Hollowness in pp at the LHC

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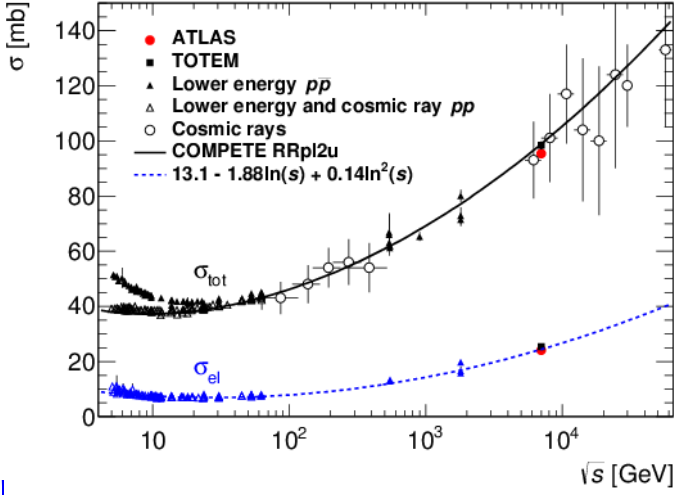
Excited QCD 2017
7-13 May 2017, Sintra, Portugal

Research with **Enrique Ruiz Arriola**

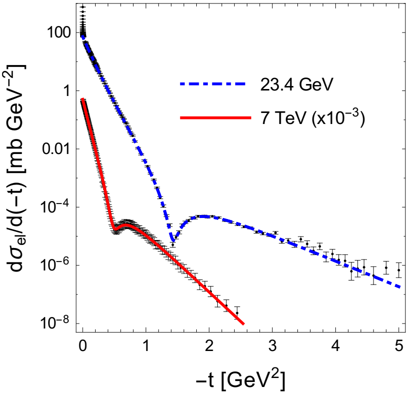
Based on [Phys.Rev. D95 (2017) 074030, arXiv:1609.05597]

pp and $p\bar{p}$ cross sections



Surprises!

Elastic scattering from ISR to LHC



Spin-averaged elastic pp scattering amplitude

Parametrization by [Fagundes 2013] based on [Barger-Phillips 1974], motivated by the Regge asymptotics:

$$\frac{|f(s,t)|}{p} = \left| \sum_n c_n(s) F_n(t) s^{\alpha_n(t)} \right| = \left| \frac{i\sqrt{A} e^{\frac{Bt}{2}}}{\left(1 - \frac{t}{t_0}\right)^4} + i\sqrt{C} e^{\frac{Dt}{2} + i\phi} \right|$$

s-dependent (real) parameters are fitted (separately) to all known differential pp cross sections for $\sqrt{s} = 23.4$, 30.5, 44.6, 52.8, 62.0, and 7000 GeV with $\chi^2/\mathrm{d.o.f} \sim 1.2 - 1.7$

$$\frac{d\sigma_{\mathrm{el}}(s,t)}{dt} = \frac{\pi}{p^2} |f(s,t)|^2$$

ρ parameter

$$\rho(s,t) = \frac{\mathrm{Re} f(s,t)}{\mathrm{Im} f(s,t)}$$

$$f(s,t) = \frac{i+\rho(s,t)}{\sqrt{1+\rho(s,t)^2}}|f(s,t)|$$

From the optical theorem

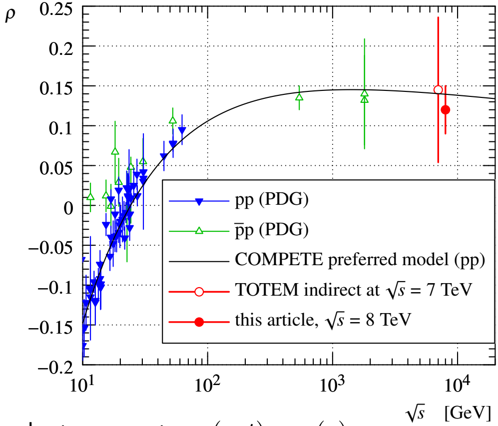
$$\sigma_{\mathrm{tot}}(s) = \frac{4\pi}{p}\mathrm{Im} f(s,0) = \frac{4\sqrt{\pi d\sigma_{\mathrm{el}}/dt|_{t=0}}}{\sqrt{1+\rho(s,0)^2}}$$

