Anisotropic hydrodynamics for conformal quark-gluon mixture

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Introduction

- Standard approach to relativistic hydrodynamics might be questioned because it is based on the gradient expansion around equilibrium. At the early stages of heavy ion collisions gradients are large and this leads to substantial modifications of pressure.
- For describing the system with large pressure anisotropies it is better to use anisotropic hydrodynamics.
- So far, anisotropic hydrodynamics has been used mostly to describe simple (one-component) fluids, but we usually deal with mixtures of quarks and gluons.
- Anisotropic hydrodynamics for mixtures was initiated in:

  W.Florkowski, R.Maj, R.Ryblewski, M.Strickland, Phys. Rev. C **87**, 034914 (2013)

  W.Florkowski, O.Madetko, Acta Phys. Polon. B **45**, 1103 (2014) where unfortunately poor agreement with the underlying kinetic theory was found.
- In this talk I present a generalised description of mixtures within anisotropic hydrodynamics.

Ideas of the presented model

- One-dimensional and boost-invariant system.
- All particles in the mixture are massless.
- We use zeroth, first and second moment of the kinetic equation in the relaxation time approximation.
- Finding new equations allows us to have a different values of the transverse momentum scale Λ for quarks and gluons.
- In this case baryon chemical potential λ_q is not zero.
- Unknown functions: $T(\tau)$, $\xi_q(\tau)$, $\xi_g(\tau)$, $\Lambda_q(\tau)$, $\Lambda_g(\tau)$.

Boltzman equation

General setup

- Boltzmann equation in the relaxation time approximation (RTA)

$$p^\mu \partial_\mu f(x,p) = C[f(x,p)] \qquad C[f] = p \cdot u \, \frac{f_{\rm eq} - f}{\tau_{\rm eq}}$$

- Distribution functions $f(x,p)$ of quarks, antiquarks and gluons (Romatschke-Strickland):

$$Q^{\pm}(x,p) = \exp\left(\frac{\pm\lambda_q - \sqrt{(p\cdot U)^2 + \xi_q (p\cdot Z)^2}}{\Lambda_q}\right)$$

$$G(x,p) = \exp\left(-\frac{\sqrt{(p\cdot U)^2 + \xi_g (p\cdot Z)^2}}{\Lambda_g}\right)$$

- Equilibrium distribution functions $f_{\rm eq}(x,p)$:

$$Q^{\pm}_{\rm eq}(x,p) = \exp\left(\frac{\pm\mu - p\cdot U}{T}\right)$$

$$G_{\rm eq}(x,p) = \exp\left(-\frac{p\cdot U}{T}\right)$$

Zeroth moment of the Boltzmann equation

- Zeroth moment of the kinetic equation describes production of the particles

$$\partial_\mu (n_q^{\pm} U^\mu) = \frac{n_{q,\mathrm{eq}}^{\pm} - n_q^{\pm}}{\tau_{\mathrm{eq}}}$$

$$\partial_\mu (n_g U^\mu) = \frac{n_{g,\mathrm{eq}} - n_g}{\tau_{\mathrm{eq}}}$$

- We introduce non-equilibrium and equilibrium densities of quarks, antiquarks and gluons

$$n_q^{\pm} = \frac{g_q}{\pi^2} \frac{e^{\pm \lambda_q/\Lambda_q} \Lambda_q^3}{\sqrt{1+\xi_q}} \quad n_{q,\mathrm{eq}}^{\pm} = \frac{g_q}{\pi^2} e^{\pm \mu/T} T^3,$$

$$n_g = \frac{g_g}{\pi^2} \frac{\Lambda_g^3}{\sqrt{1+\xi_g}}, \quad n_{g,\mathrm{eq}} = \frac{g_g}{\pi^2} T^3$$

Zeroth moment of the Boltzmann equation cont.

