From the Gribov ambiguity to confining effective models

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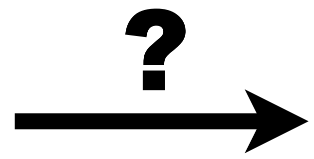
Outline

- Motivation: ***what is the effect of quark confinement in low-energy models?***
- the Gribov(-Zwanziger) approach to quantize Yang-Mills theories beyond PT
- A confining quark model
- Thermodynamics of confined quarks and magnetic pressure
- Final comments and perspectives

Motivation: the confinement problem

Fundamental degrees of freedom are *unphysical*: not part of the spectrum;



Physical spectrum of bound states dynamically generated at low energies.

[Yang-Mills gauge theories] and QCD:

$$\mathcal{L}^{\mathrm{E}}_{\mathrm{YM}} = \left[\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}\right] + \overline{\Psi}_f[\gamma_\mu D_\mu + m_f]\Psi_f$$

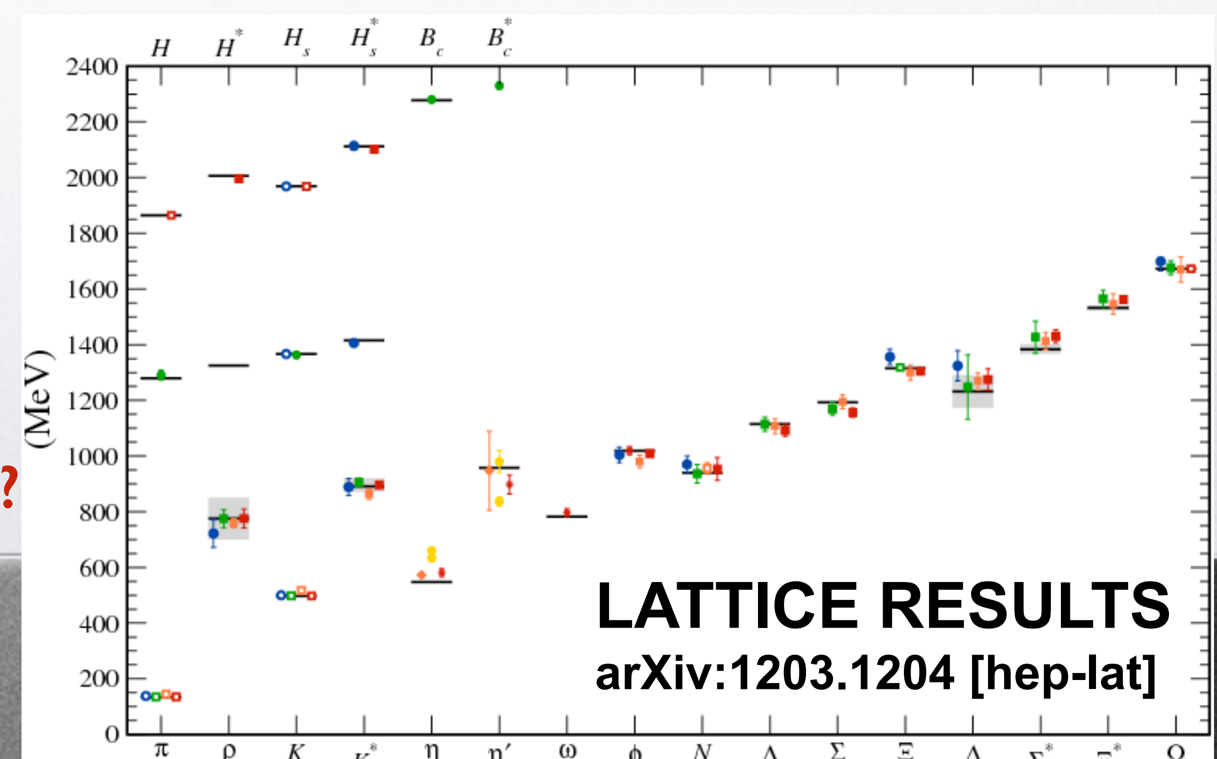
[Gluons] and quarks

[Glueballs,] baryons and mesons

➠ What is the mechanism??

➠ What happens to quarks and gluons in the IR??

➠ **How does confinement affect low-energy QCD models (effective actions) written in terms of quarks?**



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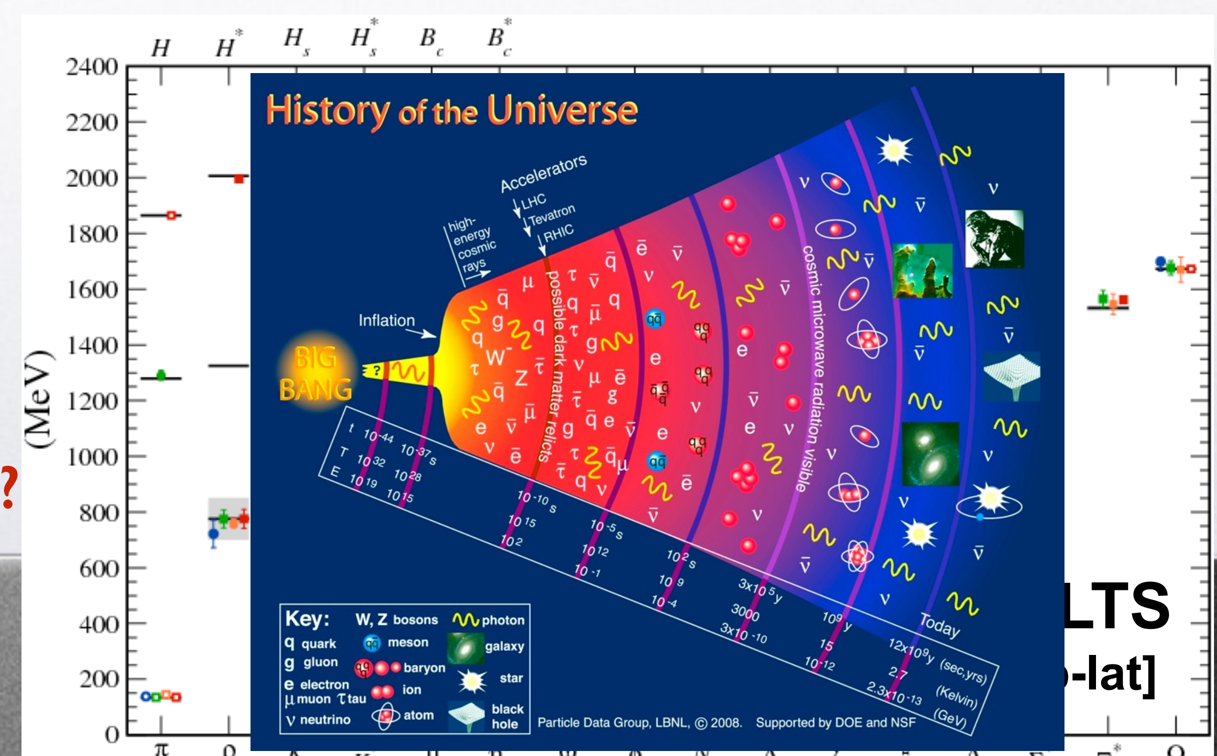

