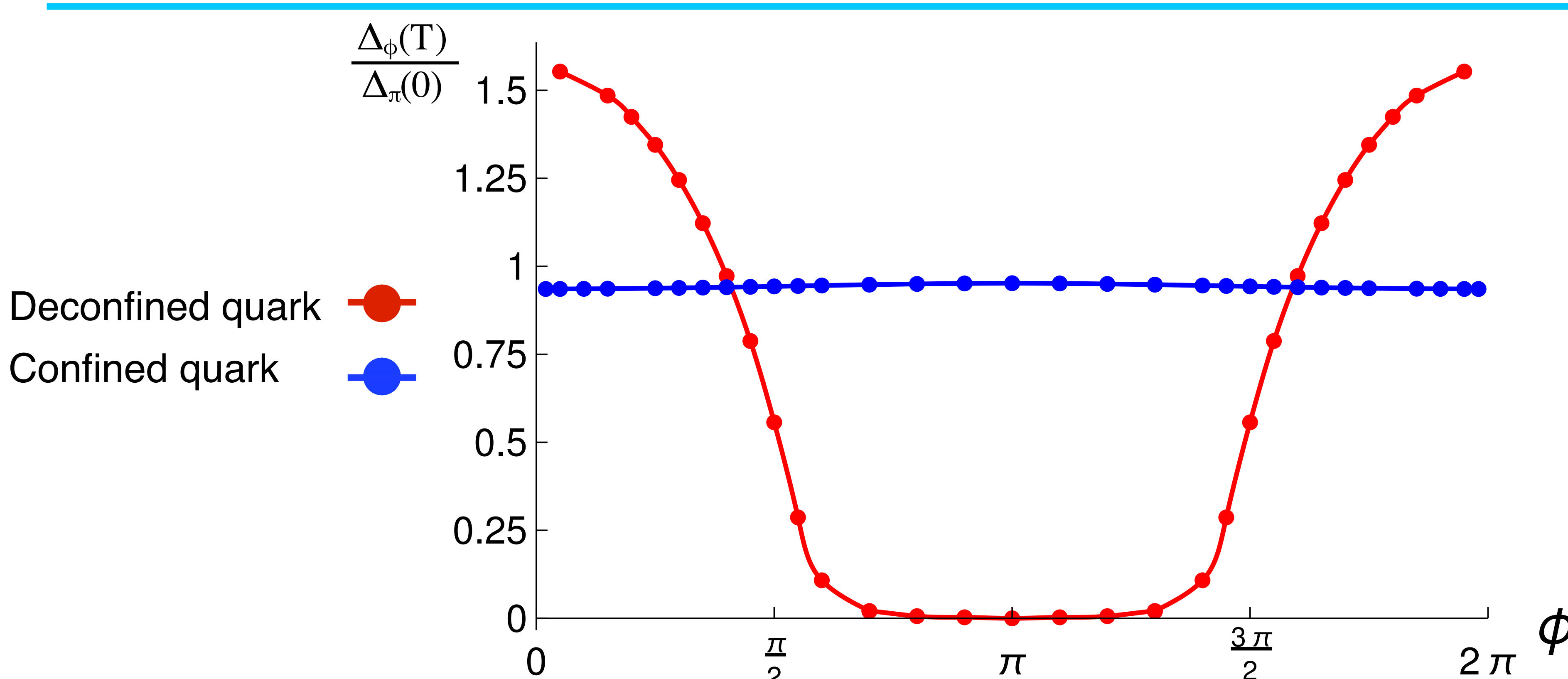
$$\Delta_\phi(T) = Z_2(Z_m)N_c\,T \sum_{\omega_n(\phi)} \int_{\vec{p}} \frac{d^3\vec{p}}{(2\pi)^3} \mathrm{Tr}[S(\vec{p},\omega_n(\phi))]$$

We introduce a phase dependence : $\omega_n = \pi T \left(2n + \frac{\phi}{\pi}\right)$

$$\Sigma_1 = \int_0^{2\pi} e^{i\phi}\, d\phi\, \Delta_\phi(T)$$

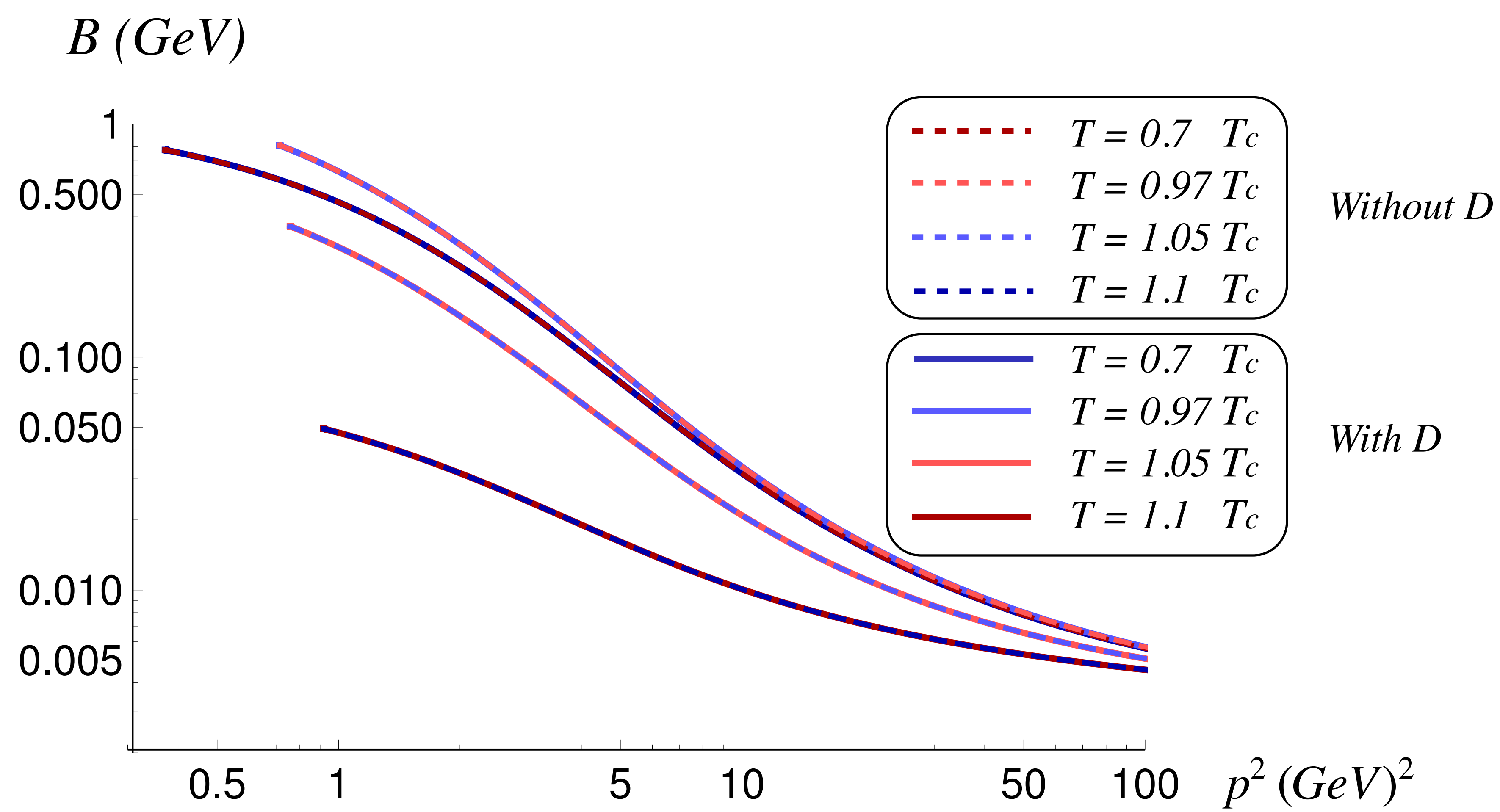
The dual quark condensate is proportional to the Polyakov Loop

[E. Bilgici, F. Bruckmann, C. Gattringer, and C. Hagen (2008)]
[C.S. Fischer (2009)]

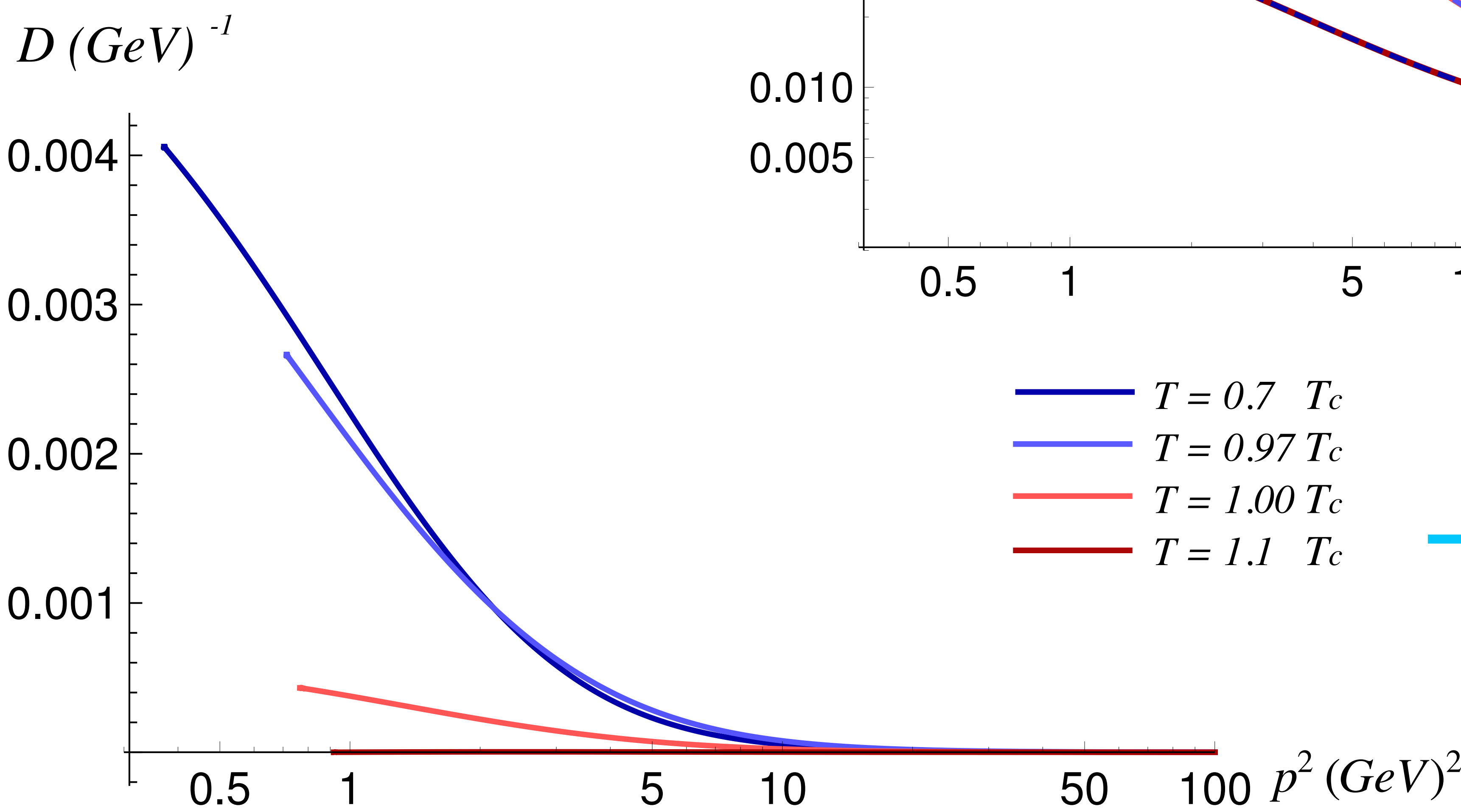


3 Quenched QCD
Effects of the *D* function

$$S^{-1}(\vec{p}, \omega_0) = A(\vec{p}, \omega_0)\, \vec{\gamma}\, \vec{p} + C(\vec{p}, \omega_0)\, \omega_0 \gamma_4 + B(\vec{p}, \omega_0) + \omega_0 \gamma_4\, \vec{p}\, \vec{\gamma}\, D(\vec{p}, \omega_0)$$



