Chern-Simons 5-form and Holographic Baryon

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1 Introduction

- Holographic QCD = Holographic dual of "QCD"
  ("QCD" is a gauge theory that flows to QCD at low energies)

- Meson effective theory turns out to be
  5 dim U(Nf) YM-CS theory in a curved space-time.

number of flavors

[Son-Stephanov 03, Sakai-S.S. 04,05]

$$S = S_{\mathrm{YM}} + S_{\mathrm{CS}}$$

$\mu,\nu = 0 \sim 3$ 5^{th} coordinate

metric

$$S_{\mathrm{YM}} = -\frac{\kappa}{2}\int_{5\mathrm{dim}} \mathrm{Tr}(F\wedge *F) \qquad ds^2 = a(z)\eta_{\mu\nu}dx^\mu dx^\nu + b(z)dz^2$$

$$S_{\mathrm{CS}} = \frac{N_c}{24\pi^2}\int_{5\mathrm{dim}} \omega_5(A) \qquad \omega_5(A) = \mathrm{Tr}\left(AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^5\right)$$

CS-term CS 5-form

Success

- reproduces a lot of properties of QCD and hadrons
  chiral symmetry breaking, confinement, phase transition, etc.

- "Derivation" of old hadron models :
  * Vector meson dominance model (interaction with photon)
  * Hidden local symmetry (model for rho, a1, …)
  * Skyrme model (model for baryon)
  * Gell-Mann – Sharp – Wagner model (model for omega meson)

- A lot of masses and couplings can be computed quite easily
  (at least in some approximation)
  and they are in reasonably good agreement with experiments.

CS-term

$$S_{\mathrm{CS}} = \frac{N_c}{24\pi^2}\int_{M_5}\omega_5(A)$$

$$\omega_5(A) = \mathrm{Tr}\left(AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^5\right)$$

$$d\omega_5(A) = \mathrm{Tr}(F^3)$$

- This term is ***not*** gauge invariant, if the manifold has boundaries:

$$\delta_\Lambda\omega_5(A) = d\omega_4^1(\Lambda, A)$$

$$\delta_\Lambda A = D_A\Lambda$$

$$\omega_4^1(A) = \mathrm{Tr}\left(\Lambda d\left(AdA + \frac{1}{2}A^3\right)\right)$$

- Reproduces the chiral anomaly in QCD

$$\delta_\Lambda S_{\mathrm{CS}} = \frac{N_c}{24\pi^2}\int_{M_4}\left(\omega_4^1(\Lambda, A)|_{z=+\infty} - \omega_4^1(\Lambda, A)|_{z=-\infty}\right)$$

with the standard identification $(\widehat{A}_-, \widehat{A}_+) = (A|_{z=-\infty}, A|_{z=+\infty})$

external $U(Nf)_L \times U(Nf)_R$ gauge fields

boundary values of the 5 dim gauge field

WZW term from CS 5-form

- pion field: $U(x^\mu) = \mathrm{P}\exp\left(-\int dz\, A_z(x^\mu, z)\right)$
- vector mesons: $A_\mu(x^\mu, z) \xrightarrow{\text{mode exp}} B_\mu^{(1)}(x^\mu),\ B_\mu^{(2)}(x^\mu),\ B_\mu^{(3)}(x^\mu),\ \cdots$

  interpreted as $\rho,\ a_1,\ \rho',\ \cdots$
- One can show

terms with vector mesons

$$S_{\mathrm{CS}} \simeq -\frac{N_c}{48\pi^2}\int_{\mathrm{4dim}} Z + \frac{N_c}{240\pi^2}\int_{\mathrm{5dim}} \mathrm{Tr}(U^{-1}dU)^5 + \cdots$$