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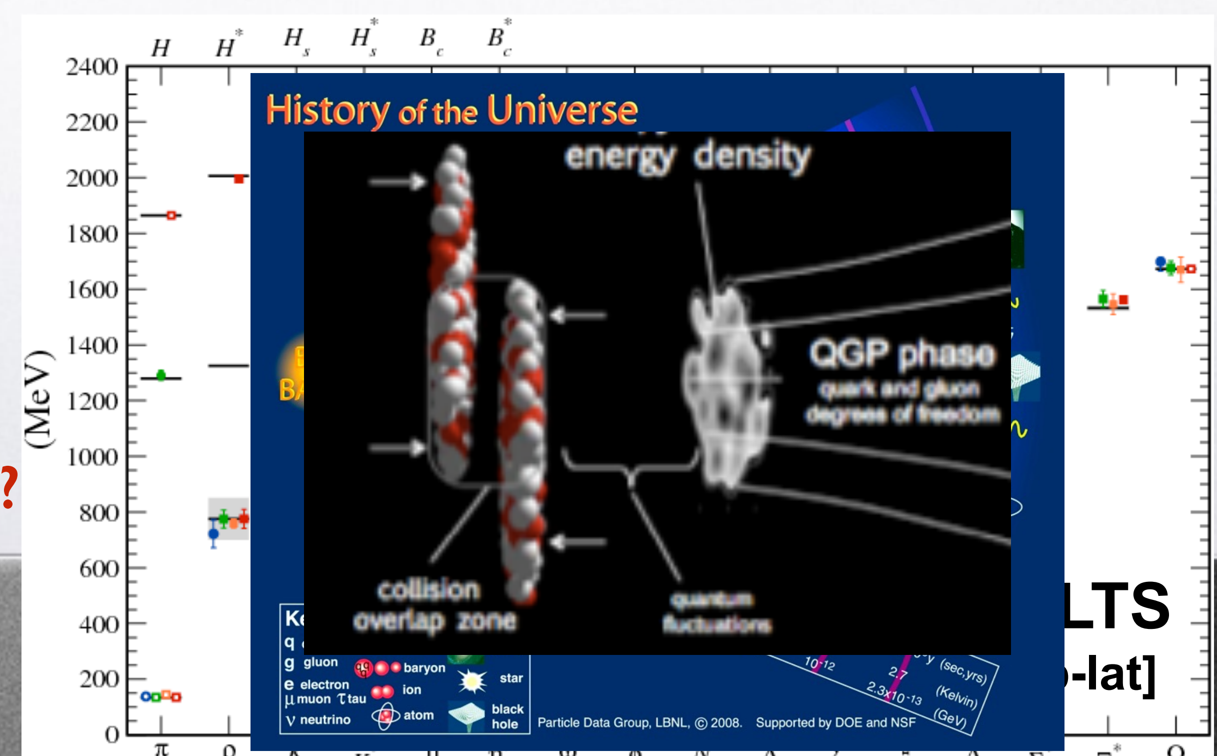

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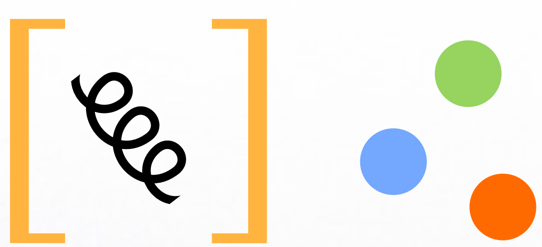

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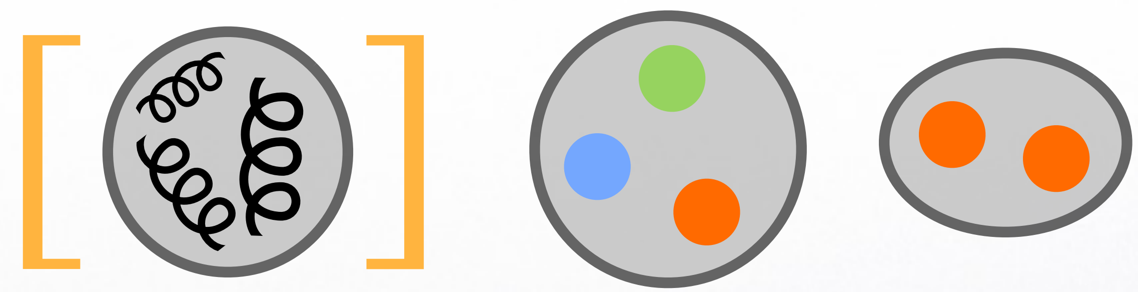

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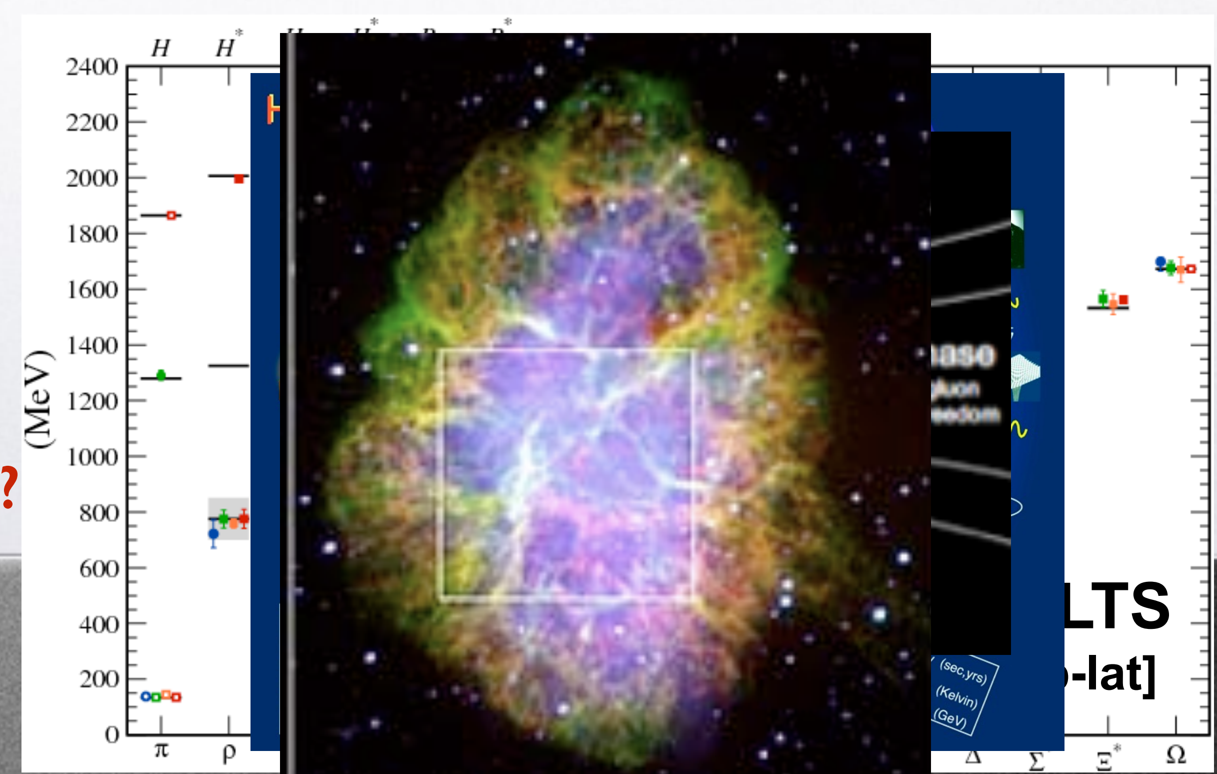


[Gribov (1978)]

The Gribov problem:

- In the Landau gauge, for instance, the theory assumes the form

$$\int \mathcal{D}A\mathcal{D}\bar{c}\mathcal{D}c\mathcal{D}b\, e^{-S_{YM}+S_{gf}}$$

$$S_{gf} = b^a\partial_\mu A^a_\mu - \bar{c}^a\mathcal{M}^{ab}c^b\,, \qquad \mathcal{M}^{ab} = -\partial_\mu\left(\delta^{ab}\partial_\mu + gf^{abc}A^c_\mu\right)$$

- Gribov copies $\rightarrow$ zero eigenvalues of the Faddeev-Popov operator $\mathcal{M}^{ab}$.

- Copies cannot be reached by small fluctuations around $A = 0$ (perturbative vacuum) $\rightarrow$ pertubation theory works.

- Once large enough gauge field amplitudes have to be considered (non-perturbative domain) the copies will show up enforcing the effective breakdown of the Faddeev-Popov procedure.