$$p^2 = (\omega_0)^2 + \vec{p}^{\,2}$$

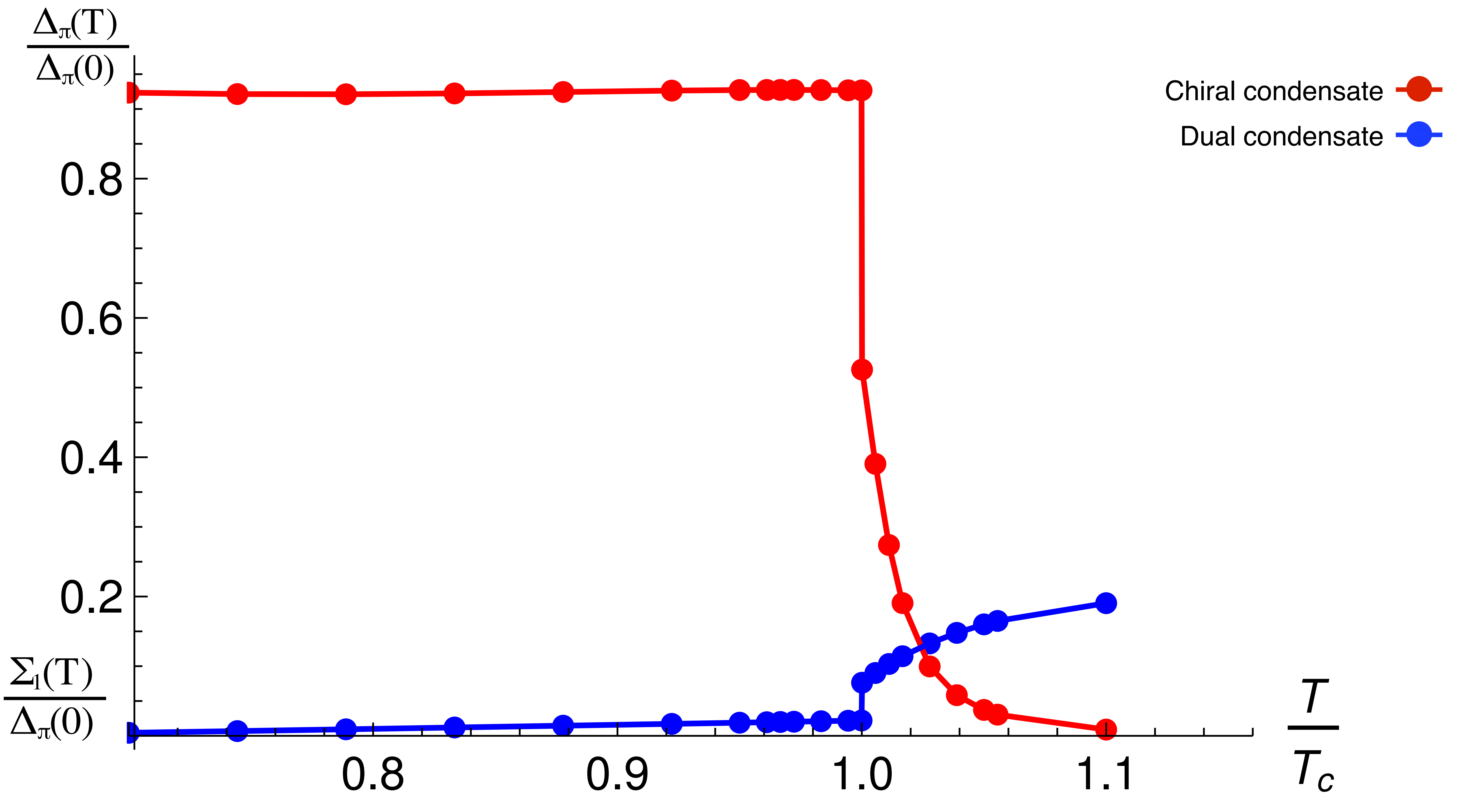


Relative contribution :

→ < 0.01 % to the *B* dressing

Order parameters

Chiral Condensate and dual condensate



Order parameters

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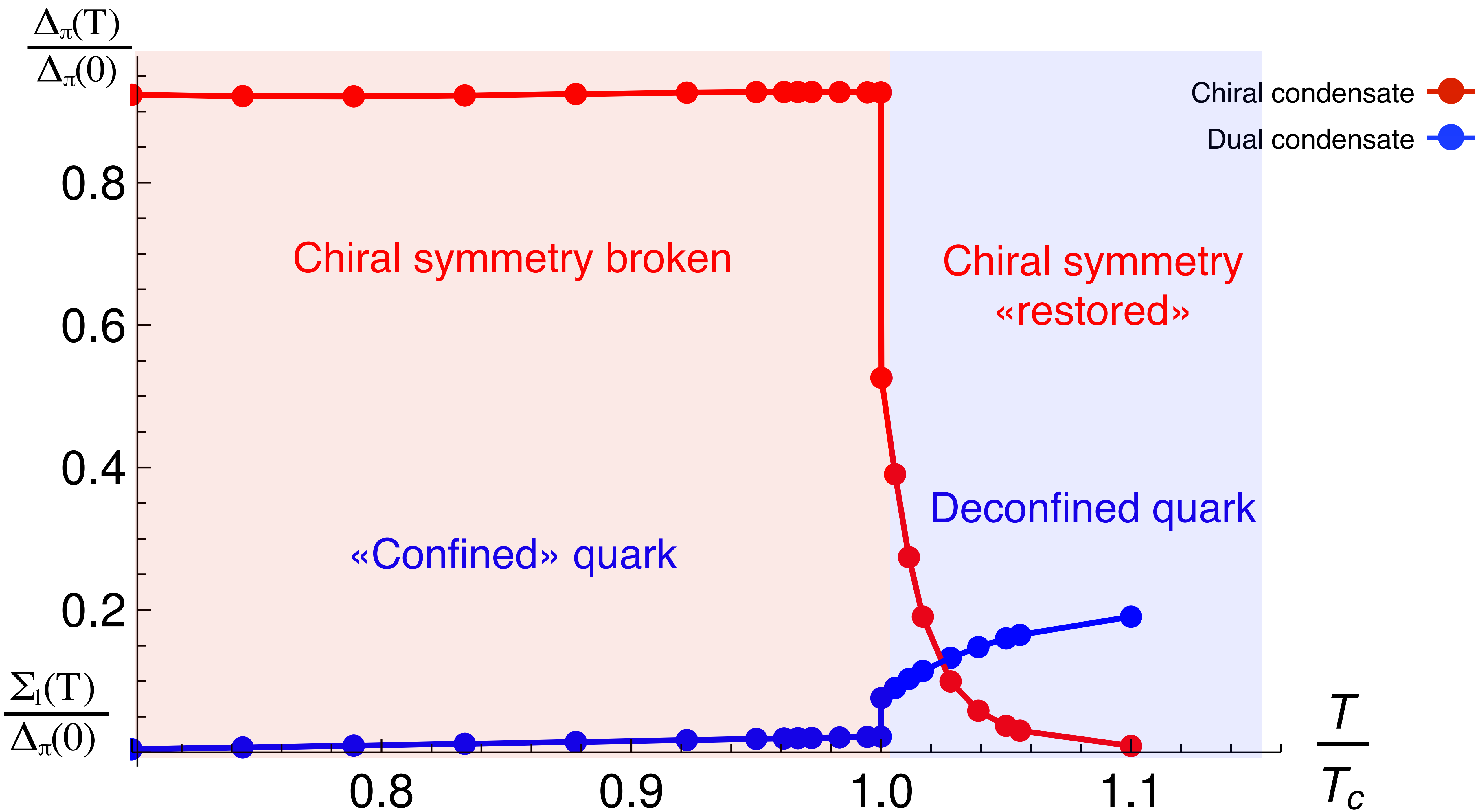
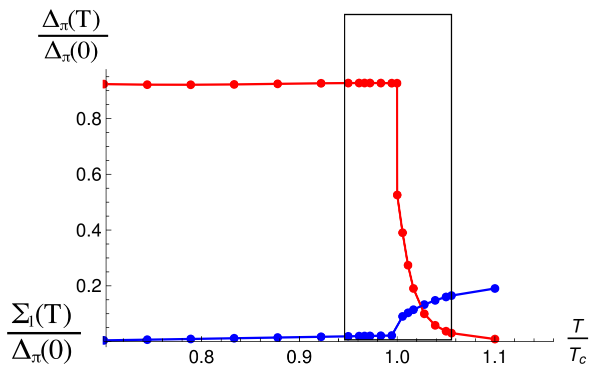


