Phase diagram and isentropic curves from the vector meson extended Polyakov quark meson model

Péter Kovács

Wigner Research Centre for Physics

kovacs.peter@wigner.mta.hu

10 May 2017

eQCD, Sintra, Portugal

Collaborators: Zsolt Szép, György Wolf

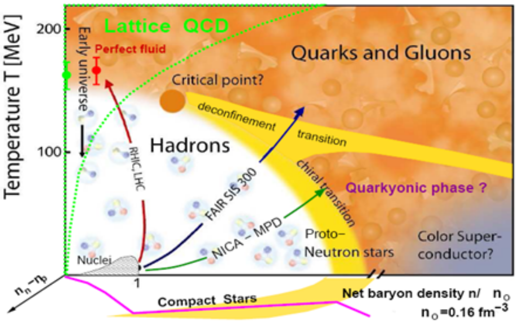
results partially based on: PRD87 (2013) no.1, 014011; PRD93 (2016) no.11, 114014

Overview

1. Introduction
   - Motivation, chiral symmetry
2. The model
   - Vector meson extended PQM model
   - Vector meson extended PQM model – Polyakov loop
3. eLSM at finite T/μ_B
   - Field equations
   - Meson masses
   - Parametrization at $T = 0$
4. Results
   - Temperature dependence of order parameters and masses
   - Pressure and derived quantities
   - Critical endpoint
   - Isentropic curves
5. Summary

Envisaged phase diagram of QCD

Effective models helps revealing the rich phase structure at large μ_B. The success of an effective model depends on: d.o.f used, implemented resummation, parametrization of the model ...



- At $\mu_B = 0$ $T_c = 153(3)$ MeV Y. Aoki,*et al.*, PLB **643**, 46 (2006)
- Is there a CEP?
- The T-dependence of thermodynamical quantities like pressure, interaction measure, quark density is known from lattice only at $\mu_B = 0$.
- At which μ_B is there the phase boundary for $T = 0$?
- In medium changes of masses and widths

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)

Chiral symmetry, chiral models

If the quark masses are zero (chiral limit) $\Longrightarrow$ QCD invariant under the following global transformation (chiral symmetry):

$$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A = SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$$

$U(1)_V \longrightarrow$ baryon number conservation (exact symmetry of nature)

$U(1)_A \longrightarrow$ connected to axial anomaly

$U(3)_L \times U(3)_R \longrightarrow$ broken down to $U(1)_V \times SU(2)_V$ if $m_u = m_d \neq m_s$
$\longrightarrow$ or to $U(1)_V$ if $m_u \neq m_d \neq m_s$ (realized in nature)

Since QCD is very hard to solve $\longrightarrow$ low energy effective models $\longrightarrow$ reflecting the global symmetries of QCD $\longrightarrow$ degrees of freedom: observable particles instead of quarks and gluons

Linear realization of the symmetry $\longrightarrow$ linear sigma model

Lagrangian I.

$\mathcal{L}$ constructed based on linearly realized global $U(3)_L \times U(3)_R$ symmetry and its explicit breaking

$$
\begin{aligned}
\mathcal{L} = {} & \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\
& + c_1(\det\Phi + \det\Phi^\dagger) + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\
& + \mathrm{Tr}\left[\left(\frac{m_1^2}{2}\mathbb{1} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger) \\
& + \bar{\Psi}i\gamma_\mu D^\mu\Psi - g_F\bar{\Psi}(\Phi_S + i\gamma_5\Phi_{PS})\Psi,
\end{aligned}
$$

$$
\begin{aligned}
D^\mu\Phi &= \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3, \Phi], \\
L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\}, \\
R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\}, \\
D^\mu\Psi &= \partial^\mu\Psi - iG^\mu\Psi, \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a.
\end{aligned}
$$

+ Polyakov loop potential

Lagrangian II.

the matter and external fields are

$$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8}(\rho_i^\mu - b_i^\mu)T_i, \quad L^\mu = \sum_{i=0}^{8}(\rho_i^\mu + b_i^\mu)T_i, \quad \Delta = \sum_{i=0}^{8}\delta_i T_i$$

$$\Psi = (u, d, s)^T$$

non strange – strange base:

$$\xi_N = \sqrt{2/3}\xi_0 + \sqrt{1/3}\xi_8,$$
$$\xi_S = \sqrt{1/3}\xi_0 - \sqrt{2/3}\xi_8, \qquad \xi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates $\langle\sigma_{N/S}\rangle \equiv \bar{\sigma}_{N/S}$

Mesonic particle content

- Vector and Axial-vector meson nonets

$$V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star +} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu \quad A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

$\rho \to \rho(770), K^\star \to K^\star(894)$
$\omega_N \to \omega(782), \omega_S \to \phi(1020)$

$a_1 \to a_1(1230), K_1 \to K_1(1270)$
$f_{1N} \to f_1(1280), f_{1S} \to f_1(1426)$

- Scalar $\left(\sim \bar{q}_i q_j\right)$ and pseudoscalar $\left(\sim \bar{q}_i \gamma_5 q_j\right)$ meson nonets

$$\Phi_S = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_0^{\star +} \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_0^{\star 0} \\ K_0^{\star -} & \bar{K}_0^{\star 0} & \sigma_S \end{pmatrix} \quad \Phi_{PS} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

unknown assignment
mixing in the $\sigma_N - \sigma_S$ sector

$\pi \to \pi(138), K \to K(495)$
mixing: $\eta_N, \eta_S \to \eta(548),\ \eta'(958)$