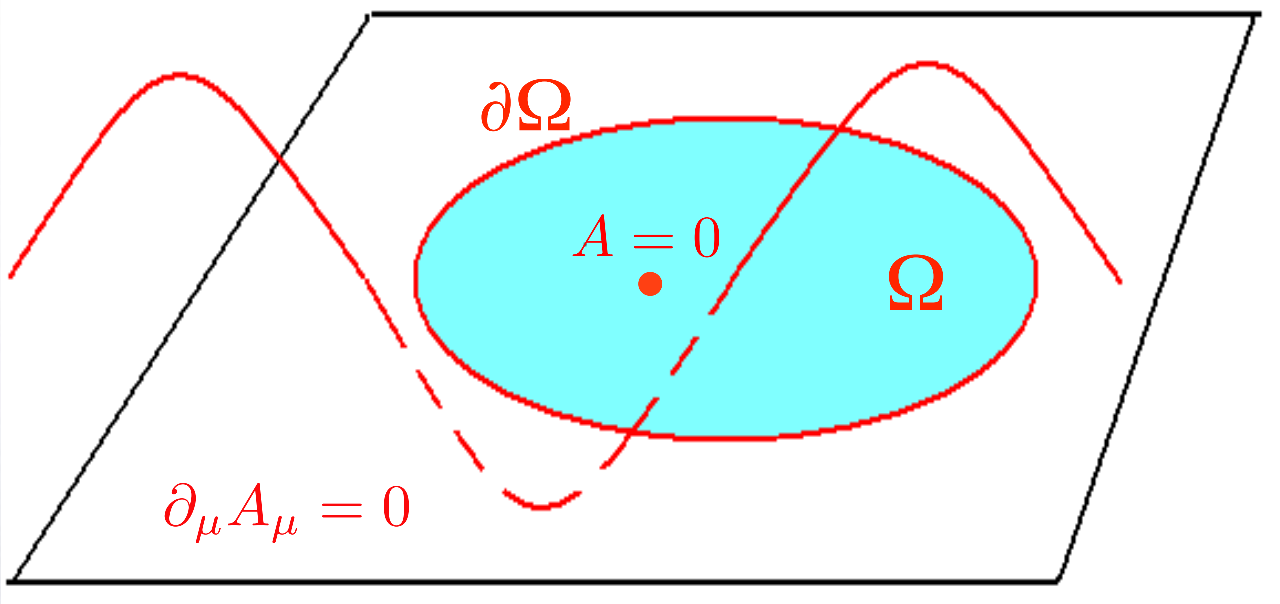
- Gribov proposed a way to eliminate (infinitesimal) Gribov copies from the integration measure over gauge fields: **the restriction to the (first) Gribov region Ω**

$$\int [DA]\delta(\partial A)\det(\mathcal{M})e^{-S_{\rm YM}} \longrightarrow \int_\Omega [DA]\delta(\partial A)\det(\mathcal{M})e^{-S_{\rm YM}} \qquad S_{\rm YM} = \frac{1}{4}\int_x F^2$$

with $\Omega = \{A^a_\mu \; ; \; \partial A^a = 0, \mathcal{M}^{ab} > 0\}$

$$\mathcal{M}^{ab} = -\partial_\mu\left(\delta^{ab}\partial_\mu + f^{abc}A^c_\mu\right) = -\partial_\mu D^a_\mu$$

(Faddeev-Popov operator)



- The **restriction** can be implemented as a **gap equation** for the vacuum energy obtained as: [Zwanziger (1989,...)]

$$Z = e^{-V\mathcal{E}(\gamma)} = \int \mathcal{D}A\, \delta(\partial A)\ \det \mathcal{M}\ e^{-\left(S_{YM} \underbrace{+\gamma^4 H(A) - \gamma^4 V D(N^2-1)}_{=:\,+\gamma^4 \mathcal{H}}\right)}$$

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Gap equation: $$\frac{\partial\mathcal{E}(\gamma)}{\partial\gamma} = 0 \Rightarrow \langle H(A)\rangle_{1PI} = VD(N^2-1)$$

$$H(A) = \int_p\int_q A^a_\mu(-p)\left(\mathcal{M}^{ab}\right)^{-1} A^b_\mu(q)$$

The Gribov-Zwanziger action

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- Using auxiliary fields, this can be cast in a *local* form: $Z = \int [\mathcal{D}\Phi]\ \delta(\partial A)\ \det \mathcal{M}\ e^{-S_{\rm GZ}}$

- The GZ theory is unstable against the formation of certain dimension 2 condensates, giving rise to a refinement of the effective IR action:

[Dudal et al (2008)]

$$S_{\rm YM} \xrightarrow[\text{Gribov restriction(UV}\to\text{IR)}]{} S_{\rm GZ} = S_{\rm YM} + \gamma^4 \mathcal{H}$$

Dynamical generation of dim.2 condensates

$$S_{\rm RGZ} = S_{\rm YM} + \gamma^4 \mathcal{H} + \frac{m^2}{2} AA - M^2 \left(\overline{\varphi}\varphi - \overline{\omega}\omega\right)$$

The Refined Gribov-Zwanziger action

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Gap equation for the Gribov param.:

$$\frac{\partial \mathcal{E}(\gamma)}{\partial \gamma} = 0 \Rightarrow \langle H(A) \rangle_{1PI} = VD(N^2 - 1)$$

The parameters *M* and *m* are obtained via minimization of an effective potential for:

$$\langle \overline{\varphi}\varphi - \overline{\omega}\omega \rangle \neq 0 \qquad \langle A^2 \rangle \neq 0$$

- Non-perturbative effects included: $(\gamma, M, m) \propto \mathrm{e}^{-\frac{1}{g^2}}$

A checklist for RGZ

- ✓ *(can be cast in a) local and renormalizable action*
- ✓ *reduces to QCD (pure gauge) at high energies*
- ✓ *consistent with gluon 'confinement': confining propagator (no physical propagation; violation of reflection positivity)*
- ✓ *consistent with lattice IR results ?*

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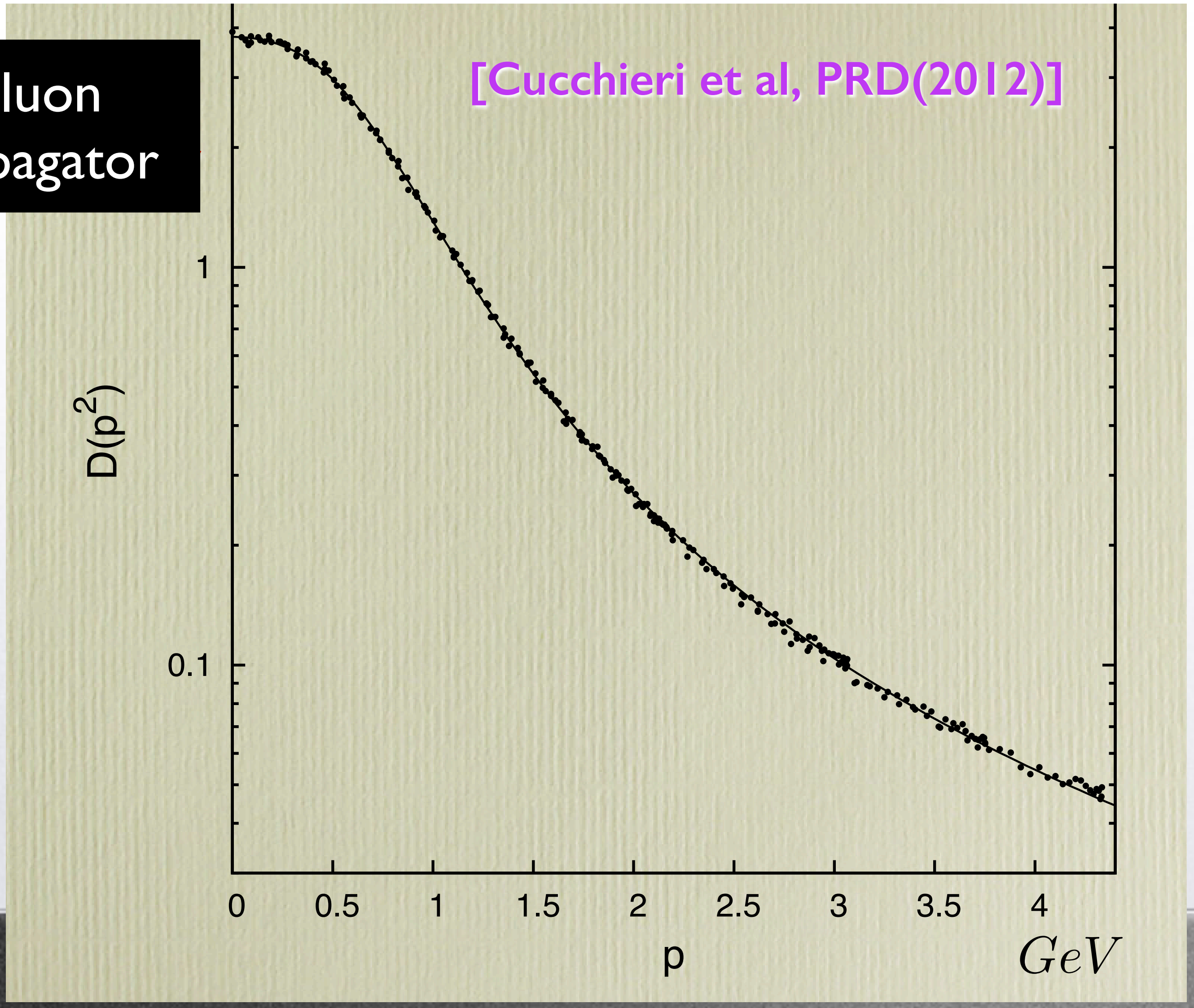
Gluon Propagator



$$\langle A^a_\mu A^b_\nu \rangle_p = \delta^{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) D(p^2)$$

$$D_{\rm fit}(p^2) = C\,\frac{p^2+s}{p^4+u^2p^2+t^2}$$

$$C = 0.56(0.01)\,,\ u = 0.53(0.04)\,\mathrm{GeV}\,,$$

$$t = 0.62(0.01)\,\mathrm{GeV}^2\,,\ u = 2.6(0.2)\,\mathrm{GeV}^2$$

poles: $m^2_\pm \ = \ (0.352 \pm 0.522i)\mathrm{GeV}^2$

$$D_{\rm RGZ}(p^2) = \frac{p^2+M^2}{p^4+(M^2+m^2)p^2+2g^2N\gamma^4}$$

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- ✓ *physical spectrum of bound states ??*