- Subtraction of the equations describing quarks and antiquarks production

$$\frac{d}{d\tau}\left(n_q^+ - n_q^-\right) + \frac{n_q^+ - n_q^-}{\tau} = \frac{n_{q,\mathrm{eq}}^+ - n_{q,\mathrm{eq}}^- - (n_q^+ - n_q^-)}{\tau_{\mathrm{eq}}}$$

$$\frac{db}{d\tau} + \frac{b}{\tau} = \frac{b_{\mathrm{eq}} - b}{\tau_{\mathrm{eq}}} \qquad b(\tau) = \frac{b_0\tau_0}{\tau}$$

- The linear combination of the equations describing particles production

$$\alpha\left(\frac{dn_q}{d\tau} + \frac{n_q}{\tau}\right) + (1-\alpha)\left(\frac{dn_g}{d\tau} + \frac{n_g}{\tau}\right) = \alpha\,\frac{n_{q,\mathrm{eq}} - n_q}{\tau_{\mathrm{eq}}} + (1-\alpha)\,\frac{n_{g,\mathrm{eq}} - n_g}{\tau_{\mathrm{eq}}}$$

Zeroth moment of the Boltzmann equation cont.

- Linear combination of the particles production finally has a following form

$$\frac{\mathbf{d}}{\mathbf{d}\tau}\left(\alpha\frac{\sqrt{\mathbf{1}+\mathbf{D^2}}\mathbf{\Lambda_q^3}}{\sqrt{\mathbf{1}+\xi_\mathbf{q}}}+(\mathbf{1}-\alpha)\frac{\mathbf{\tilde{r}}\mathbf{\Lambda_g^3}}{\sqrt{\mathbf{1}+\xi_\mathbf{g}}}\right) + \left(\frac{\mathbf{1}}{\tau}+\frac{\mathbf{1}}{\tau_\mathrm{eq}}\right)\left(\alpha\frac{\sqrt{\mathbf{1}+\mathbf{D^2}}\mathbf{\Lambda_q^3}}{\sqrt{\mathbf{1}+\xi_\mathbf{q}}}+(\mathbf{1}-\alpha)\frac{\mathbf{\tilde{r}}\mathbf{\Lambda_g^3}}{\sqrt{\mathbf{1}+\xi_\mathbf{g}}}\right) = \frac{\mathbf{T^3}}{\tau_\mathrm{eq}}\left(\alpha\sqrt{\mathbf{1}+\mathbf{D^2}/\kappa_\mathbf{q}^\mathbf{2}}+(\mathbf{1}-\alpha)\mathbf{\tilde{r}}\right) \tag{1}$$

- $\alpha = 1$ – quarks; $\alpha = 0$ – gluons; $\alpha = 1/2$ – quarks and gluons.

$$D(\tau,\Lambda_q,\xi_q) = \left(\frac{3\pi^2 b_0\tau_0\sqrt{1+\xi_q}}{2g_q\tau\Lambda_q^3}\right) \qquad \tilde{r} = \frac{g_g}{2g_q}$$

First moment of the Boltzmann equation

- First moment of the kinetic equation describes energy-momentum conservation law

$$\partial_\mu \underbrace{\int dP\, p^\nu p^\mu G}_{} = \int dP\, p^\nu C = 0 \qquad \frac{d\mathcal{E}}{d\tau} = -\frac{\mathcal{E} + \mathcal{P}_\parallel}{\tau}$$

$$T^{\mu\nu} = (\mathcal{E} + \mathcal{P}_\perp) u^\mu u^\nu - \mathcal{P}_\perp g^{\mu\nu} + (\mathcal{P}_\parallel - \mathcal{P}_\perp) V^\mu V^\nu$$

$$u^\mu = \left(\frac{t}{\tau}, 0, 0, \frac{z}{\tau}\right) \qquad V^\mu = \left(\frac{z}{\tau}, 0, 0, \frac{t}{\tau}\right)$$

- Landau matching condition alows us to find the effective temperature

$$\mathbf{T^4} = \frac{\mathbf{\Lambda_q^4}\sqrt{\mathbf{1 + D^2}}\,\mathcal{R}(\xi_\mathbf{q}) + \mathbf{\Lambda_g^4 \tilde{r}}\,\mathcal{R}(\xi_\mathbf{g})}{\sqrt{\mathbf{1 + D^2}/\kappa_\mathbf{q}^\mathbf{2}} + \mathbf{\tilde{r}}} \qquad (2)$$

$$\mathcal{R}(\xi) = \frac{1}{2(1+\xi)}\left[1 + \frac{(1+\xi)\tan^{-1}\sqrt{\xi}}{\sqrt{\xi}}\right]$$

First moment of the Boltzmann equation cont.