$$\begin{aligned} Z = {} & \mathrm{Tr}[(\hat{A}_- d\hat{A}_- + d\hat{A}_-\hat{A}_- + \hat{A}_-^3)(U^{-1}\hat{A}_+U + U^{-1}dU) - \mathrm{p.c.}] \\ & + \mathrm{Tr}[d\hat{A}_- dU^{-1}\hat{A}_+U - \mathrm{p.c.}] + \mathrm{Tr}[\hat{A}_-(dU^{-1}U)^3 - \mathrm{p.c.}] \\ & + \frac{1}{2}\mathrm{Tr}[(\hat{A}_- dU^{-1}U)^2 - \mathrm{p.c.}] + \mathrm{Tr}[U\hat{A}_-U^{-1}\hat{A}_+ dU dU^{-1} - \mathrm{p.c.}] \\ & - \mathrm{Tr}[\hat{A}_- dU^{-1}U\hat{A}_-U^{-1}\hat{A}_+U - \mathrm{p.c.}] + \frac{1}{2}\mathrm{Tr}[(\hat{A}_-U^{-1}\hat{A}_+U)^2] \end{aligned}$$

$$\boxed{\begin{array}{ccc} \hat{A}_+ & \leftrightarrow & \hat{A}_-, \\ U & \leftrightarrow & U^{-1} \end{array}}$$

reproduces the WZW term in QCD

Holographic baryon

- Baryons are "instantons" in 4 dim space

$$n_B = \frac{1}{8\pi^2}\int_{\Sigma_4} \mathrm{Tr}(F^2)$$: "instanton" number

baryon number

$$\Sigma_4 = \{(x^1, x^2, x^3, z)\}$$

follows from $S_{\mathrm{CS}} = \frac{1}{24\pi^2}\int_{\mathrm{5dim}} A^{\mathrm{U}(1)}\mathrm{Tr}(F^2) + \cdots$

- This is analogous to the Skyrme model

$$n_B = \frac{1}{24\pi^2}\int_{S^3} \mathrm{Tr}((U^{-1}dU)^3)$$: winding # of *U* on S^3

$$S^3 = \{(x^1, x^2, x^3)\} \cup \{\infty\}$$

So far it looks good …

2 Problems

1 Consider $\Sigma_4 = \{(x^1, x^2, x^3, z)\} = S^3 \times \mathbf{R}$

$$n_B = \frac{1}{8\pi^2}\int_{\Sigma_4} \mathrm{Tr}(F^2) = \frac{1}{8\pi^2}\int_{S^3}\left(\omega_3(A)|_{z=+\infty} - \omega_3(A)|_{z=-\infty}\right)$$

CS 3-form $d\omega_3(A) = \mathrm{Tr}(F^2)$

This is ***not*** consistent with $(\widehat{A}_-, \widehat{A}_+) = (A|_{z=-\infty}, A|_{z=+\infty})$

ext. gauge field boundary values

In particular, configurations with $n_B \neq 0$ and $\widehat{A}_\pm = 0$ should be allowed.

- Then, how can we relate $(\widehat{A}_-, \widehat{A}_+)$ and $(A|_{z=-\infty}, A|_{z=+\infty})$?
- If $(\widehat{A}_-, \widehat{A}_+) \neq (A|_{z=-\infty}, A|_{z=+\infty})$, the naïve CS-term does not give the correct Chiral anomaly. How can we fix it?

2 Hata and Murata (2007) pointed out that the CS-term does not lead to a constraint needed to get the correct baryon spectrum for $N_f = 3$.

→ see next

The goal of this talk is to provide a solution to these problems.

Constraint for Baryon spectrum

- Baryons in Skyrme model for $N_f = 3$

solution for $N_f=2$

To quantize fluctuations around soliton solution,

$$U^{\text{cl}} = \begin{pmatrix} U_0 & \\ & 1 \end{pmatrix}$$

$$U(x^\mu) = a(t)U^{\text{cl}}(\vec{x})a(t)^{-1}$$

pion field

collective coordinates

→ Quantum mechanics for $a(t) \in SU(3)$

$N_f = 3$

- One can show:

$$S_{\text{WZW}} = \frac{N_c}{\sqrt{3}}\text{Tr}(t_8 a^\dagger \dot{a}) \qquad t_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

This term leads to a constraint on the wave function:

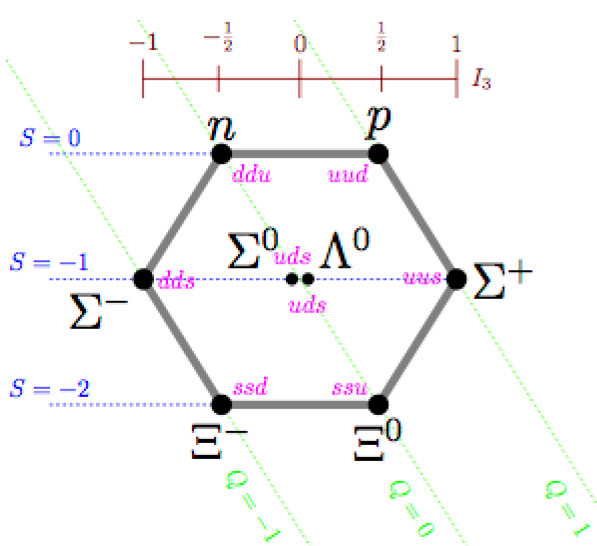
$$\psi(ae^{it_8\theta}) = \psi(a)e^{i\frac{N_c}{2\sqrt{3}}\theta}$$

Constraint for Baryon spectrum

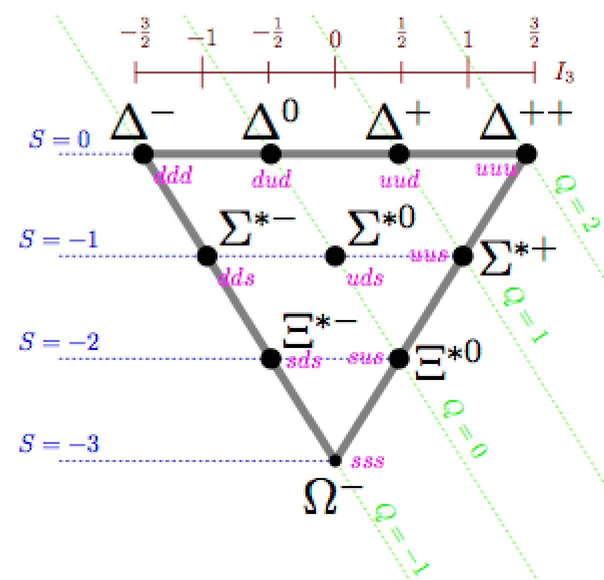
$$\psi(ae^{it_8\theta}) = \psi(a)e^{i\frac{N_C}{2\sqrt{3}}\theta}$$

It is known that the correct baryon spectrum is obtained (at least qualitatively) with this constraint.

J=1/2 states



J=3/2 states



(Figures taken from https://glenmartin.wordpress.com/2014/01/04/the-story-of-quarks-part-i/)

What about baryons in holographic QCD ?

- Baryons are solitons in 5 dim YM-CS theory

$$n_B = \frac{1}{8\pi^2}\int_{\Sigma_4} \mathrm{Tr}(F^2)$$

- Similar analysis can be done in holographic QCD, and again reduces to QM of $a(t) \in SU(3)$ (for Nf = 3).

- The CS-term should reproduce $S_{\mathrm{WZW}} = \frac{N_c}{\sqrt{3}i}\mathrm{Tr}(t_8 a^\dagger \dot{a})$

