Recent QCD-related results from Kaon physics at CERN (NA48/2 and NA62)

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on behalf of the NA48/2 and the NA62 collaboration

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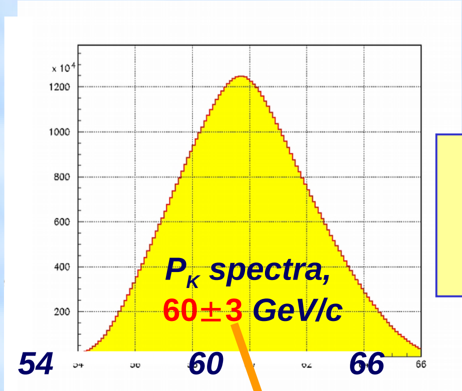
Outline

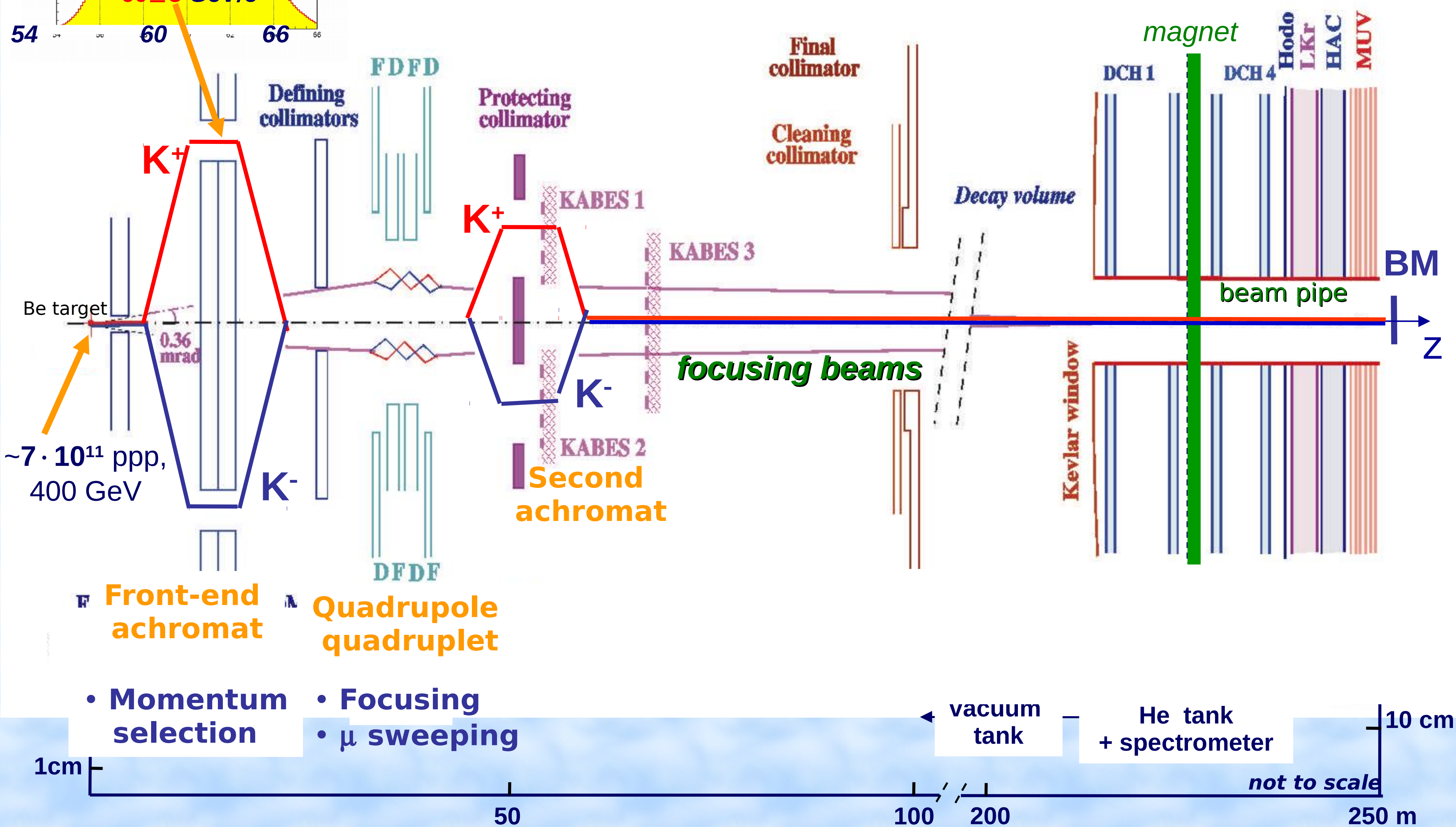
- NA48/2 experiment
- K_{l3} form factors precision measurement
- π^0 transition form factor slope measurement (NA62 experiment, 2007 data analysis)
- Conclusion

NA48/2 kaon beam



2003+2004 ~ 6 months,
~ 2 10^{11} K decays
Flux ratio: K^+/K^- ≈ 1.8

Simultaneous K^+ and K^- beams:
large charge symmetrization of experimental conditions



NA48/2 detector

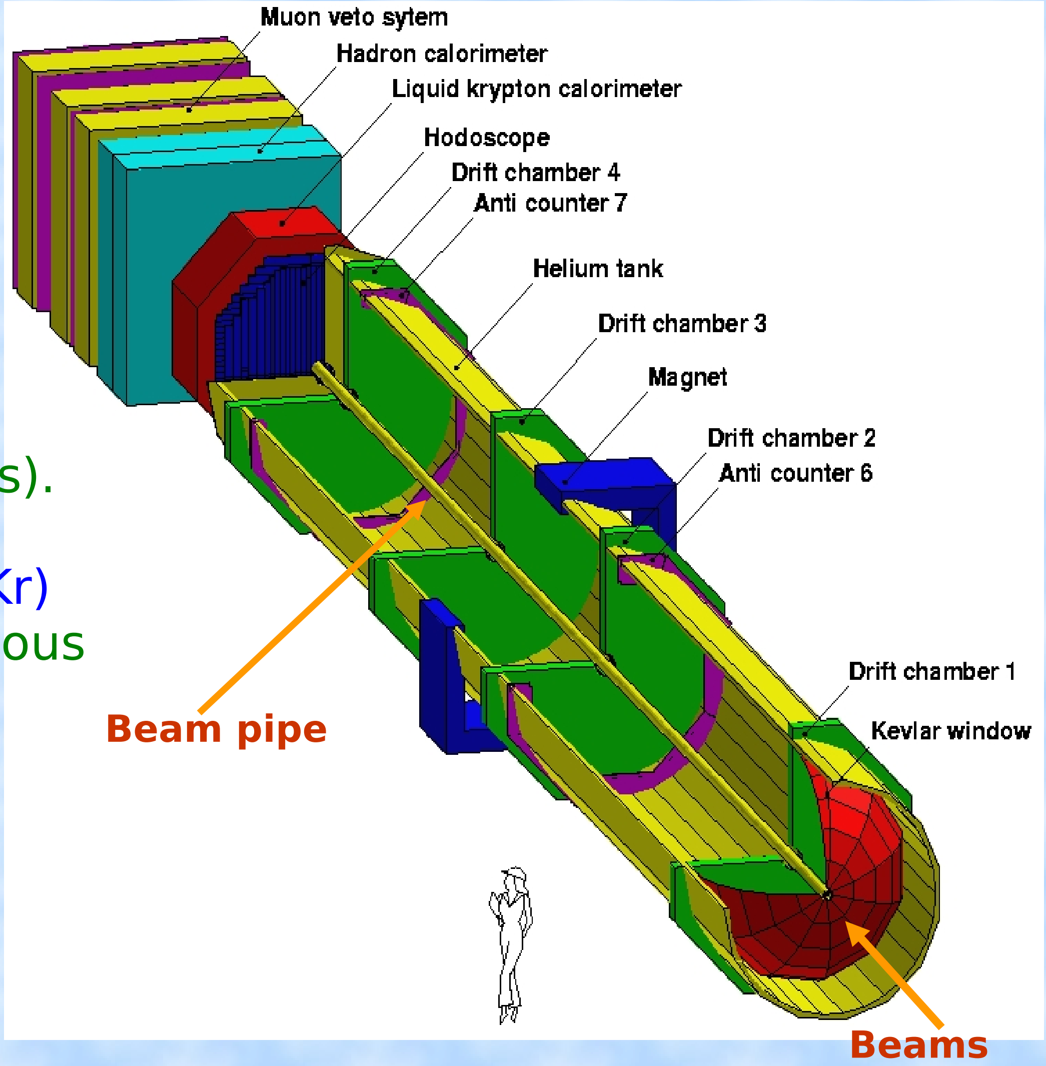
Main detector components:

- Magnetic spectrometer (4 DCHs):
  4 views/DCH inside a He tank
  $\Delta p/p = 1.02\% \oplus 0.044\%*p$
  [p in GeV/c].

- Hodoscope
  fast trigger;
  precise time measurement (150ps).

- Liquid Krypton EM calorimeter (LKr)
  High granularity, quasi-homogenious
  $\sigma_E/E = 3.2\%/E^{1/2} \oplus 9\%/E \oplus 0.42\%$
  $\sigma_x=\sigma_y=0.42/E^{1/2} \oplus 0.06$cm
  [E in GeV]. (0.15cm@10GeV).

- Hadron calorimeter, muon veto counters, photon vetoes.



$K^{\pm} \rightarrow \pi^0 l^{\pm} \nu$ (K_{l3}) form factors

exper. input for $|V_{us}|$ extraction (apart from $\Gamma(K_{l3}\gamma)$)

Without radiative effects : $\rho_0 = d^2N/(dE_l\, dE_\pi) \sim A f_+^2(t) + B f_+(t) f_-(t) + C f_-^2(t)$, where

$t = (P_K - P_\pi)^2 = M_K^2 + M_\pi^2 - 2 M_K E_\pi$

$f_-(t) = (f_+(t) - f_0(t))(m_K^2 - m_\pi^2)/t$. (just another formulation, f_0 is «scalar» and f_+ is «vector» FF),

E_l is charged lepton energy, E_π is π^0 energy (both in the kaon rest frame).

$A = M_K(2 E_l E_\nu - M_K(E_\pi^{max} - E_\pi)) + M_l^2 ((E_\pi^{max} - E_\pi)/4 - E_\nu)$

$B = M_l^2(E_\nu - (E_\pi^{max} - E_\pi)/2)$ negligible for Ke3

$C = M_l^2(E_\pi^{max} - E_\pi)/4$ negligible for Ke3

$E_\pi^{max} = (M_K^2 + M_\pi^2 - M_l^2)/(2 M_K)$

FF Parameterisation (PDG name)	f_+(t,parameters)	f_0(t,parameters)
Quadratic (linear for $\bar{f}_0(t)$)	$1+\lambda'_+ \, t/m^2_\pi + ½ \, \lambda''_+(t/m^2_\pi)^2$	$1 + \lambda'_0 \, t/m^2_\pi$
Pole	$M_V^2 / (M_V^2 - t)$	$M_S^2 / (M_S^2 - t)$
Dispersive* H(t), G(t): functions fixed from theory and other experiments. Depend on 2 (H) and 3 (G) extra external parameters known with a given* uncertainty.	$\exp((\Lambda_+ + H(t))\, t/m^2_\pi)$	$\exp((\ln[C] - G(t))\, t/(m_K^2 - m^2_\pi))$

* [V. Bernard, M. Oertel, E. Passemar, J. Stern. Phys.Rev. D80 (2009) 034034]

We use MC radiative decay generator of C.Gatti [Eur.Phys.J. C45 (2006) 417–420] provided by KLOE collaboration. It includes $f_0=f_+=1+\lambda'_+ \, t/m^2_\pi$.

Data: 16 special runs from the NA48/2 data taken in 2004 (3 days)

Trigger: 1 charged track (2 hodoscope hits) and $E_{LKr} > 10$ GeV

Registered :

- **1 track** (> 0 candidates): **$P_e >= 5$ GeV, $P_\mu >= 10$ GeV , $R_{MUV} > 30$ cm, $|X_{MUV},Y_{MUV}|<115$ cm.**
- **2 LKr clusters** (> 1 candidates): **E > 3 GeV, to closest track > 15 cm.**

Neutrino is missing, beam geometry and average momentum P_b are measured from $K_{3\pi^\pm}$

Kaon momentum reconstruction

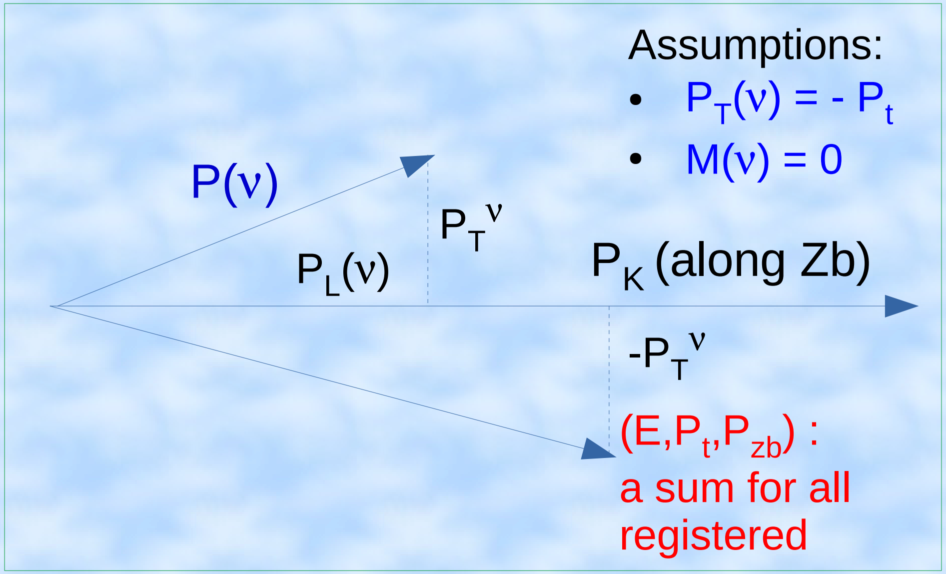
Two solutions of the quadratic equation for P_K:

$P_{1,2} = (\phi P_{Zb} \pm SQRT(\mathbf{d})) / (E^2-P_{zb}^2)$, where

$\phi = 0.5\ (M_K^2 + E^2 — P_t^2 — P_{Zb}^2)$,

$\mathbf{d} = (\phi^2 P_{Zb}^2 — (E^2 — P_{Zb}^2)(M_K^2 E^2 — \phi^2))$

When d<0, we assume d=0.

