

Heavy glueballs

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Talk prepared for:

Excited QCD 2017

7-13/5/2017, Sintra, Portugal

Outline



From quarks and gluons to hadrons

Glueball's masses

Experiments searching glueball

Glueballs below 2.6 GeV.

The vector glueball

The pseudotensor glueball

Summary

Find t glueball/1



Inspire: find t Glueball: 1393 papers (on 31/5/2016)

The screenshot shows the INSPIRE search results page for the query 'f t glueball'. The page header includes the INSPIRE logo and a welcome message. The search bar contains 'f t glueball' and the search button is labeled 'Search'. Below the search bar, there are options for 'Sort by' (earliest date) and 'Display results' (25 results). The search results section shows 1,393 records found, with the first result highlighted. The first result is titled '1. A comprehensive study of the radiative decays of J/ψ and $\psi(2S)$ to pseudoscalar meson pairs, and search for glueballs' by Sean Dobbs, A. Tomaradze, T. Xiao, and Kamal K. Seth (Northwestern U.), published in AIP Conf. Proc. 1735 (2016) 050001. The DOI is 10.1063/1.4949420 and the conference is C15-09-13 Proceedings. The search took 0.07 seconds.

Find t glueball/2



Inspire: find t Glueball: 1410 papers (on 12/11/2016)

The screenshot shows a web browser window displaying the INSPIRE search results for the query "f t glueball". The browser's address bar shows the URL: `inspirehep.net/search?ln=en&p=f+t+glueball&of=hb&action_search=Search&sf=earliestdate&so=d`. The INSPIRE logo is visible at the top left, and a welcome message is displayed in an orange banner: "Welcome to INSPIRE, the High Energy Physics information system. Please direct questions, comments or concerns to feedback@inspirehep.net".

The search results are displayed in a blue header bar with navigation links: HEP, HEPNAMES, INSTITUTIONS, CONFERENCES, JOBS, EXPERIMENTS, JOURNALS, and HELP. Below the header, the search query "f t glueball" is entered in a search box, with a "Brief format" dropdown and a "Search" button. There are also links for "Easy Search" and "Advanced Search".

Below the search box, there are options for "Sort by:" (earliest date, desc., - or rank by -) and "Display results:" (25 results, single list). A yellow banner indicates "HEP 1,410 records found 1 - 25 jump to record: 1" and "Search took 0.11 seconds."

The search results are listed as follows:

- 1. Decay of charmonium states into a scalar and a pseudoscalar glueball**
Walaa I. Eshraim (Frankfurt U., FIAS). 2016. 8 pp.
Published in *EPJ Web Conf.* **126 (2016) 04017**
DOI: [10.1051/epjconf/201612604017](https://doi.org/10.1051/epjconf/201612604017)
Conference: [C15-08-23 Proceedings](#)
[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)
[Detailed record](#)
- 2. On Slow-roll Glueball Inflation from Holography**
Lilia Anguelova. Nov 1, 2016. 9 pp.
e-Print: [arXiv:1611.00295](https://arxiv.org/abs/1611.00295) [[hep-th](#)] | [PDF](#)
[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)
[ADS Abstract Service](#)
[Detailed record](#)
- 3. Holographic QCD predictions for production and decay of pseudoscalar glueballs**
Frederic Brünner, Anton Rebhan (Vienna, Tech. U.). Oct 31, 2016. 7 pp.
e-Print: [arXiv:1610.10034](https://arxiv.org/abs/1610.10034) [[hep-ph](#)] | [PDF](#)
[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

Find t glueball /3



Inspire: find t Glueball: 1434 papers (on 6/5//2017)

The screenshot shows a web browser window with the URL `inspirehep.net/search?ln=en&p=f+t+glueball&of=hb&action_search=Search&sf=earliestdate&so=d`. The page header includes the INSPIRE logo and a navigation menu with items like HEP, HEPNAMES, INSTITUTIONS, CONFERENCES, JOBS, EXPERIMENTS, JOURNALS, and HELP. A search bar contains the query 'f t glueball' and a 'Search' button. Below the search bar, there are options for 'Sort by' (earliest date) and 'Display results' (25 results, single list). A yellow banner indicates 'HEP 1,434 records found 1 - 25 jump to record: 1' and 'Search took 0.11 seconds.' The results list includes two entries:

- 1. Casimir scaling and Yang-Mills glueballs**
Deog Ki Hong, Jong-Wan Lee, Biagio Lucini, Maurizio Piai, Davide Vadacchino. Apr 30, 2017. 11 pp.
PNUTP-17-A04
e-Print: [arXiv:1705.00286 \[hep-th\]](#) | [PDF](#)
[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)
[ADS Abstract Service](#)
[Detailed record](#)
- 2. Reply to "Comment on 'Finding the 0^{−−} Glueball'" [arXiv:1702.06634] and comment on 'Is the exotic 0^{−−} glueball a pure gluon state?'**
[arXiv:1611.08698]
Cong-Feng Qiao, Liang Tang. Apr 26, 2017.
e-Print: [arXiv:1704.08589 \[hep-ph\]](#) | [PDF](#)
[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)
[ADS Abstract Service](#)
[Detailed record](#)

The bottom of the screenshot shows a Windows taskbar with the time 6:35 PM on 5/6/2017 and several open PDF files.

Review papers



IOP PUBLISHING

JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **40** (2013) 043001 (68pp)

doi:10.1088/0954-3899/40/4/043001

TOPICAL REVIEW

The status of glueballs

Wolfgang Ochs

Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, Föhringer Ring 6,
D-80805 München, Germany

Int.J.Mod.Phys. E18 (2009) 1-49

Vincent Mathieu
Groupe de Physique Nucléaire Théorique,
Université de Mons-Hainaut,
Académie universitaire Wallonie-Bruxelles,
Place du Parc 20, BE-7000 Mons, Belgium.
vincent.mathieu@umh.ac.be

Nikolai Kochelev
Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research,
Dubna, Moscow region, 141980 Russia.
kochelev@theor.jinr.ru

Vicente Vento
Departament de Física Teòrica and Institut de Física Corpuscular,
Universitat de València-CSIC,
E-46100 Burjassot (Valencia), Spain.
vicente.vento@uv.es

Glueballs are particles whose valence degrees of freedom are gluons and therefore in their description the gauge field plays a dominant role. We review recent results in the physics of glueballs with the aim set on phenomenology and discuss the possibility of finding them in conventional hadronic experiments and in the Quark Gluon Plasma. In order to describe their properties we resort to a variety of theoretical treatments which include, lattice QCD, constituent models, AdS/QCD methods, and QCD sum rules. The review is supposed to be an informal guide to the literature. Therefore, we do not discuss in detail technical developments but refer the reader to the appropriate references.