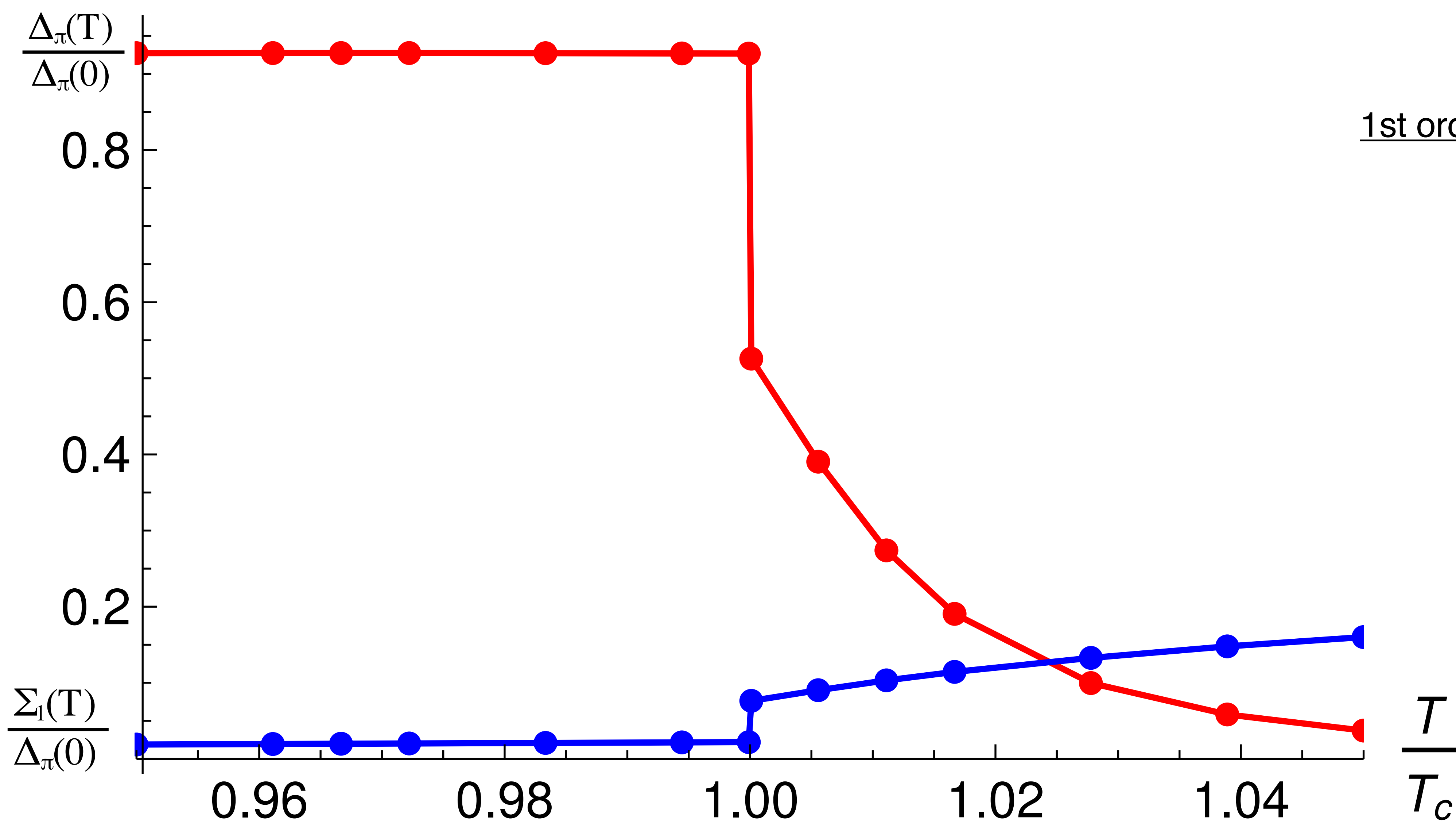
Chiral Condensate and Dual condensate

Type of transition



Chiral condensate

Dual condensate



1st order transition

QCD-like theories
Gauge group definition

- Two Color QCD

→ SU(2) for even number of degenerated quark flavors possesses a positive quark determinant

→ Expected to be second order transition for the chiral condensate
(For the quenched system)

- G_2

  - Subgroup of SO(7) which satisfies an additional cubic constraint

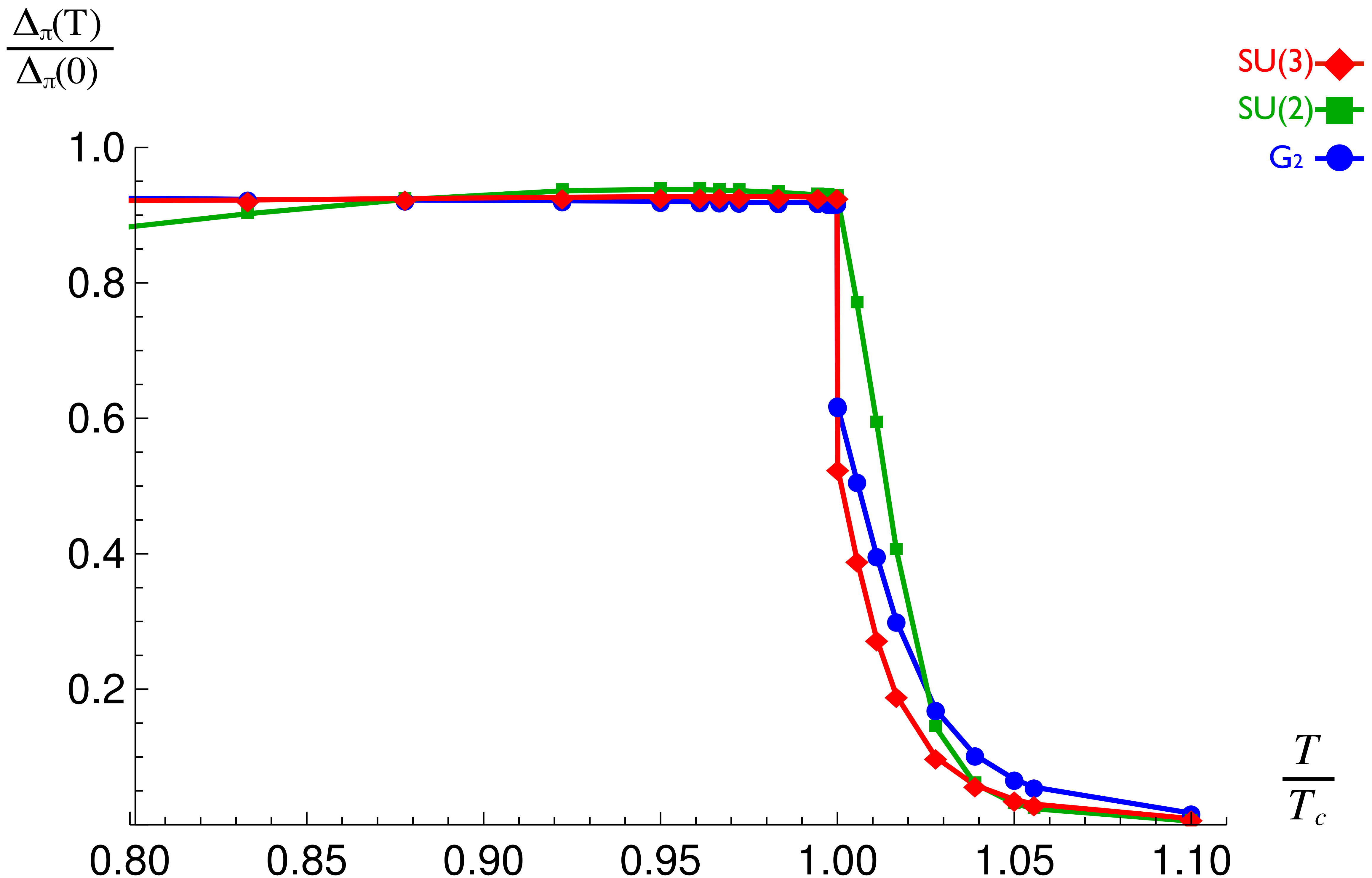
→ All representations are real, allow lattice simulation at $\mu > 0$

→ Centerless

→ Lattice simulations show a first order transition for confinement

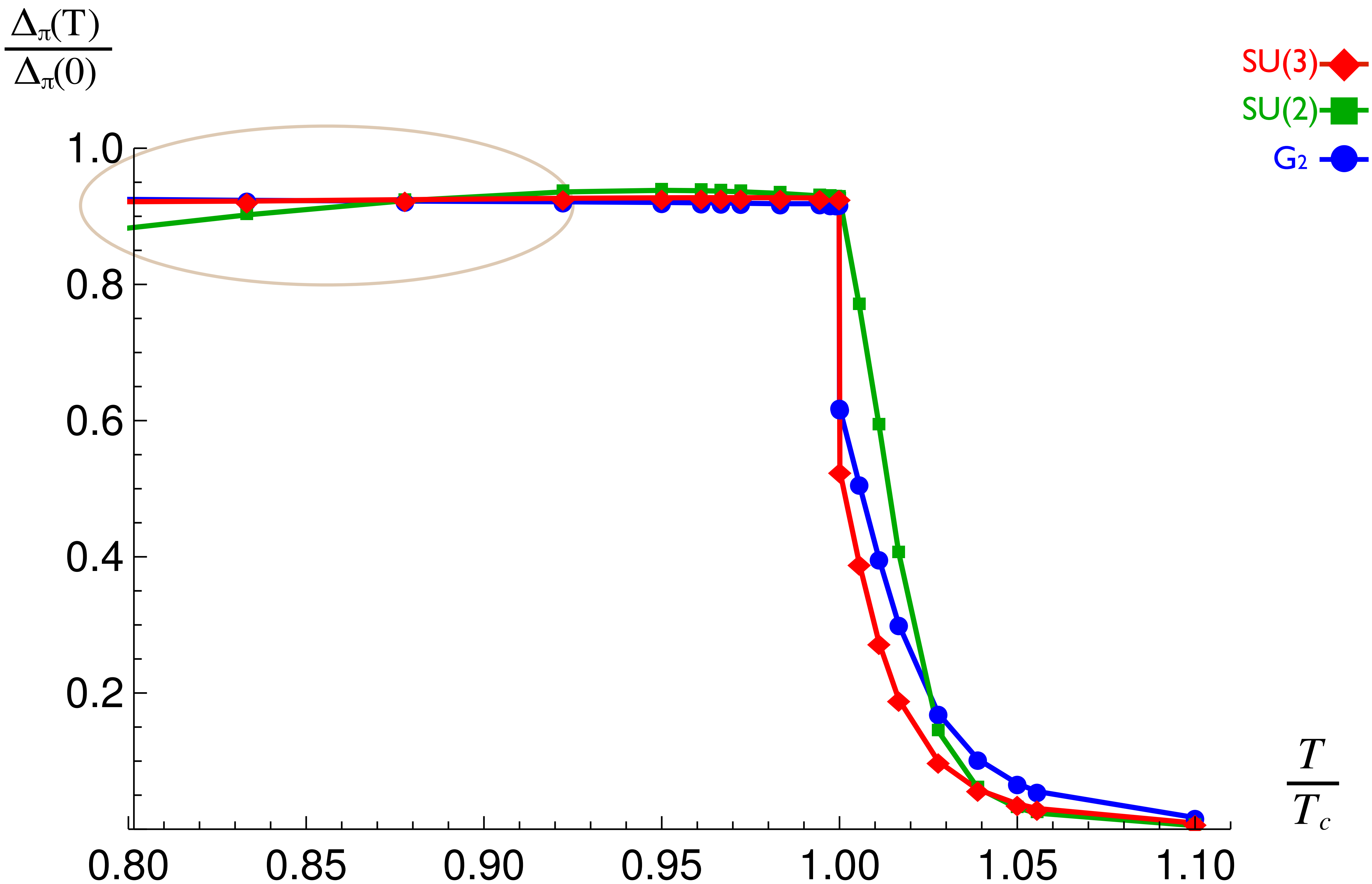
[G. Cossu, M. D'Elia, A. Di Giacomo, B. Lucini, and C. Pica (2007)]