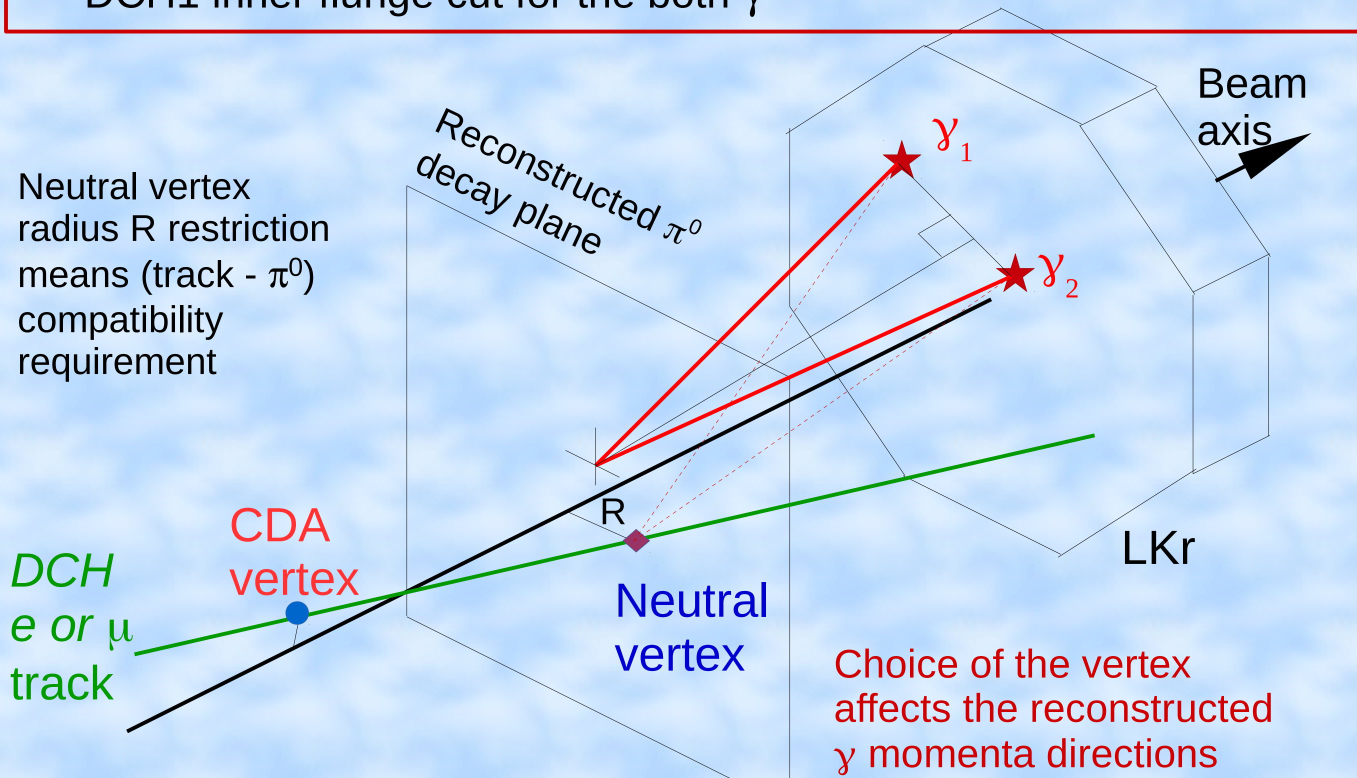


- Best P_K solution = closest $P_{1,2}$ to the average beam momentum P_b measured from $3\pi^\pm$ decays for each run is used to choose the.
- A cut: -7.5 GeV/c < $(P_K — P_b)$ < 7.5 GeV/c
- For each event, separately for K_{e3} and $K_{\mu3}$ selections, the combination with a minimum $\Delta P = |P_K — P_b|$ is the best candidate.

Selection:

π^0 :

- A pair of clusters in-time (within 5 ns) without any in-time extra clusters (to suppress BG)
- Distance between the clusters in a pair > 20 cm
- $E(\pi^0)$ > 15 GeV (for the trigger efficiency)
- Z of decay: from 2γ assuming π^0 mass («neutral Z»); Z > 200 cm downstream the last collimator
- DCH1 inner flunge cut for the both γ



Track selection and identification

- A good track in-time with the π^0 within 10 ns.
- No extra good track within 8 ns (against showers).
- If **$2.0 > E_{LKr}/P_{DCH} > 0.9$**, it is an electron of K_{e3} .
- If **$E_{LKr}/P_{DCH} < 0.9$** (for true muons it cuts nothing) and there is a MUV muon associated, it is a $K_{\mu3}$ muon.

Loose **E_{LKr}/P_{DCH}** cuts=> negligible related systematics.

Reminder: Preliminary result reported in 2012 was based on the «charged» vertex definition (from CDA between the track and the beam), that leads to high sensitivity to the exact beam shape simulation (due to the systematic shift of the vertex closer to beam).

Neutral vertex is chosen finally

(no transverse bias): $Z_{decay} = Z(\pi^0)$;

X_{decay}, Y_{decay} = impact point of reconstructed charged track on Z_{decay} plane

Final cuts

For K_{e3}

- **ν** transversal momentum with respect to beam axis $P_t >= 0.03$ GeV against $K^{\pm} \to \pi^{\pm}\pi^0$ with $\pi^{\pm}$ misidentified as *e* (when E/P > 0.9);
- $P_L(\nu)^2 = (E^{\nu})^2/c^2 - (P_t^{\nu})^2 > 0.0014\ GeV^2/c^2$
  [negative tail and zero region are difficult to simulate exactly – sensitive to beam shape]
  [**For K_{e3} only** in the region of small and negative $P_L(\nu)^2$ fit results depend on $P_L(\nu)^2$ cut]

For $K_{\mu3}$

- against the background from $K^{\pm} \to \pi^{\pm}\pi^0$ with $\pi^{\pm} \to \mu^{\pm}\bar{\nu}$
  $m(\pi^+\pi^0) < 0.47\ GeV/c^2$
  $m(\pi^+\pi^0) < (0.6 - P_t(\pi^0))\ GeV/c^2$
  $m(\mu^{\pm}\bar{\nu}) > 0.18\ GeV/c^2$ (to exclude π^+ mass region)
- a cut against $\pi^{\pm}\pi^0\boldsymbol{\pi^0}$: $(P_2-P_1)<60$ GeV
  [a difference between two P solution is large when one pion is missing]

For both K_{l3}

$$B_{ell} = \sqrt{\left(\frac{X_n - X_n^0(Z_n)}{\sigma_{Xn}(Z_n)}\right)^2 + \left(\frac{Y_n - Y_n^0(Z_n)}{\sigma_{Yn}(Z_n)}\right)^2},$$

Beam transverse elliptic variable **$B_{ell} < 11$**.
X_n, Y_n, Z_n are the reconstructed neutral vertex coordinates, X^0_n, Y^0_n, σ_{Xn}, σ_{Yn} are the reconstructed beam central positions and widths with respect to the run-dependent beam axis Zb.

Really important only $K_{2\pi}$ and $K_{3\pi}$ sources of Bg, others are negligible.

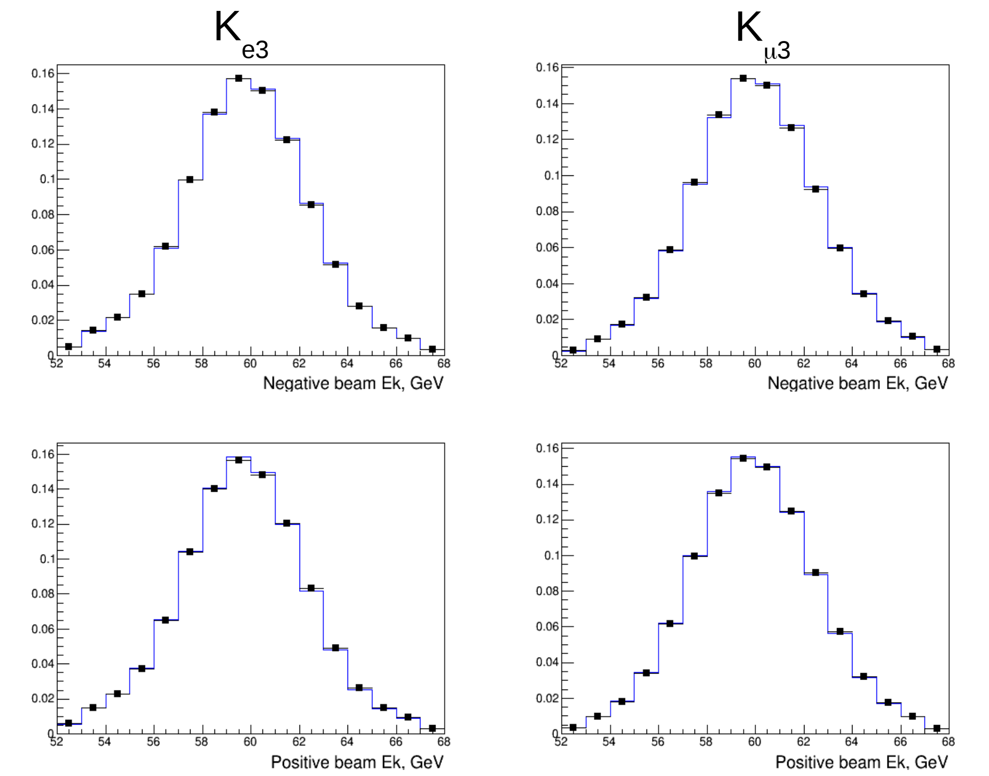
Process	Notation	Br	N_g	F_e	F_μ
$K^\pm \to \pi^\pm(\pi^0 \to 2\gamma)$	2π	20.66	393.2	0.270	0.264
$K^\pm \to \pi^\pm 2(\pi^0 \to 2\gamma)$	3π	1.761	62.5	0.286	1.833
$K^\pm \to \pi^\pm(\pi^0 \to e^+e^-\gamma)$	$2\pi D$	1.174	1.5	0.049	0.000
$K^\pm \to \pi^\pm\gamma(\pi^0 \to 2\gamma)$	$2\pi\gamma$	0.0275	35.3	0.004	0.044
$K^\pm \to \pi^0\mu^\pm\nu(\mu \to e\nu)$	$K^e_{\mu 3}$	0.03353	174.3	0.004	0.000

Table *Simulated background processes, their probabilities Br (in %), generated MC statistics N_g (in 10^6 events) and the estimated fractions F_e and F_μ (both in units of per mill) in K_{e3} and $K_{\mu 3}$ samples for the present selection.*

Reconstructed kaons energy normalized distributions (as a signal manifestation)

Histograms: MC.

Points: Data (corrected for background).



Events-weighting fit procedure

- Experimental Dalitz plot is corrected for the simulated background.
- For each fit iteration, the model Dalitz plot is filled in with an MC simulated reconstructed center-of-mass pion and lepton energies. Each event is weighted by

$$w = \rho_0(E_\pi^{true}, E_l^{true}, FF_{fit}) / \rho_0(E_\pi^{true}, E_l^{true}, FF_{MC\ enerator}),$$

where ρ_0 is the non-radiative Dalitz density formula.

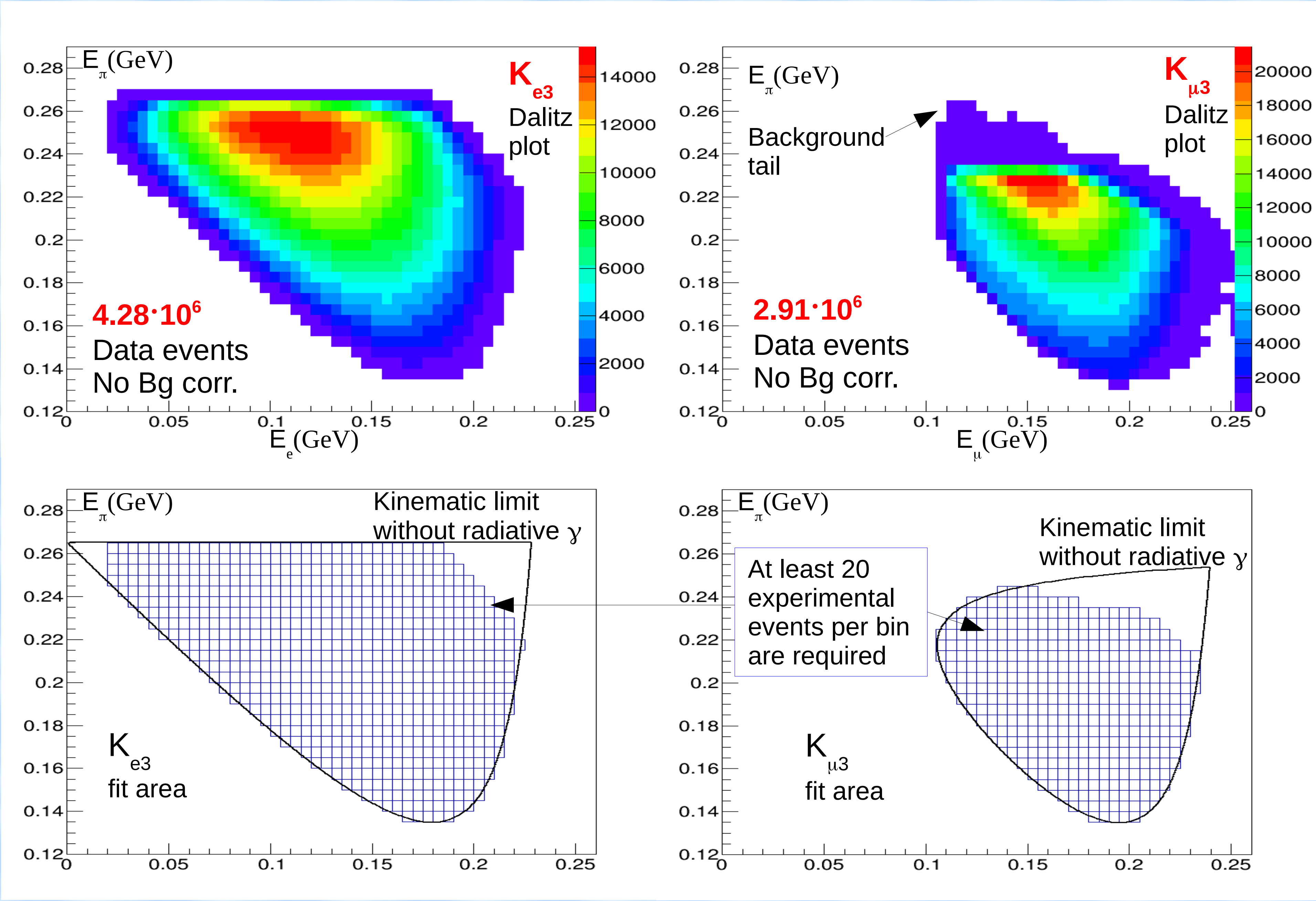
- MINUIT package is searching for the FF_{fit} parameters minimizing the standard χ^2 value:

$$\chi^2 = \Sigma_{i,j} \frac{(D_{i,j} - MC_{i,j})^2}{(\delta D_{i,j})^2 + (\delta MC_{i,j})^2},$$

where *i,j* means the Dalits plot cell indices, $D_{i,j}$ is the background-corrected experimental data content of the cell, $MC_{i,j}$ is the weighted MC bin content, and $\delta D_{i,j}$, $\delta MC_{i,j}$ are the corresponding statistical errors.