Progress in Particle and Nuclear Physics 63 (2009) 74–116

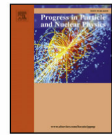


ELSEVIER

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp



Review

The experimental status of glueballs

V. Crede^a, C.A. Meyer^{b,*}

^a Florida State University, Tallahassee, FL 32306, USA

^b Carnegie Mellon University, Pittsburgh, PA 15213, USA

Previous reviews:

F.E. Close, Rep. Progress Phys. 51 (1988) 833.

C. Amsler, Rev. Modern Phys. 70 (1998) 1293. [hep-ex/9708025](http://arxiv.org/abs/hep-ex/9708025).

S. Godfrey, J. Napolitano, Rev. Modern Phys. 71 (1999) 1411. [hep-ph/9811410](http://arxiv.org/abs/hep-ph/9811410).

C. Amsler, N.A. Tornqvist, Phys. Rep. 389 (2004) 61.

A. Masoni, C. Cicalo, G.L. Usai, J. Phys. G32 (2006) R293.

Inspire: f t glueball after 28/2/2013: 93 articles. 'Soon' a new review will be needed.

Francesco Giacosa

From QCD Lagrangian to baryons and mesons

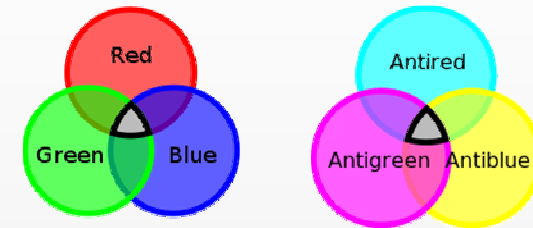


Born Giuseppe Lodovico Lagrangia
25 January 1736
Turin

Died 10 April 1813 (aged 77)
Paris

The QCD Lagrangian

Quark: u, d, s and c, b, t R, G, B



$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix}; \quad i = u, d, s, \dots$$

8 type of gluons ($R\bar{G}, B\bar{G}, \dots$)

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}$$

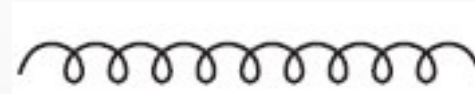
$$A_\mu^a; \quad a = 1, \dots, 8$$

Feynman diagrams of QCD

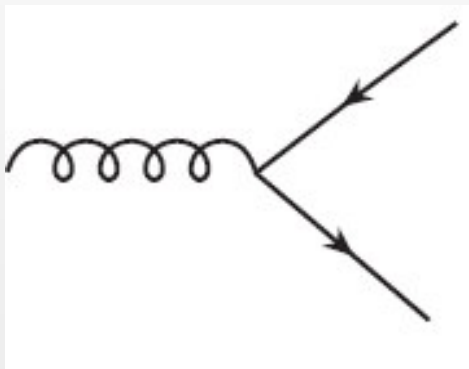
$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}$$



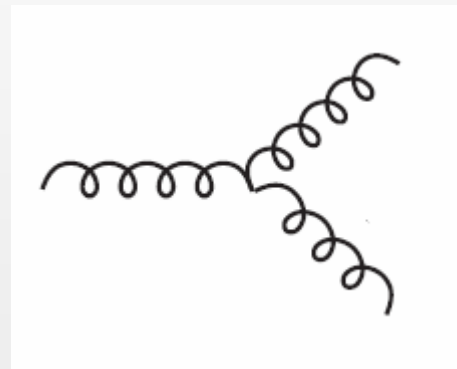
Quark



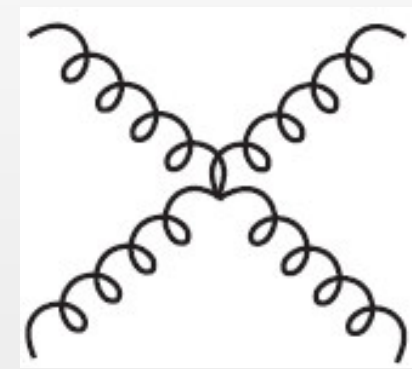
Gluon



Gluon-quark-antiquark
vertex



3-gluon vertex



4-gluon vertex

Already here one may ask: **what about bound states of gluons???**

Symmetries of QCD and breakings



SU(3)_{color}: exact. Confinement: you never see color, but only white states.

Dilatation invariance: holds only at a classical level and in the chiral limit. Broken by quantum fluctuations (**trace anomaly**) and by quark masses.

SU(3)_R × SU(3)_L: holds in the chiral limit, but is broken by nonzero quark masses. Moreover, it is **spontaneously** broken to U(3)_{V=R+L}

U(1)_{A=R-L}: holds at a classical level, but is also broken by quantum fluctuations (**axial anomaly**)

Hadrons



The QCD Lagrangian contains 'colored' quarks and gluons. However, no 'colored' state has been seen.

Confinement: physical states are white and are called hadrons.

Hadrons can be:

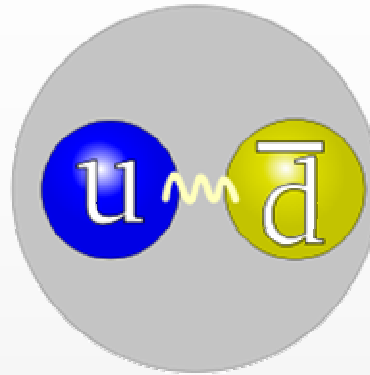
Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.

A quark-antiquark state is a conventional meson.