Glueball masses are obtained by computing two-point correlation functions of composite operators with the appropriate quantum numbers and casting them in the form of a Källén-Lehmann spectral representation.

J^{PC}	confining gluon propagator
0^{++}	2.27
2^{++}	2.34
0^{-+}	2.51
2^{-+}	2.64

[Dudal,Guimaraes,Sorella, PRL(2011), PLB(2014)]

-Lattice: (1) Y. Chen *et al.* PRD **73**, 014516 (2006)
-Flux tube model: M. Iwasaki *et al.* PRD **68**, 074007 (2003).
-Hamiltonian QCD: A. P. Szczepaniak and E. S. Swanson, PLB **577**, 61 (2003).
-AdS/CFT: K. Ghoroku, K. Kubo, T. Taminato and F. Toyoda, arXiv:1111.7032.
-AdS/CFT: H. Boschi-Filho, N. R. F. Braga JHEP 0305, 009 (2003)

J^{PC}	Lattice	Flux tube model	Hamiltonian QCD	ADS/CFT
0^{++}	1.71	1.68	1.98	1.21
2^{++}	2.39	2.69	2.42	2.18
0^{-+}	2.56	2.57	2.22	3.05
2^{-+}	3.04	–	–	–

RGZ: Correct hierarchy of masses

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✓ *(can be cast in a) local and renormalizable action*

✓ *reduces to QCD (pure gauge) at high energies*

✓ *consistent with gluon 'confinement': confining propagator (no physical propagation; violation of reflection positivity)*

✓ *consistent with lattice IR results (propagators)* [Cucchieri et al, PRD(2012)]

✓ *physical spectrum of bound states? Glueballs w/ masses compatible w/ lattice* [Dudal,Guimaraes,Sorella, PRL(2011), PLB(2014)]

✓ *thermodynamics and phase structure:*

A Polyakov-loop potential is computed and a phase transition of the right order is obtained for $SU(2)_C$! [Canfora *et al* EPJC75 (2015) 326]

Related studies: Curci-Ferrari/Landau-DeWitt massive gauges [Reinosa, Serreau, Tissier, Wschebor, PRD93 (2016) 105002, PRD92 (2015) 025021 *[heavy quarks]*, PRD91 (2015) 045035]

✓ *exact BRST invariance, with a modified BRST symmetry.*

Extension to the matter sector: COLOR confinement

- *How can this be extended to the confined matter sector?*

PROPOSAL / ANSATZ: Faddeev-Popov operator as a carrier of color confinement.

A general gauge-matter IR action reads: $$S_{\rm IR} = S_{\rm YM} + \sum_F M_F \mathcal{H}_F$$

$$\mathcal{H}_F = -g^2 \int \frac{d^D p}{(2\pi)^D} \int \frac{d^D q}{(2\pi)^D} \left(T^b\right)^{ij} F^j(-p) \left(\mathcal{M}^{-1}\right)^{bc}_{pq} \left(T^c\right)^{ik} F^k(q) \,.$$

where F^j stands for a fundamental confined field $= \{A^a_\mu, \phi^a, \psi^i\}$

➡ Can be localized, resulting in a renormalizable action that reduces to QCD (quark case) in the UV.

➡ Tree-level propagator which fits lattice data and violates positivity

➡ Can be applied to SUSY theories, like N=1 SYM. [Capri *et al*, EPJC (2014)]

➡ Can be used to estimate meson masses [Dudal, Guimaraes, LFP, Sorella, Annals Phys. (2016)]

Propagators for confined matter

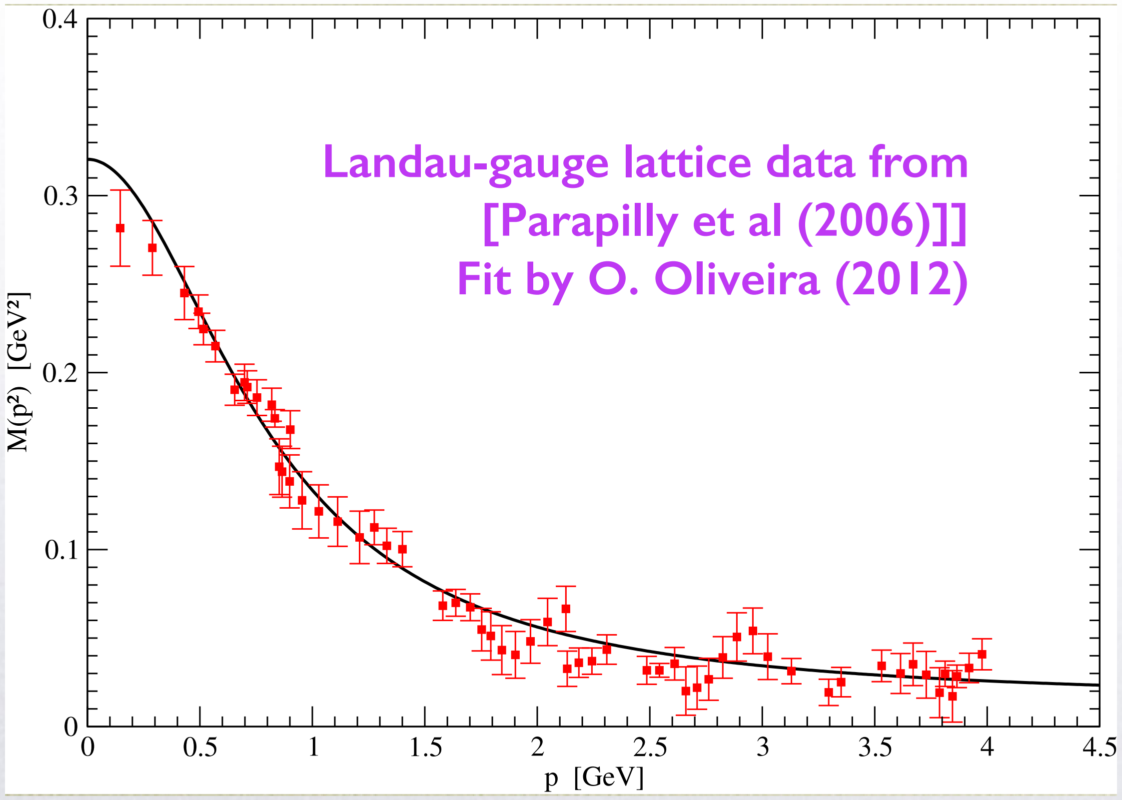
Quark case:

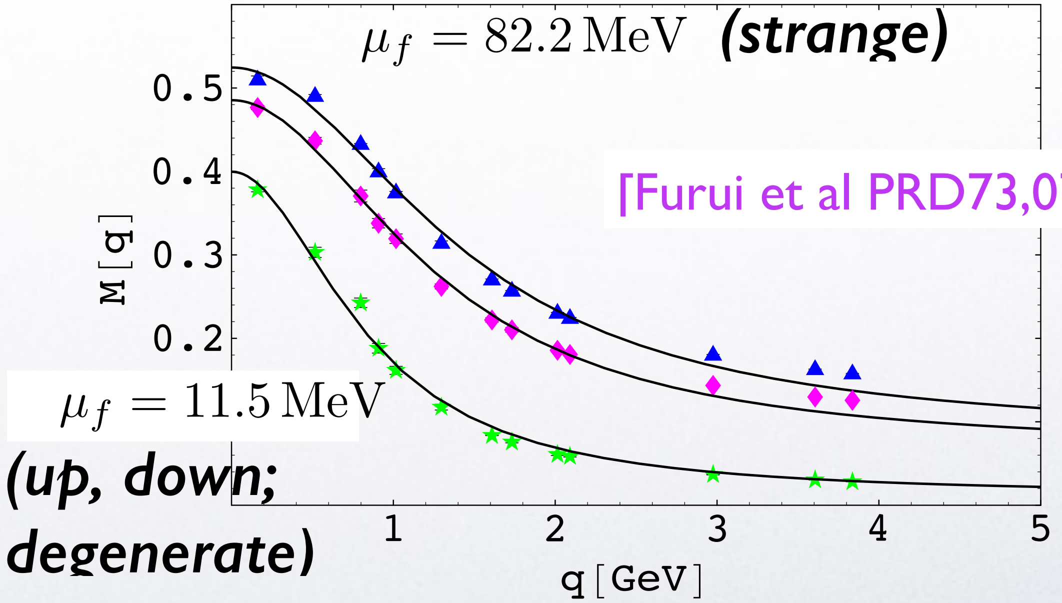
$$\langle \psi^i(k)\bar{\psi}^j(-k)\rangle = \delta^{ij}\,\frac{-ik_\mu\gamma_\mu + \mathcal{A}(k^2)}{k^2+\mathcal{A}^2(k^2)}\,,$$

where

$$\mathcal{A}(k^2) = m_\psi + \frac{g^2\sigma^3 C_F}{k^2+\mu_\psi^2}\,,$$

where m_ψ is the fermion mass and μ_ψ is the condensate mass parameter.





➡ With fit values, the propagator displays:
one real pole and two complex-conjugated ones

Thermodynamics of confined quarks

The simplest setup to test for consequences of this approach of quark confinement:

interactions enter only through the non-perturbative background/propagator

$$\mathcal{L}_{IRq} = \bar{\psi}\left[i\partial\!\!\!/ - \mathcal{M}(\partial^2)\right]\psi \qquad \mathcal{M}(\partial^2) = m_0 + \frac{M_3}{-\partial^2 + m^2}$$

- Good description of Landau-gauge lattice quark mass function
  - In the UV: reduces to free quarks
  - In the IR: positivity violation/ absence from spectrum
- Systematic improvements are possible:

  e.g. perturbative expansion in powers of g around the nontrivial vacuum!

- Even though non-perturbative, it is a quadratic model:

  Exact partition function, extracted from determinant of nonlocal operator:

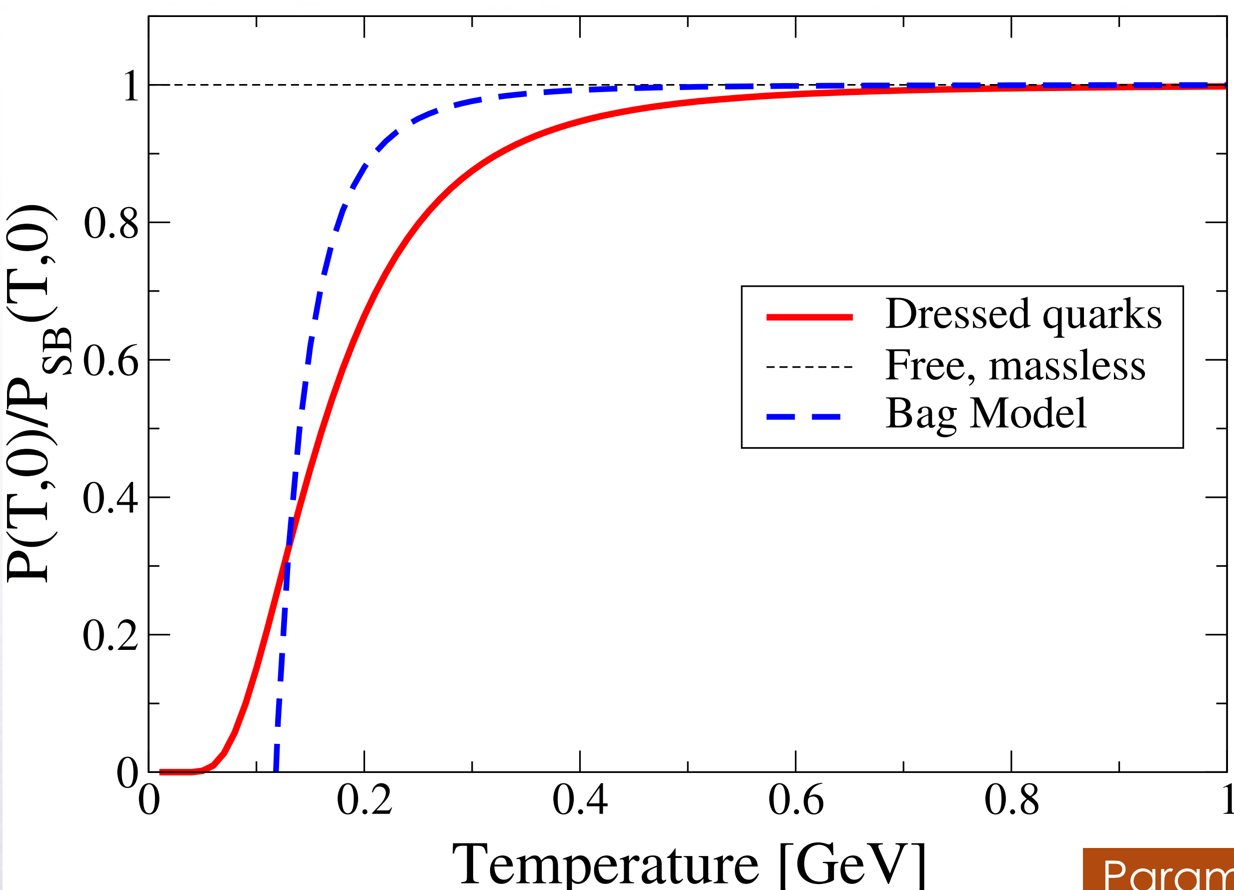
$$Z(T,\mu) = \int [D\psi][D\bar{\psi}]\,\exp\left[-S_{eff}(\psi,\bar{\psi})\right]$$

where

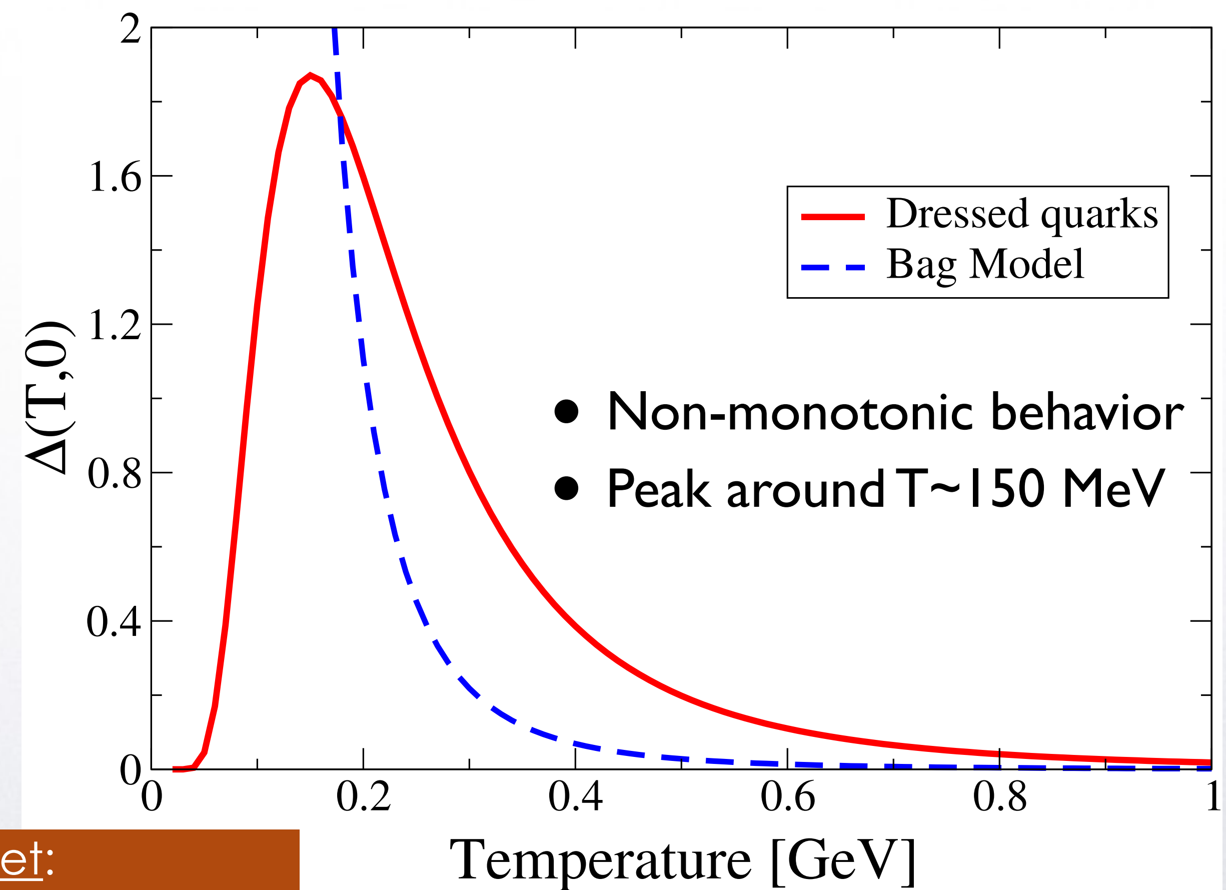
$$S_{eff}(\psi,\bar{\psi}) = \int_0^\beta d^4x\;\bar{\psi}\left[i\gamma_4(\partial_4-\mu) - i\vec{\gamma}\cdot\vec{\nabla} - \frac{M_3}{-(\partial_4-\mu)^2 - \vec{\partial}^2 + m^2}\right]\psi$$