Spontaneous symmetry breaking: $\sigma_{N/S}$ acquire nonzero expectation values $\phi_{N/S}$

fields shifted by their expectation value: $\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S}$

Structure of scalar mesons below 2 GeV

	Mass (MeV)	width (MeV)	decays
$a_0(980)$	980 ± 20	$50 - 100$	$\pi\pi$ dominant
$a_0(1450)$	1474 ± 19	265 ± 13	$\pi\eta$, $\pi\eta'$, $K\bar{K}$
$K_s(800) = \kappa$	682 ± 29	547 ± 24	$K\pi$
$K_s(1430)$	1425 ± 50	270 ± 80	$K\pi$ dominant
$f_0(500) = \sigma$	400–550	$400 - 700$	$\pi\pi$ dominant
$f_0(980)$	980 ± 20	$40 - 100$	$\pi\pi$ dominant
$f_0(1370)$	1200–1500	$200 - 500$	$\pi\pi \approx 250$, $K\bar{K} \approx 150$
$f_0(1500)$	1505 ± 6	109 ± 7	$\pi\pi \approx 38$, $K\bar{K} \approx 9.4$
$f_0(1710)$	1722 ± 6	135 ± 7	$\pi\pi \approx 30$, $K\bar{K} \approx 71$

scalar $\bar{q}q$ nonet content: 1 a_0, 1 $K_0^\star$, and 2 f_0s $\Longrightarrow$ 40 possible assignments

result of a $T = 0$ parametrization: $a_0^{\bar{q}q} \to a_0(1450)$, $K_0^{\star,\bar{q}q} \to K_0^\star(1430)$
$f_0^{L,\bar{q}q} \to f_0(1370)$, $f_0^{H,\bar{q}q} \to f_0(1710)$

D. Parganlija *et al.*, PRD87, 014011

Considering only $\bar{q}q$ states is unrealistic because most probably scalars are mixtures of $\bar{q}q$, tetraquarks and glueballs, nevertheless we will do this here.

$f_0(500)$, $f_0(980)$, $a_0(980)$, $K_0^\star(800)$ could be predominantly tetraquarks
$f_0(1710)$ could be predominantly glueball $\longrightarrow$ F. Giacosa's talk

Features of our approach

- D.O.F's: – scalar, pseudoscalar, vector, and axial-vector nonets
  – u, d, s constituent quarks ($m_u = m_d$)
  – Polyakov loop variables $\Phi, \bar{\Phi}$ with $\mathcal{U}_{\log}^{\rm YM}$ or $\mathcal{U}_{\log}^{\rm glue}$
- no mesonic fluctuations in the grand potential, only fermionic ones

$$\mathcal{Z}=e^{-\beta V\Omega(T,\mu_q)}=\int_{\rm PBC}\prod_a \mathcal{D}\xi_a \int_{\rm APBC}\prod_f \mathcal{D}q_f\mathcal{D}q_f^\dagger \exp\left[-\int_0^\beta d\tau \int_V d^3x\left(\mathcal{L}+\mu_q\sum_f q_f^\dagger q_f\right)\right]$$

approximated as $\Omega(T,\mu_q) = U_{\rm meson}^{\rm tree}(\langle M\rangle) + \Omega_{\bar{q}q}^{(0)}(T,\mu_q) + \mathcal{U}_{\log}(\Phi,\bar{\Phi})$ with $\tilde{\mu}_q=\mu_q-iG_4$

$$e^{-\beta V\Omega_{\bar{q}q}^{(0)}} = \int_{\rm APBC}\prod_{f,g}\mathcal{D}q_g\mathcal{D}q_f^\dagger \exp\left\{\int_0^\beta d\tau\int_x q_f^\dagger\left[\left(i\gamma_0\vec{\gamma}\cdot\vec{\nabla}-\frac{\partial}{\partial_\tau}+\tilde{\mu}_q\right)\delta_{fg} - \gamma_0\mathcal{M}_{fg}|_{\xi_a=0}\right]q_g\right\}$$

- quarks not coupled to the (axial)vectors $\Longrightarrow$ tree-level (axial)vector masses
- fermionic vacuum and thermal fluctuations included in the (pseudo)scalar curvature masses used to parameterize the model
- 4 coupled T/μ_B-dependent field equations for condensates: $\phi_N, \phi_S, \Phi, \bar{\Phi}$
- thermal contribution of π, K, f_0^L included in the pressure, however their curvature mass contains no mesonic fluctuations

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp\left[i \int_0^\beta d\tau\, G_4(\vec{x}, \tau)\right]$$

↪ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement transition

low T: confined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$
high T: deconfined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

- Polyakov gauge: $G_4(\vec{x}, \tau) = G_4(\vec{x})$, plus gauge rotation to diagonal form in color space
- further simplification: $\vec{x}$-independence

$$\hookrightarrow \quad L = e^{i\beta G_4} = \mathrm{diag}(a, b, c) \left(\overset{!}{\in} SU(3)^{\text{color}} \right); \quad a, b, c \in \mathbb{Z}$$

↪ use this to calculate partition function of free quarks on constant gluon background