Example of conventional quark-antiquark states: the ρ and the π mesons



Rho-meson

$$m_{\rho^+} = 775 \text{ MeV}$$

Pion

$$m_{\pi^+} = 139 \text{ MeV}$$

$$m_u + m_d \approx 7 \text{ MeV}$$

Mass generation in QCD
is a nonpert. phenomenon
based on SSB
(mentioned previously).

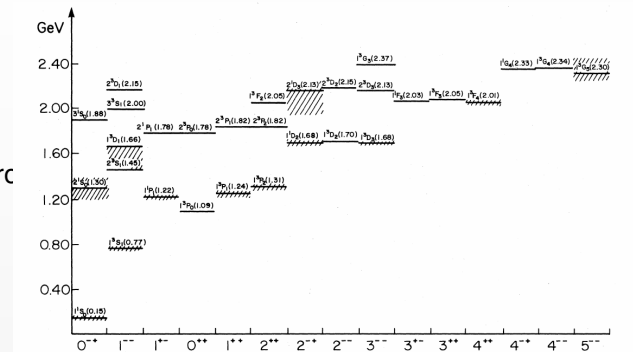
Quark model(s) and their QFT extensions



Mesons in a Relativized Quark Model with Chromodynamics
 S. Godfrey, Nathan Isgur
 Published in Phys.Rev. D32 (1985) **189-231**

Mesonic loops e.g. included into
 A Low Lying Scalar Meson Nonet in a Unitarized Meson Model
 E. van Beveren, T. A. Rijken, K. Metzger, C.~Dullemond, G.~Rupp and J. E.~Ribeirc
 Z. Phys. C **30** (1986) 615

Meson spectroscopy: too much excitement and too few excitations
 G. Rupp, S. Coito and E. van Beveren,
 Acta Phys. Polon. Supp. 5 (2012) 1007



QCD phenomenology based on a chiral effective Lagrangian
 Tetsuo Hatsuda, Teiji Kunihiro
 Phys.Rept. **247** (1994) 221-367

NJL: quark-based model with
 chiral symmetry and SSB
 chiral condensate
 Effective quark mass
 Mesons as quarkonia (pion: ok)

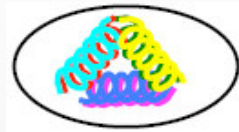
The Infrared behavior of QCD Green's functions: Confinement dynamical symmetry breaking,
 and hadrons as relativistic bound states
 Reinhard Alkofer, Lorenz von Smekal
 Phys.Rept. **353** (2001) 281

DS:
 quarks and gluons propagators
 from QCD
 Condensates
 Effective quark and gluon masses
 Spectra of mesons as quarkonia
 (pion: ok) and baryons as qqg states

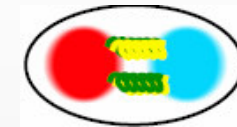
Baryons as relativistic three-quark bound states
 G. Eichmann, H.~Sanchis-Alepuz, R. Williams, R. Alkofer and C. S. Fischer,
 Progr. Part. Nucl. Phys. **91** (2016) 1

Non-conventional mesons: theoretical expectations

1) Glueballs



2) Hybrids



Compact diquark-antidiquark states



3) Four-quark states

Molecular states (a type of dynamical generation)



Companion poles (another type of dynamical generation)

Masses of glueballs

Glueball masses: bag models



A. Chodos, et al., Phys. Rev. D 9 (1974) 3471.

R.L. Jaffe, K. Johnson, Phys. Lett. B60 (1976) 201

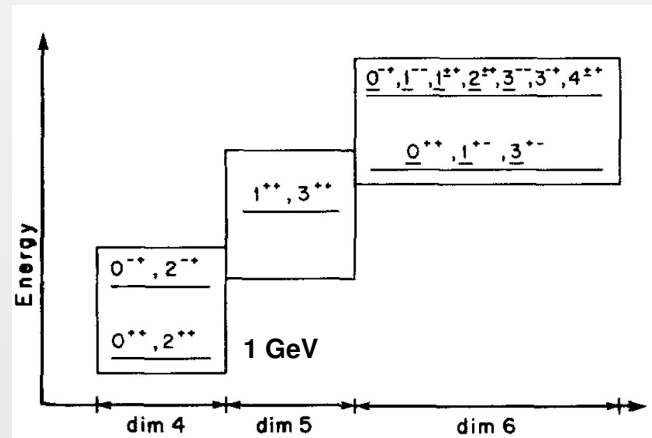
ANNALS OF PHYSICS 168, 344–367 (1986)

Qualitative Features of the Glueball Spectrum*

R. L. JAFFE, K. JOHNSON, AND Z. RYZAK[†]

*Center for Theoretical Physics, Laboratory for Nuclear Science,
and Department of Physics, Massachusetts Institute of Technology,
Cambridge, Massachusetts 02139*

Received September 13, 1985



Lattice



PHYSICAL REVIEW D **73**, 014516 (2006)

Glueball spectrum and matrix elements on anisotropic lattices

Y. Chen,^{1,2} A. Alexandru,² S.J. Dong,² T. Draper,² I. Horváth,² F.X. Lee,^{3,4} K.F. Liu,² N. Mathur,^{2,4} C. Mornik,⁵ M. Peardon,⁶ S. Tamhankar,² B.L. Young,⁷ and J.B. Zhang⁸

¹Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, People's Republic of China

²Department of Physics & Astronomy, University of Kentucky, Lexington, Kentucky 40506, USA

³Center for Nuclear Studies, Department of Physics, George Washington University, Washington, D.C. 20052 USA