- Energy-momentum conservation law has a form

$$\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon + P_L}{\tau},$$

- It leads to the following equation

$$\frac{\mathbf{d}}{\mathbf{d}\tau}\left[\mathbf{\Lambda_q^4}\sqrt{\mathbf{1}+\mathbf{D^2}}\,\mathcal{R}(\xi_\mathbf{q}) + \tilde{\mathbf{r}}\mathbf{\Lambda_g^4}\mathcal{R}(\xi_\mathbf{g})\right] = \frac{\mathbf{2}}{\tau}\left[\mathbf{\Lambda_q^4}\sqrt{\mathbf{1}+\mathbf{D^2}}\,(\mathbf{1}+\xi_\mathbf{q})\mathcal{R}'(\xi_\mathbf{q}) + \tilde{\mathbf{r}}\mathbf{\Lambda_g^4}(\mathbf{1}+\xi_\mathbf{g})\mathcal{R}'(\xi_\mathbf{g})\right] \quad (3)$$

$$\mathcal{R}(\xi) = \frac{1}{2(1+\xi)}\left[1 + \frac{(1+\xi)\tan^{-1}\sqrt{\xi}}{\sqrt{\xi}}\right]$$

Second moment of the Boltzmann equation

- Second moment of the kinetic equation in the relaxation time approximation

$$\frac{d}{d\tau}\ln\Theta_I + \theta - 2\theta_I - \frac{1}{3}\sum_J\left[\frac{d}{d\tau}\ln\Theta_J + \theta - 2\theta_J\right]$$
$$= \frac{1}{\tau_{\rm eq}}\left[\frac{\Theta_{\rm eq}}{\Theta_I} - 1\right] - \frac{1}{3}\sum_J\left\{\frac{1}{\tau_{\rm eq}}\left[\frac{\Theta_{\rm eq}}{\Theta_J} - 1\right]\right\}$$

W.Florkowski, L.Tinti, Phys. Rev. C **89**, 034907 (2014)

- Variables θ_I are equal

$$\theta_X = \theta_Y = 0, \qquad \theta_Z = -1/\tau, \qquad \theta = 1/\tau.$$

- One-dimensional case

$$\frac{d}{d\tau}\ln\Theta_X - \frac{d}{d\tau}\ln\Theta_Z - \frac{2}{\tau} = \frac{\Theta_{\rm eq}}{\tau_{\rm eq}}\left[\frac{1}{\Theta_X} - \frac{1}{\Theta_Z}\right]$$

Second moment of the Boltzmann equation cont.

Quarks and antiquarks

- Non-equilibrium functions Θ_I^q:

$$\Theta_X^q = \Theta_Y^q = \frac{8 g_q \Lambda_q^5}{\pi^2 (1+\xi_q)^{1/2}} \sqrt{1+D^2}$$

$$\Theta_Z^q = \frac{8 g_q \Lambda_q^5}{\pi^2 (1+\xi_q)^{3/2}} \sqrt{1+D^2}$$

- Equilibrium functions $\Theta_{I,\mathrm{eq}}^q$

$$\Theta_{X,\mathrm{eq}}^q = \Theta_{Y,\mathrm{eq}}^q = \Theta_{Z,\mathrm{eq}}^q = \frac{8 g_q T^5}{\pi^2} \sqrt{1 + D^2/\kappa_q^2}.$$