Effects of Polyakov loops on FD statistics

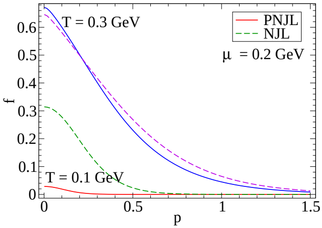
Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p-\mu_q) \longrightarrow f_\Phi^+(E_p) = \frac{\left(\bar\Phi + 2\Phi e^{-\beta\left(E_p-\mu_q\right)}\right) e^{-\beta\left(E_p-\mu_q\right)} + e^{-3\beta\left(E_p-\mu_q\right)}}{1+3\left(\bar\Phi + \Phi e^{-\beta\left(E_p-\mu_q\right)}\right) e^{-\beta\left(E_p-\mu_q\right)} + e^{-3\beta\left(E_p-\mu_q\right)}}$$

$$f(E_p+\mu_q) \longrightarrow f_\Phi^-(E_p) = \frac{\left(\Phi + 2\bar\Phi e^{-\beta\left(E_p+\mu_q\right)}\right) e^{-\beta\left(E_p+\mu_q\right)} + e^{-3\beta\left(E_p+\mu_q\right)}}{1+3\left(\Phi + \bar\Phi e^{-\beta\left(E_p+\mu_q\right)}\right) e^{-\beta\left(E_p+\mu_q\right)} + e^{-3\beta\left(E_p+\mu_q\right)}}$$

$\Phi, \bar\Phi \to 0 \Longrightarrow f_\Phi^\pm(E_p) \to f(3(E_p \pm \mu_q))$ $\quad \Phi, \bar\Phi \to 1 \Longrightarrow f_\Phi^\pm(E_p) \to f(E_p \pm \mu_q)$

three-particle state appears: mimics confinement of quarks within baryons



the effect of the Polyakov loop is more relevant for $T < T_c$

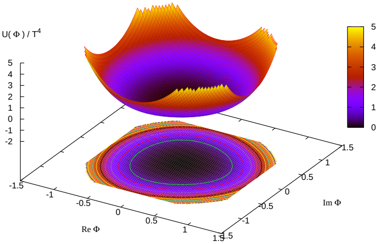
at $T = 0$ there is no difference between models with and without Polyakov loop: $\Theta(3(\mu_q - E_p)) \equiv \Theta((\mu_q - E_p))$
H. Hansen et al., PRD75, 065004

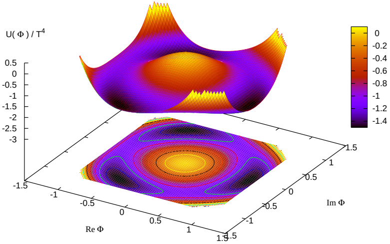
Polyakov loop potential

"Color confinement"
$\langle\Phi\rangle = 0 \longrightarrow$ no breaking of $\mathbb{Z}_3$

"Color deconfinement"
$\langle\Phi\rangle \neq 0 \longrightarrow$ spontaneous breaking of $\mathbb{Z}_3$

H. Hansen et al., PRD75, 065004 (2007)





Form of the potential:

- Polynomial: $U_{\text{YM}}^{\text{Poly}}$
- Logarithmic: U_{YM}
- Improved Polyakov loop potential (logarithmic): U_{glue}

Form of the potential

I.) Simple polynomial potential invariant under $\mathbb{Z}_3$ and charge conjugation: R.D.Pisarski, PRD 62, 111501

$$\frac{\mathcal{U}^{\rm YM}_{\rm poly}(\Phi,\bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2$$

with $\quad b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2} + a_3\frac{T_0^3}{T^3}$

II.) Logarithmic potential coming from the $SU(3)$ Haar measure of group integration K. Fukushima, Phys. Lett. **B591**, 277 (2004)

$$\frac{\mathcal{U}^{\rm YM}_{\rm log}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi\bar{\Phi} + b(T)\ln\left[1-6\Phi\bar{\Phi}+4\left(\Phi^3+\bar{\Phi}^3\right)-3\left(\Phi\bar{\Phi}\right)^2\right]$$

with $\quad a(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2}, \qquad b(T) = b_3\frac{T_0^3}{T^3}$

$\mathcal{U}^{\rm YM}\left(\Phi,\bar{\Phi}\right)$ models the free energy of a pure gauge theory
$\longrightarrow$ the parameters are fitted to the pure gauge lattice data

Improved Polyakov loop potential

Previous potentials describe successfully the first order phase transition of the pure $SU(3)$ Yang – Mills

$\hookrightarrow$ taking into account the gluon dynamics (quark polarization of gluon propagator) $\longrightarrow$ QCD glue potential

$\hookrightarrow$ can be implemented by changing the reduced temperature

$$t_{\text{glue}} \equiv \frac{T - T_c^{\text{glue}}}{T_c^{\text{glue}}}, \quad t_{\text{YM}} \equiv \frac{T^{\text{YM}} - T_c^{\text{YM}}}{T_c^{\text{YM}}}$$

$$t_{\text{YM}}(t_{\text{glue}}) \approx 0.57 t_{\text{glue}}$$

$$\frac{\mathcal{U}^{\text{glue}}}{T^4}(\Phi, \bar{\Phi}, t_{\text{glue}}) = \frac{\mathcal{U}^{\text{YM}}}{(T^{\text{YM}})^4}(\Phi, \bar{\Phi}, t_{\text{YM}}(t_{\text{glue}}))$$

L. M. Haas *et al.*, PRD 87, 076004 (2013)

Four coupled field equations are obtained by extremizing the grand potential

$$\Omega(T,\mu_q) = U^{\text{tree}}_{\text{meson}}(\langle M\rangle) + \Omega^{(0)\text{vac}}_{\bar{q}q} + \Omega^{(0)T}_{\bar{q}q}(T,\mu_q) + \mathcal{U}_{\log}(\Phi,\bar{\Phi})$$

using $\frac{\partial \Omega_H}{\partial \phi_N} = \frac{\partial \Omega_H}{\partial \phi_S} = \frac{\partial \Omega_H}{\partial \Phi} = \frac{\partial \Omega_H}{\partial \bar{\Phi}} = 0$ $\quad E_f^{\pm}(p) = E_f(p) \mp \mu_q, \quad E_f^2(p) = p^2 + m_f^2$