⁴Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA

⁵Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA

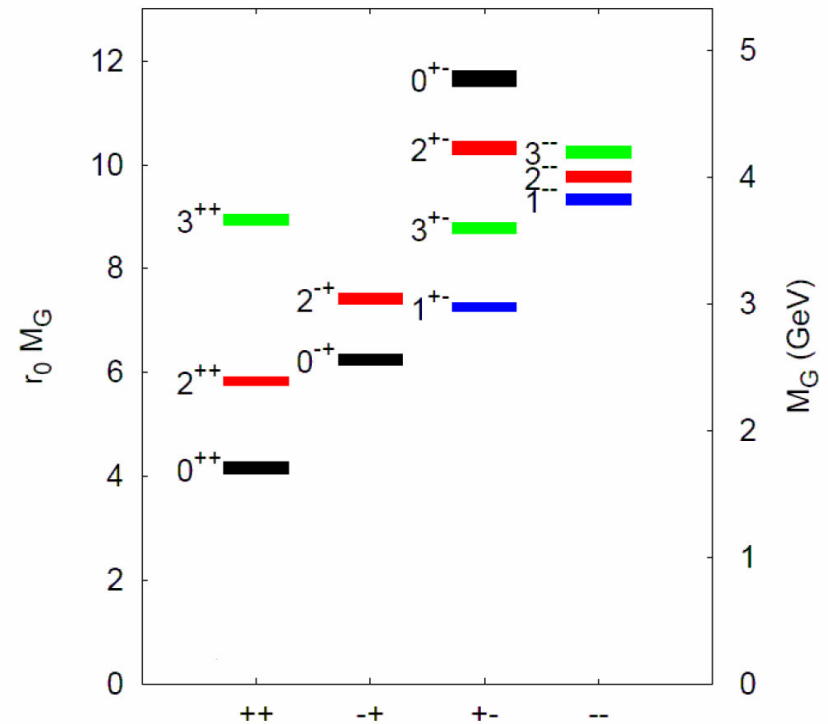
⁶School of Mathematics, Trinity College, Dublin, Dublin 2, Ireland

⁷Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA

⁸CSSM and Department of Physics, University of Adelaide, Adelaide, SA 5005, Australia

(Received 13 October 2005; published 26 January 2006)

The glueball-to-vacuum matrix elements of local gluonic operators in scalar, tensor, and pseudoscalar channels are investigated numerically on several anisotropic lattices with the spatial lattice spacing ranging from 0.1–0.2 fm. These matrix elements are needed to predict the glueball branching ratios in J/ψ radiative decays which will help identify the glueball states in experiments. Two types of improved local gluonic operators are constructed for a self-consistent check and the finite-volume effects are studied. We find that lattice spacing dependence of our results is very weak and the continuum limits are reliably extrapolated, as a result of improvement of the lattice gauge action and local operators. We also give updated glueball masses with various quantum numbers.



Quoted by the PDG in the 'Quark Model' review.

See also: Gregory et al, **JHEP 1210 (2012) 170**

Towards the glueball spectrum from unquenched lattice QCD

Conclusions and future prospects: “*The most conservative interpretation of our results is that the masses in terms of lattice representations are broadly consistent with results from quenched QCD. We do not see any evidence for large unquenching effects, however a definitive calculation requires a continuum extrapolation, and the inclusion of fermionic operators.*”

Francesco Giacosa

Experiments searching glueballs

Glueball production and decays:



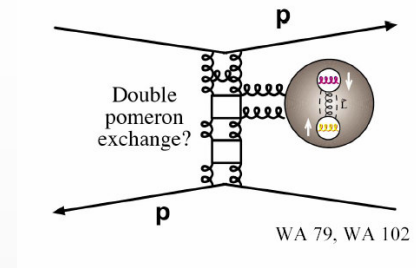
Glueballs should be found in gluon-rich processes
(such as J/ψ decays, proton-antiproton fusion, ...)

Glueballs should couple to all quark flavors with similar strength

Moreover, glueballs should have a suppressed (but nonzero!) decay into photons.

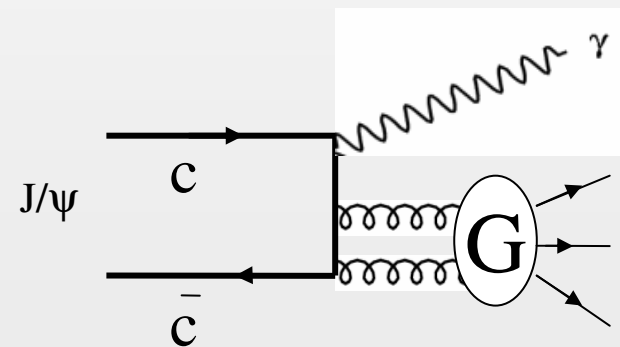
Hadronic experiments

Proton-proton
(WA79, WA102, LHC, RHIC)

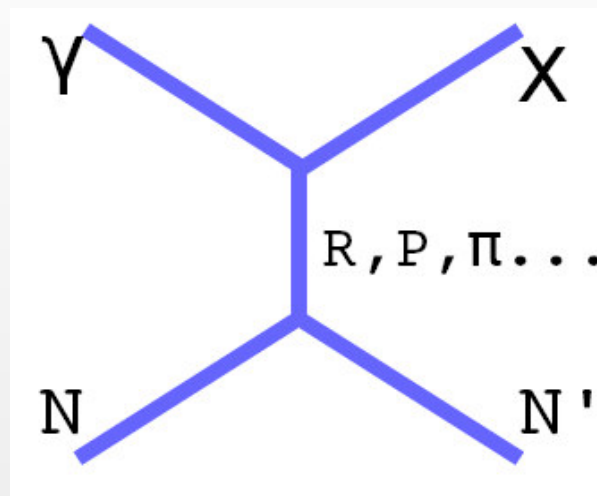


Electron-positron (with strange-antistrange,
charm-anticharm or bottom-antibottom formation)