At least 20 data events per cell are required in the fit area, so χ^2 works well.

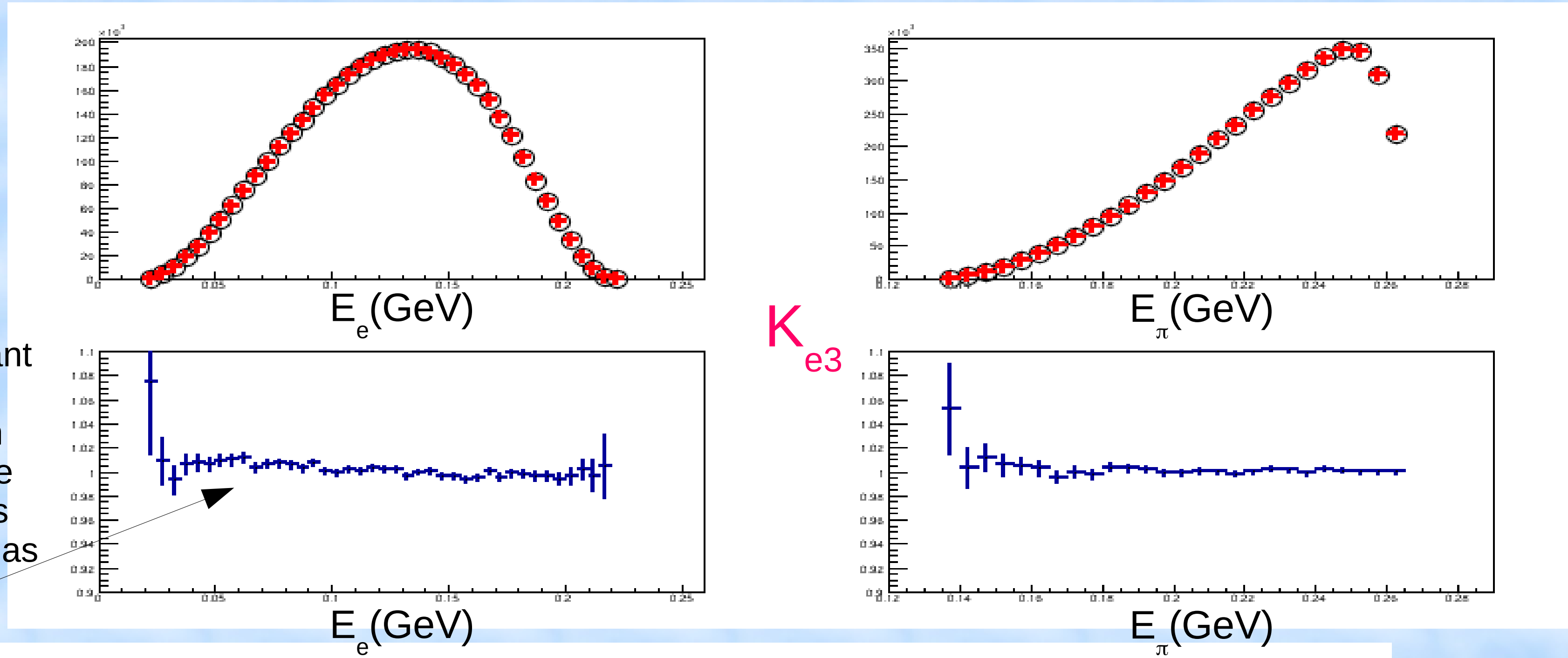
Experimental Dalitz plots and fits areas (5x5 MeV cells)

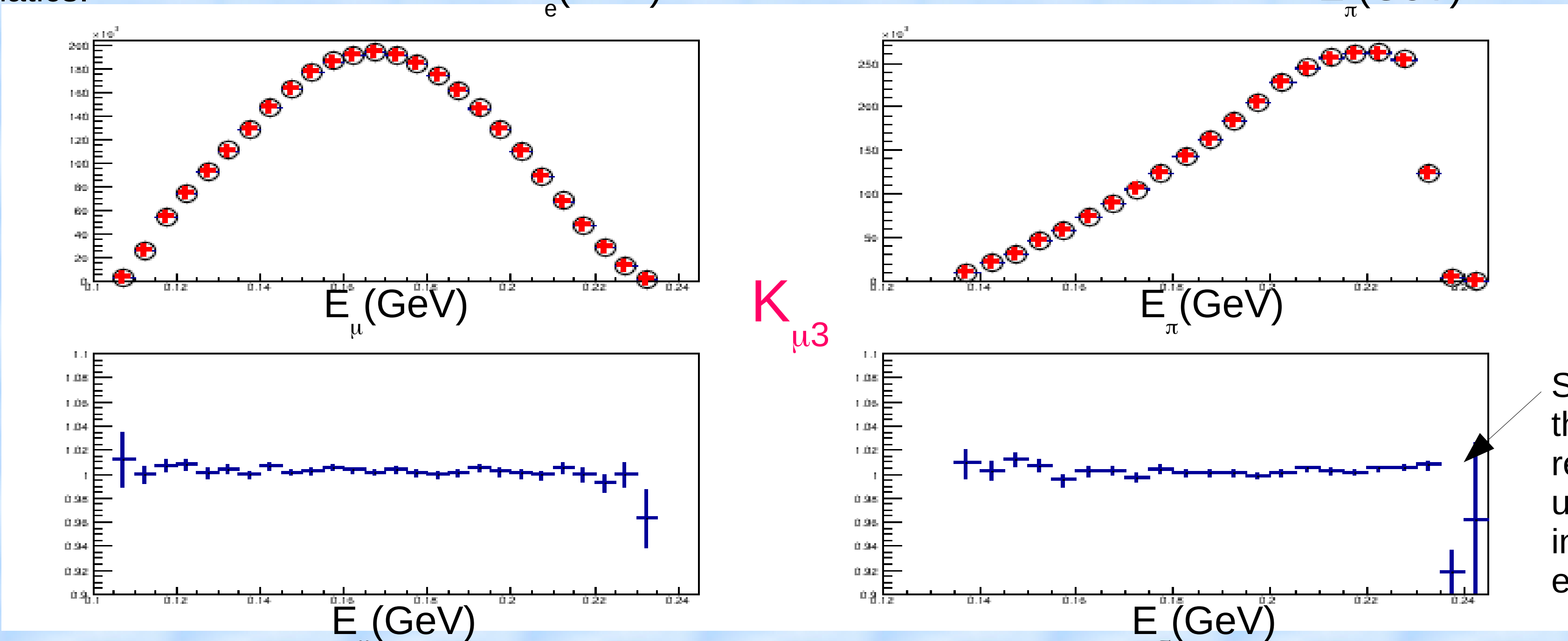


Dalitz plot projections

- Data-Bg
- MC fit result (quadr.)
- (Data-Bg)/MC

Marginally significant slope within the radiative correction precision. Radiative effect uncertainty is taken into account as a contribution to systematics.





Small deviation in the Bg area. Bg-related uncertainty is included into syst. error.

Results for the joint K_{l3} analysis

Analysis has been performed:

- For K_{e3}
- For $K_{\mu3}$
- For the combined K_{l3} result:

A joint fits are done minimizing

$$\chi^2(K_{e3}) + \chi^2(K_{\mu3})$$

with a common set of fit parameters.

Quadratic parameterization (in units of 10^{-3})

$\chi^2/\text{ndf} = 1004.6/1073$

Correlation coefficients

	$\lambda''_+(K_{l3})$	$\lambda_0(K_{l3})$
$\lambda'_+(K_{l3})$	-0.954	-0.076
$\lambda''_+(K_{l3})$		0.035

	$\lambda'_+(K_{l3})$	$\lambda''_+(K_{l3})$	$\lambda_0(K_{l3})$
Central values	23.35	1.73	14.90
Stat. error	0.75	0.29	0.55
Beam scattering	0.90	0.35	0.45
LKr nonlinearity	0.19	0.03	0.35
LKr scale	0.66	0.15	0.08
Background	0.07	0.03	0.04
Trigger	0.20	0.10	0.45
Accidentals	0.23	0.08	0.08
Acceptance	0.24	0.07	0.01
Pk average	0.04	0.01	0.24
Pk spectra	0.01	0.00	0.04
Neutrino P cut	0.18	0.04	0.03
Binning	0.08	0.02	0.16
Resolution	0.00	0.02	0.14
Radiative	0.22	0.01	0.06
Syst. error	1.23	0.41	0.80
Total error	1.44	0.50	0.97

Pole parameterization (in MeV)

$\chi^2/ndf = 1001.1/1074$

	$m_V(K_{l3})$	$m_S(K_{l3})$
Central values	894.3	1185.5
Stat. error	3.2	16.6
Beam scattering	0.1	27.2
LKr nonlinearity	1.7	14.3
LKr scale	3.9	3.6
Background	0.1	0.6
Trigger	0.7	12.9
Accidentals	0.5	0.0
Acceptance	0.7	3.3
Pk average	0.3	8.8
Pk spectra	0.1	1.5
Neutrino P cut	1.0	0.7
Binning	0.4	5.1
Resolution	0.8	4.3
Radiative	2.7	2.7
Syst. error	5.4	35.5
Total error	6.3	39.2

Correlation = - 0.278

Dispersion parameterization (in units of 10^{-3})

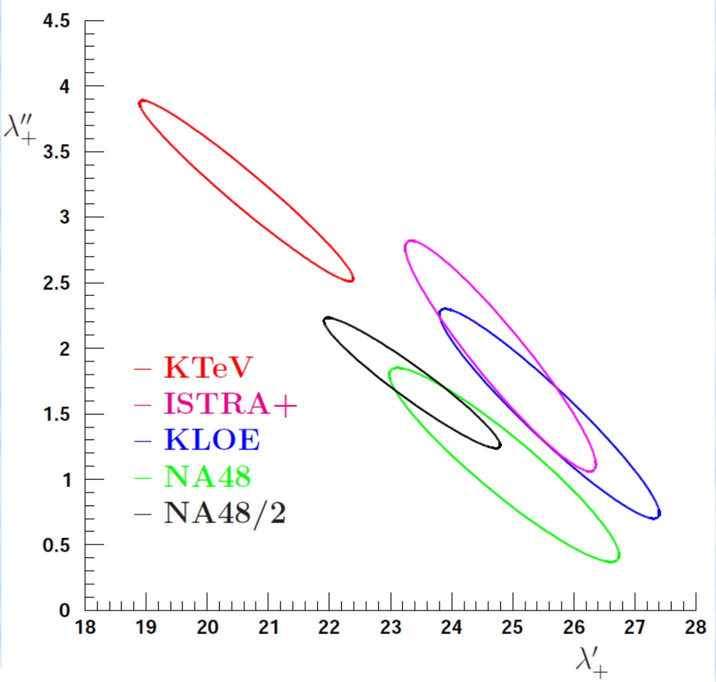
$\chi^2/ndf = 998.3/1074$ (the best)

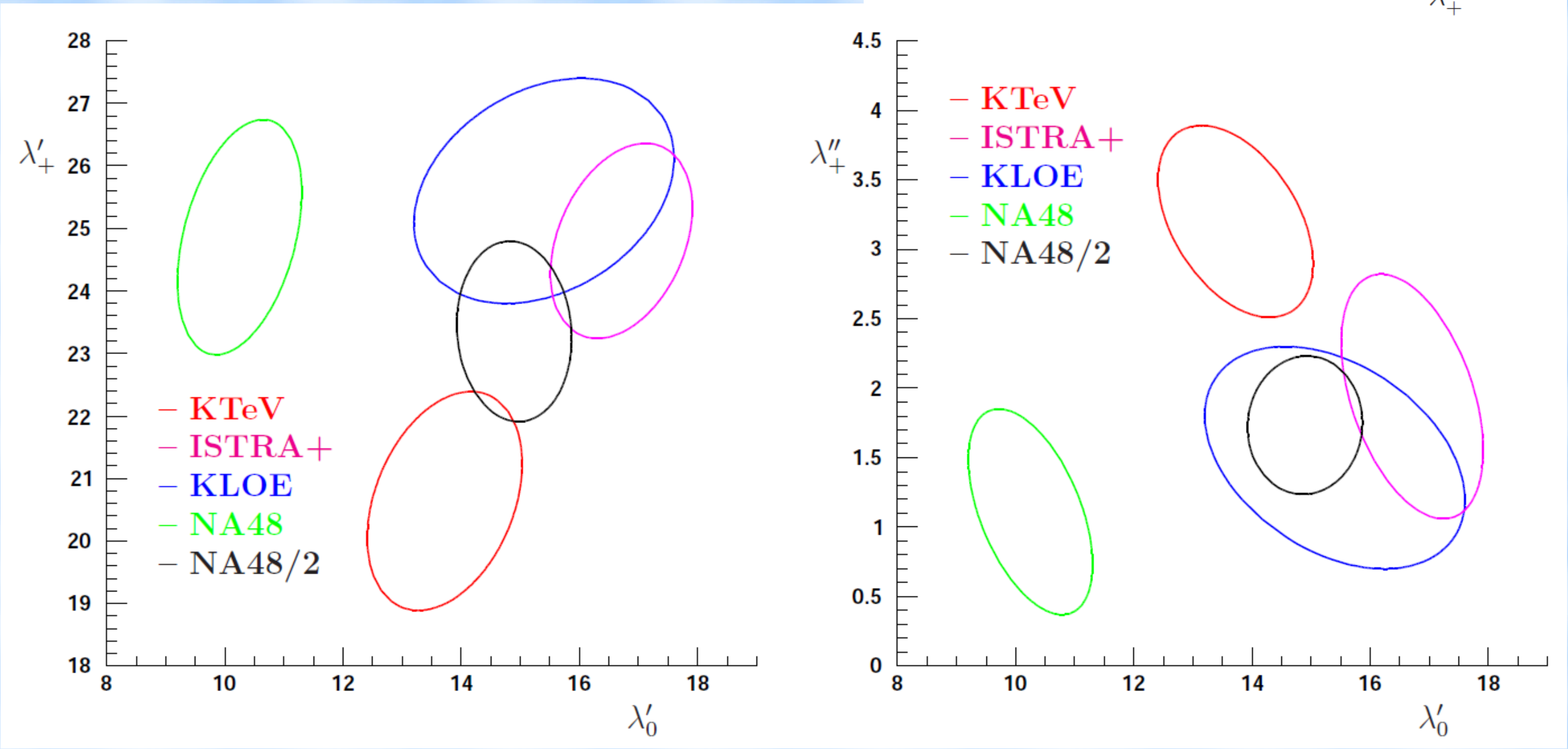
	$\Lambda_+(K_{l3})$	$ln[C](K_{l3})$
Central values	22.67	189.12
Stat. error	0.18	4.91
Beam scattering	0.01	8.39
LKr nonlinearity	0.10	4.04
LKr scale	0.23	0.88
Background	0.00	0.14
Trigger	0.04	3.73
Accidentals	0.03	0.01
Acceptance	0.04	0.92
Pk average	0.02	2.63
Pk spectra	0.00	0.44
Neutrino P cut	0.06	0.16
Binning	0.03	1.46
Resolution	0.05	1.28
Radiative	0.16	0.75
Parameterization	0.44	3.04
Syst. error	0.55	11.09
Total error	0.58	12.13

Correlation = - 0.035

Joint K_{l3} results comparison for quadratic parameterization

1σ ellipses rather than 68%
for better visibility





NA62 Experiment in 2007

The detectors was the same as NA48/2

The main goal:

$R_K = \Gamma(K_{e2})/\Gamma(K_{\mu 2})$ measurement.