- Thermodynamic quantities are non-trivial as well as stable for all temperatures.
  ***Qualitative** behavior is compatible with lattice data for a thermal crossover.*

- Pressure:



- Trace Anomaly ("interaction measure"):



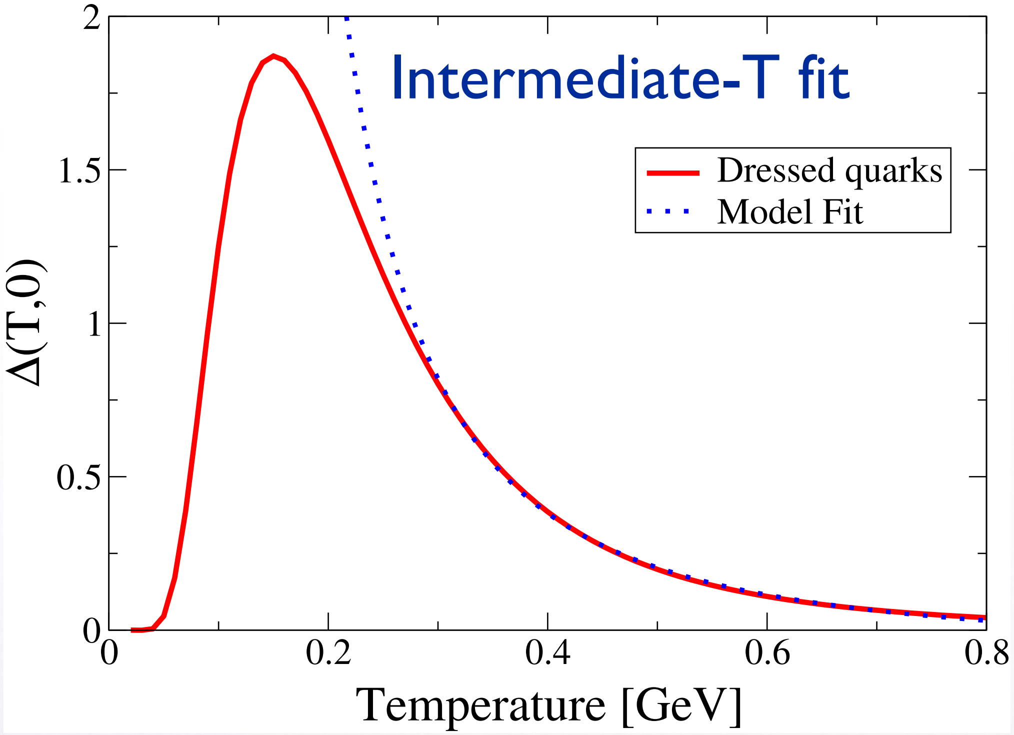
Parameter set:

$$M_3 = 0.196\ GeV^3$$
$$m^2 = 0.639\ GeV^2$$
$$m_0 = 0.014\ GeV$$

(From: lattice quark Propagator fits at $T = 0$)

[Guimaraes,Mintz,LFP, PRD92 (2015) 085029]

- A bag model fit for intermediate temperatures above the peak (T = [300,800] MeV) reveals an effective negative pressure (bag constant) naturally present in the model:



$$\Delta_{\text{fit}}(T) = a + \frac{b}{T^2} + \frac{4B}{T^4}$$

$$a = -0.069$$

$$b = 0.062\ \text{GeV}^2$$

$$B = (141\ \text{MeV})^4$$

Another indication of built-in confinement?