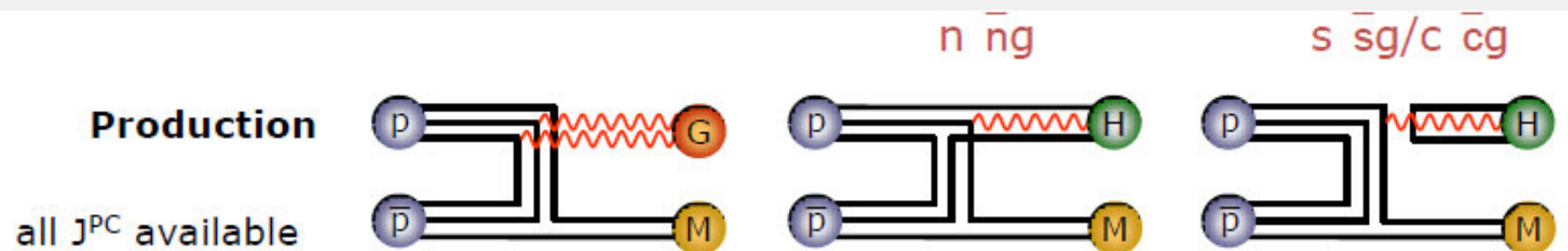
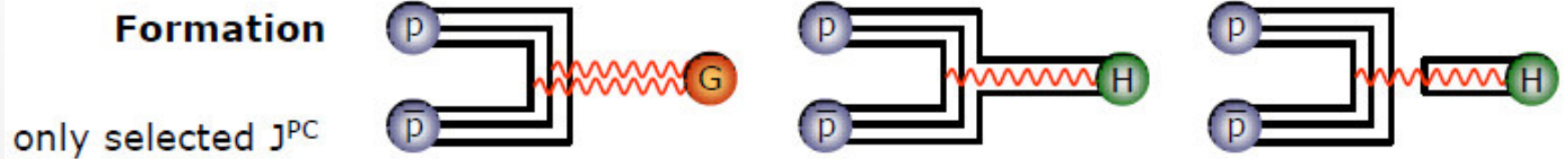
(Belle, Belle II, Babar,
BES, KLOE, KLOEII,...)



Photoproduction: Compass at CERN (also with pion instead of the photons)
GlueX AND CLAS12 at JLab (start soon)

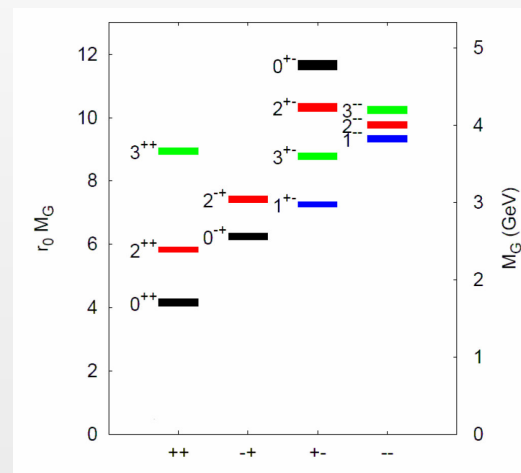


Proton-antiproton (Lear, Fermilab, and in the future: PANDA)



PANDA mass range: from 2.5 to 4.5 GeV

The scalar glueball



Mixing pattern

Above 1 GeV one has two quark-antiquark states and a bare glueball.

$$\sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$$

$$\bar{s}s$$

Glueball: gg

They mix to form the 3 resonances on the right.

Note:
 $a_0(980)$ $k(800)$ $f_0(980)$ $f_0(500)$
 are regarded as non-quarkonium objects

$f_0(1370)$

$$J^{PC} = 0^{+}(0^{++})$$

See also the mini-reviews on scalar mesons under $f_0(500)$ (see the index for the page number) and on non- $q\bar{q}$ candidates in PDG 06, Journal of Physics **G33** 1 (2006).

$f_0(1370)$ T-MATRIX POLE POSITION

Note that $\Gamma \approx 2 \text{Im}(\sqrt{s_{\text{pole}}})$.

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
1200-1500			OUR ESTIMATE

$f_0(1500)$

$$J^{PC} = 0^{+}(0^{++})$$

See also the mini-reviews on scalar mesons under $f_0(500)$ (see the index for the page number) and on non- $q\bar{q}$ candidates in PDG 06, Journal of Physics **G33** 1 (2006).

$f_0(1500)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1504 ± 6				OUR AVERAGE Error includes scale factor of 1.3. See the ideogram below.

$f_0(1500)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
109 ± 7				OUR AVERAGE

$f_0(1710)$

$$J^{PC} = 0^{+}(0^{++})$$

See our mini-review in the 2004 edition of this Review, Physics Letters **B592** 1 (2004). See also the mini-review on scalar mesons under $f_0(500)$ (see the index for the page number).

$f_0(1710)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1723 ± 6				OUR AVERAGE Error includes scale factor of 1.6. See the ideogram below.

$f_0(1710)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
139 ± 8				OUR AVERAGE Error includes scale factor of 1.1.

G in the extended Linear Sigma Model (eLSM)



Hadronic model based on the symmetries of QCD: scalar glueball G built in from the very beginning (trace anomaly)

Nonet	L	S	J^{PC}	Current	Assignment
P	0	0	0^{-+}	$P_{ij} = \frac{1}{\sqrt{2}} \bar{q}_j i \gamma^5 q_i$	π, K, η, η'
S	1	1	0^{++}	$S_{ij} = \frac{1}{\sqrt{2}} \bar{q}_j q_i$	$a_0(1450), K_0^*(1430), f_0(1370), f_0(1510)$
V^μ	0	1	1^{--}	$V_{ij}^\mu = \frac{1}{\sqrt{2}} \bar{q}_j \gamma^\mu q_i$	$\rho(770), K^*(892), \omega(785), \phi(1024)$
A^μ	1	1	1^{++}	$A_{ij}^\mu = \frac{1}{\sqrt{2}} \bar{q}_j \gamma^5 \gamma^\mu q_i$	$a_1(1230), K_{1,A}, f_1(1285), f_1(1420)$