1) $$-\frac{1}{T^4}\frac{d\,U(\Phi,\bar{\Phi})}{d\Phi} + \frac{6}{T^3}\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\left(\frac{e^{-\beta E_f^-(p)}}{g_f^-(p)} + \frac{e^{-2\beta E_f^+(p)}}{g_f^+(p)}\right) = 0$$

2) $$-\frac{1}{T^4}\frac{dU(\Phi,\bar{\Phi})}{d\bar{\Phi}} + \frac{6}{T^3}\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\left(\frac{e^{-\beta E_f^+(p)}}{g_f^+(p)} + \frac{e^{-2\beta E_f^-(p)}}{g_f^-(p)}\right) = 0$$

3) $$m_0^2\phi_N + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi_N^3 + \lambda_1\phi_N\phi_S^2 - \frac{c_1}{\sqrt{2}}\phi_N\phi_S - h_{0N} + \frac{3}{2}g_F\left(\langle\bar{q}_u q_u\rangle_T + \langle\bar{q}_d q_d\rangle_T\right) = 0$$

4) $$m_0^2\phi_S + (\lambda_1 + \lambda_2)\phi_S^3 + \lambda_1\phi_N^2\phi_S - \frac{\sqrt{2}}{4}c_1\phi_N^2 - h_{0S} + \frac{3}{\sqrt{2}}g_F\langle\bar{q}_s q_s\rangle_T = 0$$

renormalized fermion tadpole: $\quad m_{u,d} = \frac{g_F}{2}\phi_N$ and $m_s = \frac{g_F}{\sqrt{2}}\phi_S$

$$\langle\bar{q}_f q_f\rangle_T = 4m_f\left[-\frac{m_f^2}{16\pi^2}\left(\frac{1}{2} + \ln\frac{m_f^2}{M_0^2}\right) + \int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_f(p)}\left(f_f^-(p) + f_f^+(p)\right)\right]$$

Curvature masses

$$\mathcal{M}^2_{i,ab} = \frac{\partial^2 \Omega(T,\mu_f)}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\Big|_{\rm min} = m^2_{i,ab} + \Delta_0 m^2_{i,ab} + \Delta_T m^2_{i,ab},$$

$m^2_{i,ab} \longrightarrow$ tree-level mass matrix,
$\Delta_{0/T} m^2_{i,ab} \longrightarrow$ fermion vacuum/thermal fluctuation,

$$\Delta_0 m^2_{i,ab} = \frac{\partial^2\Omega^{\rm vac}_{q\bar q}}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\Big|_{\rm min} = -\frac{3}{8\pi^2}\sum_{f=u,d,s}\left[\left(\frac{3}{2}+\log\frac{m_f^2}{M^2}\right)m^{2(i)}_{f,a}m^{2(i)}_{f,b} + m_f^2\left(\frac{1}{2}+\log\frac{m_f^2}{M^2}\right)m^{2(i)}_{f,ab}\right],$$

$$\Delta_T m^2_{i,ab} = \frac{\partial^2\Omega^{\rm th}_{q\bar q}}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\Big|_{\rm min} = 6\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_f(p)}\left[(f_f^+(p)+f_f^-(p))\left(m^{2(i)}_{f,ab}-\frac{m^{2(i)}_{f,a}m^{2(i)}_{f,b}}{2E_f^2(p)}\right)\right.$$
$$\left. + (B_f^+(p)+B_f^-(p))\frac{m^{2(i)}_{f,a}m^{2(i)}_{f,b}}{2TE_f(p)}\right],$$

where $m^{2(i)}_{f,a} \equiv \partial m_f^2/\partial\varphi_{i,a}$, $m^{2(i)}_{f,ab} \equiv \partial^2 m_f^2/\partial\varphi_{i,a}\partial\varphi_{i,b}$

Determination of the parameters

14 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F)$ $\longrightarrow$ determined by the min. of χ^2:

$$\chi^2(x_1,\dots,x_N) = \sum_{i=1}^{M}\left[\frac{Q_i(x_1,\dots,x_N) - Q_i^{\exp}}{\delta Q_i}\right]^2 ,$$

$(x_1,\dots,x_N) = (m_0, \lambda_1, \lambda_2, \dots)$, $Q_i(x_1,\dots,x_N)$ $\longrightarrow$ from the model, $Q_i^{\exp}$ $\longrightarrow$ PDG value, $\delta Q_i = \max\{5\%, \text{PDG value}\}$
multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Curvature masses $\rightarrow$ 16 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- Pseudocritical temperature T_c at $\mu_B = 0$