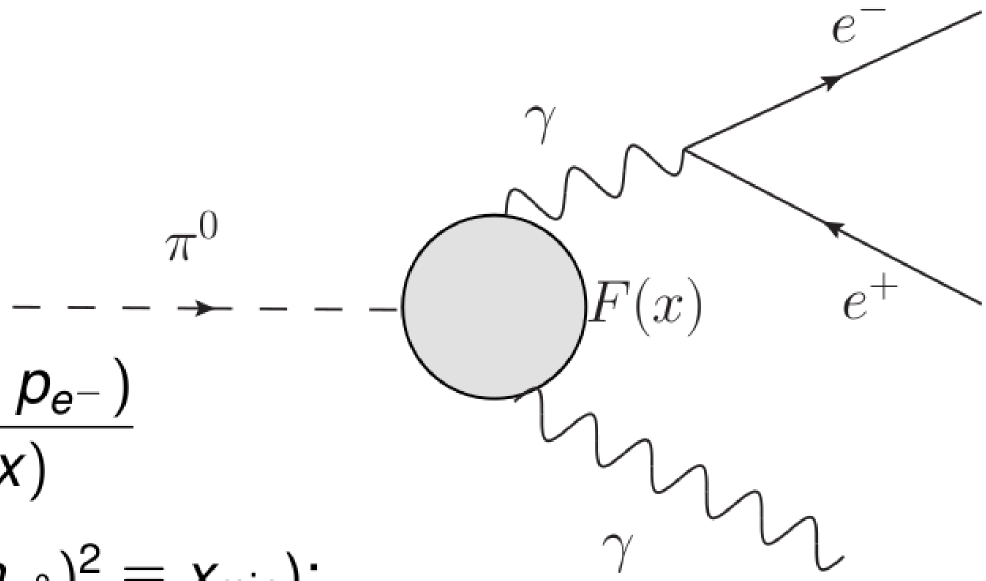
Phys. Lett. B 719 (2013) 326

$K^{\pm}$ beam momentum (74 ± 2) GeV/c

Main trigger: electron from K_{e2}

Efficient for $\pi^0{}_D$ decay

Dalitz Decay: $\pi^0 \to e^+e^-\gamma$



- π^0_D decay – kinematic variables x, y:

$$x = \frac{(p_{e^+} + p_{e^-})^2}{m^2_{\pi^0}}, \quad y = \frac{2\, p_{\pi^0}.\,(p_{e^+} - p_{e^-})}{m^2_{\pi^0}(1-x)}$$

- Differential decay width ($r^2 = (2m_e/m_{\pi^0})^2 \equiv x_{\text{min}}$):

Transition Form Factor (TFF)

$$\frac{1}{\Gamma(\pi^0_{2\gamma})}\frac{d^2\Gamma(\pi^0_D)}{dx\,dy} = \frac{\alpha}{4\pi}\frac{(1-x)^3}{x}\left(1 + y^2 + \frac{r^2}{x}\right)\,(1+\delta(x,y))\,|F(x)|^2$$

$$F(x) \approx 1 + a\,x, \qquad a:\ \textit{TFF slope parameter}$$

- π^0 TFF slope measurement at NA62 (kaon decay experiment)
  - $K^\pm \to \pi^\pm\pi^0$ decay: source of tagged π^0 decays (BR($K_{2\pi}$) $\approx$ 21%)
  - NA62 in 2007: data taking conditions optimized for $e^\pm$ from $K^\pm \to e^\pm\nu_e$

    $\to$ Large and clean sample of $K^\pm \to \pi^\pm\pi^0$; $\pi^0 \to \gamma\, e^+e^-$ decays

π^0 TFF Slope: NA62 Result

Fit procedure:

- Split reconstructed Dalitz x data into equal population bins
- Compare data with simulation (constant TFF slope: $a_{\rm sim} = 0.032$)
  $\rightarrow$ To obtain simulated x distribution, corresponding to different a slope: re-weight simulated events with weight $w(a) = (1 + a\, x_{\rm true})^2 / (1 + a_{\rm sim}\, x_{\rm true})^2$
- Minimise $\chi^2(a)$ Data/Simulation comparison wrt a

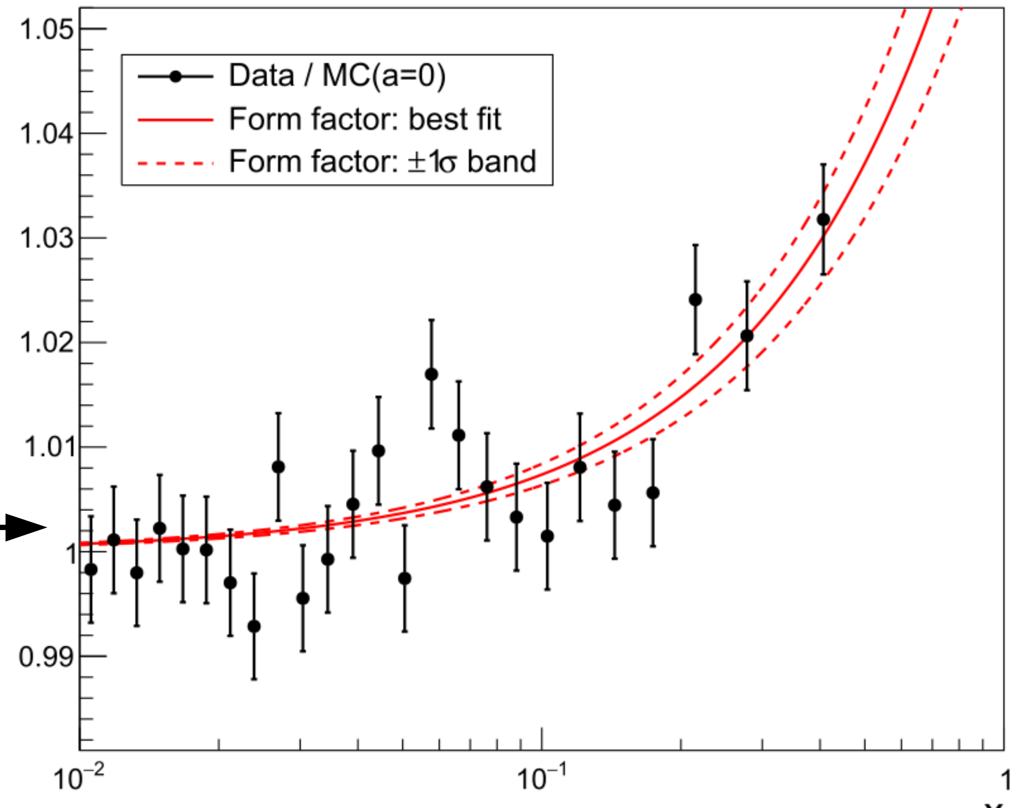
Fit result:

$$a = (3.68 \pm 0.48_{\rm stat}) \times 10^{-2}$$

χ^2/n.d.f = 54.8/49 p-value: 0.26

Fit result illustration

$\rightarrow$ Data / Simulation(a=0) ratio
$\rightarrow$ 25 equal population bins
$\rightarrow$ Points are in bin barycenters



π^0_D: Sources of Uncertainty

Source	$\delta a(\times 10^{-2})$
Statistical - Data	0.48
Statistical - MC	0.18
Spectrometer momentum scale	0.16
Spectrometer resolutuion	0.05
Lkr calibration	0.04
Beam momentum spectrum simulation	0.03
Calorimeter trigger inefficiency	0.06
Accidental background	0.15
Particle misidentification	0.06
Neglected π^0_D sources	0.01
Higher order radiative contributions	<0.01

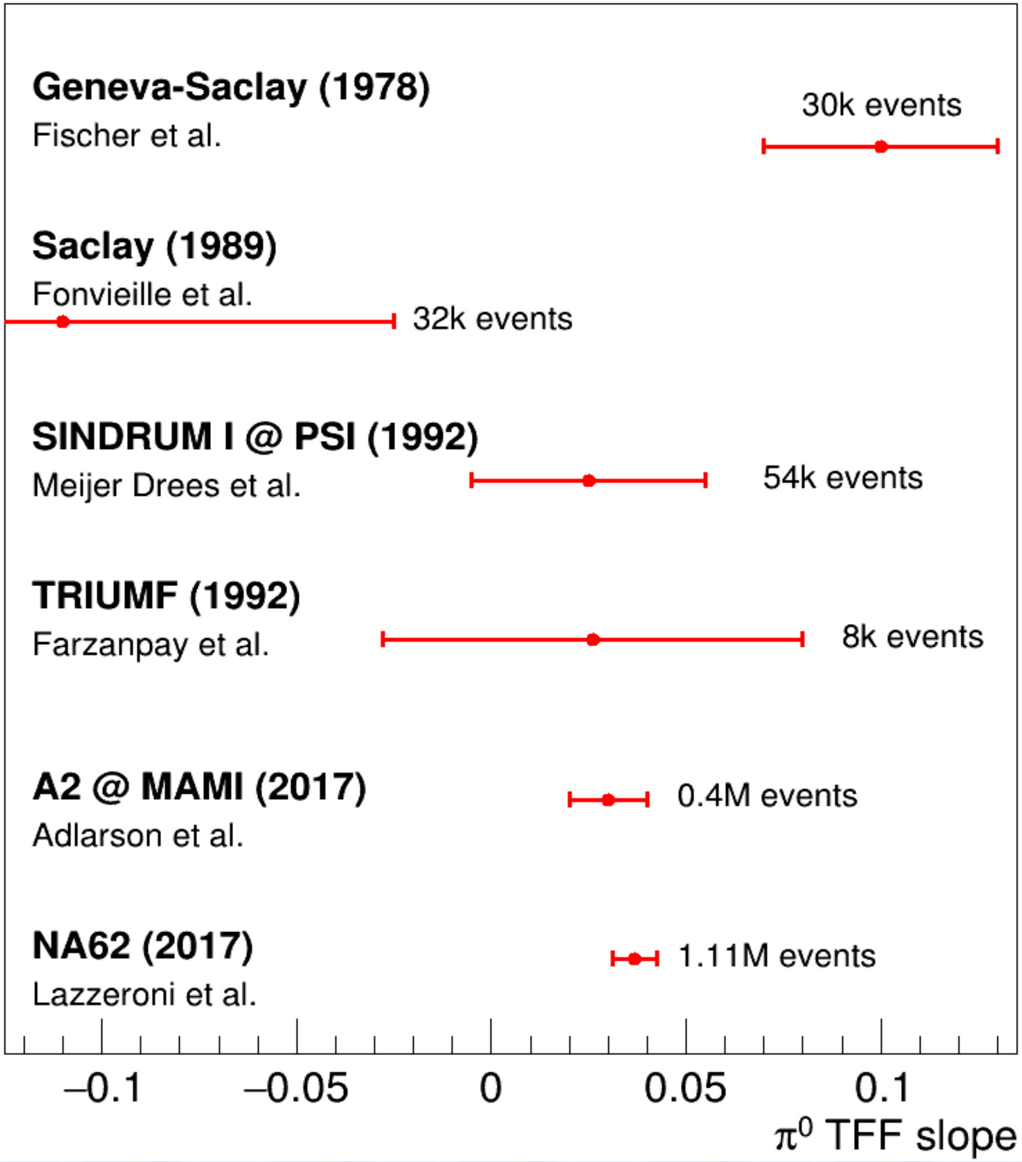
NA62 result on π^0_D TFF slope parameter:

$$a = (3.68 \pm 0.51_{stat} \pm 0.25_{syst}) \times 10^{-2} = (3.68 \pm 0.57) \times 10^{-2}$$

C. Lazzeroni, et al, NA62 Collaboration [Physics Letters B 768 (2017), pages 38-45.]

π^0 TFF Slope: World Data

π^0 TFF Slope Measurements from π^0_D



TFF slope theory expectations:

K. Kampf et al., EPJ C46 (2006), 191.
Chiral perturbation theory:
$a = (2.90 \pm 0.50) \times 10^{-2}$

M. Hoferichter et al., EPJ C74 (2014), 3180.
Dispersion theory:
$a = (3.07 \pm 0.06) \times 10^{-2}$

T. Husek et al., EPJ C75 (2015) 12, 586.
Two-hadron saturation (THS) model:
$a = (2.92 \pm 0.04) \times 10^{-2}$

CELLO measurement:

H. J. Behrend et al., Z. Phys. C49 (1991), 401.
Extrapolation of space-like momentum region data fit to VMD model:
$a = (3.26 \pm 0.26_{\text{stat}}) \times 10^{-2}$

Conclusion

- K_{l3} form factors measurement is performed by NA48/2 on the basis of 2004 run selected $4.28 \cdot 10^6$ (K_{e3}) and $2.91 \cdot 10^6$ ($K_{\mu3}$) events. Result is competetive with the other ones in $K_{\mu3}$ mode, and a smallest error in K_{e3} has been reached, that gives us also the most precise combined K_{l3} result. For the first time both K^+ and K^- K_{e3} decays were studied together.