Chiral multiplet	Current	$U_R(3) \times U_L(3)$	P	C
$\Phi = S + iP$	$\sqrt{2} \bar{q}_{R,j} q_{L,i}$	$U_L \Phi U_R^\dagger$	Φ^\dagger	Φ^t
$R^\mu = V^\mu - A^\mu$	$\sqrt{2} \bar{q}_{R,j} \gamma^\mu q_{R,i}$	$U_R R^\mu U_R^\dagger$	L_μ	$L^{t\mu}$
$L^\mu = V^\mu + A^\mu$	$\sqrt{2} \bar{q}_{L,j} \gamma^\mu q_{L,i}$	$U_L R^\mu U_L^\dagger$	R_μ	$R^{t\mu}$

$$\begin{aligned}
 \mathcal{L} = & \frac{1}{2} (\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda^2} \left(G^4 \ln \left| \frac{G}{\Lambda} \right| - \frac{G^4}{4} \right) + \text{Tr} [(D^\mu \Phi)^\dagger (D_\mu \Phi)] \\
 & - m_0^2 \left(\frac{G}{G_0} \right)^2 \text{Tr} [\Phi^\dagger \Phi] - \lambda_1 (\text{Tr} [\Phi^\dagger \Phi])^2 - \lambda_2 \text{Tr} [(\Phi^\dagger \Phi)^2] \\
 & + \left(\frac{G}{G_0} \right)^2 \text{Tr} \left[\left(\frac{m_1^2}{2} + \Delta \right) ((L^\mu)^2 + (R^\mu)^2) \right] \\
 & - \frac{1}{4} \text{Tr} [(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \text{Tr} [H (\Phi^\dagger + \Phi)] \\
 & + c_1 [\det(\Phi) - \det(\Phi^\dagger)]^2 + \frac{h_1}{2} \text{Tr} [\Phi^\dagger \Phi] \text{Tr} [L_\mu L^\mu + R_\mu R^\mu] \\
 & + h_2 \text{Tr} [\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] + 2h_3 \text{Tr} [\Phi R_\mu \Phi^\dagger L^\mu]
 \end{aligned}$$

D. Parganlija, P. Kovacs, G. Wolf, F. G., D. H. Rischke,
 Phys. Rev. D 87 (2013) no.1, 014011
 [arXiv:1208.0585 [hep-ph]].

eLSM: fit (11 parameters, 21 exp. quantities)

Error from PDG or 5% of exp.
Scalar-isoscalar sector not
included.

$$\chi_{red}^2 = 1.2$$

Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
m_{K^*}	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^*}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho \rightarrow \pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^* \rightarrow K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi \rightarrow \bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1 \rightarrow \rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1 \rightarrow \pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420) \rightarrow K^*K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^* \rightarrow K\pi}$	285 ± 12	270 ± 80

arXiv:1208.0585

Scalar glueball in the eLSM



PHYSICAL REVIEW D **90**, 114005 (2014)

Is $f_0(1710)$ a glueball?

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(Received 26 August 2014; published 2 December 2014)

We study the three-flavor chirally and dilatation invariant extended linear sigma model with (pseudo) scalar and (axial-)vector mesons as well as a scalar dilaton field whose excitations are interpreted as a glueball. The model successfully describes masses and decay widths of quark-antiquark mesons in the low-energy region up to 1.6 GeV. Here we study in detail the vacuum properties of the scalar-isoscalar $J^{PC} = 0^{++}$ channel and find that (i) a narrow glueball is only possible if the vacuum expectation value of the dilaton field is (at tree level) quite large (i.e. larger than what lattice QCD and QCD sum rules suggest) and (ii) only solutions in which $f_0(1710)$ is predominantly a glueball are found. Moreover, the resonance $f_0(1370)$ turns out to be mainly $(\bar{u}u + \bar{d}d)/\sqrt{2}$ and thus corresponds to the chiral partner of the pion, while the resonance $f_0(1500)$ is mainly $\bar{s}s$.

DOI: [10.1103/PhysRevD.90.114005](https://doi.org/10.1103/PhysRevD.90.114005)

PACS numbers: 12.39.Fe, 12.39.Mk, 12.40.Yx, 13.25.Jx

Unique assignment is found:

$$\begin{pmatrix} \mathbf{f}_0(1370) \\ \mathbf{f}_0(1500) \\ \mathbf{f}_0(1710) \end{pmatrix} = \begin{pmatrix} \mathbf{0.91} & \mathbf{-0.24} & \mathbf{0.33} \\ \mathbf{0.30} & \mathbf{0.94} & \mathbf{-0.17} \\ \mathbf{-0.27} & \mathbf{0.26} & \mathbf{0.93} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ \bar{s}s \\ \mathbf{Glueball: gg} \end{pmatrix}$$

Clear outcome: $f_0(1710)$ is predominantly gluonic

Lattice result on J/Psi decay into glueball



PRL **110**, 021601 (2013)

PHYSICAL REVIEW LETTERS

week ending
11 JANUARY 2013

Scalar Glueball in Radiative J/ψ Decay on the Lattice

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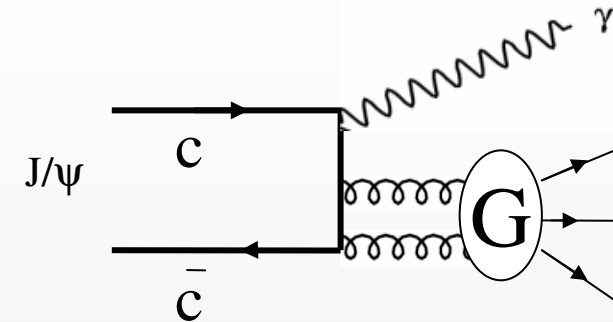
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(Received 5 June 2012; published 10 January 2013)

The form factors in the radiative decay of J/ψ to a scalar glueball are studied within quenched lattice QCD on anisotropic lattices. The continuum extrapolation is carried out by using two different lattice spacings. With the results of these form factors, the partial width of J/ψ radiatively decaying into the pure gauge scalar glueball is predicted to be 0.35(8) keV, which corresponds to a branching ratio of $3.8(9) \times 10^{-3}$. By comparing with experiments, our results indicate that $f_0(1710)$ has a larger overlap with the pure gauge glueball than other related scalar mesons.