Chiral condensate



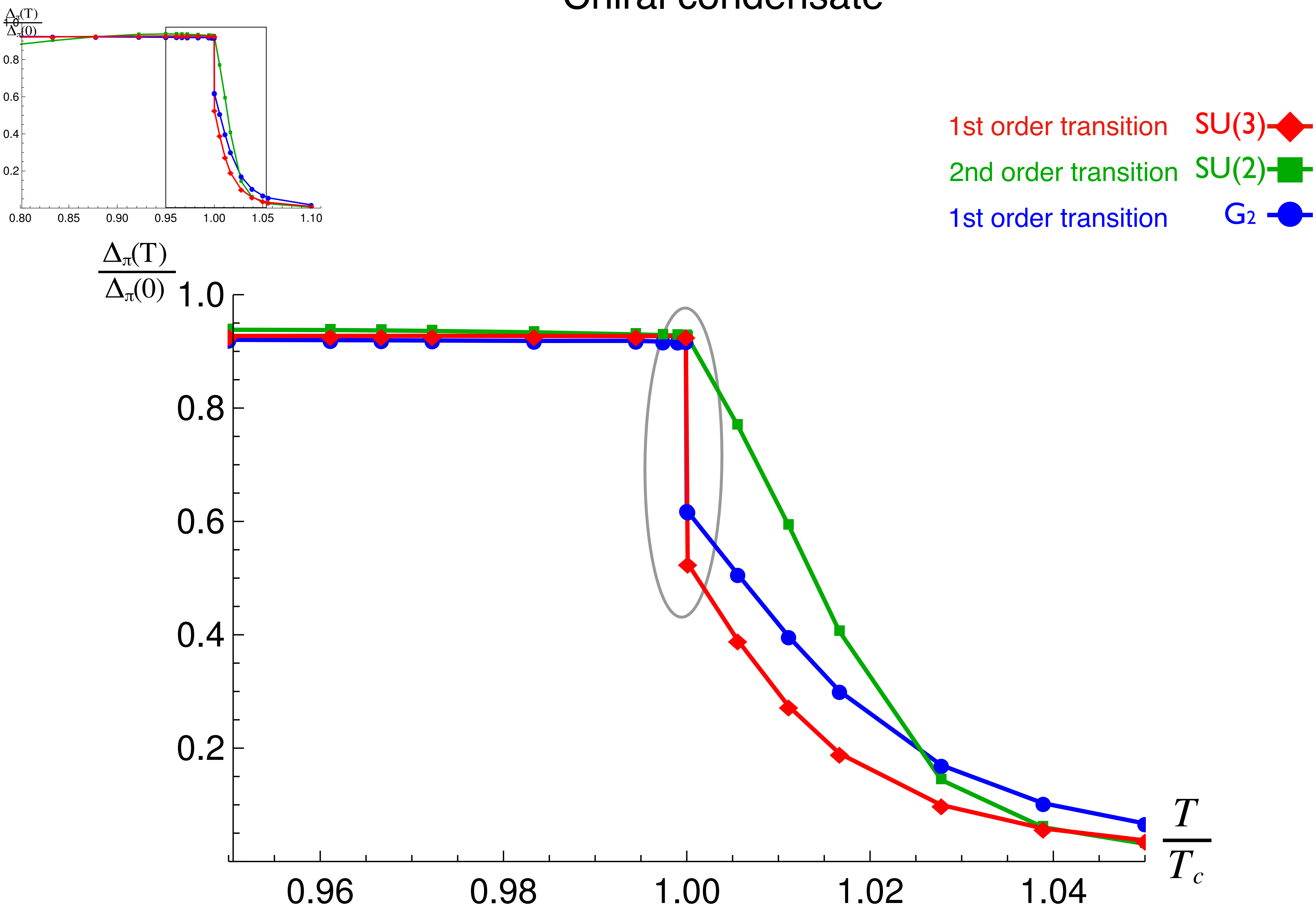
Order parameters

Chiral condensate



Order parameters

Chiral condensate



Order parameters

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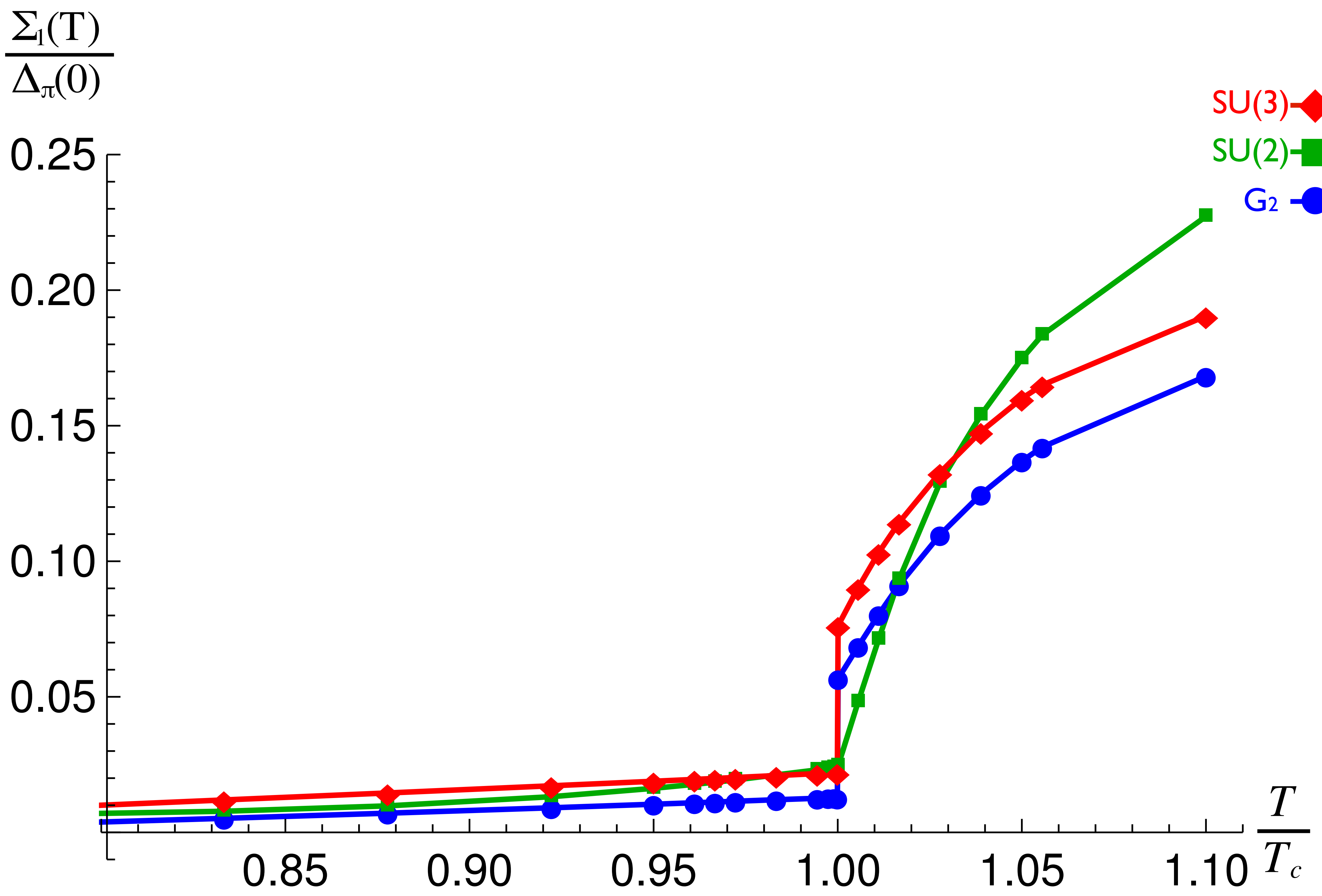