  However, Hata and Murata claimed

$$S_{\mathrm{CS}} = 0 \quad \textit{!!}$$

- They proposed a new CS-term

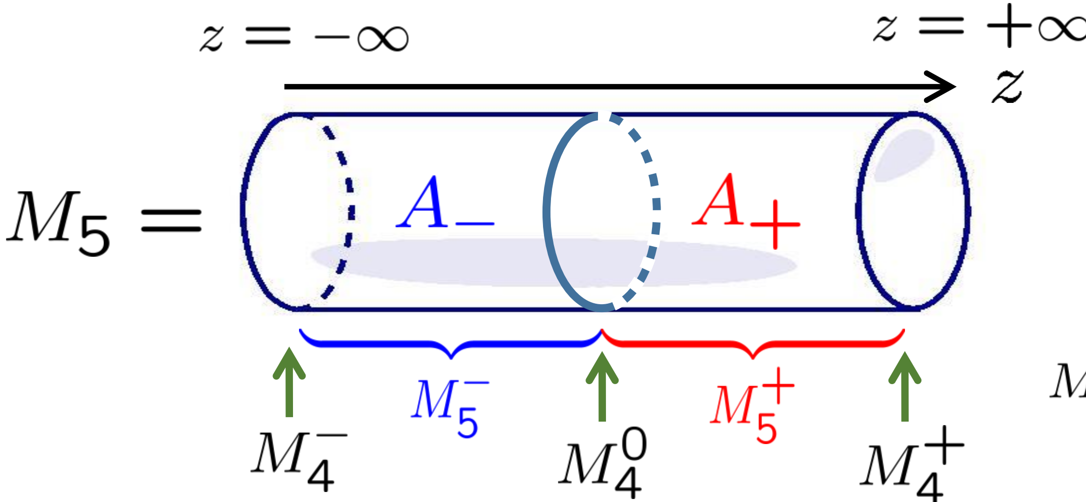
$$S_{\mathrm{CS}}^{\mathrm{HM}} = \frac{N_c}{24\pi^2}\int_{M_6} \mathrm{Tr}(F^3) \qquad \partial M_6 = M_5$$

  However, it doesn't reproduce the chiral anomaly of QCD.

What should we do?

3 Proposal

- Consider the following situation:



time

$$M_4^+ \simeq M_4^- \simeq M_4^0 \simeq S^1 \times S^3$$

gluing condition

$$A_+ = A_-^h \equiv hA_-h^{-1} + hdh^{-1} \quad \text{on} \quad M_5^- \cap M_5^+ \simeq M_4^0 \times (-\epsilon, \epsilon)$$

(we take $\epsilon \to 0$)

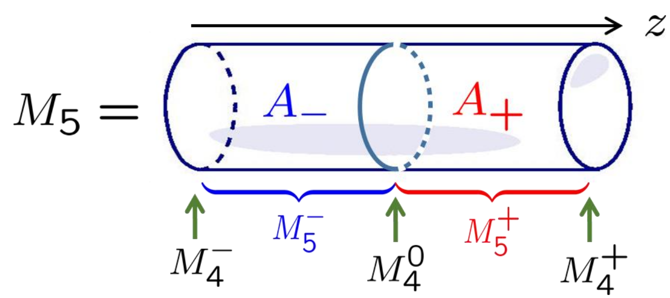
- We identify the external gauge fields as $\hat{A}_\pm = A_\pm|_{z=\pm\infty}$

new

Then, one can show (for $\hat{A}_\pm = 0$)

$$n_B = \frac{1}{8\pi^2}\int_{S^3\times\{z\}} \mathrm{Tr}(F^2) = \frac{1}{24\pi^2}\int_{S^3} \mathrm{Tr}\left((hdh^{-1})^3\right)$$ = winding # of *h* on S^3

New CS-term



- Note that

$$S_{\text{CS}}^{\text{naive}} \equiv \frac{N_c}{24\pi^2}\left(\int_{M_5^-}\omega_5(A_-) + \int_{M_5^+}\omega_5(A_+)\right)$$ does not make sense.

This is not invariant under "local" gauge transformations (gauge transformation that are trivial at the boundaries)

$$\delta_\Lambda S_{\text{CS}}^{\text{naive}} = \frac{N_c}{24\pi^2}\int_{M_4^0}\left(\omega_4^1(\Lambda, A_-) - \omega_4^1(\Lambda, A_+)\right) \neq 0 \quad !$$