From the PDG (decay of the j/ψ): the radiative decays into $f_0(1710)$ are larger than into $f_0(1500)$.

$$\gamma f_0(1710) \rightarrow \gamma K \bar{K} \quad (8.5 \quad {}^{+1.2}_{-0.9}) \times 10^{-4}$$

$$\gamma f_0(1710) \rightarrow \gamma \pi \pi \quad (4.0 \quad \pm 1.0) \times 10^{-4}$$

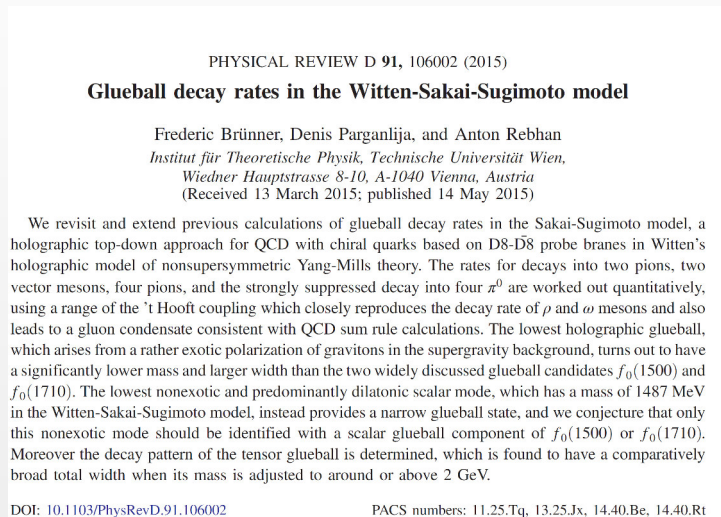
$$\gamma f_0(1710) \rightarrow \gamma \omega \omega \quad (3.1 \quad \pm 1.0) \times 10^{-4}$$

$$\gamma f_0(1710) \rightarrow \gamma \eta \eta \quad (2.4 \quad {}^{+1.2}_{-0.7}) \times 10^{-4}$$

$$\gamma f_0(1500) \rightarrow \gamma \pi \pi \quad (1.01 \quad \pm 0.32) \times 10^{-4}$$

$$\gamma f_0(1500) \rightarrow \gamma \eta \eta \quad (1.7 \quad {}^{+0.6}_{-1.4}) \times 10^{-5}$$

Decays of the scalar glueball in holography



$f_0(1710)$ fits well into the picture.

Total width calculated to be about 100 MeV.

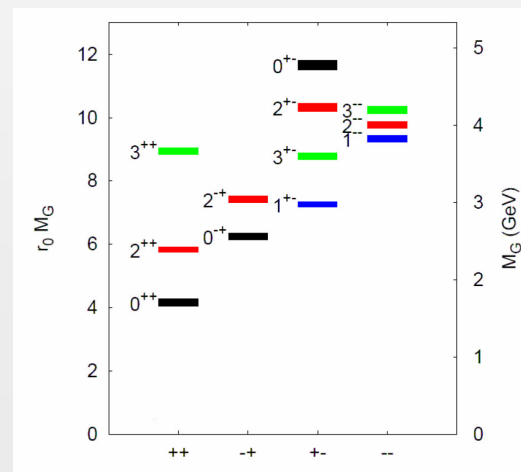
Alternative direction to study glueballs.

Summary: scalar glueball - where do we stay?



- Various scenarios exist
- Lattice's help concerning the width of the scalar glueball is expected.
- Decays into gamma-gamma of $f_0(1500)$ and $f_0(1710)$ (both not seen yet). Clarification of $f_0(1370) \rightarrow \gamma\gamma$ and $a_0(1450) \rightarrow \gamma\gamma$ (seen but no branching ration yet).
- In general, an improvement of the experimental knowledge (such as $a_0(1450) \rightarrow \omega\pi\pi$) is welcome

Tensor glueball



Tensor glueball



Candidate: $f_J(2220)$

It is not on the Regge trajectories

V. V. Anisovich, JETP Lett. 80, 715 (2004) [Pisma Zh. Eksp. Teor. Fiz. 80, 845 (2004)] [arXiv:hep-ph/0412093];
V. V. Anisovich, M. A. Matveev, J. Nyiri and A. V. Sarantsev, arXiv:hep-ph/0506133.

No decay into photon-photon

Two-pion/two-kaon ratio in agreement with flavor blindness.

Details and further refs. in:

C. Amsler and N. A. Tornqvist, Phys. Rept. 389, 61 (2004).

F. G. T. Gutsche, V. E. Lyubovitskij and A. Faessler,
"Decays of tensor mesons and the tensor glueball in an effective field approach,"
Phys. Rev. D 72 (2005) 114021 [hep-ph/0511171].

$$\mathcal{L}_{\text{eff}}^G = c_{GPP} G_{\mu\nu} \langle \Theta_P^{\mu\nu} \rangle + c_{GVV} G_{\mu\nu} \langle \mathcal{V}^\mu \mathcal{V}^\nu \rangle$$

$$\pi\pi : \bar{K}K : \eta\eta : \eta'\eta' = 1 : 0.79 : 0.17 : 0 : 0.001$$

$$\rho\rho : \bar{K}^*K^* : \omega\omega : \omega\phi : \phi\phi = 1 : 0.84 : 0.32 : 0 : 0.11$$

Tensor glueball



$f_J(2220)$

$$I^G(J^{PC}) = 0^+(2^{++} \text{ or } 4^{++})$$

OMITTED FROM SUMMARY TABLE

Needs confirmation. See our mini-review in the 2004 edition of this Review, PDG 04.

$f_J(2220)$ WIDTH

VALUE (MeV)	CL%	EVTS	DOCUMENT ID	TECN
23_{-}^{+}	$\frac{8}{7}$	OUR AVERAGE		

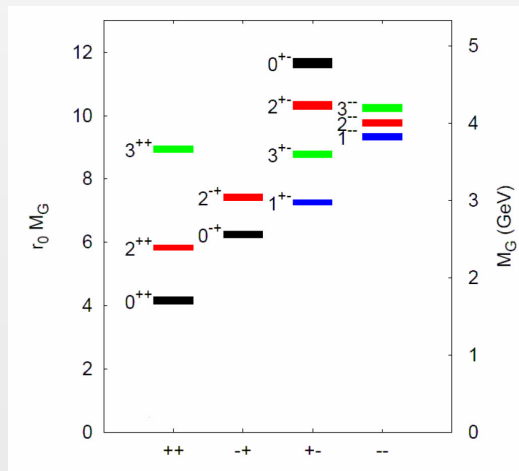
$f_J(2220)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
2231.1 ± 3.5	OUR AVERAGE			