or

$$\rho(s)^2 \equiv \rho(s,0)^2 = \frac{16\pi \frac{d\sigma_{\mathrm{el}}(s,t)}{dt}|_{t=0}}{\sigma_{\mathrm{tot}}(s)^2} - 1$$

Up to a sign $\rho(s,0)$ determined from the measured cross sections (sign may be determined from the interference with the Coulomb amplitude)

ρ parameter from the experiment



We take a t-independent parameter $\rho(s,t) = \rho(s)$
Results similar for the Bailly et al. parametrization

$$\rho(s,t) = \frac{\rho(s)}{1 - t/t_0(s)}$$

$t_0(s)$ – position of the diffractive minimum

How well it works?

$\sqrt{s}$ [GeV]	$\sigma_{\rm el}$ [mb]	$\sigma_{\rm in}$ [mb]	$\sigma_{\rm T}$ [mb]	B [GeV^{-2}]	ρ
23.4	6.6	31.2	37.7	11.6	0.00
exp.	6.7(1)	32.2(1)	38.9(2)	11.8(3)	0.02(5)
200	10.0	40.9	50.9	14.4	0.13
exp.			54(4)	16.3(25)	
7000	25.3	73.5	98.8	20.5	0.140
exp.	25.4(11)	73.2(13)	98.6(22)	19.9(3)	0.145(100)

(B is the slope parameter)

Eikonal approximation

$$f(s,t) = \sum_{l=0}^{\infty} (2l+1) f_l(p) P_l(\cos\theta)$$
$$= \frac{p^2}{\pi} \int d^2b \, h(\vec{b},s) \, e^{i\vec{q}\cdot\vec{b}} = 2p^2 \int_0^\infty bdbJ_0(bq)h(b,s)$$

$t = -\vec{q}^2$, $q = 2p\sin(\theta/2)$, $bp = l + 1/2 + \mathcal{O}(s^{-1})$, $P_l(\cos\theta) \to J_0(qb)$,
hence the amplitude in the impact-parameter representation becomes

$$h(b,s) = \frac{i}{2p}\left[1 - e^{i\chi(b)}\right] = f_l(p) + \mathcal{O}(s^{-1})$$

The eikonal approximation works well for $b < 2$ fm and $\sqrt{s} > 20$ GeV

Procedure: $f(s,t) \to h(b,s) \to \chi(b)$...

Eikonal approximation 2

The standard formulas for the total, elastic, and total inelastic cross sections read

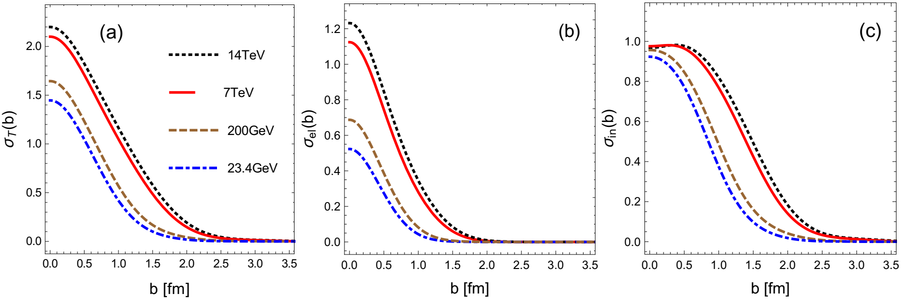
$$\sigma_T = \frac{4\pi}{p}\mathrm{Im} f(s,0) = 4p\int d^2b \mathrm{Im} h(\vec{b},s) = 2\int d^2b \left[1 - \mathrm{Re}\, e^{i\chi(b)}\right]$$
$$\sigma_{\mathrm{el}} = \int d\Omega |f(s,t)|^2 = 4p^2 \int d^2b |h(\vec{b},s)|^2 = \int d^2b |1 - e^{i\chi(b)}|^2$$
$$\sigma_{\mathrm{in}} \equiv \sigma_T - \sigma_{\mathrm{el}} = \int d^2b \sigma_{\mathrm{in}}(b) = \int d^2b \left[1 - e^{-2\mathrm{Im}\chi(b)}\right]$$