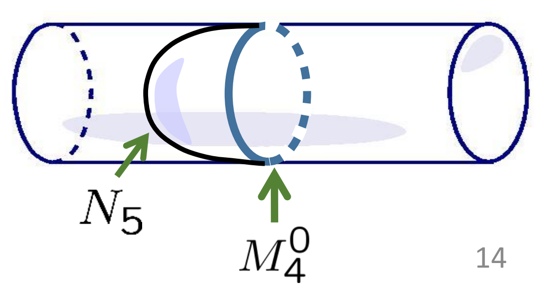
- Our proposal

$$\boxed{S_{\text{CS}}^{\text{new}} = S_{\text{CS}}^{\text{naive}} + \frac{N_c}{24\pi^2}\left(\frac{1}{10}\int_{N_5}\text{Tr}(\tilde{h}d\tilde{h}^{-1})^5) + \int_{M_4^0}\alpha_4(dh^{-1}h, A_-)\right)}$$

N_5 is 5dim manifold satisfying $\partial N_5 = M_4^0$

$\tilde{h} : N_5 \to SU(N_f)$ s.t. $\tilde{h}|_{M_4^0} = h$

$\alpha_4(V, A) \equiv \frac{1}{2}\text{Tr}\left(V(A^3 - AF - FA) + \frac{1}{2}VAVA + V^3A\right)$



Consistency check

$$S_{\text{CS}}^{\text{new}} = S_{\text{CS}}^{\text{naive}} + \frac{N_c}{24\pi^2}\left(\frac{1}{10}\int_{N_5} \text{Tr}(\tilde{h}d\tilde{h}^{-1})^5) + \int_{M_4^0} \alpha_4(dh^{-1}h, A_-)\right)$$

One can show the following:

- ✓ • reduces to S_{CS} when h is topologically trivial
- ✓ • invariant under the "local" gauge transformation
- ✓ • reproduces the correct chiral anomaly in QCD

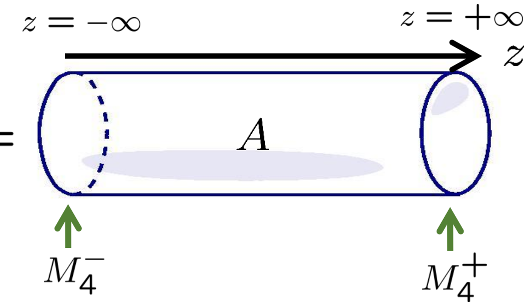
Other useful expressions

- Suppose the gauge field *A* is globally well-defined on M_5

  Then, $A|_{z=\pm\infty}$ and the external gauge fields $\widehat{A}_\pm$ are related by

$$\boxed{A|_{z=\pm\infty} = \widehat{A}_\pm^{h_\pm} \quad \text{with} \quad h_\pm : M_4^\pm \to U(N_f)}$$

$M_5 =$

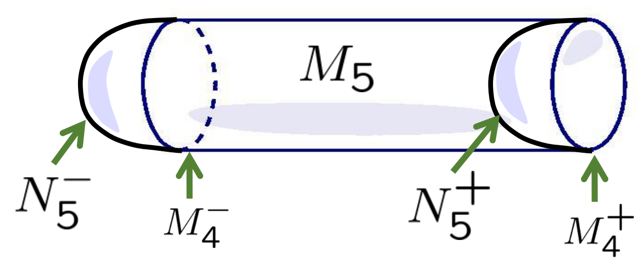


- The CS-term is

$$\boxed{S_{\mathrm{CS}}^{\mathrm{new}} = S_{\mathrm{CS}}(A) + \frac{N_c}{24\pi^2}\sum_{\epsilon=\pm}\epsilon\left(\frac{1}{10}\int_{N_5^\epsilon}\mathrm{Tr}(\widetilde{h}_\epsilon^{-1}d\widetilde{h}_\epsilon)^5) - \int_{M_4^\epsilon}\alpha_4(dh_\epsilon^{-1}h_\epsilon, \widehat{A}_\epsilon)\right)}$$

$N_5^\pm$ is 5dim manifold satisfying $\partial N_5^\pm = M_4^\pm \quad (N_5^\pm \simeq D \times S^3)$

$\widetilde{h}_\pm : N_5^\pm \to U(N_f)$ s.t. $\widetilde{h}_\pm|_{M_4^\pm} = h_\pm$