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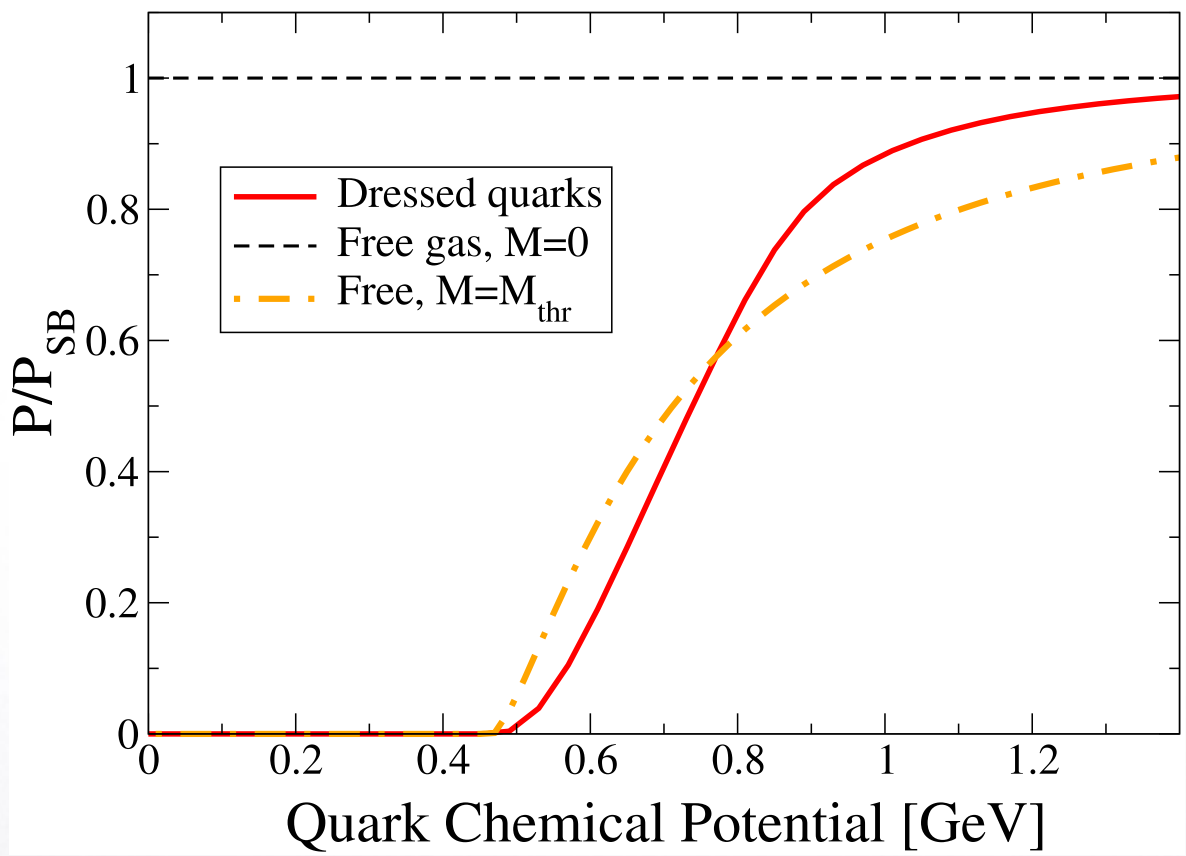
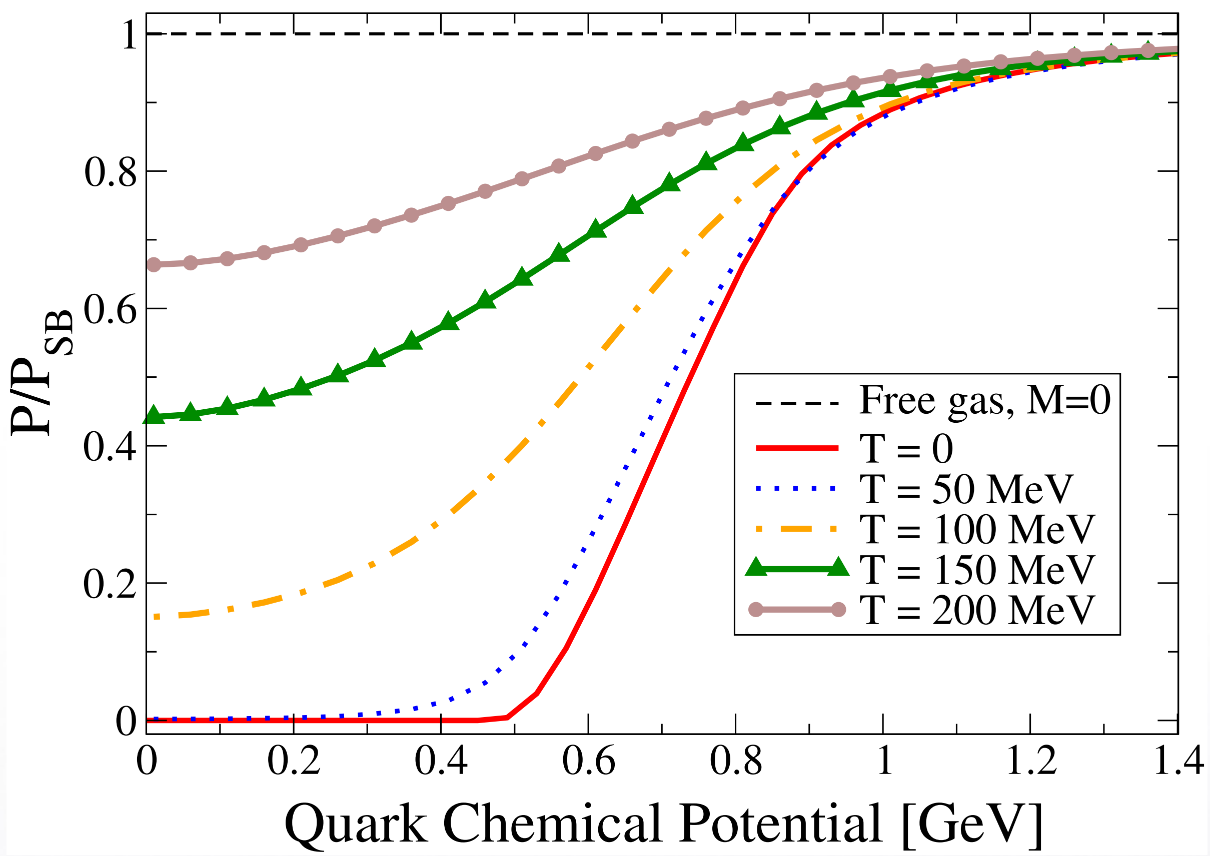
(From: lattice quark Propagator fits at T = 0)

Cold and dense thermodynamics

- Pressure @ T = 0:



- Pressure for nonzero T: *thermal excitation*



- gluons suppressed: better approximation, but no comparison with lattice available (Sign problem)
- dynamically-generated threshold mass?
- Silver Blaze problem: ok!
- No phase transition in this approximation (T- and μ-independent parameters)

[Guimaraes,Mintz,LFP, PRD92 (2015) 085029]

- smoothening of the transition
- shift of inflection point to lower μ's.

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$m_0 = 0.014\ GeV$ ⟶ **heavier world!**

(From: lattice quark Propagator fits at T = 0)

Introducing an Abelian background field

[Mintz,LFP, *in prep.*]

- Minimal coupling prescription: $\partial_\mu \mapsto D_\mu = \partial_\mu - i q_f A_\mu$ $\qquad A_\mu = (0, -By/2, Bx/2, 0)$

- In the case of the nonlocal propagator, the preservation of U(1) symmetry requires that all derivatives become covariant ones (*this can be formally checked in the local setup!*):

$$\mathcal{L}_{IRq}(B) = \bar{\psi}_i^\alpha \left[i(\not{D})_{ij}^{\alpha\beta} - \delta_{ij}\delta^{\alpha\beta} \left(\frac{M_3}{-D^2 + m^2} + m_0 \right) \right] \psi_j^\beta$$

- As in the thermal case, the determinant of the nonlocal operator may be decomposed in determinants of the standard form:

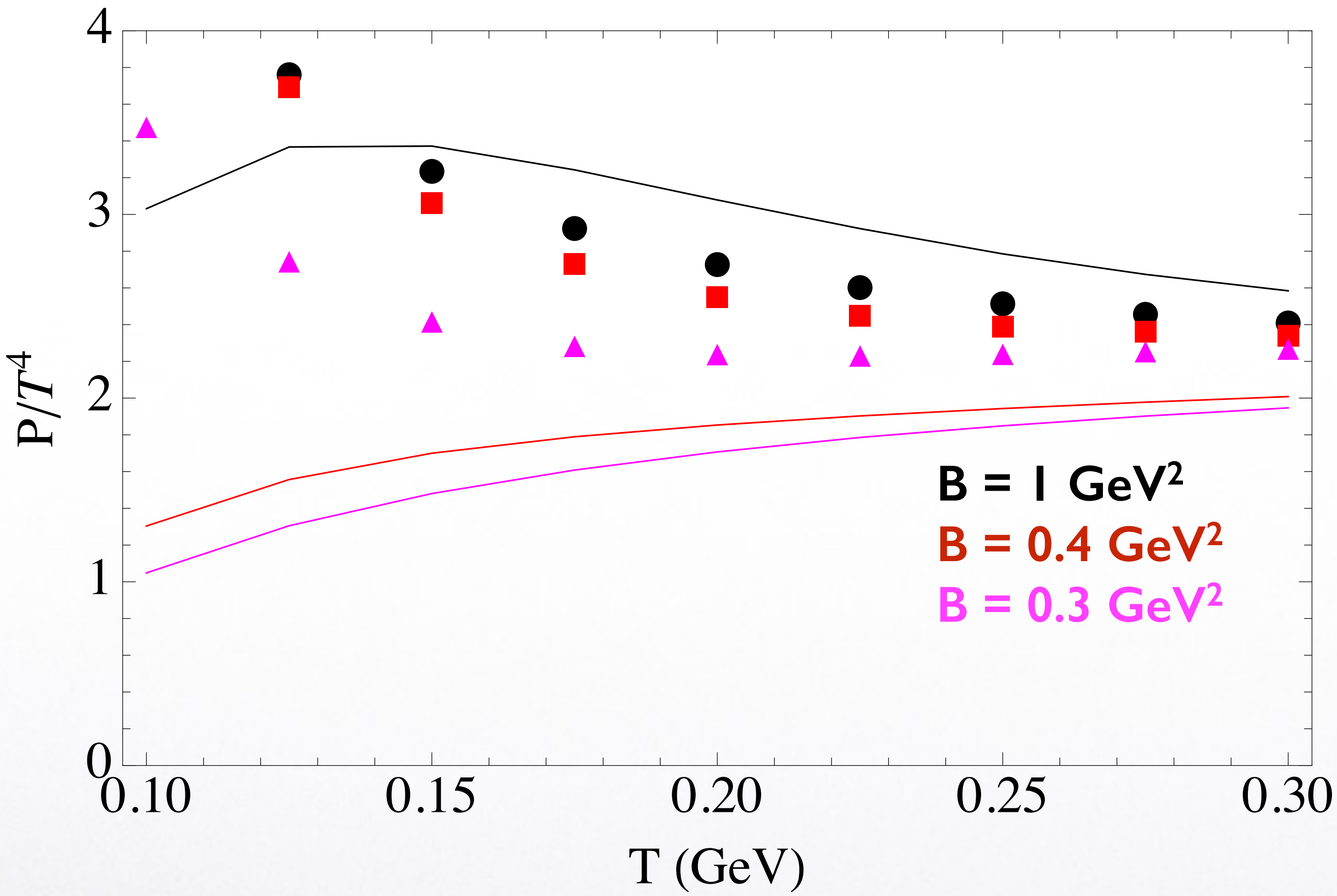
$$\det(-D^2 + K)$$

whose eigenvalues are the usual Landau levels:

$$\lambda(K) = \omega_n^2 + p_z^2 + K + |q_f| B(2l+1), \qquad l \in \mathbb{Z}_+$$

