Excited QCD 2017

# The Topological Susceptibility via the Gribov horizon?

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# Content

# Important properties of QCD



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# Important properties of QCD

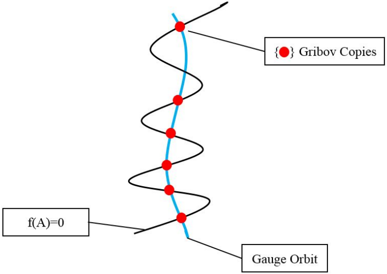


There are two important properties of QCD that are decisive for determining its observable particle spectrum:

- confinement
- chiral symmetry breaking

# Gribov problem

- Gribov[1] showed that the Faddeev-Popov construction is not valid at the non-perturbative level.




[1] V. N. Gribov, Nucl. Phys. B **139** (1978) 1.

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Consequently, Gribov copies imply that:

- we are overcounting equivalent gauge configurations, since we have more than one configuration for each gauge orbit,
- the Faddeev-Popov measure is ill-defined, since there are zero-modes of the Faddeev-Popov operator when considering the infinitesimal copies ($det\ M = 0$).

# Gribov region



$$\Omega = \{A^a_\mu;\ \partial_\mu A^a_\mu = 0, \quad \mathcal{M}^{ab}(A) = -\partial_\mu D^{ab}_\mu(A) > 0\}. \tag{1}$$

[2] G. Dell'Antonio and D. Zwanziger, Nucl. Phys. B **326** (1989) 333.

# Gribov region



$$\Omega = \{A^a_\mu;\ \partial_\mu A^a_\mu = 0, \quad \mathcal{M}^{ab}(A) = -\partial_\mu D^{ab}_\mu(A) > 0\}. \tag{1}$$

- Landau gauge, $\partial_\mu A^a_\mu = 0$,
- Hermitian Faddeev-Popov operator,

$$\mathcal{M}^{ab}(A) = -\delta^{ab}\partial^2 + gf^{abc}(A)^c_\mu \partial_\mu, \tag{2}$$

  is positive. Inside the Gribov region, there are no infinitesimal copies, since $\mathcal{M}^{ab}(A) > 0$;
- it is convex, bounded and intersected by each gauge orbit[2]
- Its boundary, $\partial\Omega$, is called the first Gribov horizon and there, the first null eigenvalue of $\mathcal{M}^{ab}(A)$ (i.e. the first zero-mode of Faddeev-Popov operator) appears.

[2] G. Dell'Antonio and D. Zwanziger, Nucl. Phys. B **326** (1989) 333.

# RGZ action



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$$S_{YM} = \frac{1}{4} \int d^4x F^a_{\mu\nu} F^a_{\mu\nu} , \tag{5}$$

$S_{FP}$ is the Landau gauge Faddeev-Popov action,

$$S_{FP} = \int d^4x \left( ib^a \, \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu (A) c^b \right) , \tag{6}$$

# RGZ action



The RGZ (Refined Gribov-Zwanziger) action is [3] [4] [5]

[3] D. Dudal, S. P. Sorella, N.Vandersickel and H. Verschelde, Phys. Rev. D **77** (2008) 071501.

[4] D. Dudal, J. A. Gracey, S. P. Sorella, N. Vandersickel and H. Verschelde, Phys. Rev. D **78** (2008) 065047.

[5] D. Dudal, S. P. Sorella and N. Vandersickel, Phys. Rev. D **84** (2011) 065039.