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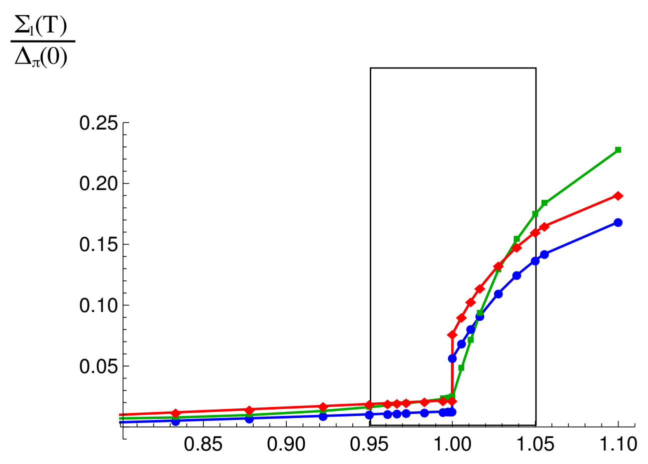
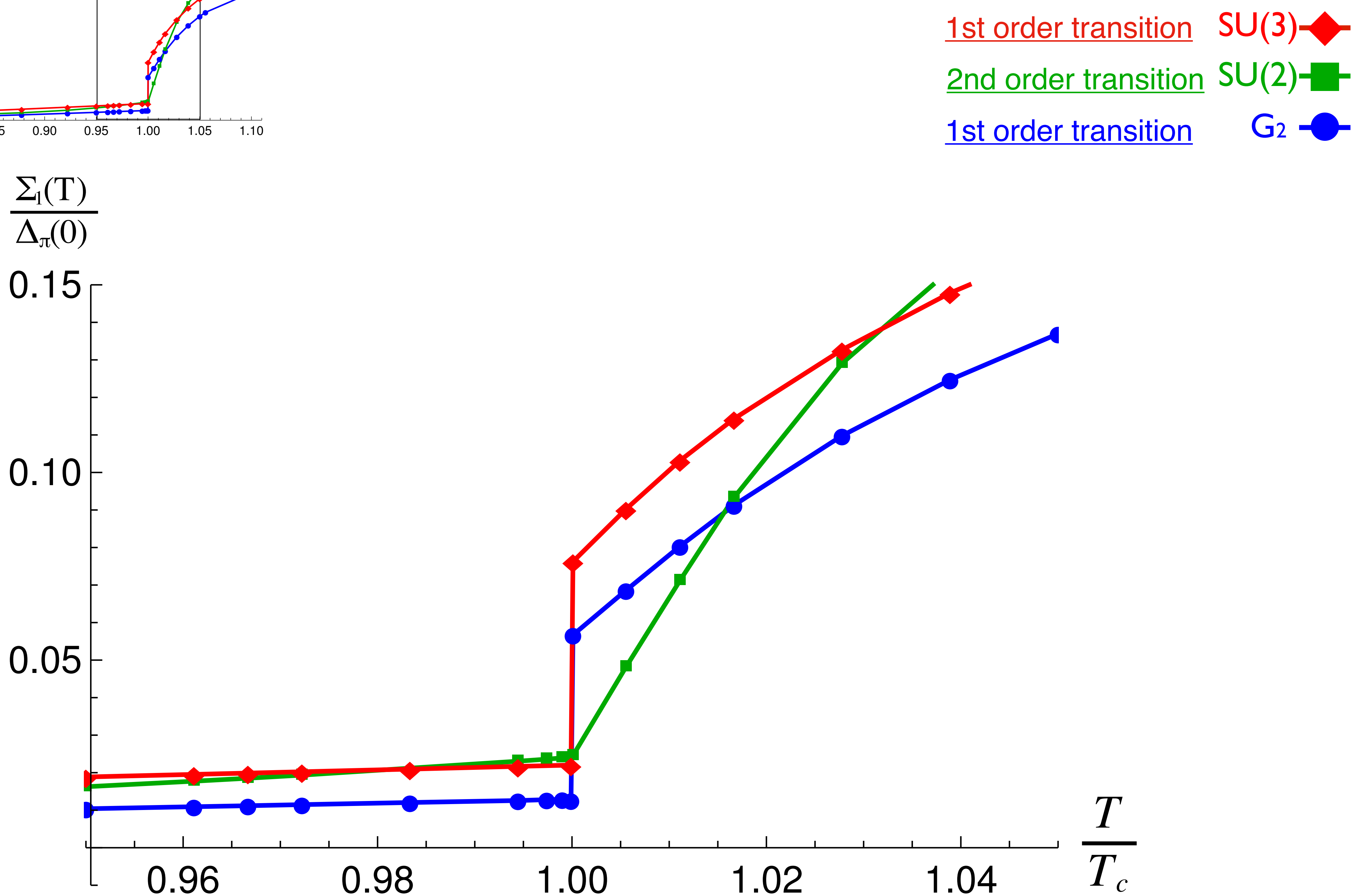
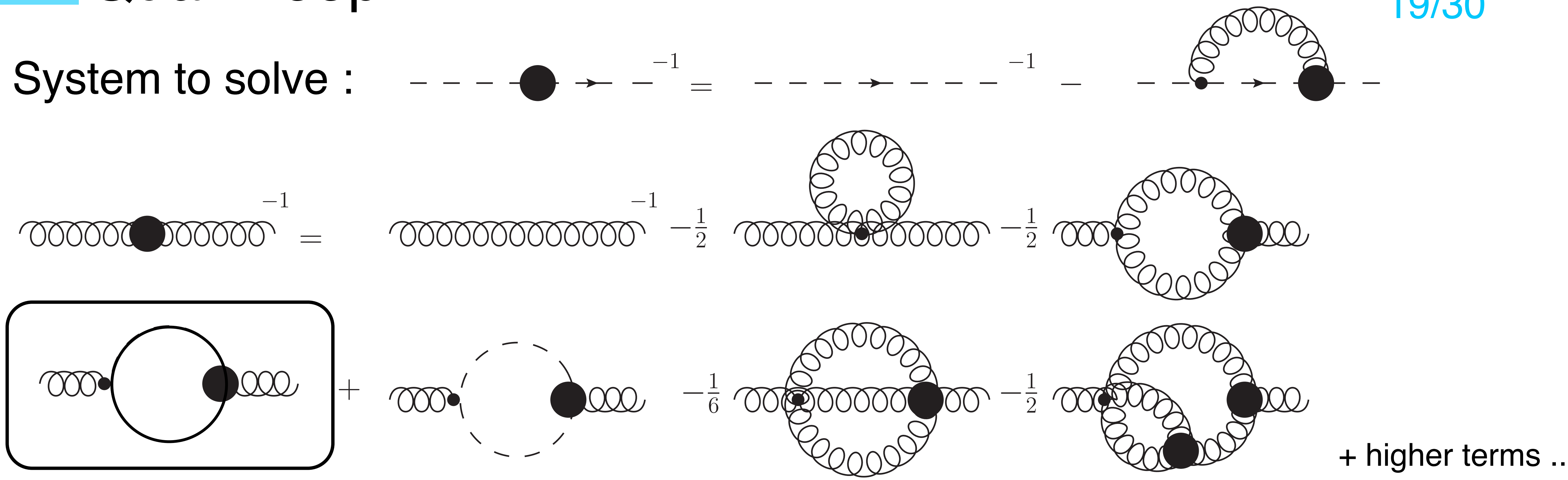


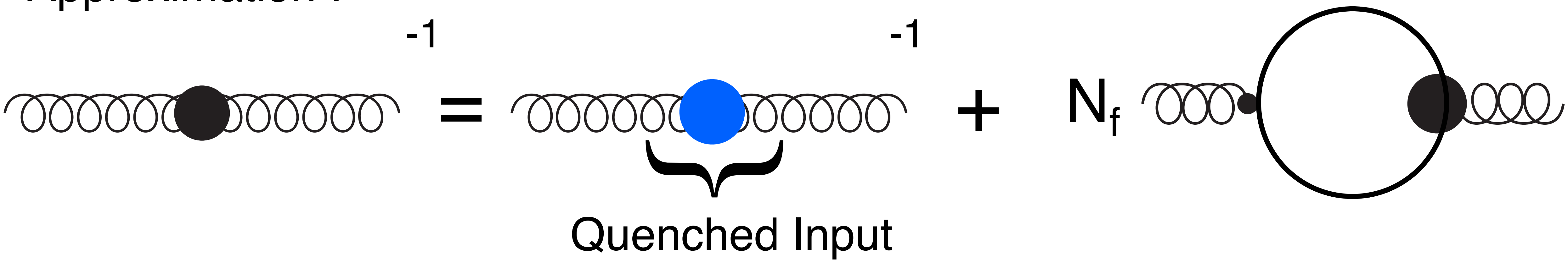


Quark loop

- System to solve :



- Approximation :

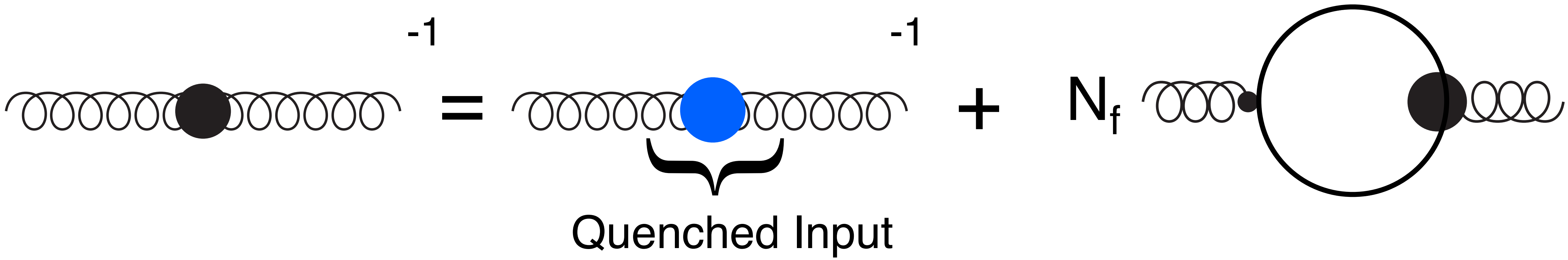


[C.S Fischer , J. Luecker (2012)]

→ Neglect all indirect quark contributions in the gluon dressing

→ Remove spurious divergence with a generalized Brown-Pennington projector

- Adding the quark loop

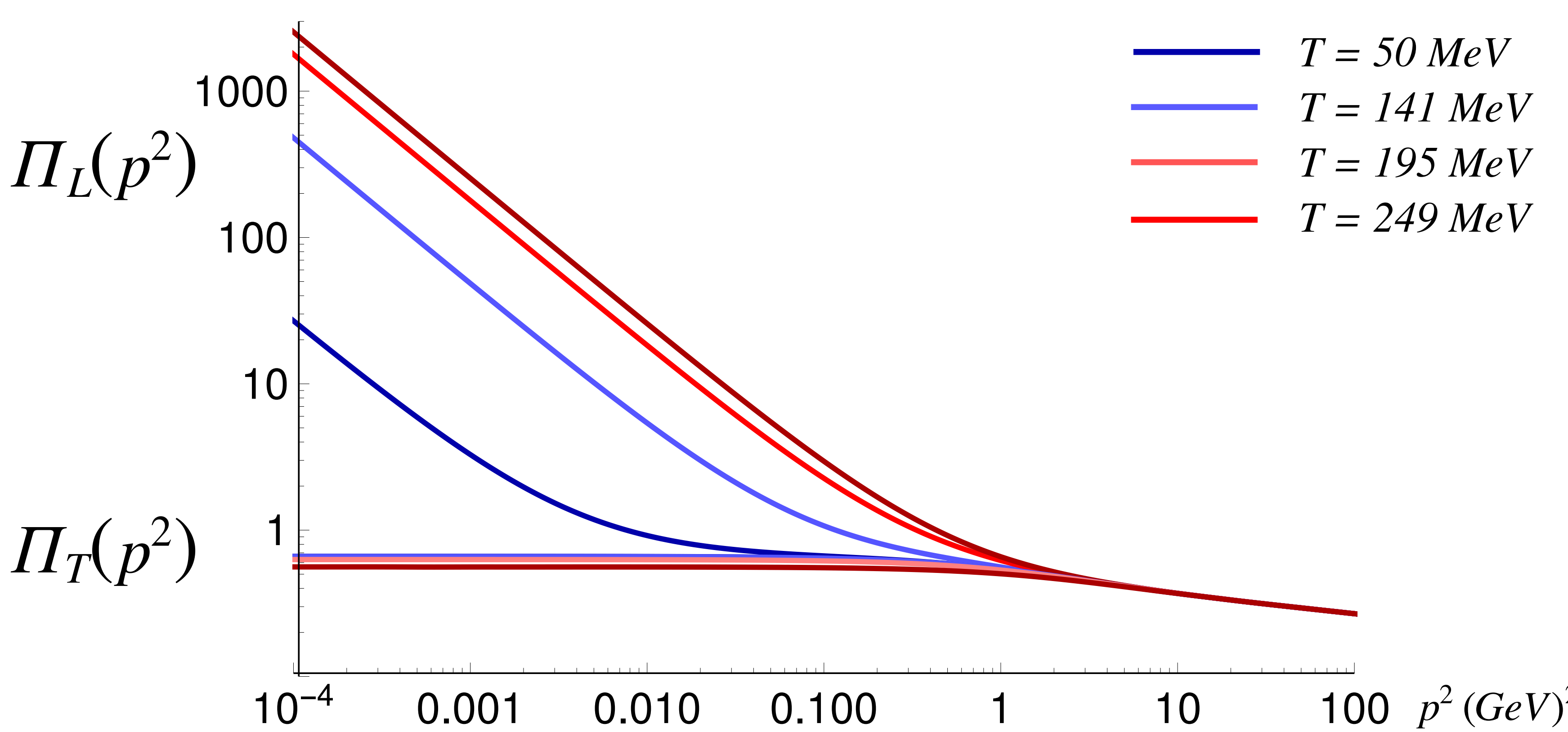


[C.S Fischer , J. Luecker (2012)]