- Finally:

$$\frac{\mathbf{d}}{\mathbf{d}\tau} \ln\left(\frac{\mathbf{\Lambda_q^5}}{(\mathbf{1}+\xi_\mathbf{q})^{\mathbf{1/2}}} \sqrt{\mathbf{1}+\mathbf{D^2}} \right) - \frac{\mathbf{d}}{\mathbf{d}\tau} \ln\left(\frac{\mathbf{\Lambda_q^5}}{(\mathbf{1}+\xi_\mathbf{q})^{\mathbf{3/2}}} \sqrt{\mathbf{1}+\mathbf{D^2}} \right) - \frac{\mathbf{2}}{\tau}$$
$$= \frac{\mathbf{T^5}}{\tau_{\mathrm{eq}} \mathbf{\Lambda_q^5}} \xi_\mathbf{q} (\mathbf{1}+\xi_\mathbf{q})^{\mathbf{1/2}} \frac{\sqrt{\mathbf{1}+\mathbf{D^2}/\kappa_\mathbf{q}^\mathbf{2}}}{\sqrt{\mathbf{1}+\mathbf{D^2}}}. \quad (4)$$

Second moment of the Boltzmann equation cont.

Gluons

- Non-equilibrium functions Θ_I^g:

$$\Theta_X^g = \Theta_Y^g = \frac{4 g_g \Lambda_g^5}{\pi^2 (1+\xi_g)^{1/2}}$$

$$\Theta_Z^g = \frac{4 g_g \Lambda_g^5}{\pi^2 (1+\xi_g)^{3/2}}$$

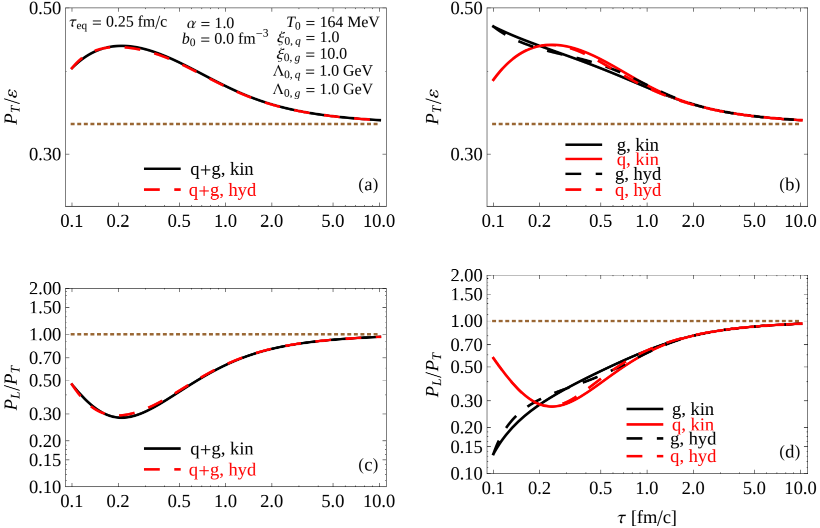
- Equilibrium functions $\Theta_{I,\mathrm{eq}}^g$

$$\Theta_{X,\mathrm{eq}}^g = \Theta_{Y,\mathrm{eq}}^g = \Theta_{Z,\mathrm{eq}}^g = \frac{4 g_g T^5}{\pi^2}$$

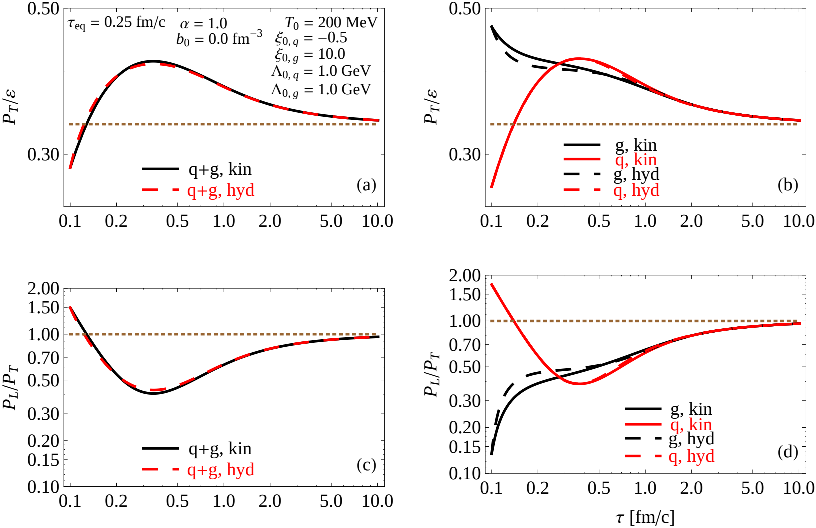
- It leads to the lat equation:

$$\frac{d}{d\tau} \ln\left(\frac{\Lambda_g^5}{(1+\xi_g)^{1/2}}\right) - \frac{d}{d\tau} \ln\left(\frac{\Lambda_g^5}{(1+\xi_g)^{3/2}}\right) - \frac{2}{\tau} = \frac{T^5}{\tau_{\mathrm{eq}} \Lambda_g^5} \xi_g (1+\xi_g)^{1/2} \tag{5}$$

Results for Oblate-Oblate system



Results for Oblate-Prolate system



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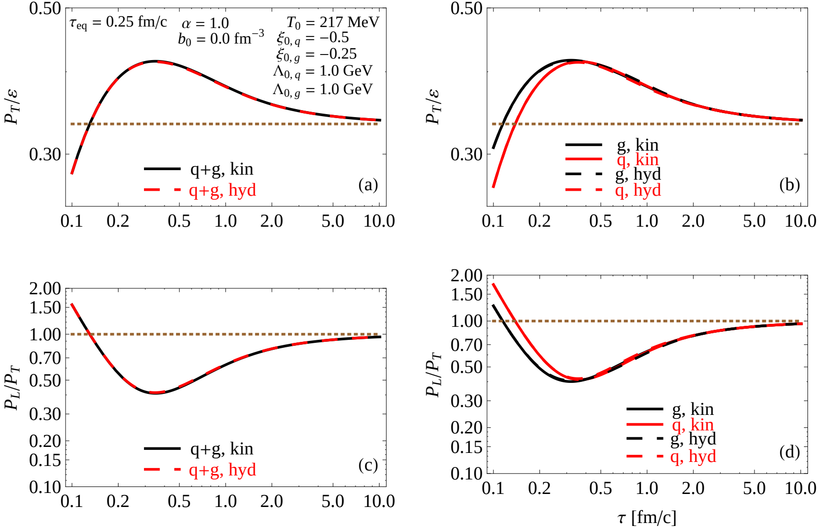
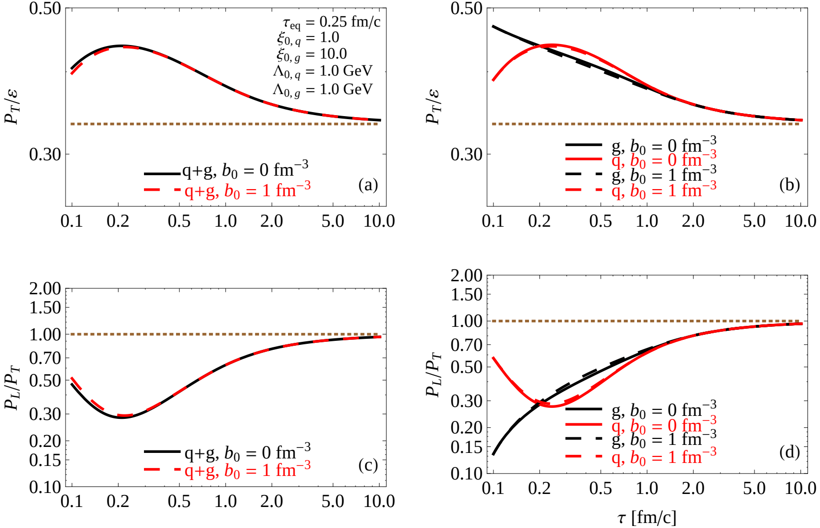
Results for Prolate-Prolate system