  The difference with our preliminary result shown in 2012 on the conference talks is due to the beam scattering component, that is much more problematic for the charged vertex definition used in 2012 than for the neutral vertex used for the final result.

- π^0 transition form factor (TFF) slope parameter is measured using the NA62 experiment data set from 2007. About 1.11 million fully reconstructed $\pi^0 \rightarrow \gamma e^+ e^-$ Dalitz decays were selected and studied. The obtained result: $a = (3.68 \pm 0.57) \times 10^{-2}$. The measured value is compatible with the theoretical predictions, the central value is $\sim 1\sigma$ above the Vector Meson Dominance expectation. The result can be also interpreted as the first direct measurement that confirmed a positive π^0 TFF slope value with a significance exceeding 6σ.

Spares

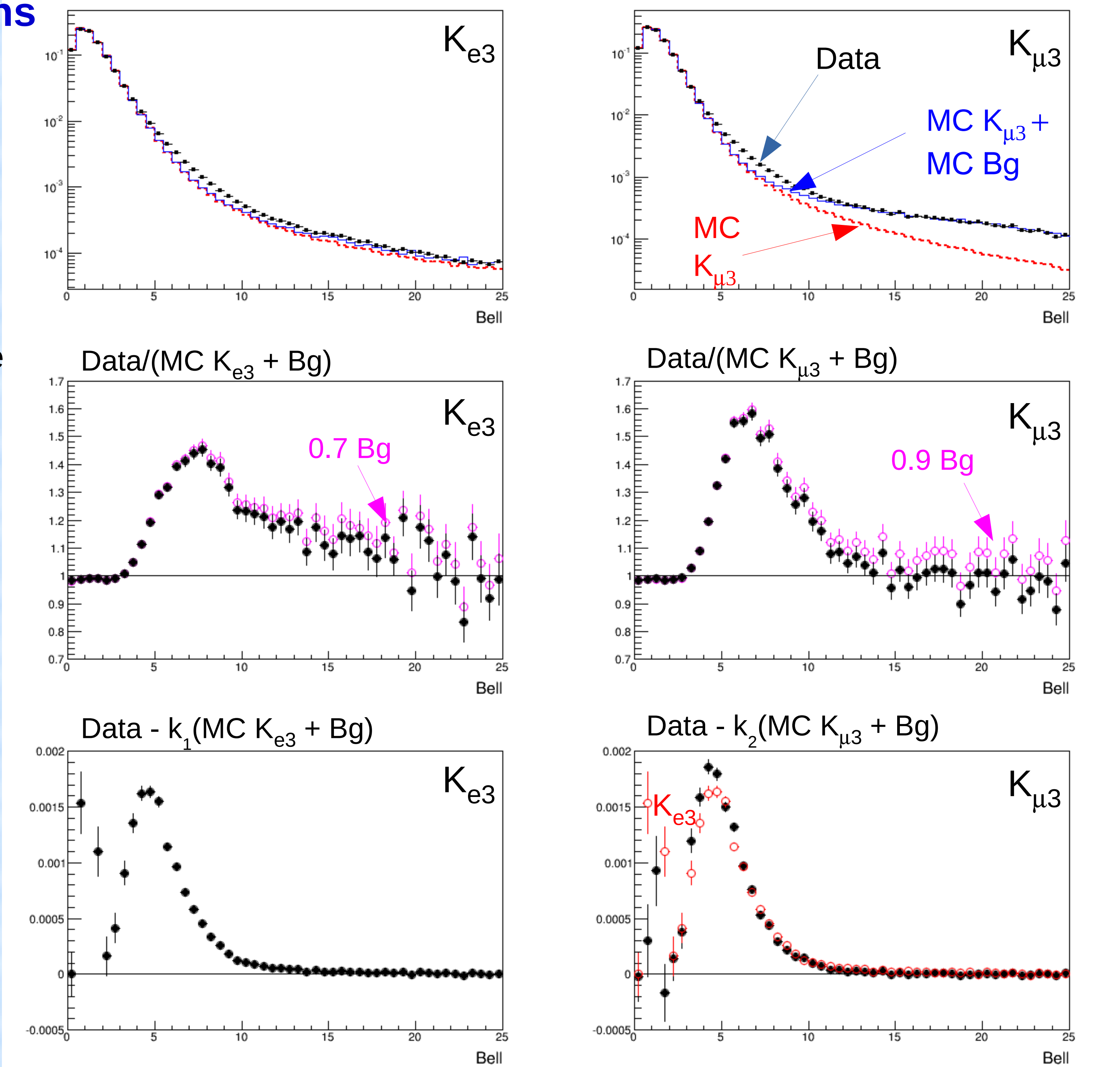
B_{ell} distributions in a wide area

~ 3σ range is relatively well simulated as well as the very far tail.

But the discrepance near ~5-10 is not described by the known background.

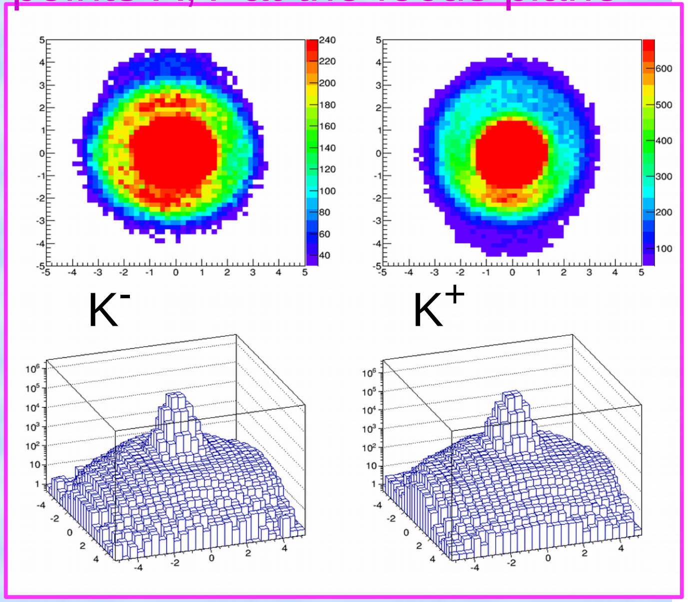
Sensitivity to the background variation at the very far tail (>20) is used to measure the Bg-related systematic uncertainty.

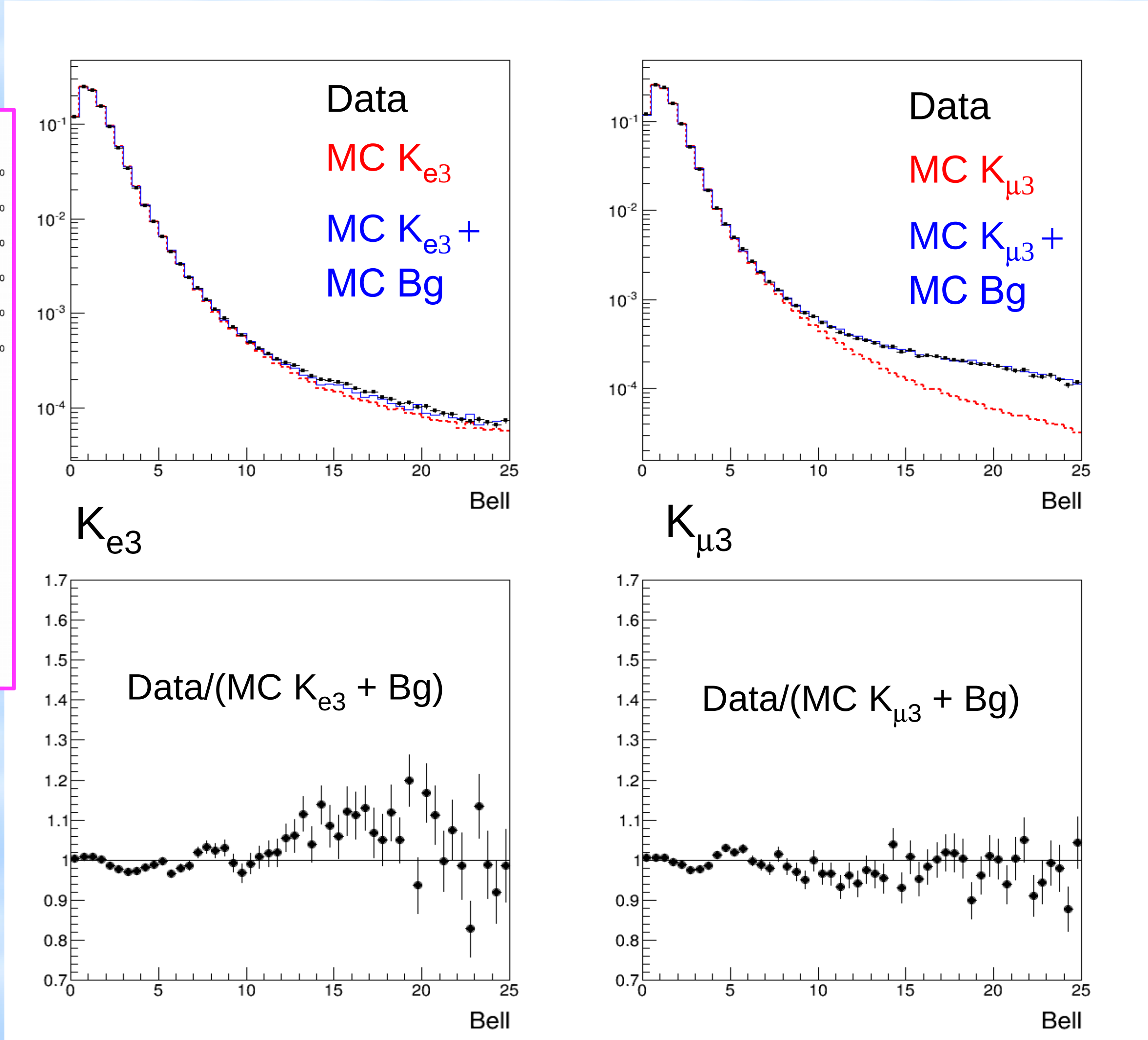
It looks like a small wide component of the beam, that becomes negligible for B_{ell} > 11. For wider cuts final results are stable.



B_{ell} distributions with the modified MC beam (systematics)

Data $3\pi^{\pm}$ decay: Kaon impact points X,Y at the focus plane





Focused scattering simulated in MC: 3% of beam kaons are additionally scattered into a series of rings with a different radius at focus > 2.2 cm.

This MC simplified modification is not used for the FF central values extraction (only for systematics estimate). So we need a wide radius cut to avoid the acceptance distortion, and also we need a vertex reconstruction, that is not too sensitive to the transverse general shift of the decay — it is a Neutral vertex rather than CDA.

Selection

Min bias **trigger**: 1 track and E_{LKr} > 10 GeV ((sevt->trigWord >> 11) & 1)

N of good clusters > 1 :

- LKr standard nonlinearity correction for Data clusters (user_lkrcalcor_SC)
- LKr small final nonlinearity correction for MC clusters, extracted from $\pi^+\pi^0\pi^0$
  (see April 2007 talk of Di Lella and Madigozhin)
- LKr scale corrections from K_{e3} E/P (different for Data and MC, sub-permill precision)
- Cluster status <= 4
- Cluster energy >= 3 GeV
- Distance to dead cell >= 2 cm
- Radius at LKr >= 15 cm
- In standard LKr acceptance
- Distance to any in-time (within 10 ns) track impact point at LKr >= 15 cm
- Distance to any another in-time (within 5 ns) cluster >= 10 cm

In Monte Carlo everithing is in-time

N of good tracks > 0 :

- Pe >= 5 GeV, Pμ >= 10 GeV (muon case cut applied after identification)
- Track momenta α,β corrections both for data and MC
- If there is the associated LKr cluster, its cluster status <=4
- Track quality >= 0.6
- Distance to dead cell >= 2 cm
- Radius at every DCH(1,2,3,4) >= 15 cm
- **Reject DCH tracks with 0 cm < X(DCH4) < 6 cm && Y(DCH4)>0** (inefficient band)
- **$K_{\mu3}$ DCH track: for all 3 MUV planes R_{MUV} > 30 cm, $|X_{MUV},Y_{MUV}|$<115 cm.**
- LKr impact point is in LKr acceptance

π^0 selection

- Check all the pairs of good in-time (within 5 ns) clusters
- Calculate π^0 time t_π (average of two γ ones) and reject the combination, if there is a good extra cluster in 5 nanoseconds around t_π (to suppress $\pi^+\pi^0\pi^0$ and showers).
- Make the projectivity correction for the experimental data and MC.
- Reject the pair, if the distance between the clusters is < 20 cm
- $E_{\pi 0}$ > 15 GeV (for trigger efficiensy: trigger E LKr > 10 GeV).
- Calculate Z_n from two γ, assuming π^0 mass
- -1600 cm $< Z <$ 9000 cm
- DCH flunge gamma cut for the both γ

Track selection and identification

For each found good π^0 check all the good tracks:

- In-time with π^0 (within 10 ns)
- There is no extra good track within 8 ns around the track time (against showers).
- If 2.0 > E/P > 0.9, it is an electron (K_{e3})
- If E/P < 0.9 and there is a muon associated, it is a muon ($K_{\mu 3}$)

First iteration decay vertex position:

- $Z_{decay} = Z\ (\pi^0)$
- X_{decay}, Y_{decay} = impact point of reconstructed charged track on the transversal plane, defined by Z_{decay}

Blue field correction:

With the «first iteration vertex», we implement the Blue field correction, obtain corrected track slopes and recalculate vertex X,Y again.

Beam position correction:

We know the position of beam axis in space (it is always displaced slightly from the nominal Z axis). For the CMC tuning, these positions were measured for each run from $3\pi^{\pm}$ data many years ago.

We use these data to calculate all the relevant values with respect to the current run beam axis rather than with respect to nominal Z arrow. First of all, we calculate the vertex (x,y) with respect to the beam center X_b,Y_b at this Z_n.

Vertex position cut (very wide):

$$\mathrm{SQRT}\left(((X-a_X(Z))/\sigma_x(Z))^2 + ((Y-a_Y(Z))/\sigma_Y(Z))^2 \right) < \mathbf{11.0}$$

Here a_X, a_Y, σ_X and σ_Y are the functions of Z and represent the average position and width of the beam with respect to standard ($3\pi^{+-}$) beam position.

They are obtained by Gaussian fit ($\pm$1.2 cm around maximum) for Z slices, separately for MC and Data, for X and Y and for positive and negative beams. Then these points are parametrised as functions of Z by polinomes of 5-th degree of Z.