The inelasticity profile

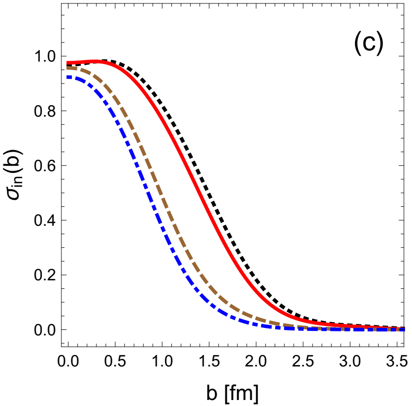
$$\sigma_{\mathrm{in}}(b) = 4p\mathrm{Im} h(b,s) - 4p^2|h(b,s)|^2$$

satisfies $0 \leq \sigma_{\mathrm{in}}(b) \leq 1$ (unitarity)

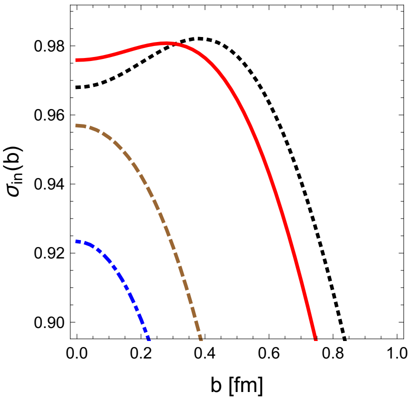
Cross sections in the b representation



Dip in the inelasticity profile

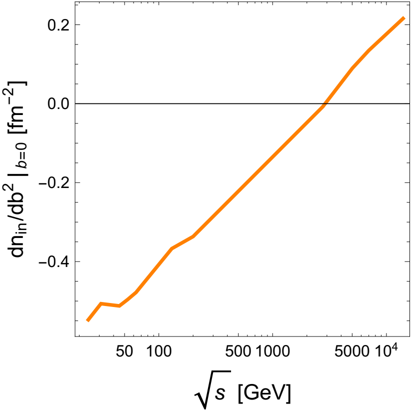


Dip in the inelasticity profile



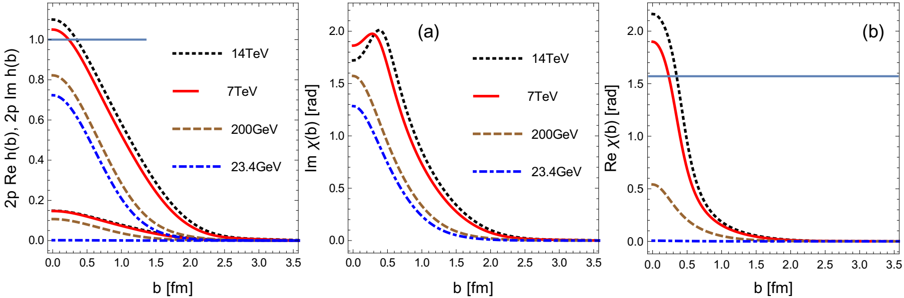
Curious effect!