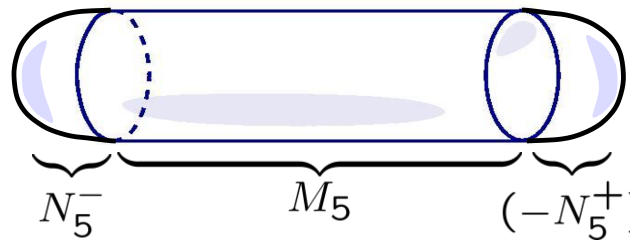
- Note that $\widetilde{M}_5 = N_5^- \cup M_5 \cup (-N_5^+)$ is a 5 dim mfd without bdry

  Suppose

  there exists M_6 s.t. $\partial M_6 = \widetilde{M}_5$

  Suppose A can be extended to M_6

  $\widehat{A}_\pm$ can be extended to $N_5^\pm$



  Then, one can show

$$\boxed{S_{\rm CS}^{\rm new} = \frac{1}{24\pi^2}\left(\int_{M_6} {\rm Tr} F^3 + \int_{N_5^+} \omega(\widehat{A}_+) - \int_{N_5^-} \omega(\widehat{A}_-)\right)}$$

Hata-Murata's proposal

With this expression, it is easy to show that it is invariant under the "local" gauge tr. and reproduces chiral anomaly in QCD.

Constraint for the baryon spectrum

- Collective coordinates are introduced as follows

$A_0 = 0$ gauge [Hata-Sakai-S.S.-Yamato 2007]

$$A_M = V A_M^{\text{cl}} V^{-1} + V \partial_M V^{-1} \qquad (M = 1, 2, 3, z)$$

classical solution

with $A_M^{\text{cl}} \to h_\pm^{\text{cl}}(\vec{x}) \partial_M h_\pm^{\text{cl}-1}(\vec{x}) \qquad (z \to \pm\infty)$

Then, the collective coordinates $a(t) \in SU(3)$ are hidden in the boundary values of V as

$$V \to h_\pm^{\text{cl}}(\vec{x}) a(t) h_\pm^{\text{cl}-1}(\vec{x}) \qquad (z \to \pm\infty)$$

- Adding the kinetic term S_{YM} and imposing the EOM for A_0, we obtain the expected term (for $N_f = 3$, $n_B = 1$):

$$S_{\text{YM}} + S_{\text{CS}}^{\text{new}} = \frac{N_c}{\sqrt{3} i} \text{Tr}(t_8 a^\dagger \dot{a}) + \cdots$$

4 Summary and outlook

- We proposed a new CS term that seems to work:

$$S_{\mathrm{CS}}^{\mathrm{new}} = S_{\mathrm{CS}}^{\mathrm{naive}} + \frac{N_c}{24\pi^2}\left(\frac{1}{10}\int_{N_5}\mathrm{Tr}(\tilde{h}d\tilde{h}^{-1})^5) + \int_{M_4^0}\alpha_4(dh^{-1}h, A_-)\right)$$

- **<u>To do list</u>**
  - Find a classical solution and compute the baryon mass
  - Compute the currents and discuss the static properties of the baryons.
  - Include quark mass to make more serious analysis.

See [Hata-Murata 07, Hashimoto-Iizuka-Ishii-Kadoh 09, …]

Thank you

Backup slides

Key features:

- 4 dim QFT ↔ higher dim theory with gravity

In our case,

4 dim QCD ↔ String theory in 10 dim curved space-time with probe D-branes

theory of quarks and gluons

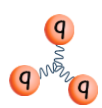
theory of open and closed strings

meson ↔

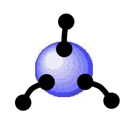
open string

glueball ↔

closed string



baryon ↔



D-brane (soliton)

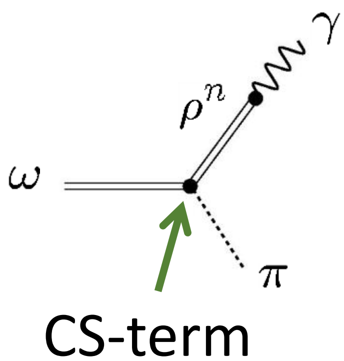
- Global symmetry in QFT ↔ Gauge symmetry in gravity theory