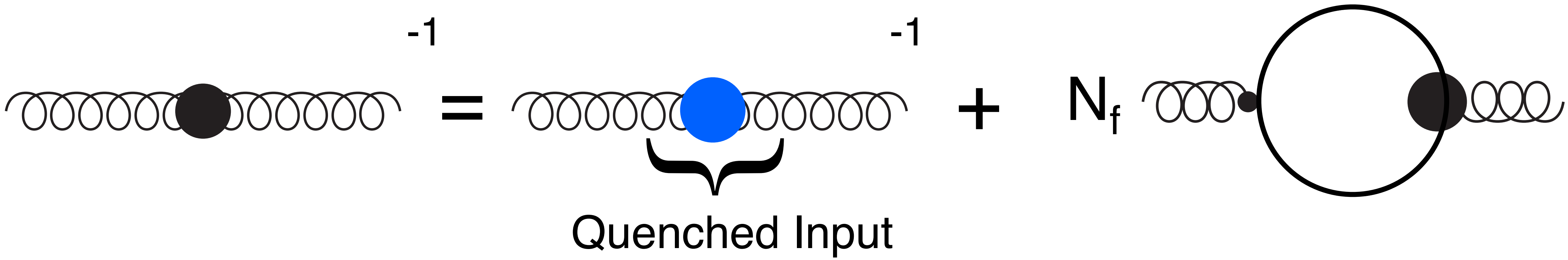
$$\Pi_L(p)p^2 \xrightarrow{p \to 0} (m_{th})^2$$