Slope of the inelasticity profile



Transition occurs around $\sqrt{s} = 3$ TeV

Amplitude and eikonal phase



$$2p\mathrm{Im}\,h(b) = 1 - e^{-\mathrm{Im}\chi(b)}\cos\mathrm{Re}\chi(b)$$
$$2p\mathrm{Re}\,h(b) = e^{-\mathrm{Im}\chi(b)}\sin\mathrm{Re}\chi(b)$$

$\chi(b) > \pi/2$ changes qualitatively the picture

Condition for hollowness

In our model

$$\frac{d\sigma_{\rm in}(b)}{db^2} = 2p\frac{d{\rm Im}\, h(b)}{db^2}\left[1-(1+\rho^2)2p{\rm Im}\, h(b)\right],$$

which is negative at the origin if

$$2p{\rm Im}h(0) > \frac{1}{1+\rho^2} \sim 1.$$

Since $\rho = 0.14$ at the LHC, the departure of $1/(1+\rho^2)$ from 1 is $\sim 2\%$.
Also

$$\frac{d\sigma_{\rm in}(b)}{db^2} = 2e^{-2{\rm Im}\chi(b)}\frac{d{\rm Im}\chi(b)}{db^2},$$

thus the appearance of the dip in $\sigma_{\rm in}(b)$ is accompanied with the dip in ${\rm Im}\chi(b)$

Gaussian model

[adaptation of the model by Dremin, 2014]

$$\mathrm{Im}(2p\,h(p)) = Ae^{-\frac{2b^2}{2B}}$$

$$A = \frac{4\sigma_{\rm el}}{\left(1+\rho^2\right)\sigma_{\rm tot}}, \quad B = \frac{\left(1+\rho^2\right)\sigma_{\rm tot}^2}{16\pi\sigma_{\rm el}}$$

Curvature of inelasticity profile at the origin

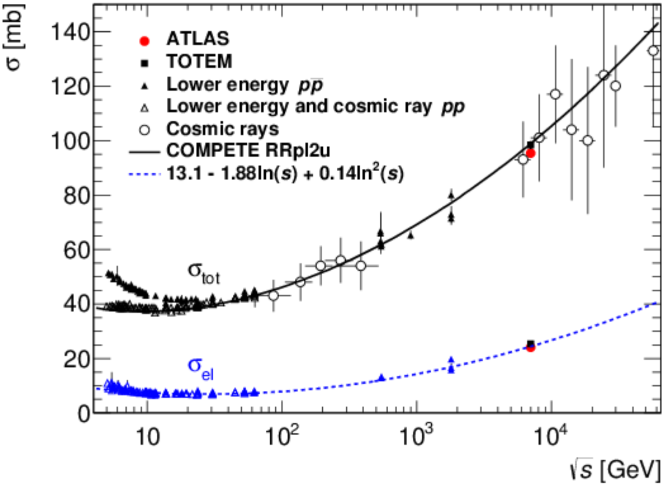
$$\frac{1}{2}\left.\frac{d^2\sigma_{\rm in}(b)}{db^2}\right|_{b=0} = \frac{64\pi\sigma_{\rm el}^2(4\sigma_{\rm el}-\sigma_{\rm tot})}{\left(\rho^2+1\right)^2\sigma_{\rm tot}^4}$$

– curvature changes sign when $\sigma_{\rm el} = \frac{1}{4}\sigma_{\rm tot}$!

Value at the origin:

$$\sigma_{\rm in}(0) = \frac{8\sigma_{\rm el}}{(1+\rho^2)\sigma_{\rm tot}}\left(1-2\frac{\sigma_{\rm el}}{\sigma_{\rm tot}}\right)$$

Cross sections



$\sigma_{\rm el}$ grows relatively faster than $\sigma_{\rm tot}$ $\rightarrow$ ratio goes above $1/4$ as s increases!