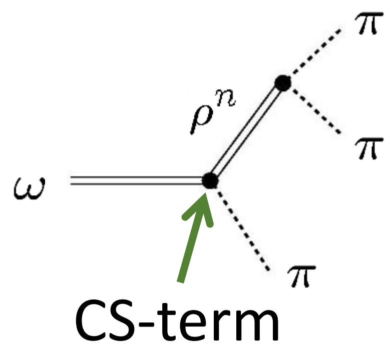
In our case,

Chiral symmetry	associated external gauge fields		boundary values of the 5 dim gauge field
$U(N_f)_L \times U(N_f)_R$	$(\widehat{A}_-, \widehat{A}_+)$	↔	$(A\vert_{z=-\infty}, A\vert_{z=+\infty})$

GSW model

- The relevant diagrams for $\omega \to \pi^0\gamma$, $\omega \to \pi^0\pi^+\pi^-$





→ Exactly the same structure as that in the **GSW model** !

[Gell-Mann – Sharp – Wagner 1962]

The CS-term seems to work
even for interactions with vector mesons!

Quantization

[Adkins-Nappi-Witten 1983]

1. Pick a soliton solution $U^{\mathrm{cl}}(\vec{x})$ representing a baryon at $\vec{x} = \vec{0}$

   pion field

   position of the baryon

   $$U(x^\mu) = a\,U^{\mathrm{cl}}(\vec{x} - \vec{X})\,a^{-1} \qquad (a \in SU(N_f))$$

   is again a solution with the same energy.

   $(\vec{X}, a)$: collective coordinates

2. Promote $(\vec{X}, a)$ to be time dependent variables and insert

   $U(x^\mu) = a(t)\,U^{\mathrm{cl}}(\vec{x} - \vec{X}(t))\,a(t)^{-1}$ into the action.

   $$S_{\mathrm{Skyrme}}(U) = \int dt\, L(\vec{X}, a, \dot{\vec{X}}, \dot{a})$$ → Quantum mechanics for $(\vec{X}, a)$

3. Solve the Schrödinger eq. and compute whatever you want (energy, magnetic moments, charge radius, etc.) for the baryon states.

This program was pursued in the N_f=2 case by Adkins-Nappi-Witten (1983)

Results:

Quantity	Prediction	Experiment
M_{N}	input	939 MeV
M_{Δ}	input	1232 MeV
F_{π}	129 MeV	186 MeV
$\langle r^2 \rangle^{1/2}_{I=0}$	0.59 fm	0.72 fm
$\langle r^2 \rangle^{1/2}_{M,I=0}$	0.92 fm	0.81 fm
μ_{p}	1.87	2.79
μ_{n}	−1.31	−1.91
$\left\lvert \frac{\mu_{\mathrm{p}}}{\mu_{\mathrm{n}}} \right\rvert$	1.43	1.46
g_{A}	0.61	1.23
$g_{\pi\mathrm{NN}}$	8.9	13.5
$g_{\pi\mathrm{N}\Delta}$	13.2	20.3
$\mu_{\mathrm{N}\Delta}$	2.3	3.3

Summary of the problems

ext. gauge field boundary values

1. Find the relation between $(\widehat{A}_-, \widehat{A}_+)$ and $(A|_{z=-\infty}, A|_{z=+\infty})$,

   s.t. configurations with $n_B \neq 0$ and $\widehat{A}_\pm = 0$ are allowed,

   and reduces to the standard identification

   $(\widehat{A}_-, \widehat{A}_+) = (A|_{z=-\infty}, A|_{z=+\infty})$ for the cases without baryons.

2. Find a consistent CS-term

   s.t. the chiral anomaly in QCD is reproduced,

   $S_{\mathrm{WZW}} = \frac{N_c}{\sqrt{3}i} \mathrm{Tr}(t_8 a^\dagger \dot{a})$ is reproduced,

   and reduces to the standard one $S_{\mathrm{CS}} = \frac{N_c}{24\pi^2} \int_{5\mathrm{dim}} \omega_5(A)$

   for the cases without baryons.