Final stage of the selection

- $P_L(\nu)^2 > 0.0014\ GeV^2$ for K_{e3} only
- Quadratic equation for P_K is solved, if no solutions, the combination is taken with zero discriminant. With the above $P_L(\nu)^2$ requirement, such a cases are rare for K_{e3}.
- Average beam momentum P_b measured from $3\pi^\pm$ decays for each run is used to choose the best P_K solution (closest to P_b from two ones).
- $-7.5\ GeV/c < (P_K — P_b) < 7.5\ GeV/c$

- For $K_{\mu3}$, the cut against $K^\pm \to \pi^\pm\pi^0$ with $\pi^\pm \to \mu^\pm\bar{\nu}$:

$m(\pi^+\pi^0) < 0.47\ GeV$ and $m(\pi^+\pi^0) < (0.6 — P_t(\pi^0))\ GeV$;

- For $K_{\mu3}$, one more cut against $K^\pm \to \pi^\pm\pi^0$ with $\pi^\pm \to \mu^\pm\bar{\nu}$: $m(\mu^\pm\bar{\nu}) > 0.18\ GeV$;
- For $K_{\mu3}$ only: a cut against $\pi^\pm\pi^0\pi^0$: $(P_2-P_1) < 60\ GeV$ <=> in terms of P_K equation discriminant squared $\mathbf{d = ((P_2-P_1)/2)^2}$: $d < 900\ GeV^2$;
- For K_{e3}, the **ν** transversal momentum with respect to beam axis must be

$P_t >= 0.03\ GeV$: a cut against $K^\pm \to \pi^\pm\pi^0$ with $\pi^\pm$ misidentified as *e* (when E/P > 0.9).

In every event, separately for K_{e3} and $K_{\mu3}$, the combination with the minimum $\Delta P = |P_K — P_b|$ is choosen as the best candidate.

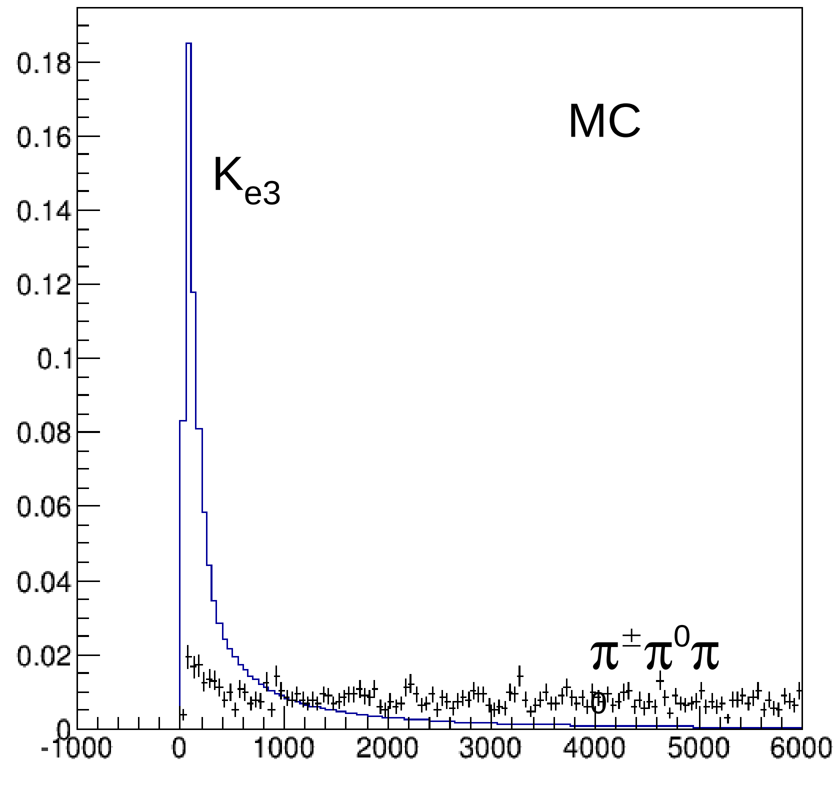
A complex nature of $(P_L^\nu)^2$ - dependent K_{e3} systematic effect

1) Mismeasurent of decay transversal coordinates happens (in the neutral vertex case it also involves the LKr clusters mismeasurement).
2) As a consequense, a small mismeasurement of transversal $(P_t^\nu)^2$
3) As a consequense, a small mismeasurement of $(P_L^\nu)^2 = (E^\nu)^2 — (P_t^\nu)^2$
4) As a consequence, a small mismeasurement of $D = ((P^K_1 — P^K_2)/2)^2$
5) When D itself is small or negative, even small D mismeasurement is relatively not small.
6) Distorted D changes in a different way the probability of the «best» P^K choise (we take the closest to average true $<P^K>$) for different vertex definitions and for MC and Data, depending on true P^K spectrum. The wrong choise may also depend on the correlations between true P^K and the transversal decay coordinates.
7) Mistake in P^K choise from two options may be not small, it is of the order of spectrum width (few GeV), and it leads to relatively big mismeasurement of Dalits plot variables, especially for E_π^*.

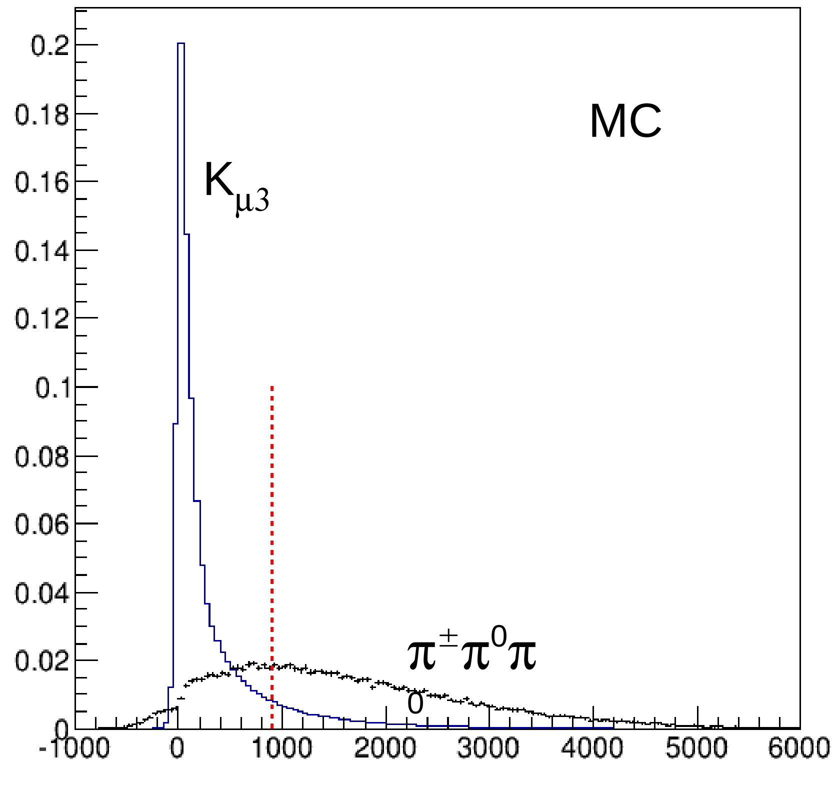
- Correct simulation of this effect seems to be difficult, we have only a simple beam correction for the scattered component.

- But we know, where the problem is concentrated (small $(P_L^\nu)^2$), so we just cut the problematic region.

For $K_{\mu 3}$ only: a cut against $\pi^{\pm}\pi^0\pi^0$: $(P_2-P_1)<60$ GeV <=> $\mathbf{D = ((P_2-P_1)/2)^2}$ < 900 GeV^2



D, Gev^2/c^2



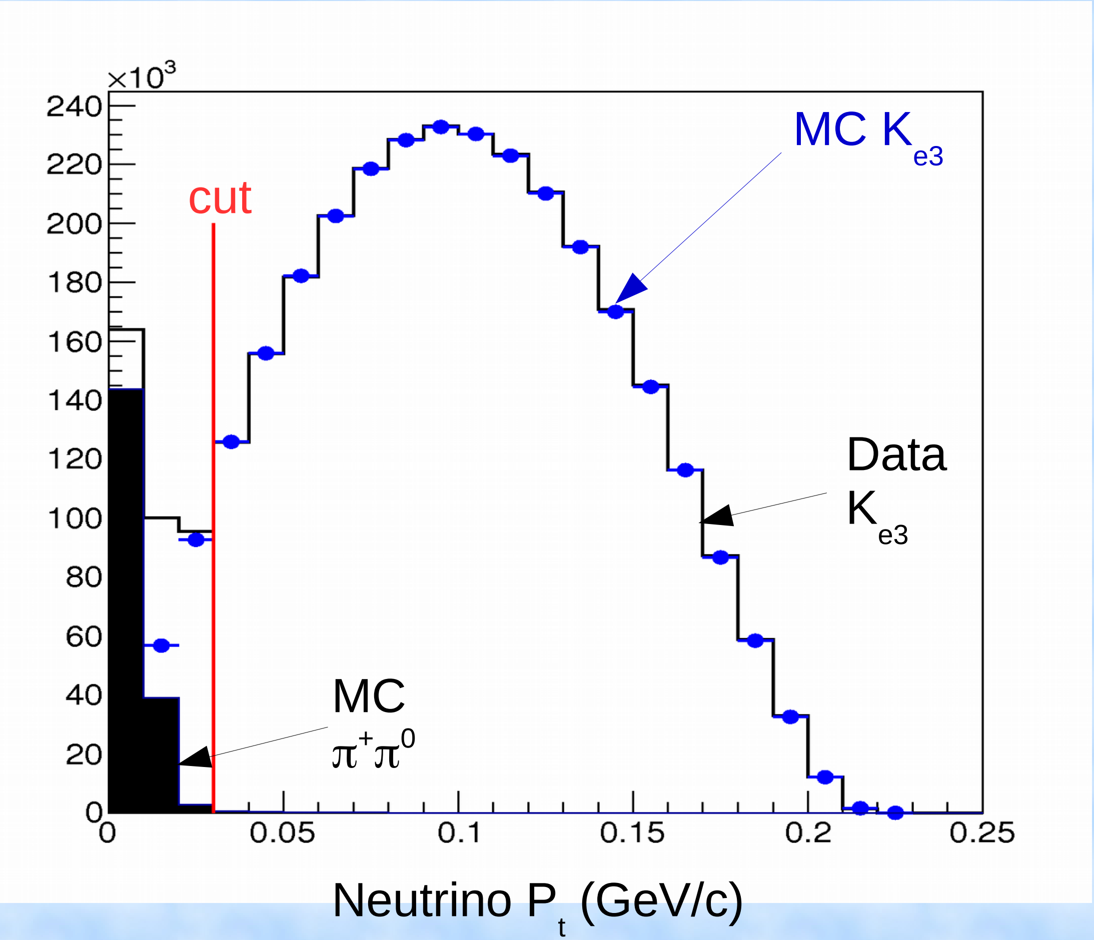
D, Gev^2/c^2

Equally normalized distributions of signal and background events are shown in order to check that the cut is doing its work in both cases.

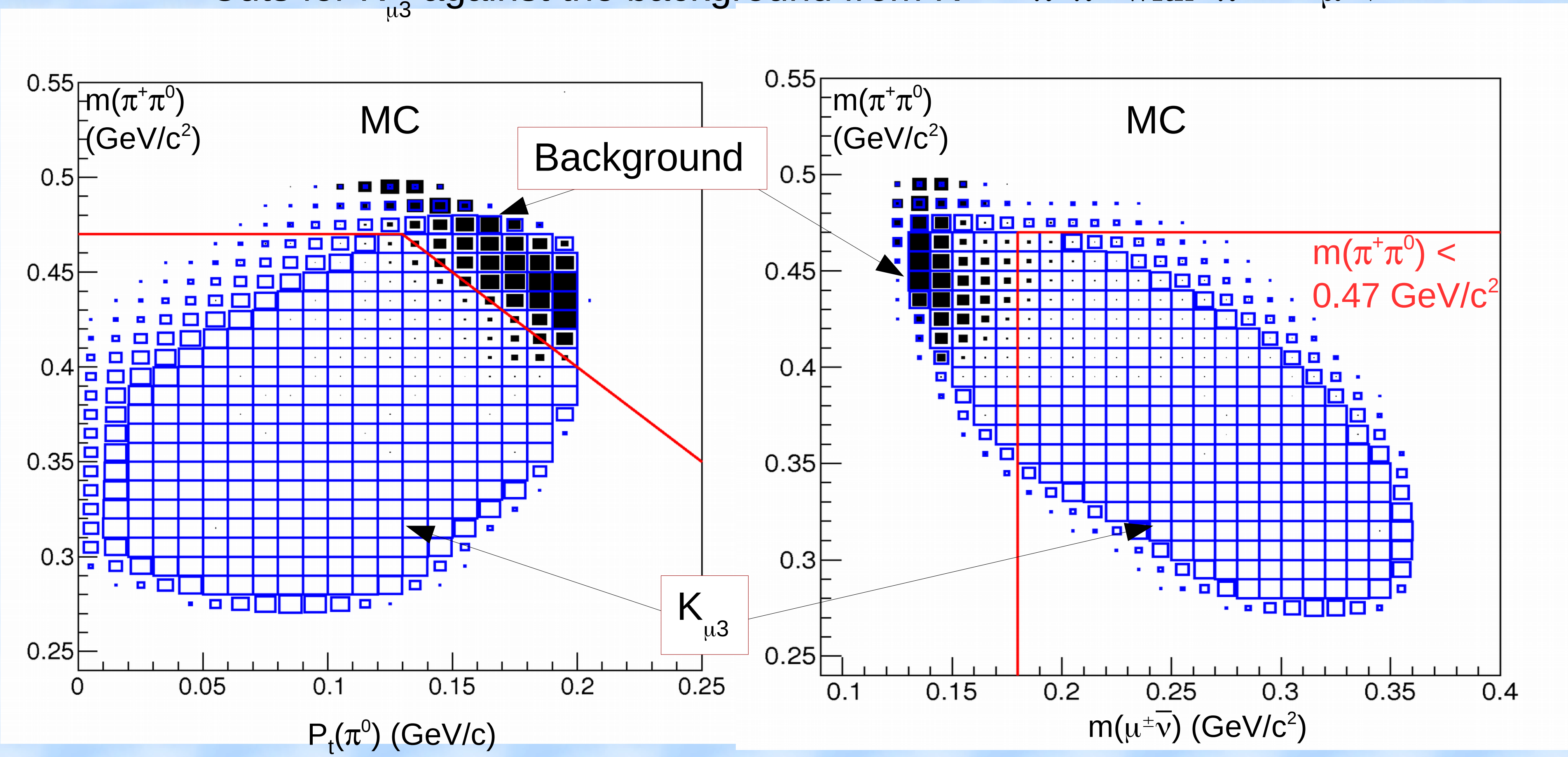
But the absolute K_{e3} background level is much smaller than for $K_{\mu 3}$.
So we don't use this cut for K_{e3} and save some experimental statistics.