No classical folding of absorptive parts

$$
\begin{aligned}
\sigma_{\rm in}(b) &\propto \int d^2b_1 d^2b_2 \rho(\vec{b}_1+\vec{b}/2) w(\vec{b}_1-\vec{b}_2)\rho(\vec{b}_2-\vec{b}/2) \\
&= \int d^3b_1 d^3b_2 \rho(\vec{b}_1) w(\vec{b}_1-\vec{b}_2)\rho(\vec{b}_2) \\
&- \frac{1}{2}\int d^3b_1 d^3b_2 [\vec{b}\cdot\nabla\rho(\vec{b}_1)] w(\vec{b}_1-\vec{b}_2)[\vec{b}\cdot\nabla\rho(\vec{b}_2)] + \dots,
\end{aligned}
$$

$\rightarrow$ $\sigma_{\rm in}(b)$ would necessarily have a local maximum at $b=0$, in contrast to the phenomenological result

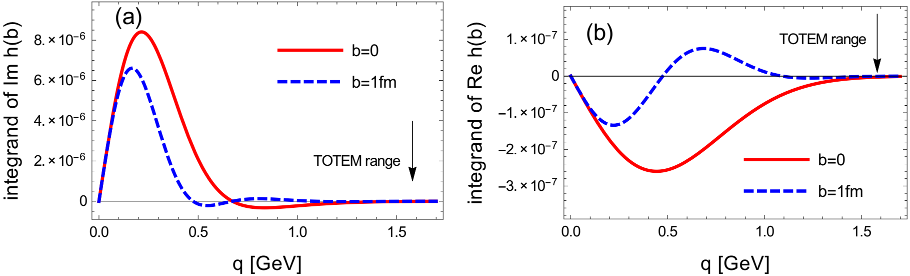
$\rightarrow$ not possible to obtain hollowness classically by folding the absorptive parts from constituents

Conclusions

- Hollowness transition inferred from the parametrization of the data, seen in $\sigma_{in}(b)$ for $s > 3$ TeV
- Quantum effect, related to compositeness of the proton and the gradual rise of the real part of the eikonal phase above $\pi/2$
- Effect impossible to obtain incoherently by folding the absorptive parts from constituents $\rightarrow$ change of paradigm used in many models
- Hot-spot model [Alba Soto+Albacete 2016] – a dynamic realization

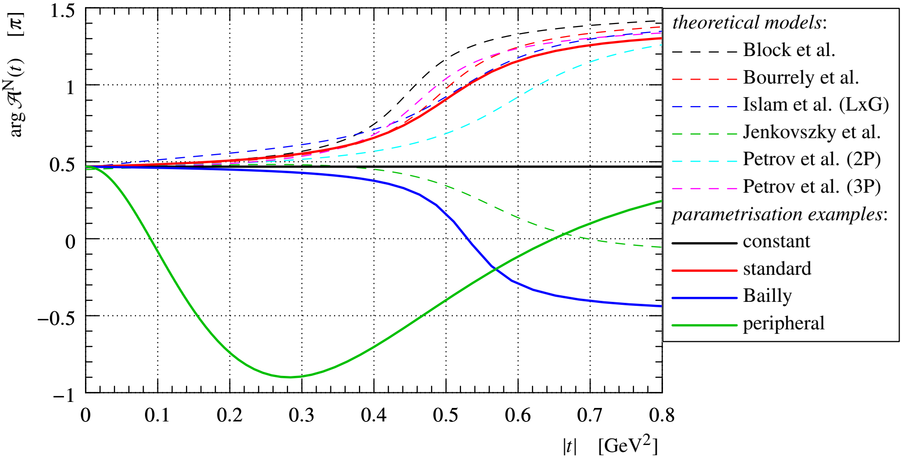
- 2D $\rightarrow$ 3D greatly magnifies the hollowness effect (flat in 2D $\rightarrow$ hollow in 3D), interpretation via optical potential [ERA+WB, 2016]
- Qualitatively similar hollowness effect appears in low-energy ($\sim$500 keV) n-A scattering – less absorption for head-on collisions than for peripheral! $\mathrm{Im}V(r) \sim d/dr\,\mathrm{Re}V(r)$

Fourier-Bessel transform



(TOTEM extends far enough)

$\rho(s,t)$



from a TOTEM analysis