Result of the parametrization

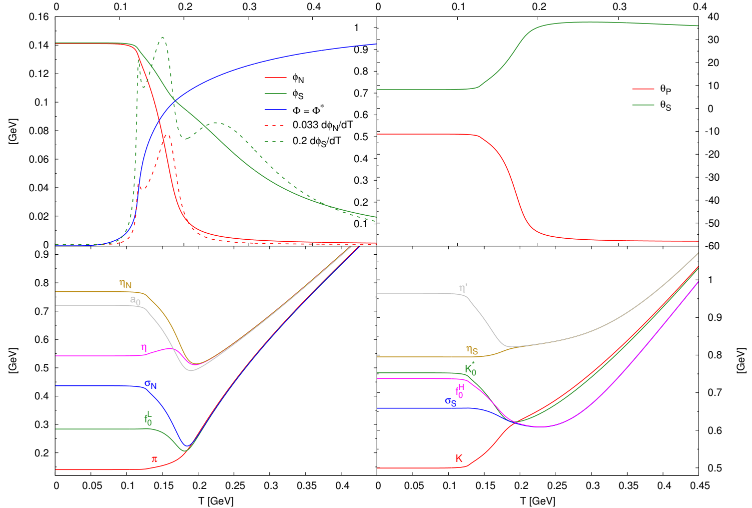
- 40 possible assignments of scalar mesons to the scalar nonet states
- 3 values of M_0 are used $\Longrightarrow$ 120 cases to investigate
  for each case $5\cdot 10^4 - 10^5$ configurations are used for the χ^2 minimization
- lowest χ^2 obtained for $M_0 = 0.3$ GeV $\qquad \chi^2 = 18.57$ and $\chi^2_{\text{red}} \equiv \frac{\chi^2}{N_{\text{dof}}} = 1.16$
  assignment: $a_0^{\bar{q}q} \to a_0(980)$, $K_0^{\star,\bar{q}q} \to K_0^\star(800)$, $f_0^{L,\bar{q}q} \to f_0(500)$, $f_0^{H,\bar{q}q} \to f_0(980)$

  problems: $m_{a_0} < m_{K_0^\star}$, $m_{f_0^{H/L}}$ too light
- by minimizing also for M_0 we obtain using $\mathcal{U}^{\text{YM}}_{\log}(\Phi,\bar{\Phi})$ with $T_0 = 182$ MeV:

Parameter	Value	Parameter	Value
ϕ_N [GeV]	0.1411	g_1	5.6156
ϕ_S [GeV]	0.1416	g_2	3.0467
m_0^2 [GeV^2]	2.3925$_{E-4}$	h_1	27.4617
m_1^2 [GeV^2]	6.3298$_{E-8}$	h_2	4.2281
λ_1	-1.6738	h_3	5.9839
λ_2	23.5078	g_F	4.5708
c_1 [GeV]	1.3086	M_0 [GeV]	0.3511
δ_S [GeV^2]	0.1133		

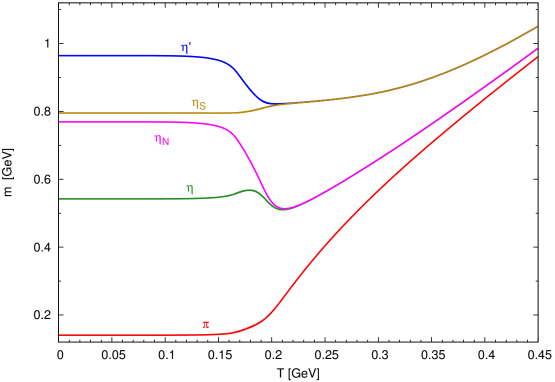
The presented results are obtained with this set of parameters.

T dependence of masses, condensates, mixing angles



- ch. partners (π, f_0^L), (η, a_0) and $(K, K_0^\star)$ become degenerate at high T
- $U(1)_A$ not restored, axial partners (π, a_0) and (η, f_0^L) not become degenerate

Mass pattern in the η, η' sector



☛ our pattern: $m_\eta \leq m_{\eta_N} < m_{\eta_S} \leq m_{\eta'}$ and similarly $m_{f_0^L} \leq m_{\sigma_N} < m_{\sigma_S} \leq m_{f_0^H}$ and also a_0 degenerates with η

in contrast to the pattern obtained w/o the inclusion of (axial)vector mesons

Schaefer & Wagner, PRD79, 014018 (QM) and Tiwari, PRD88, 074017 (PQM)

☛ in the FRG study of Rennecke & Schaefer, arXiv:1610.08748 (w/o (axial)vector mesons)

– LPA: a_0-meson degenerates with η'-meson

– LPA'+Y: a_0-meson degenerates with η-meson

Calculation of thermodynamical quantities

pressure: $p(T,\mu_q) = \Omega_{\mathrm{H}}(T=0,\mu_q) - \Omega_{\mathrm{H}}(T,\mu_q)$

entropy density: $s = \dfrac{\partial p}{\partial T}$, quark number density: $\rho_q = \dfrac{\partial p}{\partial \mu_q}$

energy density: $\epsilon = -p + Ts + \mu_q \rho_q$, speed of sound: $c_s^2 = \dfrac{\partial p}{\partial \epsilon}$,

We include mesonic thermal 1-loop contribution to the pressure:

$$p_{\mathrm{meson}} = -\Omega_{meson}^{\text{1-loop},T} = -n_b T \int \frac{d^3q}{(2\pi)^3} \ln\left(1 - e^{-\beta E_b(q)}\right)$$

where, $E_b(q) = \sqrt{q^2 + m_b^2}$, meson multiplicities : $n_\pi=3, n_K=4, n_{f_0^L}=1$

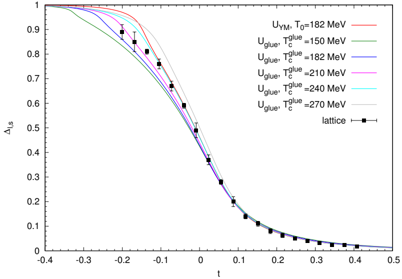
comparing with the lattice $\longrightarrow$

subtracted condensate: $\Delta_{l,s} = \dfrac{\Phi_N - \frac{h_N}{h_S}\cdot\Phi_S\big|_T}{\Phi_N - \frac{h_N}{h_S}\cdot\Phi_S\big|_{T=0}}$

scaled interaction measure: $I/T^4 = (\epsilon - 3p)/T^4$

t-dependence of the condensates compared to lattice results

The subtracted chiral condensate



– subtracted chiral condensate:

$$\Delta_{l,s} = \frac{\left(\Phi_N - \frac{h_N}{h_S}\cdot\Phi_S\right)\Big|_T}{\left(\Phi_N - \frac{h_N}{h_S}\cdot\Phi_S\right)\Big|_{T=0}}$$