For K_{e3}, the **ν** transversal momentum with respect to beam axis must be P_t >= 0.03 GeV.

It is a cut against $K^{\pm} \to \pi^{\pm}\pi^{0}$ with $\pi^{\pm}$ misidentified as *e* (when E/P > 0.9).



Cuts for $K_{\mu 3}$ against the background from $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}$ with $\pi^{\pm} \rightarrow \mu^{\pm}\bar{\nu}$



$m(\pi^{+}\pi^{0}) < 0.47$ GeV/c^2 and
$m(\pi^{+}\pi^{0}) < (0.6 - P_t(\pi^{0}))$ GeV/c^2

$m(\mu^{\pm}\bar{\nu}) > 0.18$ GeV/c^2
(to exclude π^{+} mass region)

K_{e3} requirement: $P_L(\nu)^2 > 0.0014\ GeV^2$

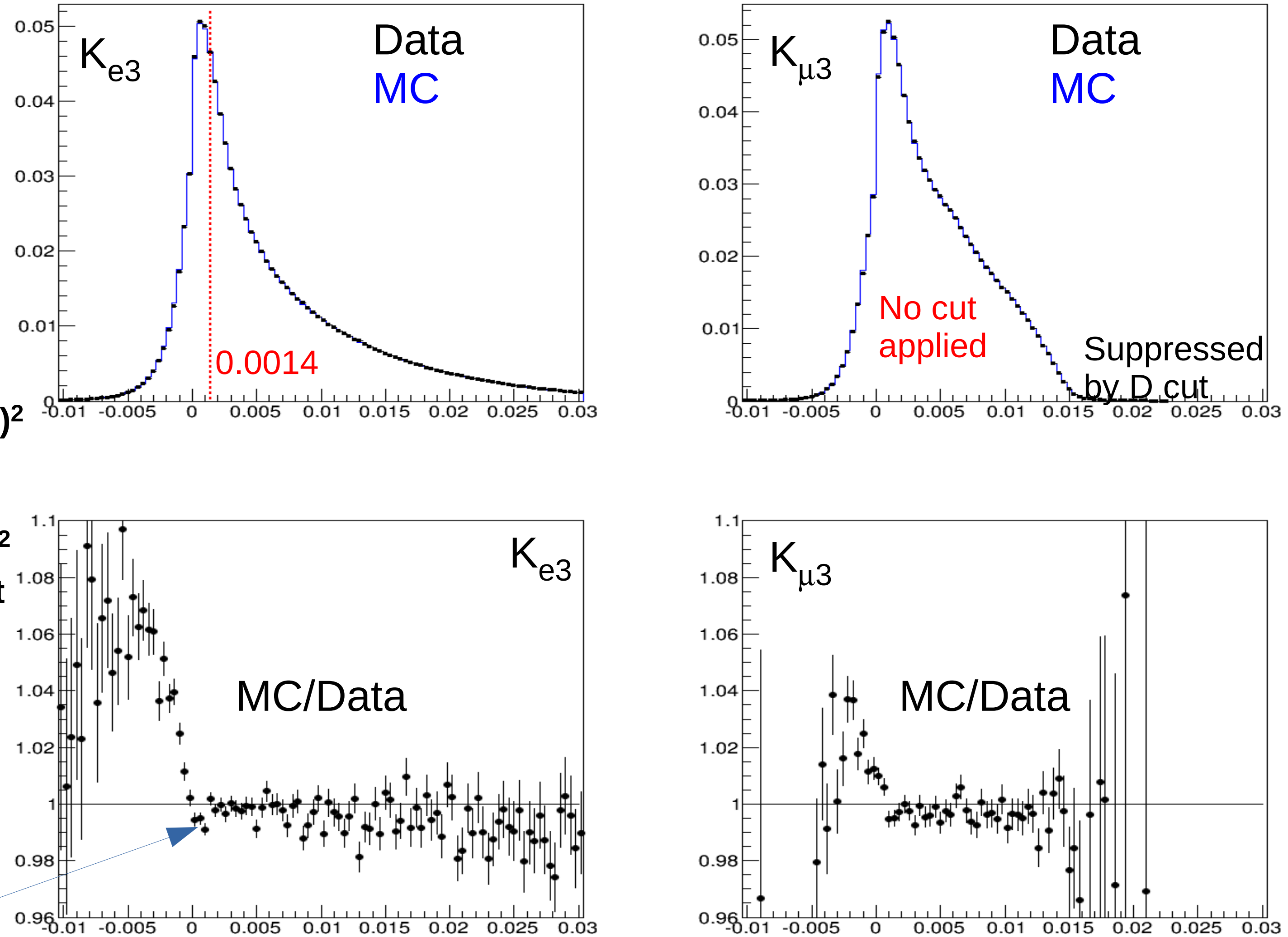
$(P_L^{\nu})^2$ normalized distributions

(Data and MC with background)

$P_L(\nu)^2 = (E^{\nu})^2 - (P_t^{\nu})^2$ negative tail is difficult to simulate precisely, as it depends on the beam transverse shape (scattering) via P_t^{ν}.

For K_{e3} only the region of small and negative $P_L(\nu)^2$ induces a systematic FF uncertainty ($P_L(\nu)^2$ dependence), that is avoided by this cut.

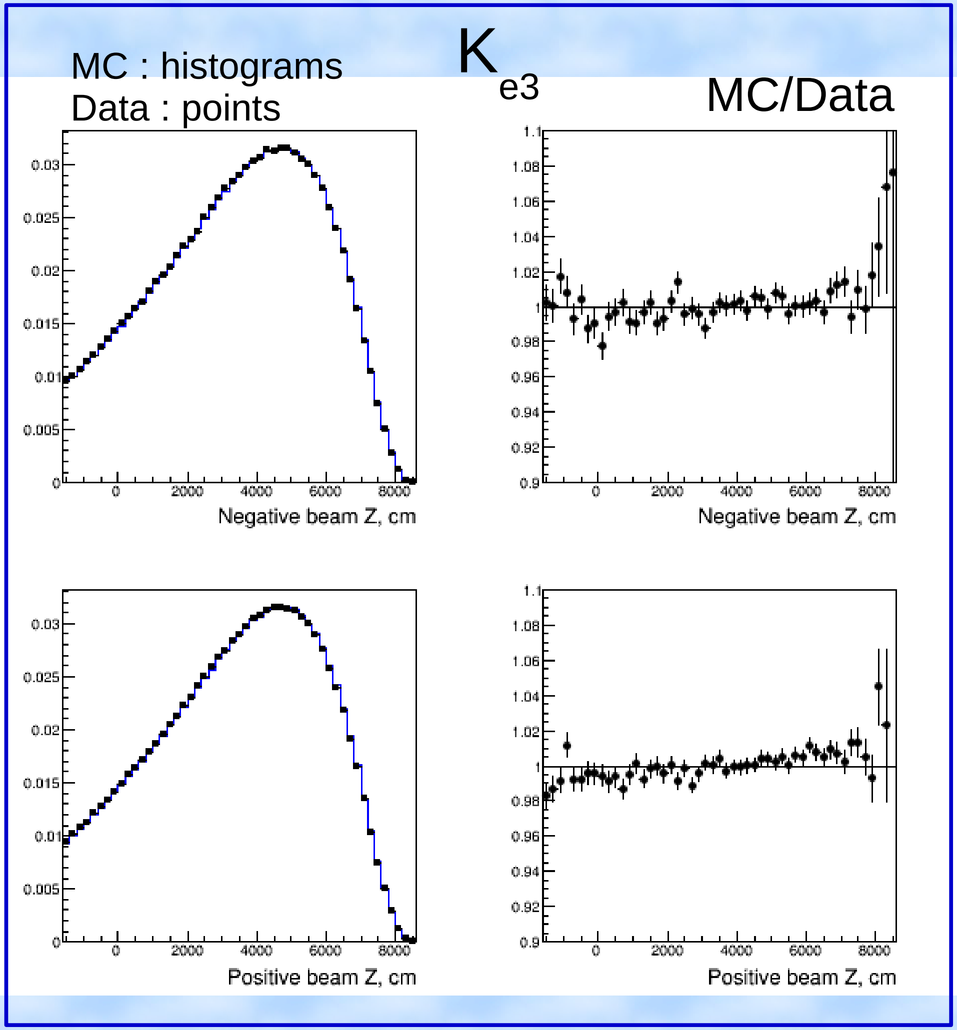
Peak sharpness residual mismatch is used to check $(P_L^{\nu})^2$ resolution systematics related to this cut.

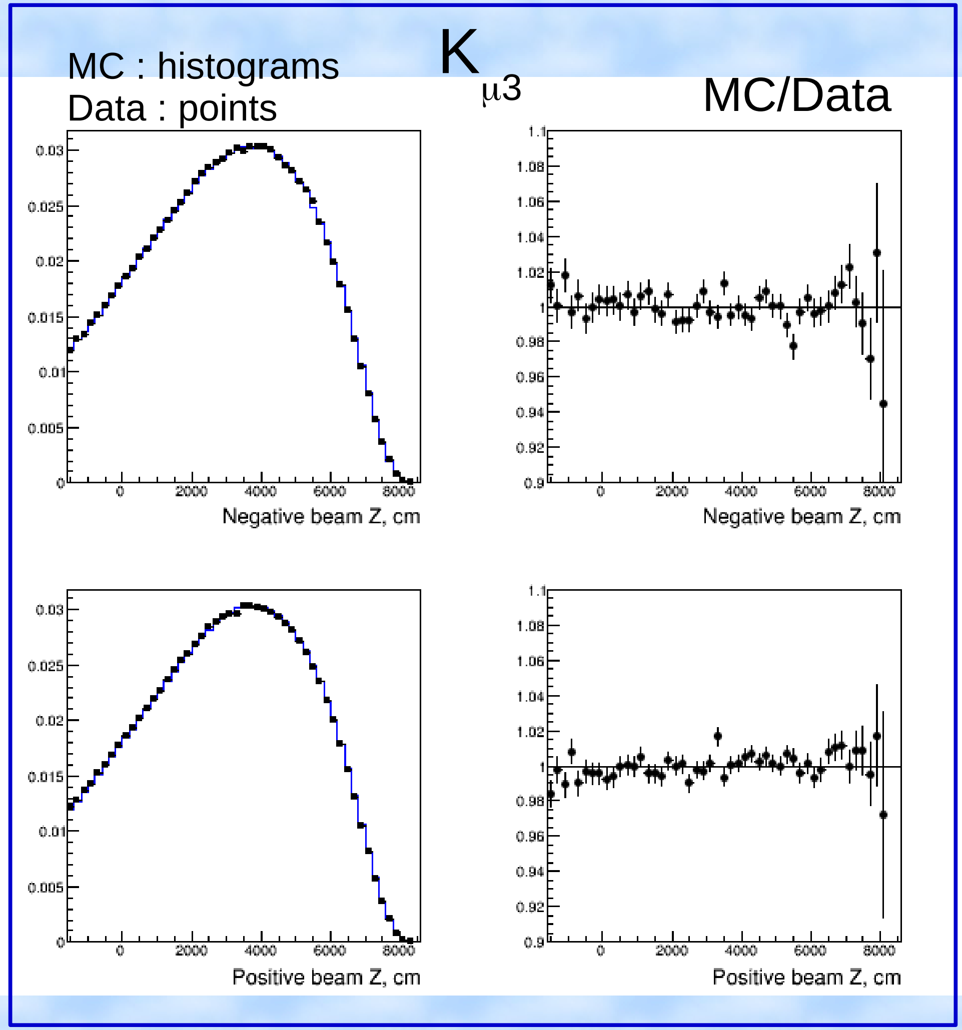


$(P_L^{\nu})^2$ (GeV²)

$(P_L^{\nu})^2$ (GeV²)

Neutral Z normalized distributions comparison





Residual discrepance (~1%) is taken into account as a contribution to systematic uncertainty = variation of final result due to the change of geometrical acceptance by the factor of 1.002, that corrects the K_{e3} differences.

Experimental systematics

Contribution	*Approach to the uncertainty calculation*
Beam scattering	Effect of the additional beam fraction imitating the beam scattering
LKr nonlinearity	Effect of the final nonlinearity correction
LKr scale	Effect of the LKr scale shift allowed by Data/MC electron E/P peak
Background	Effect of the background contribution change within B_{ell} distribution tails Data/MC agreement. It absorbs the PDG branching fraction errors
Trigger efficiency	Effect of the measured quadratically smoothed trigger efficiency (~100%)
Accidentals	Effect of the time windows doubling for clusters and tracks acceptance
Acceptance	Effect of small transversal detector cuts increasing for MC, that (over) corrects Z distributions
P_K average	Effect of beam $\langle P_K \rangle$ possible mismeasurement
P_K spectra	Effect of the MC true P_K spectra variation within the agreement of measured MC/Data P_K spectra
Neutrino P cut	Effect of the artificial $(P_L^{\nu})^2$ resolution variation within $(P_L^{\nu})^2$ peak sharpness MC/Data agreement
Binning	Effect of the bins doubling for the both Dalitz plot dimensions
Resolution	Difference between the main events weighting approach and the acceptance correction technique that is more sensitive to resolution

External contributions to systematic uncertainty.

Contribution	*Approach to the uncertainty calculation*
Radiative correction precision	Effect of the theoretical uncertainty in the radiative Dalitz plot corrections in terms of one-dimensional slopes.
Parameterization for Dispersive fits	100 fits with the independently sampled 5 external parameters known with a given uncertainty.

The full analysis is performed and form factor parameters are extracted:

- For K_{e3}
- For $K_{\mu 3}$
- For the combined K_{l3} result: A joint fits are done by minimizing of the sum $\chi^2(K_{e3}) + \chi^2(K_{\mu 3})$ with a common set of fit parameters. This is repeated also for each of the systematic uncertainty studies.