Debye Screening
of the chromo-electric
charge

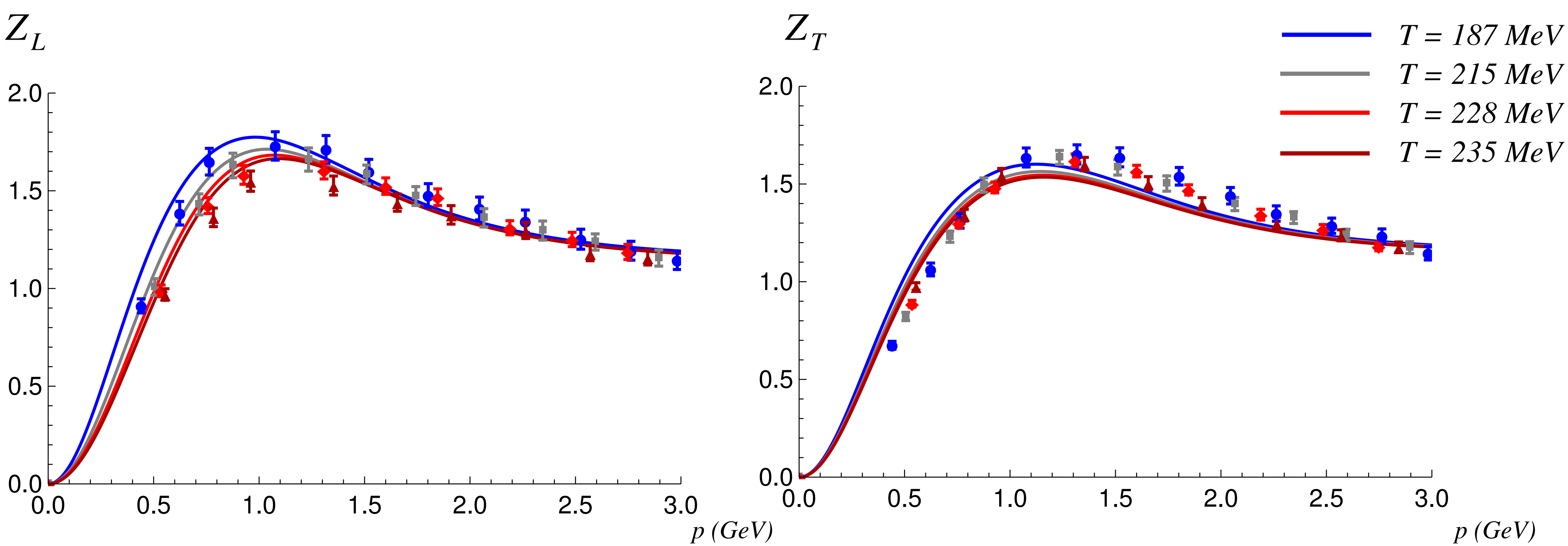
$$\Pi_T(p)p^2 \xrightarrow{p \to 0} 0$$



- Adding the quark loop



First computation made by : *[C.S Fischer , J. Luecker (2012)]*

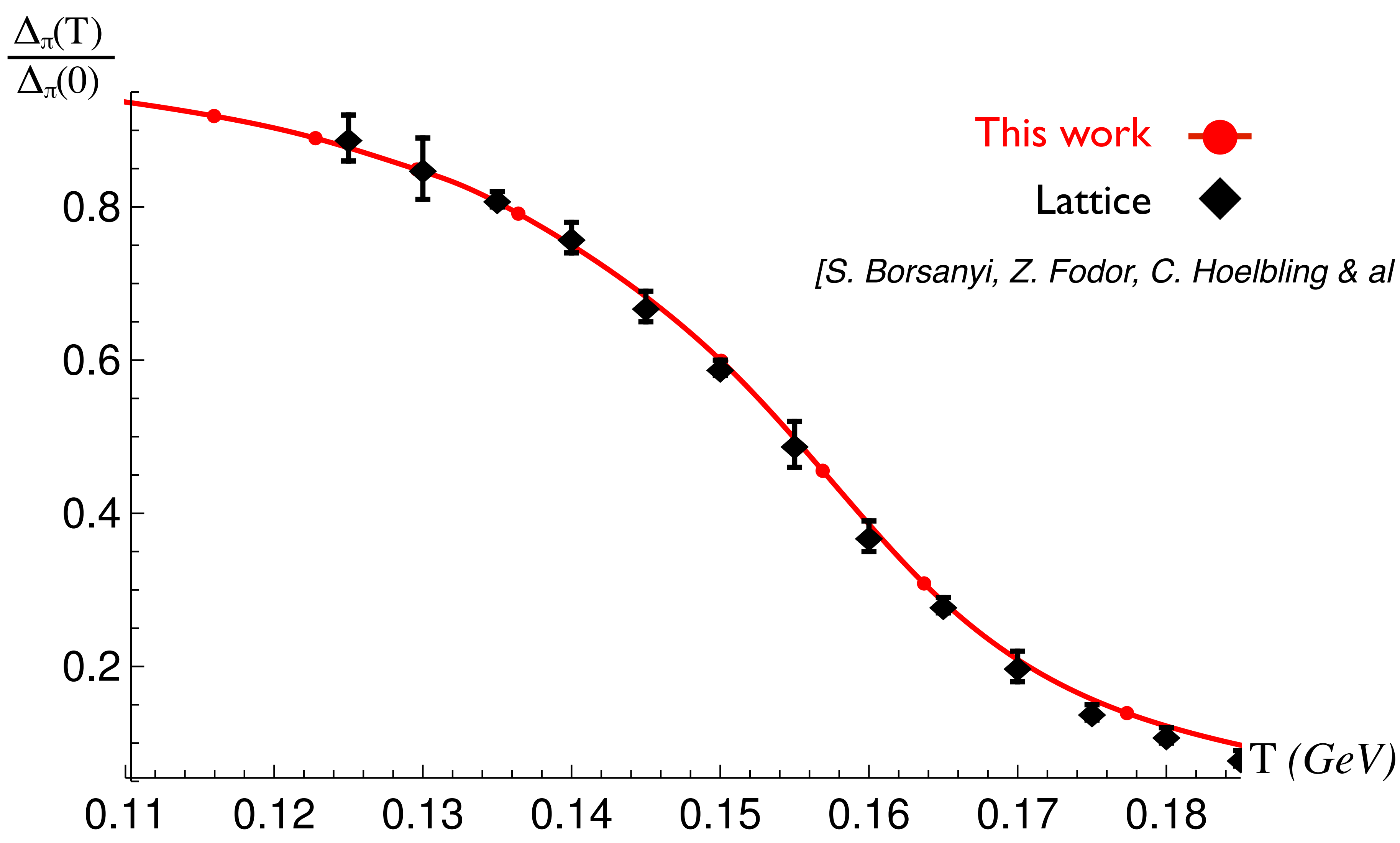


Compared to : *[R.Aouane, F. Burger E.-M. Illgenfritz & al (2012)]*

Chiral condensate

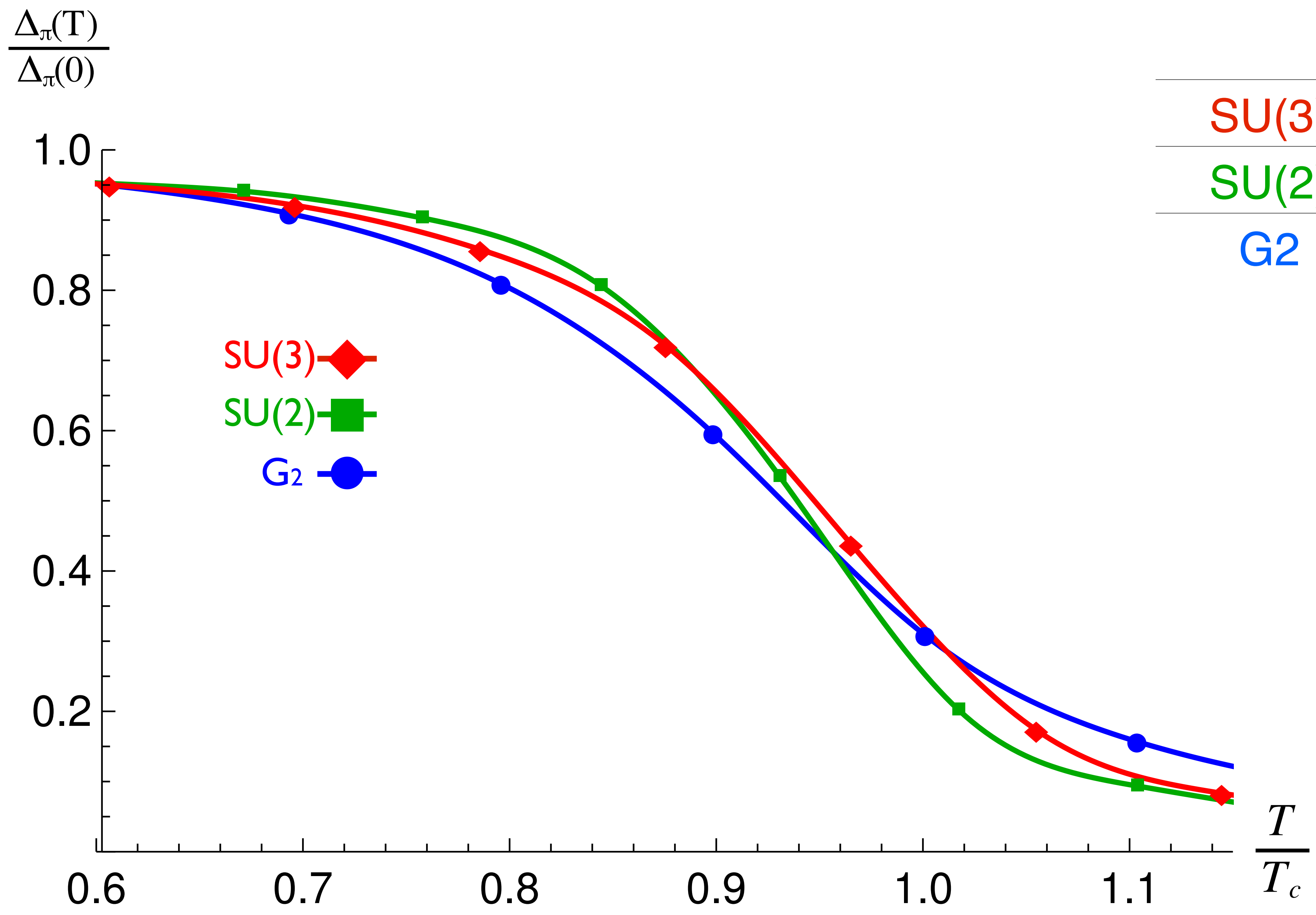
For 2 light flavors and a strange quark, comparison with lattice simulation is possible

First computation made by : *[C.S Fischer , J. Luecker (2012)]*



We can find a set of parameters which reproduce lattice results

Chiral condensate

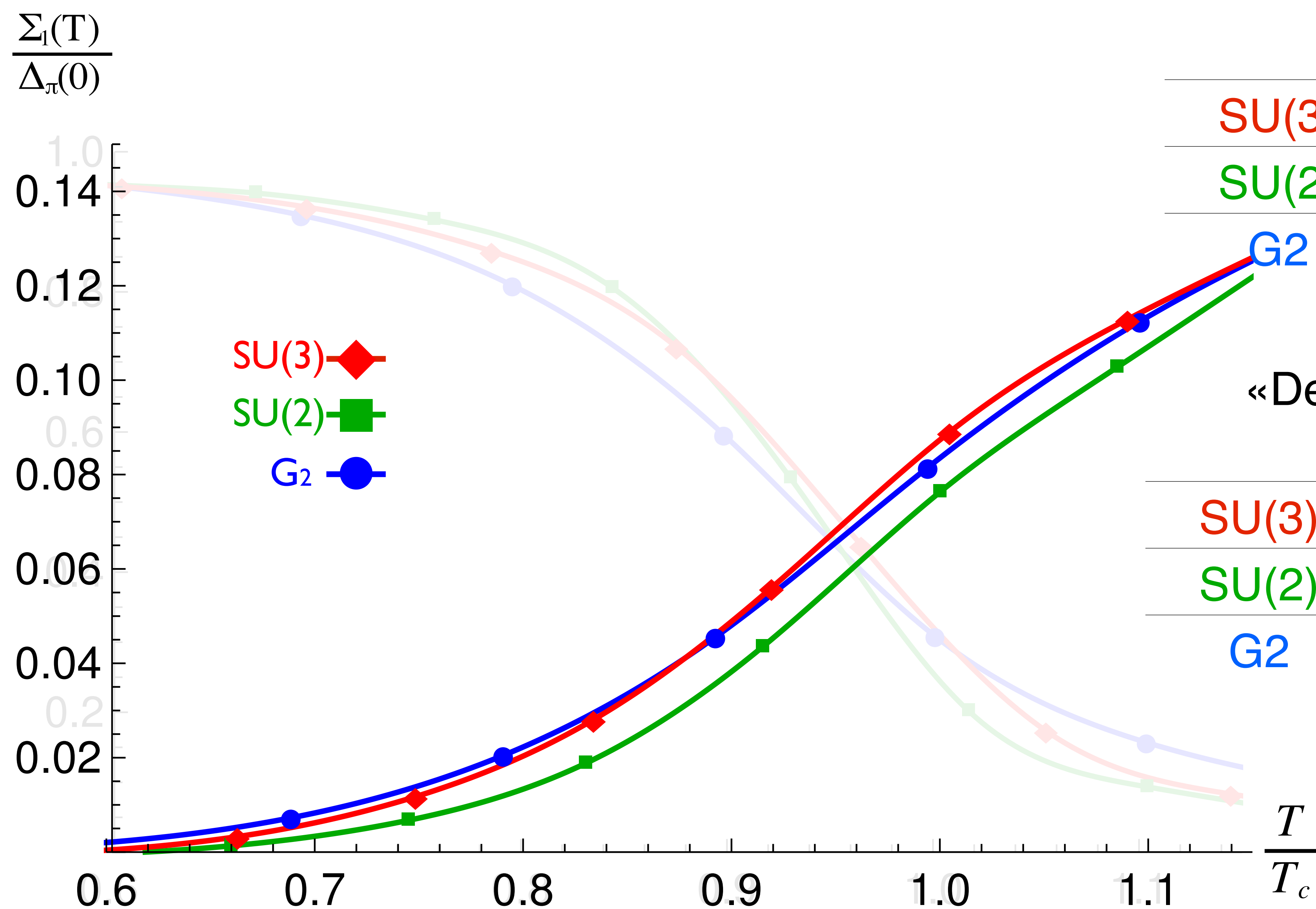


Chiral «restoration» Tc (MeV)

	quenched	2 flavors
SU(3)	277	202
SU(2)	303	229
G2	255	163

Order parameters

Dual condensate



Chiral «restoration» $Tc\ (MeV)$

	quenched	2 flavors
SU(3)	277	202
SU(2)	303	229
G2	255	163

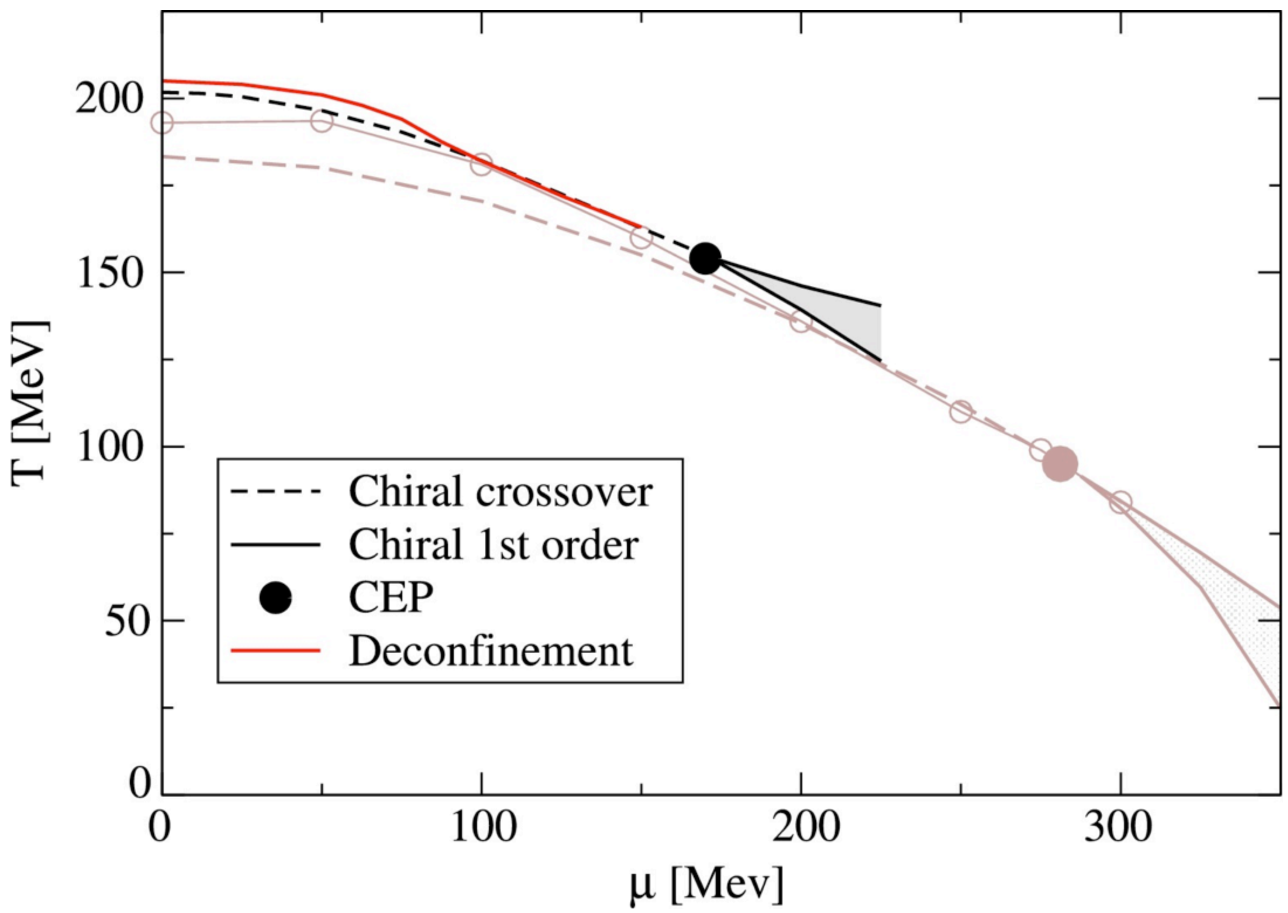
«Deconfinement» (MeV)

	quenched	2 flavors
SU(3)	277	212
SU(2)	303	233
G2	255	164

The confinement/deconfinement transitions and chiral transitions occur approximatively at the same temperatures