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\begin{aligned}
S_{RGZ} \quad = \quad & \int d^4x \left[\bar{\varphi}^{ac}_\mu \partial_\nu D^{ab}_\nu \varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu \partial_\nu (D^{ab}_\nu \omega^{bc}_\mu) - g(\partial_\nu \bar{\omega}^{an}_\mu) f^{abc} D^{bm}_\nu c^m \varphi^{cn}_\mu \right] \\
& -\gamma^2 g \int d^4x \left[ f^{abc} A^a_\mu \varphi^{bc}_\mu + f^{abc} A^a_\mu \bar{\varphi}^{bc}_\mu + \frac{d}{g}(N_c^2 - 1)\gamma^2 \right] \\
& + \frac{m^2}{2} \int d^4x A^a_\mu A^a_\mu + M^2 \int d^4x (\bar{\varphi}^{ab}_\mu \varphi^{ab}_\mu - \bar{\omega}^{ab}_\mu \omega^{ab}_\mu).
\end{aligned}
\tag{7}
$$

[3] D. Dudal, S. P. Sorella, N.Vandersickel and H. Verschelde, Phys. Rev. D **77** (2008) 071501.

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# The local gauge invariance of $A^h_\mu$



The configuration $A^h_\mu$ is a non-local power series in the gauge field, obtained by minimizing the functional $f_A[u]$ along the gauge orbit of $A_\mu$ [6] [7] [8] , with

$$
\begin{aligned}
f_A[u] &\equiv \min_{\{u\}} \mathrm{Tr} \int d^4x\, A^u_\mu A^u_\mu, \\
A^u_\mu &= u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u. \qquad (8)
\end{aligned}
$$

One finds that a local minimum is given by

$$
\begin{aligned}
A^h_\mu &= \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}\right)\phi_\nu \,, \qquad \partial_\mu A^h_\mu = 0 \,, \\
\phi_\nu &= A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \frac{1}{\partial^2}\partial A\right] + O(A^3). \qquad (9)
\end{aligned}
$$

6 G. Dell'Antonio and D. Zwanziger, Nucl. Phys. B **326** (1989) 333.

7 P. van Baal, Nucl. Phys. B **369** (1992) 259.

8 M. Lavelle and D. McMullan, Phys. Rept. **279** (1997) 1.

# The local gauge invariance of $A_\mu^h$



We set

$$A_\mu^h = (A^h)_\mu^a T^a = h^\dagger A_\mu^a T^a h + \frac{i}{g} h^\dagger \partial_\mu h, \tag{10}$$

while

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The local gauge invariance of $A_\mu^h$ under a gauge transformation $u \in SU(N)$ is now immediately clear from

$$h \to u^\dagger h\,, \quad h^\dagger \to h^\dagger u\,, \quad A_\mu \to u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u. \tag{12}$$

# BRST invariance



The action $S$ enjoys an exact nilpotent BRST invariance, $sS = 0$, if we assign the following BRST transformation rules to all fields,

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b\,, \quad sc^a = \frac{g}{2} f^{abc} c^b c^c\,, \\
s\bar{c}^a &= ib^a\,, \quad sb^a = 0\,. \\
sh^{ij} &= -igc^a (T^a)^{ik} h^{kj} \\
s\varphi^{ab}_\mu &= 0\,, \quad s\omega^{ab}_\mu = 0\,, \\
s\bar{\omega}^{ab}_\mu &= 0\,, \quad s\bar{\varphi}^{ab}_\mu = 0\,, \\
s\tau^a &= 0. \qquad (13)
\end{aligned}
$$

# Gluon propagator



$$D_{\mu\nu}(p) = D(p)P_{\mu\nu}(p) + L(p)\frac{p_\mu p_\nu}{p^2}, \tag{14}$$

with the transverse form factor $D(p)$ (at tree level),

$$D(p) \;=\; \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + M^2 m^2 + \lambda^4}. \tag{15}$$

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$$L(p) = \frac{\alpha}{p^2}, \tag{16}$$

with

$$P_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}, \qquad L_{\mu\nu}(p) = \frac{p_\mu p_\nu}{p^2} \tag{17}$$

the transversal and longitudinal projectors.

# Topological charge density



In Euclidean space-time, we have the classical instanton solutions, describing in Minkowski space-time the tunneling between the degenerate vacuum states with different Chern-Simons charge[9],

$$X = \int d^3 x K_0, \tag{18}$$

with $K_0$ the temporal component of topological Chern-Simons current,

[9] D. E. Kharzeev, Int. J. Mod. Phys. A **31** (2016) no.28n29, 1645023.