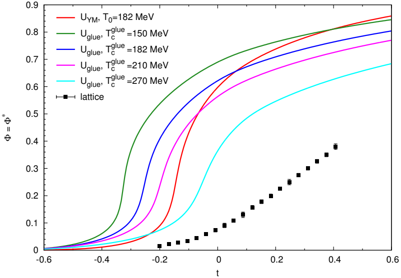
– $U_{\log}^{\rm glue}$ with $T_c^{\rm glue}\in(210,240)$ MeV gives good agreement with the lattice result of

Borsányi *et al.*, JHEP 1009, 073 (2010)

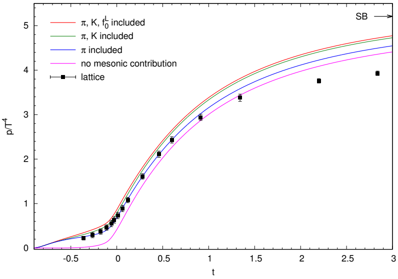
– lattice result shows a very smooth transition
– our result is completely off
– renormalization of the Polyakov loop could explain part of the discrepancy

Andersen *et al.*, PRD92, 114504

Polyakov loop expectation values

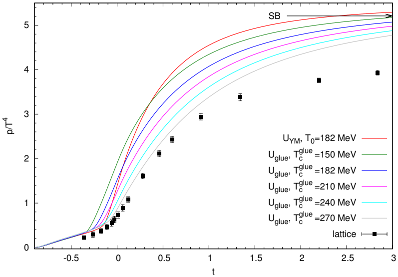


Normalized pressure and the effects of meson contributions



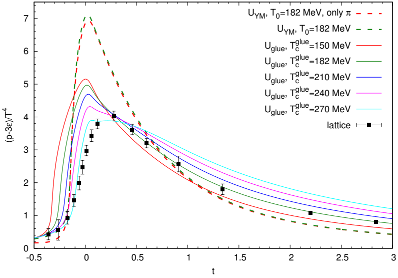
– we used $U_{\rm glue}$ with $T_c^{\rm glue} = 270$ MeV
– pions dominate the pressure at small T
– contribution of the kaons is important
– at high T the pressure overshoots the lattice data of Borsányi *et al.*, JHEP 1011, 077 (2010)

– overshooting increases with decreasing $T_c^{\rm glue}$

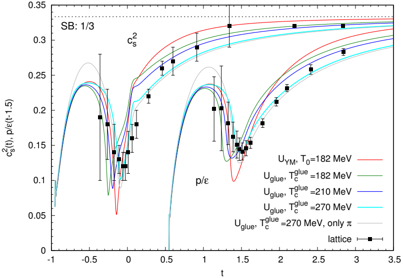


Scaled interaction measure, speed of sound and p/ϵ

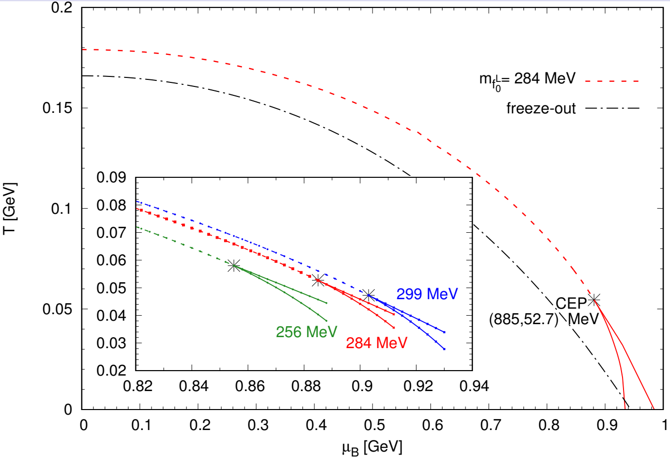
Scaled interaction measure



Speed of sound and p/ϵ



$T - \mu_B$ Phase Diagram



– we used $U_{\log}^{\text{glue}}$ with $T_c^{\text{glue}} = 210$ MeV

– freeze-out curve from Cleymans *et al.*, J.Phys.G 32, S165 (2006)

– curvature κ at $\mu_B = 0$ obtained from the fit $\frac{T_c(\mu_B)}{T_c(\mu_B{=}0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B{=}0)}\right)^2$

$\kappa = 0.0193$ obtained, close to the lattice value $\kappa = 0.020(4)$ of Cea *et al.*,PRD93, 014507

CEP obtained with other methods

- Dyson-Schwinger equation

$(\mu_B, T)_{\text{CEP}} \approx (3.0, 0.9)\, T_c(\mu = 0)$ Roberts *et al.*, PRL106 (2011) 172301

$(\mu_B, T)_{\text{CEP}} \approx (660, 97)$ MeV for $N_f = 2 + 1$

Gutierrez *et al.*, J.Phys. G41 (2014) 075002

$(\mu_B, T)_{\text{CEP}} = (504, 115)$ MeV for $N_f = 2 + 1$

Fischer *et al.*, PRD90 (2014) 034022

- FRG study of Rennecke & Schaefer, arXiv:1610.08748

$(\mu_B, T)_{\text{CEP}}$ =	(795,44) MeV	LPA
	(765,46) MeV	LPA + Y
	(705,61) MeV	LPA' + Y