[Mintz,LFP, *in prep.*]

- Thermal Pressure @ different B's (N_f = 2):



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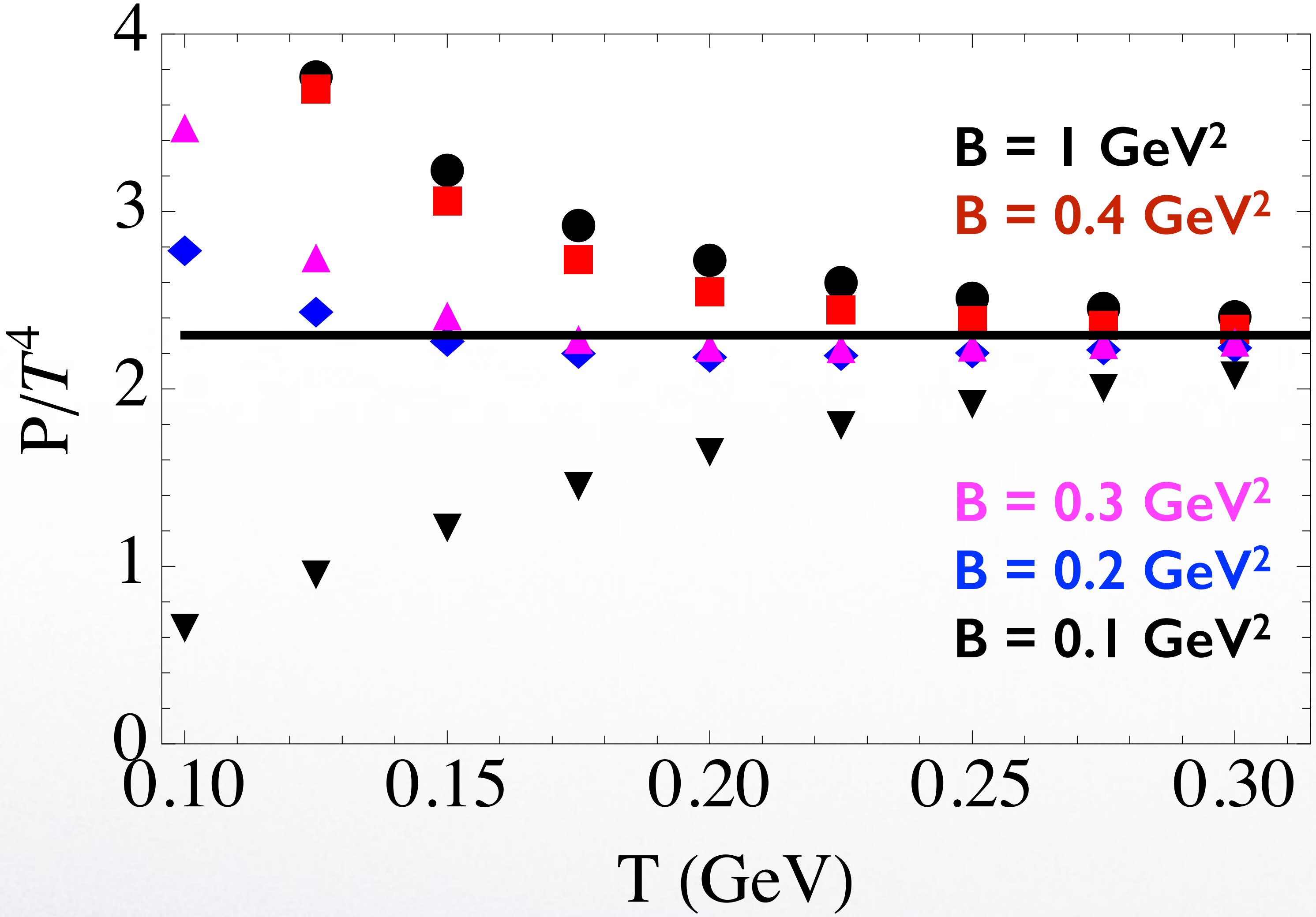
(From: lattice quark Propagator fits at T = 0)

⟶ **heavier world!**

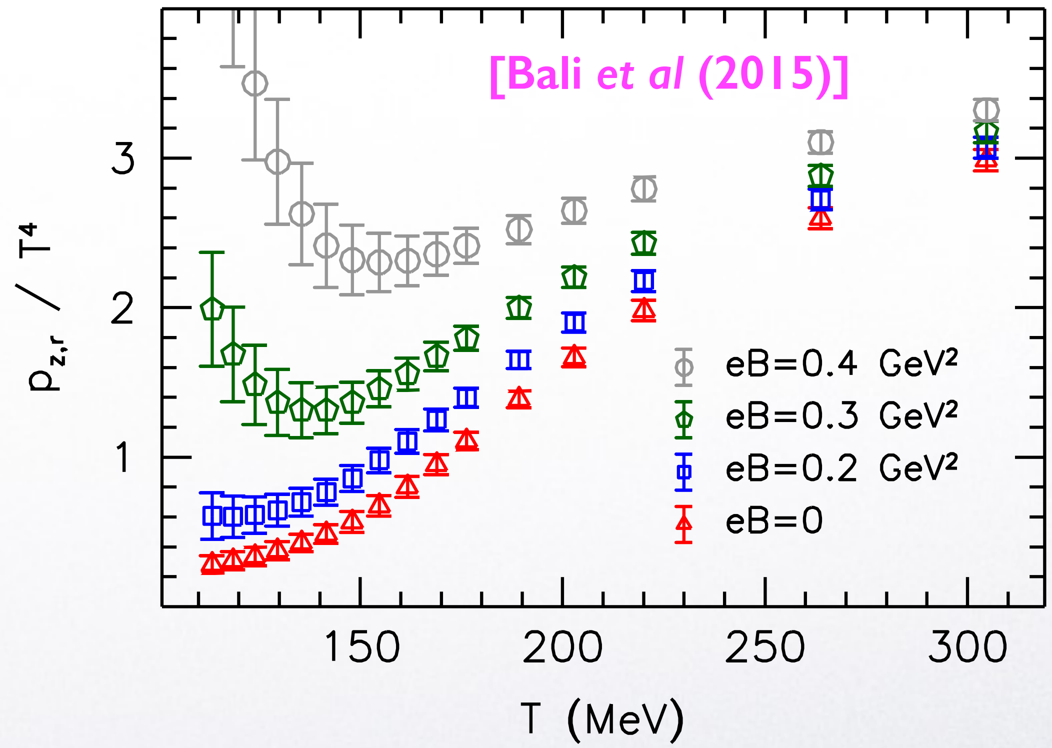
- larger confinement effects for lower temperature: may affect critical region
- expect larger corrections for light quarks:
  for large current masses, propagator becomes closer to a constant
  (~ usual 'constituent' quark)

[Mintz,LFP, *in prep.*]

- Thermal Pressure @ different B's:



- DISCLAIMERS: $N_f = 2$, heavier quarks, no gluons!



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Final comments

- ***The mechanism of confinement is still not understood.***

- The *Gribov problem* is present and should profoundly affect the IR regime of gauge-fixed non-Abelian gauge theories, being possibly connected with *confinement*.

- The *RGZ framework* represents a consistent scenario to study the non-perturbative IR physics and has provided *interesting results for the gluon sector*.

- We have proposed an extension of this scenario to the case of quark confinement, including a tree-level propagator compatible with Landau-gauge lattice data. Results for the thermodynamics of the model in the simplest approximation display consistent physical behavior for all T, mu, and B investigated.

- <u>Perspectives:</u> chiral condensate (current-mass dependence or couple to mesons) and phase structure; Polyakov loop through background-field method; dressed vertices.

- <u>Many caveats (very distant perspectives! :)), of course:</u> dynamical origin of matter horizon, physical operators, unitarity… Recent advances on gauge invariance!

[Capri *et al.*, PRD94 (2016) 065009; 025035; PRD95 (2017) 045011]

Thank you for your attention!