Results for $b \neq 0$



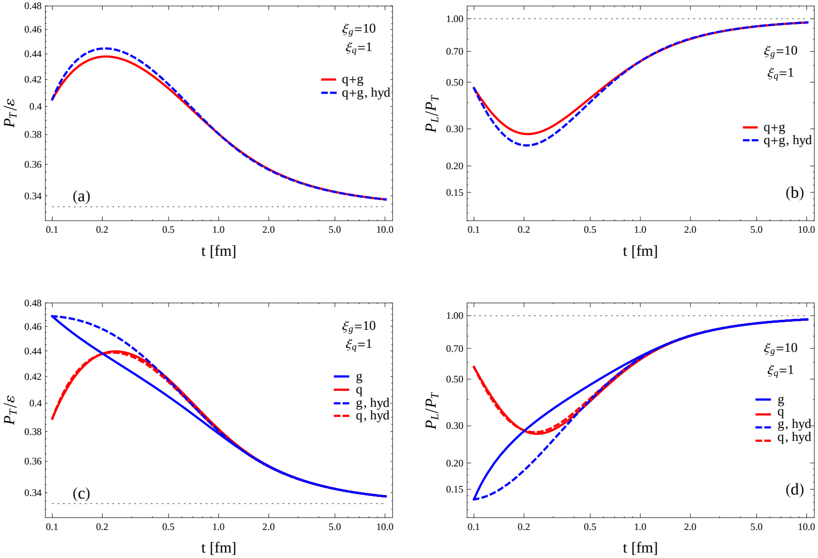
Summary

- We have build a model, based on the zeroth, first and the second moments of the kinetic equation for a mixture of quark and gluon fluids. Model allows to find $T(\tau)$, $\xi_q(\tau)$, $\xi_g(\tau)$, $\Lambda_q(\tau)$, $\Lambda_g(\tau)$ functions.
- Anisotropic hydrodynamics works very well in the case of mixture of quark and gluon fluids.
- In comparison with previous papers, new formulation of anisotropic hydrodynamics allows to have a different values of Λ parameter for quarks and gluons.
- We have found a very good agreement between anisotropic hydrodynamics and kinetic theory.

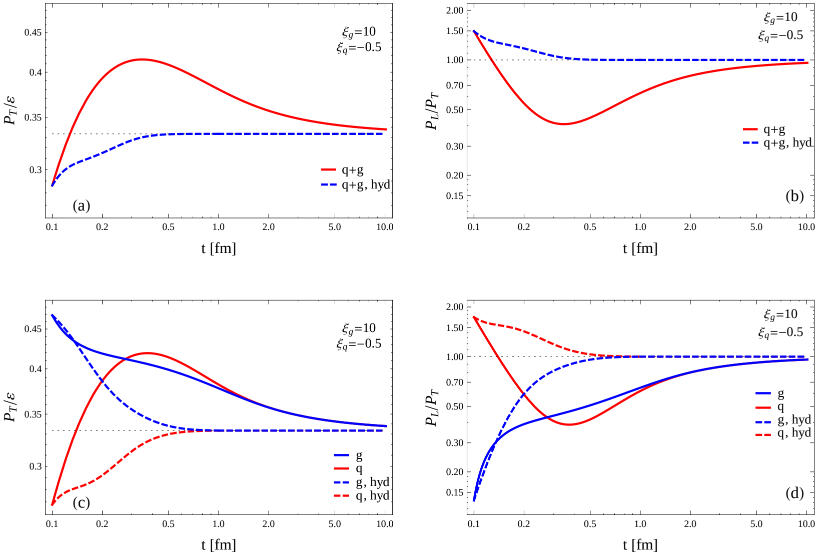
Thank you

Backup slides

Results for Oblate-Oblate system



Results for Oblate-Prolate system



Results for Prolate-Prolate system