Dependence of μ_B^{CEP} on the width of the susceptibility at $\mu = 0$

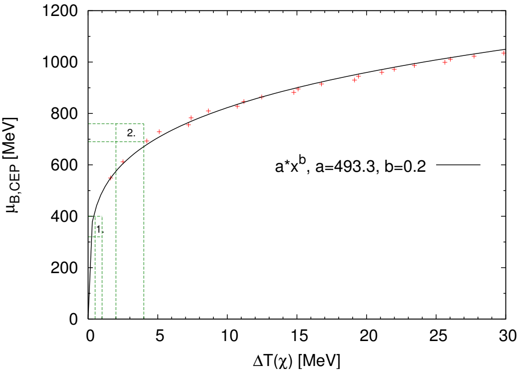
cf. P. Kovács, PhD thesis

Lattice results (fixed a):
$(\mu_B, T_c)^{CEP} = (725 \pm 35, 160 \pm 3.5)$MeV
$m_\pi \approx 2m_\pi^{phys} \longrightarrow$ shown as '2.' on fig.
Fodor & Katz, JHEP 0203:014,2002

$(\mu_B, T_c)^{CEP} = (360 \pm 40, 162 \pm 3)$MeV

$m_\pi = m_\pi^{phys} \longrightarrow$ shown as '1.' on fig.

Fodor & Katz, JHEP 0404:050,2004

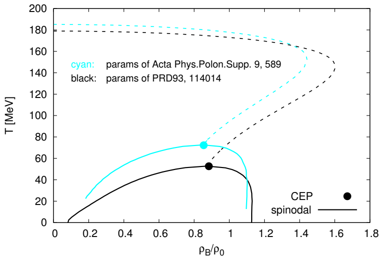


- estimation by S. D. Katz: $\Delta T_c(\chi_{\bar\psi\psi}) \approx 0.5 - 1$ MeV and $\Delta T_c(\chi_{\bar\psi\psi}) \approx 2 - 4$ MeV
- $T_c(\chi_{\bar\psi\psi}) \approx 28$ MeV at the physical point in the continuum limit Aoki *et al.* (2006)

$\Longrightarrow$ higher μ_B^{CEP} can be expected in continuum limit

$T - \rho_B$ Phase Diagram

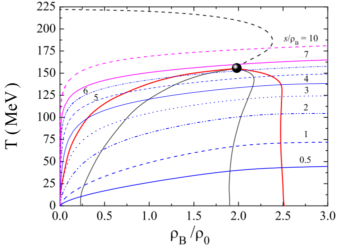
our model



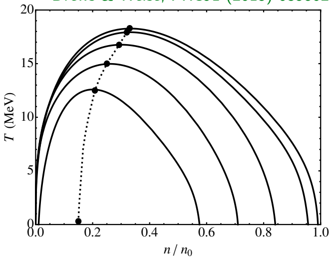
☛ the $T - \rho_B$ phase diagram of our model is closer to that of the nuclear liquid-gas PT than to those obtained in other chiral models

PNJL with vector interaction

P. Costa, PRD93 (2016) 114035

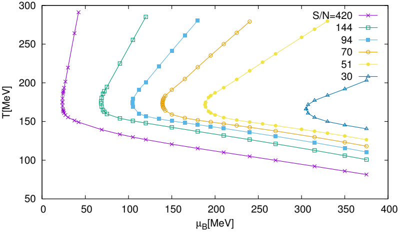


Drews & Weise, PRC91 (2015) 035802



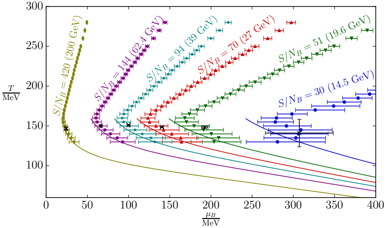
Isentropic trajectories in the $T - \mu_{\rm B}$ plane (I.)

our model, where $\mu_B^{\rm CEP} > 850\text{MeV}$



lattice (analytic continuation)

Günther *et al.*, arXiv:1607.02493



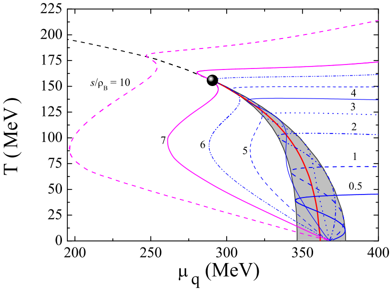
☛ same qualitative behavior of the isentropic trajectories for $\mu_B \leq 400$ MeV

$\Longrightarrow$ indication that in the lattice result there is no CEP in this region of μ_B

Isentropic trajectories in the $T - \mu_B$ plane (II.)