LKr Nonlinearity

Use 2004 $\pi^0\pi^0\pi^{+-}$ data (done for cusp analysis):

22 < E (π^0_1) < 26 GeV
$E(\pi^0_2) < E(\pi^0_1)$
$E(\gamma)^{max} < 0.55\ E(\pi^0)$ for both π^0

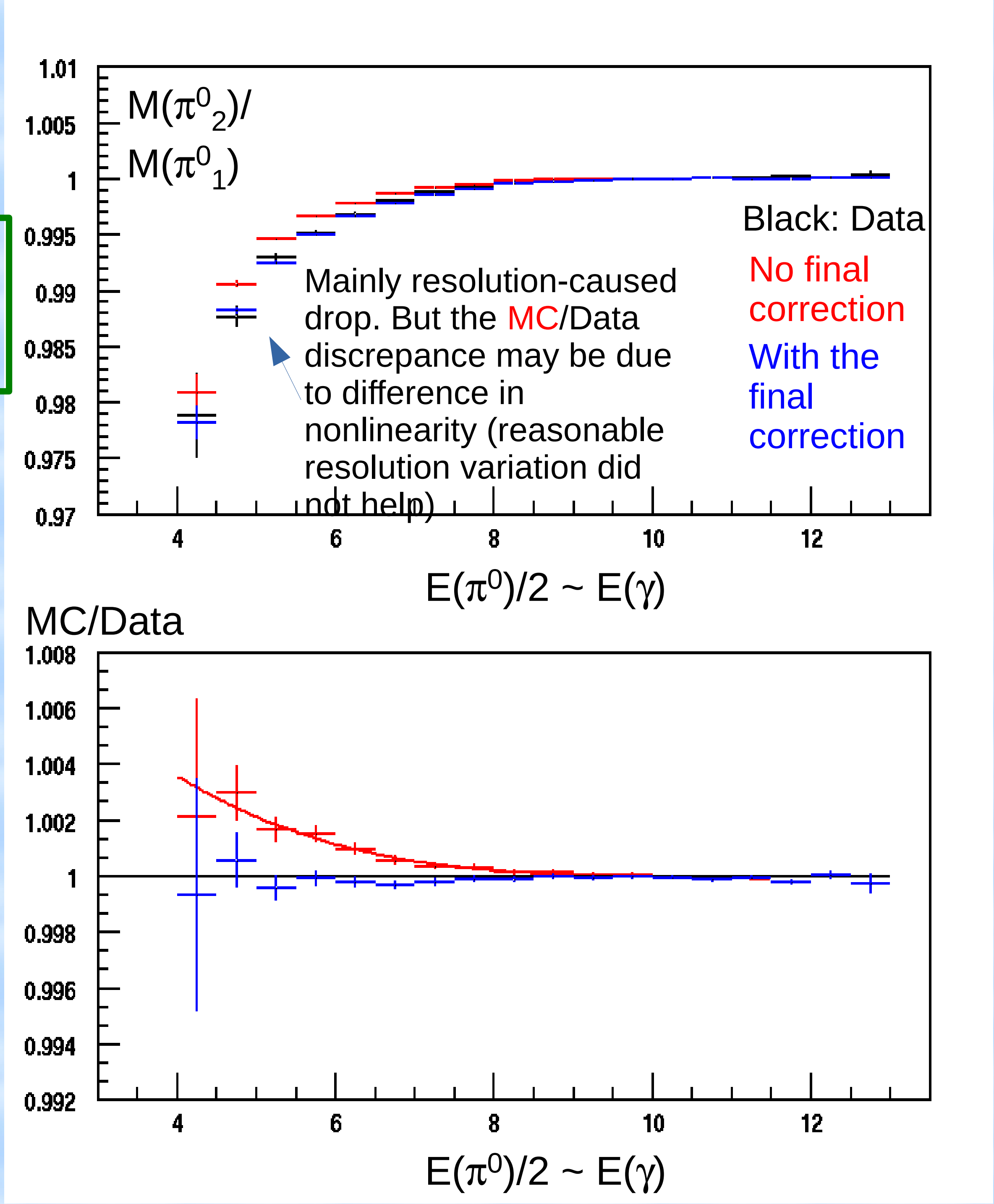
Final correction for MC:

P_0 1.0170
P_1 -0.48025E-02
P_2 0.45538E-03
P_3 -0.14474E-04

E: cluster energy in GeV

$f=P_0+P_1E+P_2E^2+P_3E^3$
if(f > 1) E= E/f

100% of the final correction effect is taken as the nonlinearity-related uncertainty.



Blue field correction:

With the «first iteration vertex», we implement the Blue field correction, obtain corrected track slopes and recalculate vertex X,Y again.

Beam position correction:

We know the position of beam axis in space (it is always displaced slightly from the nominal Z axis). For the CMC tuning, these positions were measured for each run from $3\pi^{\pm}$ data many years ago.

We use these data to calculate all the relevant values with respect to the current run beam axis rather than with respect to nominal Z arrow. First of all, we calculate the vertex (x,y) with respect to the beam center X_b, Y_b at this Z_n.

Vertex position cut (very wide):

$$\mathrm{SQRT}\left(((X-a_X(Z))/\sigma_x(Z))^2 + ((Y-a_Y(Z))/\sigma_Y(Z))^2 \right) < \mathbf{11.0}$$

Here a_X, a_Y, σ_X and σ_Y are the functions of Z and represent the average position and width of the beam with respect to standard ($3\pi^{+-}$) beam position.

They are obtained by Gaussian fit ($\pm$1.2 cm around maximum) for Z slices, separately for MC and Data, for X and Y and for positive and negative beams. Then these points are parametrised as functions of Z by polinomes of 5-th degree of Z.

Results for K_{e3} and $K_{\mu3}$

$\chi^2/NDF(K_{e3})$: 609.4/687

$\chi^2/NDF(K_{\mu3})$: 391.2/384

Quadratic parameterization
(in units of 10^{-3})

	$\lambda'_+(K_{e3})$	$\lambda''_+(K_{e3})$	$\lambda'_+(K_{\mu3})$	$\lambda''_+(K_{\mu3})$	$\lambda_0(K_{\mu3})$
Central values	23.52	1.60	23.32	2.14	14.33
Stat. error	0.78	0.30	3.08	1.06	1.11
Beam scattering	0.90	0.32	0.25	0.12	0.58
LKr nonlinearity	0.28	0.01	2.85	0.73	0.93
LKr scale	0.68	0.12	0.83	0.18	0.14
Background	0.07	0.04	0.26	0.05	0.04
Trigger	0.27	0.13	0.67	0.23	0.12
Accidentals	0.24	0.08	0.01	0.00	0.01
Acceptance	0.28	0.08	0.85	0.23	0.25
Pk average	0.06	0.01	0.20	0.07	0.32
Pk spectra	0.00	0.00	0.12	0.04	0.00
Neutrino P cut	0.18	0.04	0.00	0.00	0.00
Binning	0.05	0.00	0.11	0.05	0.15
Resolution	0.01	0.02	1.44	0.46	0.39
Radiative	0.20	0.01	0.15	0.03	0.06
Syst. error	1.29	0.39	3.50	0.96	1.25
Total error	1.51	0.49	4.67	1.43	1.67

Correlation
-0.927

Correlation

	$\lambda''_+(K_{\mu3})$	$\lambda_0(K_{\mu3})$
$\lambda'_+(K_{\mu3})$	-0.969	0.851
$\lambda''_+(K_{\mu3})$		-0.810

Pole parameterization (in units of 10^{-3})

χ^2/NDF(K_{e3}): 609.3/688

χ^2/NDF($K_{\mu3}$): 388.0/385

	$m_V(K_{e3})$	$m_V(K_{\mu3})$	$m_S(K_{\mu3})$
Central values	896.8	879.1	1196.4
Stat. error	3.4	8.1	18.1
Beam scattering	1.4	7.6	22.6
LKr nonlinearity	3.5	9.6	6.2
LKr scale	5.3	4.1	2.2
Background	0.4	1.5	0.7
Trigger	0.8	0.1	12.7
Accidentals	0.5	0.0	0.3
Acceptance	1.3	2.4	1.0
Pk average	0.3	0.2	9.0
Pk spectra	0.1	0.0	1.6
Neutrino P cut	1.2	0.0	0.0
Binning	0.7	0.5	4.5
Resolution	0.6	2.2	1.0
Radiative	3.2	0.8	1.6
Syst. error	7.6	13.5	28.8
Total error	8.3	15.7	34.0

Correlation 0.320

Dispersion parameterization (in units of 10^{-3})

χ^2/NDF(K_{e3}): 609.1/688

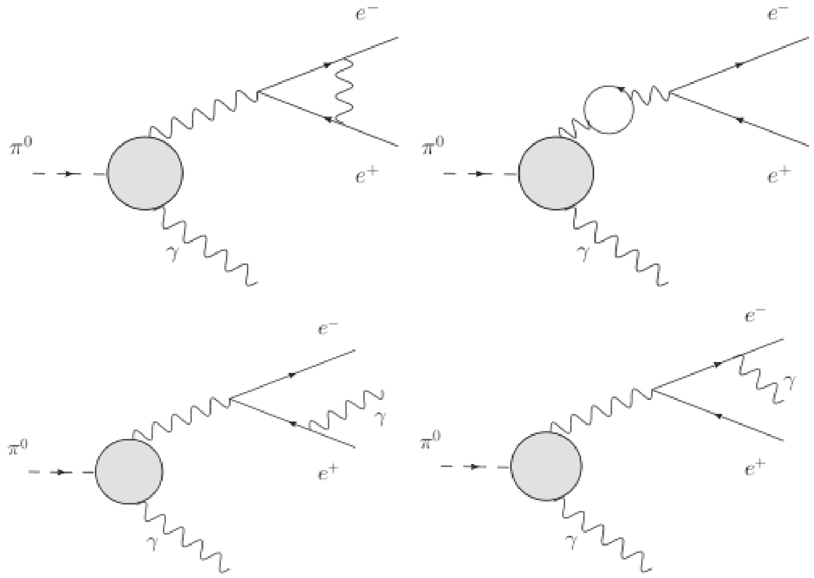
χ^2/NDF($K_{\mu3}$): 385.8/385

	$\Lambda_+(K_{e3})$	$\Lambda_+(K_{\mu3})$	$ln[C](K_{\mu3})$
Central values	22.54	23.55	186.68
Stat. error	0.20	0.50	5.12
Beam scattering	0.09	0.48	7.05
LKr nonlinearity	0.20	0.60	2.08
LKr scale	0.31	0.26	0.50
Background	0.02	0.10	0.15
Trigger	0.04	0.01	3.62
Accidentals	0.03	0.00	0.09
Acceptance	0.08	0.16	0.35
Pk average	0.02	0.01	2.62
Pk spectra	0.00	0.00	0.46
Neutrino P cut	0.07	0.00	0.00
Binning	0.04	0.03	1.24
Resolution	0.03	0.10	0.50
Radiative	0.18	0.05	0.49
Parameterization	0.44	0.49	2.95
Syst. error	0.62	0.97	9.23
Total error	0.65	1.10	10.55

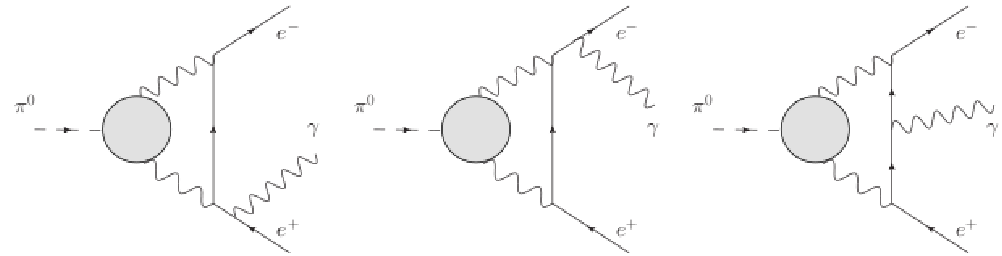
Correlation 0.408

π_D^0: Radiative Corrections

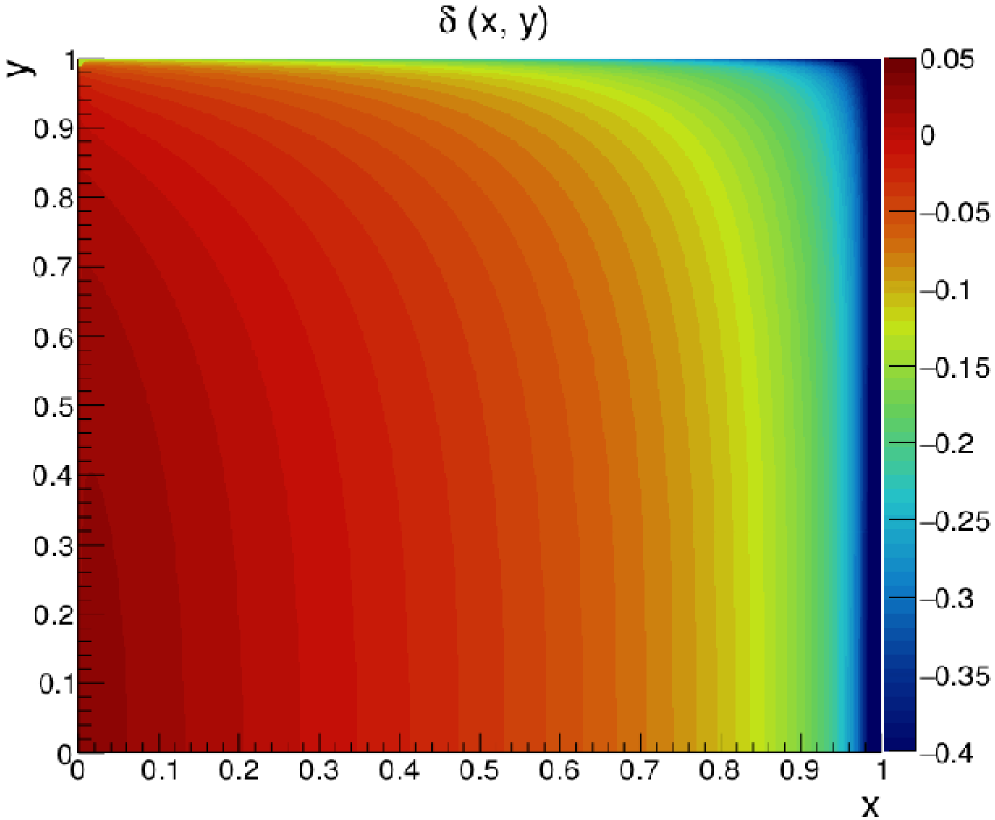
Mikaelian and Smith
Phys.Rev. D5 (1972) 1763



Husek, Kampf and Novotny
Phys.Rev. D92 (2015) 5, 054027



$$\frac{d^2\Gamma}{dx\,dy} = \left(\frac{d^2\Gamma}{dx\,dy}\right)_0 (1+\delta(x,y))$$



→ Corrections included in the simulation
→ Radiative photon emission simulated

Reconstructed x Dalitz variable and Acceptances of the $K_{2\pi D}$