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$$K_\mu = \frac{g^2}{16\pi^2}\epsilon_{\mu\nu\rho\sigma} A_{\nu,a}\left(\partial^\rho A^{\sigma,a} + \frac{g}{3} f^{abc} A_b^\rho A_c^\sigma\right). \tag{19}$$

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This current is related to the topological charge density,

$$Q(x) = \partial_\mu K_\mu = \frac{g^2}{32\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu}. \tag{20}$$

[9] D. E. Kharzeev, Int. J. Mod. Phys. A **31** (2016) no.28n29, 1645023.

# Veneziano ghost



Witten and Veneziano suggested that the vacuum topology fluctuations can be captured by the occurrence of an unphysical mass pole[10 11], the Veneziano ghost, in the topological current correlator

[10] E. Witten, Nucl. Phys. B **156** (1979) 269.

[11] G. Veneziano, Nucl. Phys. B **159** (1979) 213.

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$$p_\mu p_\nu \left\langle K_\mu K_\nu \right\rangle_{p=0} \neq 0. \quad (21)$$

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Thus, the Veneziano solution was to assume that

$$K_{\mu\nu}(p) = i \int d^4x \; e^{ipx} \left\langle K_\mu(x) K_\nu(0) \right\rangle \overset{p^2 \sim 0}{\sim} -\frac{\chi^4}{p^2} g_{\mu\nu}, \tag{22}$$

where $\chi^4 \geq 0$ is the topological susceptibility of pure Yang-Mills theory.

[10] E. Witten, Nucl. Phys. B **156** (1979) 269.

[11] G. Veneziano, Nucl. Phys. B **159** (1979) 213.

# The “glost” of Kharzeev-Levin



An effective ghost-gluon vertex $\Gamma_\mu(q, p)$ was postulated, and then it was found that a dynamically corrected gluon propagator (the “glost”),

$$\mathcal{G}_{\mu\nu}(p^2) = \frac{p^2}{p^4 + \chi^4}\delta_{\mu\nu}, \tag{23}$$

solves the Dyson-Schwinger equation, when using only this coupling[12] [13] in the deep infrared. Immediately, we notice that there is an inconsistency between (14) and (23), indicating that the propagator (23) is incompatible with BRST symmetry.

[12] D. E. Kharzeev and E. M. Levin, Phys. Rev. Lett. **114** (2015) 24, 242001.

[13] D. Dudal and M. S. Guimaraes, Phys. Rev. D **93** (2016) no.8, 085010.

# Topological susceptibility $\chi^4$



The topological susceptibility $\chi^4$ can be written as

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The topological susceptibility $\chi^4$ can be written as

$$\chi^4 = - \lim_{p^2 \to 0} p_\mu p_\nu \langle K_\mu K_\nu \rangle \geq 0. \qquad (24)$$

# Källén-Lehmann spectral density and $\chi^4$



Let us show this also removes any ambiguity imposed by the subtraction procedure. We may in general set

$$\begin{aligned} \langle K_\mu(p) K_\nu(-p) \rangle &= \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \mathcal{K}_\perp(p^2) + \frac{p_\mu p_\nu}{p^2} \mathcal{K}_\parallel(p^2) \\ &\equiv \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \int_0^\infty d\tau \frac{\rho_\perp(\tau)}{\tau + p^2} + \frac{p_\mu p_\nu}{p^2} \int_0^\infty d\tau \frac{\rho_\parallel(\tau)}{\tau + p^2}, \end{aligned} \tag{25}$$

based on Euclidean invariance. Then, we already find that

$$\hat{\mathcal{Q}}(p^2) = -p^2 \mathcal{K}_\parallel(p^2) = -p^2 \int_0^\infty d\tau \frac{\rho_\parallel(\tau)}{\tau + p^2} \tag{26}$$

and thus

$$-\chi^4 = \lim_{p^2 \to 0} p^2 \mathcal{K}_\parallel(p^2) = \lim_{p^2 \to 0} p^2 \int_0^\infty d\tau \frac{\rho_\parallel(\tau)}{\tau + p^2}. \tag{27}$$