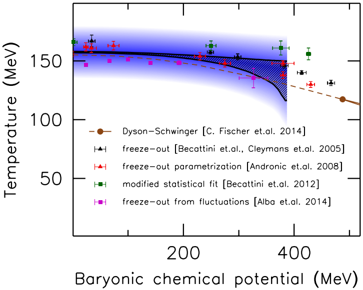
PNJL with vector interaction

P. Costa, PRD93 (2016) 114035



lattice (analytic continuation)

Bellwied *et al.*, PLB751 (2015) 559



☛ effective models show a different behavior of the isentropic trajectories close to CEP compared to the small μ_B case

☛ no indication for CEP from the continuum extrapolated lattice results analytically continued to the $\mu_B \leq 400$ MeV region

Summary

- The thermodynamics of the extended PQM model was studied with similar parameterization as in Parganlija *et al.*, PRD 87, 014011.
- 40 possible assignments of scalars to the nonet states were investigated. Lowest χ^2: all scalar masses below 1 GeV
- For the best set of parameters a CEP was found in the $\mu_B - T$ plane. A self-consistent treatment of quarks would most probably decrease μ_B^{CEP} and increase T_c^{CEP}.
- T and μ_B dependence of various thermodynamical observables measured on the lattice is qualitatively reproduced with an improved Polyakov loop potential.
- Comparison of isentropic curves with lattice results suggests that CEP with small μ_B value is unfavorable ($\mu_B^{CEP} < 400$ MeV)
- The model and the approximation used could be improved by:
  - including tetraquarks for a more reliable vacuum phenomenology
  - coupling the constituent quarks to the (axial)vectors
  - including mesonic fluctuations
  - treating the quarks self-consistently.

Backup slides

Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra is not $\longrightarrow$ SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \bar{\sigma}_{N/S} \qquad \bar{\sigma}_{N/S} \equiv <\sigma_{N/S}>$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\mathrm{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)]$:

$$\begin{aligned}
\eta_N - f_{1N}^\mu &: \quad -g_1 \bar{\sigma}_N f_{1N}^\mu \partial_\mu \eta_N, \\
\pi - a_1^\mu &: \quad -g_1 \bar{\sigma}_N (a_1^{\mu +} \partial_\mu \pi^- + a_1^{\mu 0} \partial_\mu \pi^0) + \mathrm{h.c.}, \\
\eta_S - f_{1S}^\mu &: \quad -\sqrt{2} g_1 \bar{\sigma}_S f_{1S}^\mu \partial_\mu \eta_S, \\
K_S - K_\mu^\star &: \quad \frac{ig_1}{2} (\sqrt{2}\bar{\sigma}_S - \bar{\sigma}_N)(\bar{K}_\mu^{\star 0} \partial^\mu K_S^0 + K_\mu^{\star -} \partial^\mu K_S^+) + \mathrm{h.c.}, \\
K - K_1^\mu &: \quad -\frac{g_1}{2} (\bar{\sigma}_N + \sqrt{2}\bar{\sigma}_S)(K_1^{\mu 0} \partial_\mu \bar{K}^0 + K_1^{\mu +} \partial_\mu K^-) + \mathrm{h.c.}.
\end{aligned} \tag{1}$$

Mixing in the extended model

Mixing in the $N-S$ sector for σ and π $\longrightarrow$ $(m_\sigma^2)_{NS} \neq 0$, $(m_\pi^2)_{NS} \neq 0$ $\rightarrow$ resolved 2 dim. orthog. transf.

Mixing between nonets $\longrightarrow$ $\rho_a^\mu \leftrightarrow \sigma$ and $b_a^\mu \leftrightarrow \pi$
Resolved by the following field shifts:

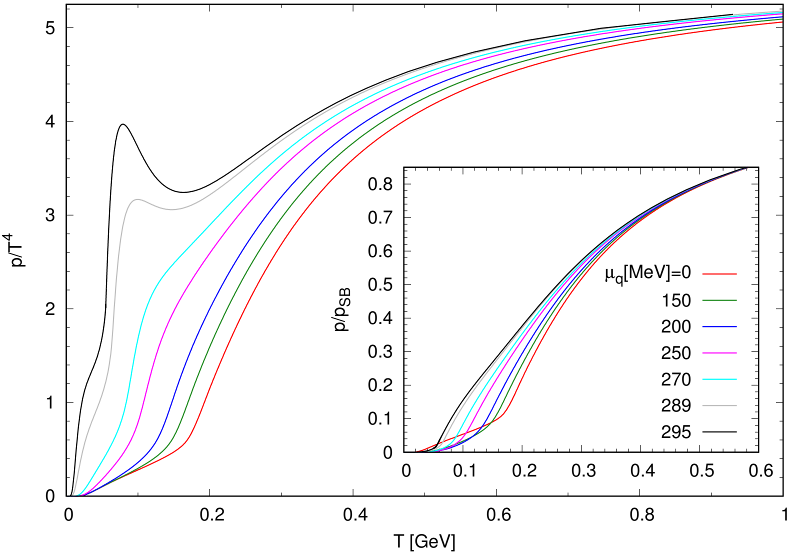
$$
\begin{aligned}
f_{1N/S}^\mu &\longrightarrow f_{1N/S}^\mu + w_{f_{1N/S}} \partial^\mu \eta_{N/S}, \\
a_1^{\mu+,0} &\longrightarrow a_1^{\mu+,0} + w_{a_1} \partial^\mu \pi^{+,0}, (+\text{h.c.}) \\
K_1^{\mu+,0} &\longrightarrow K_1^{\mu+,0} + w_{K_1} \partial^\mu K^{+,0}, (+\text{h.c.}) \\
K^{\star\mu+,0} &\longrightarrow K^{\star\mu+,0} + w_{K^\star} \partial^\mu K_S^{+,0} (+\text{h.c.})
\end{aligned}
$$

Vanishing of the crossterms $\longrightarrow$ determination of the w_i's

After these shifts, π, η_N, η_S, K, and K_S are not canonically normalized $\longrightarrow$ field renormalization: Z_π, Z_{η_N}, Z_{η_S}, Z_K, Z_{K_S}

(for details see PRD87 (2013) no.1, 014011)

Normalized pressure as a function of t at different μ_q's



Quark susceptibility and density versus t at different μ_q's