- Shift ω_n to $\omega_n + i\ \mu$
- For 2 light flavors,

SU(3)



CEP : (171, 154) MeV

[C.S Fischer , J. Luecker (2012)]

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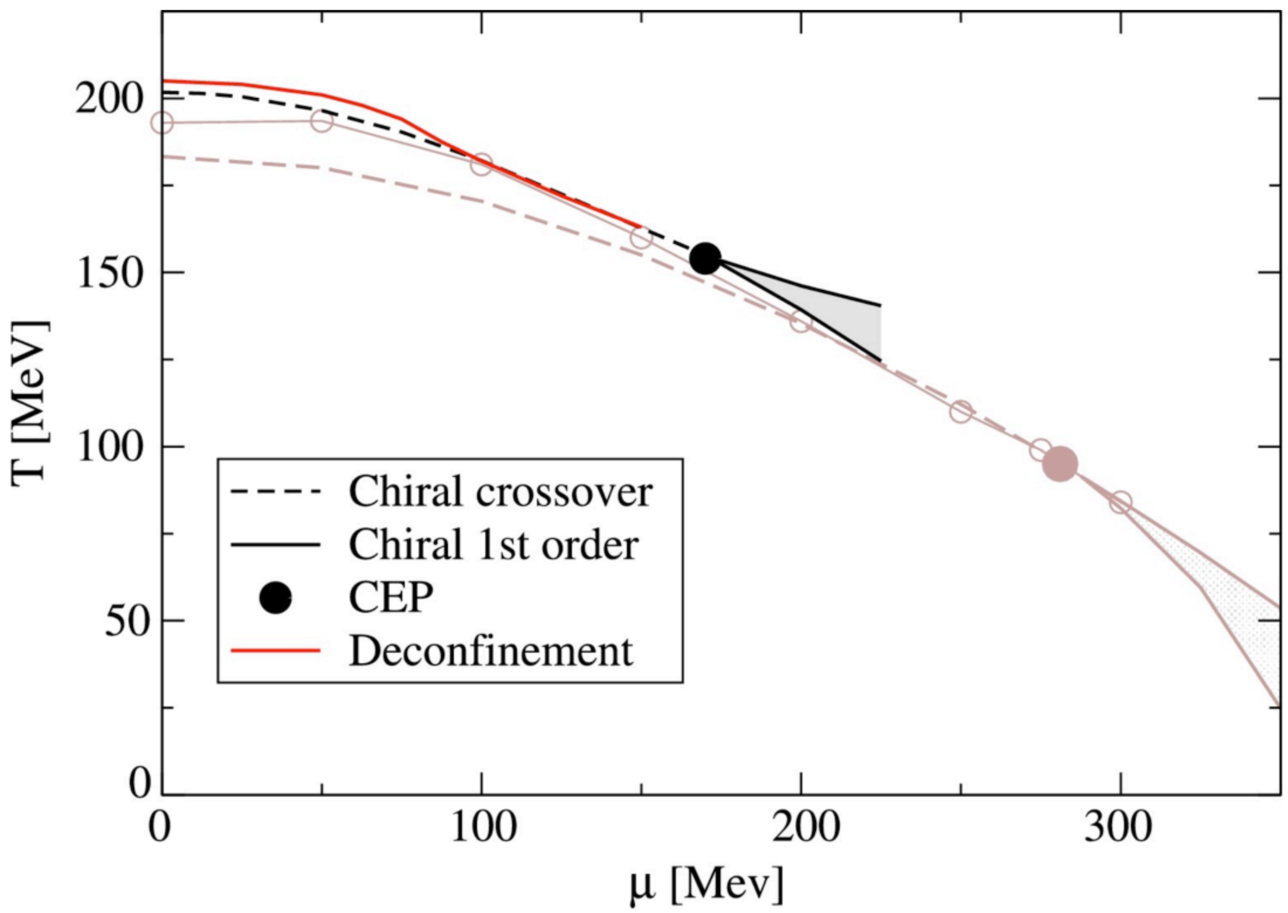
- For 2 light flavors,

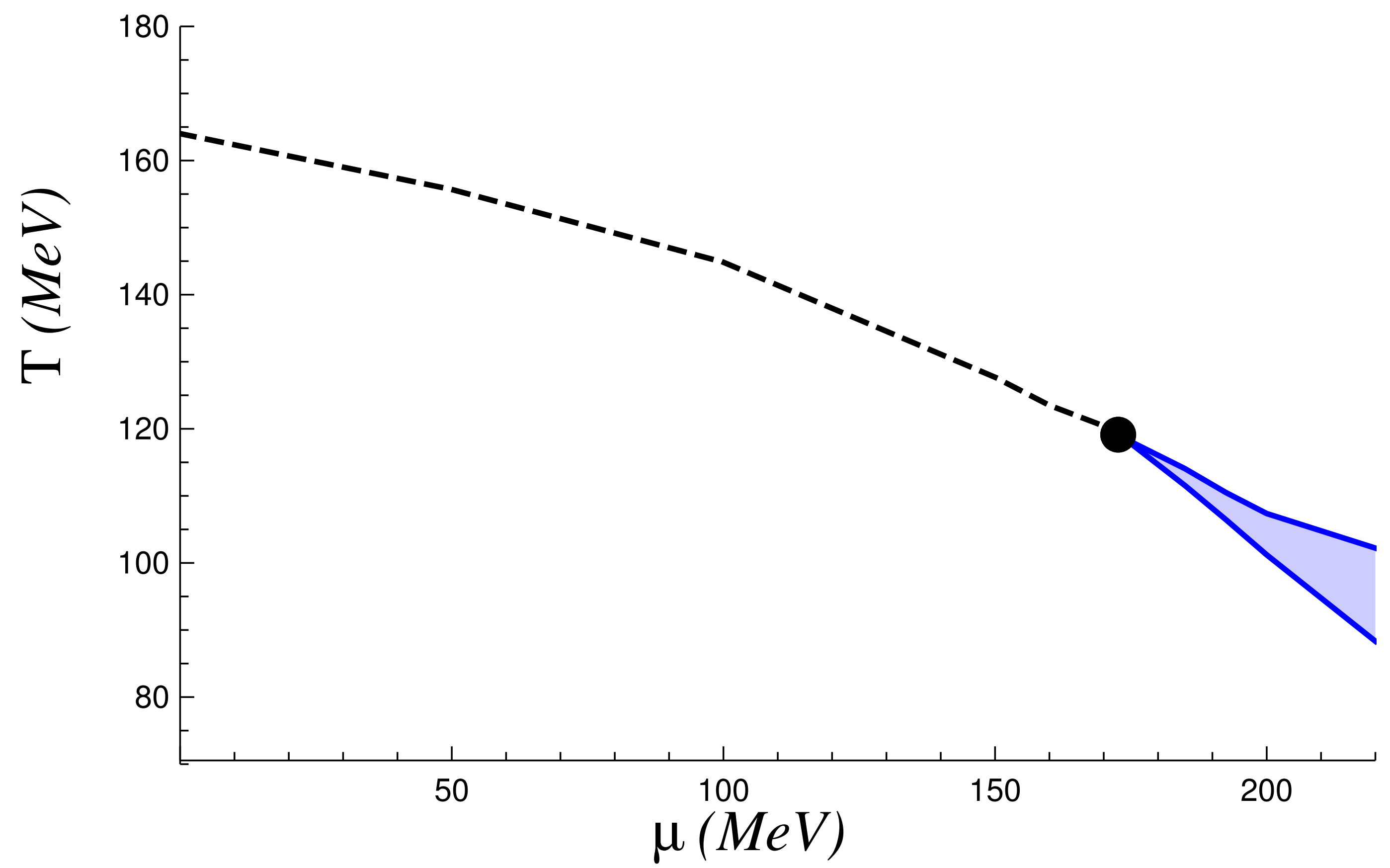
SU(3)



CEP : (171, 154) MeV

[C.S Fischer , J. Luecker (2012)]

Preliminary G_2



CEP : (174, 119) MeV

6 Conclusion

- The *D* dressing function can be neglected in this truncation at μ = 0
- The quenched results show the expected behavior

→ The truncation can be generalised for differents gauge-groups

- Ansatz for the gluon dressing

→ G_2 is good choice for an evaluation of the truncation effects in medium

→ Parameter dependence of our quark-gluon vertex

6 Conclusion

- The *D* dressing function can be neglected in this truncation at $\mu = 0$
- The quenched results show the expected behavior

→ The truncation can be generalised for differents gauge-groups

→ G_2 is good choice for an evaluation of the truncation effects in medium

→ Parameter dependence of our quark-gluon vertex

- An unquenching procedure is possible

→ The qualitative behaviour of the order parameters is respected

- The qualitative behaviour remains the same for differents quark-gluon vertex parameters
- The (pseudo)-critical temperature for chiral and deconfinement are close to each other

→ The (pseudo)-critical temperature depends on the quark-gluon vertex parameters

6 Conclusion

- The *D* dressing function can be neglected in this truncation at $\mu = 0$
- The quenched results show the expected behavior

→ The truncation can be generalised for differents gauge-groups

→ G_2 is good choice for an evaluation of the truncation effects in medium

→ Parameter dependence of our quark-gluon vertex

- An unquenching procedure is possible

→ The qualitative behaviour of the order parameters is respected

→ The (pseudo)-critical temperature depend on the quark-gluon vertex parameters

Thank you

- Software used :
  - ***CrasyDSE***

M.Q. Huber and M. Mitter, "CrasyDSE: A Framework for solving Dyson-Schwinger equations," Comput.phys.commun., vol. 183, pp. 2441–2457, 2012

  - ***DoFun***

M.Q. Huber and J. Braun, "Algorithmic derivation of functional renormalization group equations and Dyson-Schwinger equations," Comput.phys.commun., vol. 183, pp. 1290–1320, 2012

R. Alkofer, M.Q. Huber and K. Schwencher "Algorithmic derivation of Dyson-Schwinger equations," Comput.Phys.Commun. 180 965-976 , 2009