# Källén-Lehmann spectral density and $\chi^4$



From dimensional analysis, it is clear that this time we only need 2 subtractions ($\rho_{||}(\tau) \sim \tau$ for $\tau \to \infty$), so a finite result is guaranteed from

$$\mathcal{K}_{||}(p^2) = b_0 + b_1 p^2 + p^4 \int_0^\infty d\tau \frac{\rho_{||}(\tau)}{(\tau + p^2)\tau^2} \tag{28}$$

and thus

$$-\chi^4 = \lim_{p^2 \to 0} p^2 \left( b_0 + b_1 p^2 + p^4 \int_0^\infty d\tau \frac{\rho_{||}(\tau)}{(\tau + p^2)\tau^2} \right), \tag{29}$$

with $b_{0,1}$ subtraction constants. Obviously, we can rewrite (29) as

$$-\chi^4 = \lim_{p^2 \to 0} p^6 \int_0^\infty d\tau \frac{\rho_{||}(\tau)}{(\tau + p^2)\tau^2}. \tag{30}$$

# The spectral density associated with the Källén-Lehmann representation



We temporarily rewrite the RGZ gluon propagator as

$$D(p^2) = \frac{p^2 + M_1^2}{p^4 + M_2^2 p^2 + M_3^4}, \tag{31}$$

we obtain

$$\rho_{||}(\tau) = -2A_+A_- \frac{g^4(N^2-1)}{2^{2d+5}\pi^{7/2}\Gamma(\frac{d-1}{2})} \frac{\left(\tau^2 - 4b^2 - 4a\tau\right)^{(d-1)/2}}{\tau^{d/2}}, \tag{32}$$

for $\tau \geq \tau_c = 2(a + \sqrt{a^2 + b^2})$, where

$$a = \frac{M_2^2}{2}, \qquad b = \frac{\sqrt{4M_3^4 - M_2^4}}{2}. \tag{33}$$

In MOM scheme:

$$D(p^2 = \mu^2) = \frac{1}{\mu^2}. \tag{34}$$

# $g^2(\mu)$ in MOM scheme



The proper renormalization factor $Z$ is thus given by, at scale $\mu$,

$$D(p^2) = Z \frac{p^2 + M_1^2}{p^4 + M_2^2 p^2 + M_3^4}, \tag{35}$$

with

$$Z = \frac{1}{\mu^2} \frac{\mu^4 + M_2^2 \mu^2 + M_3^4}{\mu^2 + M_1^2}. \tag{36}$$

The gluon propagator we will use is to be renormalized in MOM scheme at scale $\mu$, so the $g^2$ present becomes

$$g^2(\mu) = \frac{1}{\beta_0 \log\left(\frac{\mu^2}{\Lambda^2_{\text{MOM}}}\right)}, \qquad \beta_0 = \frac{11}{3} \frac{N}{16\pi^2}. \tag{37}$$

We use $\Lambda^{N=2}_{\text{MOM}} \approx 628$ MeV and $\Lambda^{N=3}_{\text{MOM}} \approx 425$ MeV [14].

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[14] P. Boucaud, F. De Soto, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene, J. Rodriguez-Quintero, Phys. Rev. **D79** (2009) 014508.

# Padé approximation



- At the to be considered scales $\mu$, relative to the MOM scale, after which we “extrapolate” to the deep infrared using the described Padé analysis.
- We approximated (30) with the [3,1] Padé rational function in variable $p^2$.
- We opted to do the Padé approximation around $p^2 = P^2$.

# The spectral density in MOM scheme in $SU(3)$



For $N = 3$, the spectral density thence reads

$$\rho_{||}(\tau) = -2A_+A_- \frac{g^4(\mu)Z^2}{2^9\pi^4} \frac{\left(\tau^2 - 4b^2 - 4a\tau\right)^{3/2}}{\tau^2}. \quad (38)$$

Using the lattice obtained values $M_1^2 = 4.473\ \text{GeV}^2$; $M_2^2 = 0.704\ \text{GeV}^2$; $M_3^4 = 0.3959\ \text{GeV}^4$ [15],we get

$$a = 0.352\ \text{GeV}^2, \qquad b = 0.522\ \text{GeV}^2, \qquad 2A_+A_- = 31.719. \quad (39)$$

[15] O. Oliveira and P. J. Silva, Phys. Rev. D **86** (2012) 114513.

# The spectral density in MOM scheme in $SU(3)$

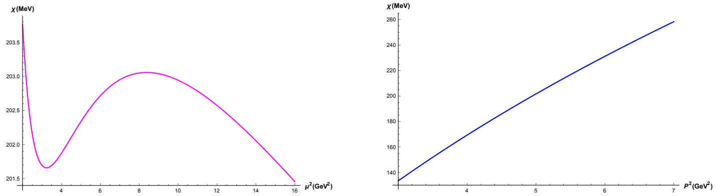


Figure: Topological susceptibility $\chi$ for variable $\mu^2$ and fixed $P^2 = 5$ GeV$^2$ (left) and for variable $P^2$ and fixed $\mu^2 = 3.230$ GeV$^2$ (left) $\mu^2$ ($SU(3)$ case).

# The spectral density in MOM scheme in $SU(2)$



For $N = 2$, the spectral density thence reads

$$\rho_{||}(\tau) = -2A_+A_- \frac{3g^4(\mu)Z^2(\mu)}{2^{12}\pi^4} \frac{\left(\tau^2 - 4b^2 - 4a\tau\right)^{3/2}}{\tau^2}. \qquad (40)$$

Following the same procedure as for $N = 3$, we get the graphs of FIG. 2 in the $N = 2$ case. Here, we used $M_1^2 = 2.508$ GeV$^2$; $M_2^2 = 0.590$ GeV$^2$; $M_3^4 = 0.518$ GeV$^4$ [16], yielding

$$a = 0.295 \text{ GeV}^2, \qquad b = 0.657 \text{ GeV}^2, \qquad 2A_+A_- = 6.176. \qquad (41)$$

[16] A. Cucchieri, D. Dudal, T. Mendes and N. Vandersickel, Phys. Rev. D **85** (2012) 094513

# The spectral density in MOM scheme in $SU(2)$



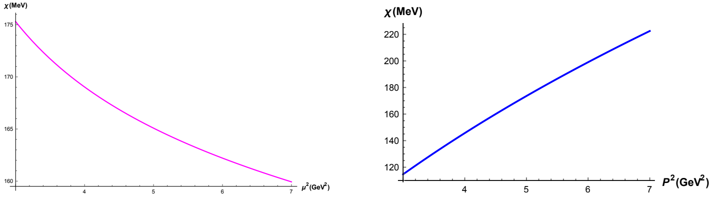


Figure: Topological susceptibility $\chi$ for variable $\mu^2$ and fixed $P^2 = 5$ GeV$^2$ (left) and for variable $P^2$ and fixed $\mu^2 = 3.330$ GeV$^2$ (left) $\mu^2$ ($SU(2)$ case).

# Conclusion



- In an attempt to get estimates for the topological susceptibility, we developed a particular Padé rational function approximation based on the Källén-Lehmann spectral integral representation of the topological current correlation function.

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# Conclusion



- In an attempt to get estimates for the topological susceptibility, we developed a particular Padé rational function approximation based on the Källén-Lehmann spectral integral representation of the topological current correlation function.
- We can estimate a range of values for the topological susceptibility $\chi^4$ qualitatively compatible with lattice data.
- In order to improve upon this crude estimates, we plan to include the next order correction in future work. Notice this will be computationally challenging, thanks to the significantly enlarged set of vertices in the now considered Gribov-Zwanziger action for the linear covariant gauge.

Obrigada!