$f_J(2220)$ DECAY MODES

Mode	Fraction (Γ_i/Γ)
$\Gamma_1 \quad \pi\pi$	seen
$\Gamma_2 \quad \pi^+\pi^-$	seen
$\Gamma_3 \quad K\bar{K}$	seen
$\Gamma_4 \quad p\bar{p}$	
$\Gamma_5 \quad \gamma\gamma$	not seen
$\Gamma_6 \quad \eta\eta'(958)$	seen
$\Gamma_7 \quad \phi\phi$	not seen
$\Gamma_8 \quad \eta\eta$	not seen

Pseudoscalar glueball



Pseudoscalar glueball

INSTITUTE OF PHYSICS PUBLISHING

JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **32** (2006) R293–R335

doi:10.1088/0954-3899/32/9/R01

TOPICAL REVIEW

The case of the pseudoscalar glueball

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Published 10 August 2006

Online at stacks.iop.org/JPhysG/32/R293

Abstract

Glueballs represent a key requirement of quantum chromodynamics as a non-Abelian field theory. Their search provides one of the strongest motivations for meson spectroscopy. The first glueball candidate was identified in 1980 in the J/Ψ radiative decay. Its discovery actually dates back to 1963 and for four decades about 30 experiments, using six different production mechanisms, were dedicated to studying the pseudoscalar states lying in the 1.4–1.5 GeV/ c^2 mass region. Today, the presence of two pseudoscalar states and an axial vector can be considered as established (see 2004 edition of the *Review of Particle Properties*). Assuming that $\eta(1295)$ is established and the nonet filled, the lower mass pseudoscalar, $\eta(1405)$, becomes a supernumerary and shows the properties of a non- $\bar{q}q$ state. Here, we review the experimental effort related to this long search, which can be considered a sort of paradigm for light-quark spectroscopy.



Pseudoscalar glueball



The bare states

$$\sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$$

$$\bar{s}s$$

Glueball: gg

mix and form the 3 resonances.
Usually, in such studies the glueball turns out to sit mostly in the eta(1405) state.

Conflict with Lattice!

$\eta(1295)$	$I^G(J^{PC}) = 0^+(0^{-+})$
See also the mini-review under $\eta(1405)$	
$\eta(1295)$ MASS	
<small>VALUE (MeV)</small>	<small>EVTS</small> <small>DOCUMENT ID</small> <small>TECN</small> <small>COMMENT</small>
1294 ± 4 OUR AVERAGE	Error includes scale factor of 1.6. See the ideogram below.
$\eta(1405)$	$I^G(J^{PC}) = 0^+(0^{-+})$
A REVIEW GOES HERE – Check our WWW List of Reviews	
$\eta(1405)$ MASS	
<small>VALUE (MeV)</small>	<small>DOCUMENT ID</small>
1408.8 ± 1.8 OUR AVERAGE	Includes data from the 2 datablocks that follow this one. Error includes scale factor of 2.1. See the ideogram below.
$\eta(1475)$	$I^G(J^{PC}) = 0^+(0^{-+})$
See also the $\eta(1405)$.	
$\eta(1475)$ MASS	
$K\bar{K}\pi$ MODE ($K^*(892)$ K dominant)	
<small>VALUE (MeV)</small>	<small>EVTS</small> <small>DOCUMENT ID</small> <small>TECN</small> <small>COMMENT</small>
1476 ± 4 OUR AVERAGE	Error includes scale factor of 1.3. See the ideogram below.

The pseudoscalar glueball: predictions from the eLSM

$$\mathcal{L}_{\tilde{G}\text{-mesons}}^{int} = ic_{\tilde{G}\Phi} \tilde{G} \left(\det\Phi - \det\Phi^\dagger \right)$$

Quantity	Value
$\Gamma_{\tilde{G} \rightarrow KK\eta} / \Gamma_{\tilde{G}}^{tot}$	0.049
$\Gamma_{\tilde{G} \rightarrow KK\eta'} / \Gamma_{\tilde{G}}^{tot}$	0.019
$\Gamma_{\tilde{G} \rightarrow \eta\eta\eta} / \Gamma_{\tilde{G}}^{tot}$	0.016
$\Gamma_{\tilde{G} \rightarrow \eta\eta\eta'} / \Gamma_{\tilde{G}}^{tot}$	0.0017
$\Gamma_{\tilde{G} \rightarrow \eta\eta'\eta'} / \Gamma_{\tilde{G}}^{tot}$	0.00013
$\Gamma_{\tilde{G} \rightarrow KK\pi} / \Gamma_{\tilde{G}}^{tot}$	0.46
$\Gamma_{\tilde{G} \rightarrow \eta\pi\pi} / \Gamma_{\tilde{G}}^{tot}$	0.16
$\Gamma_{\tilde{G} \rightarrow \eta'\pi\pi} / \Gamma_{\tilde{G}}^{tot}$	0.094



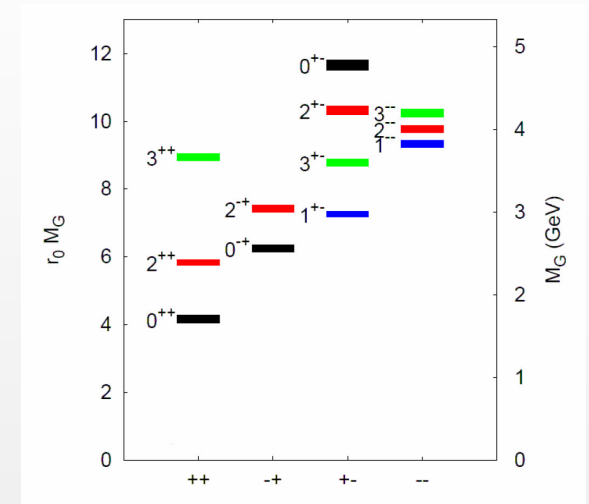
$$\Gamma_{\tilde{G} \rightarrow \pi\pi\pi} = 0$$

Future experimental search, e.g. at BES and PANDA

Details in:

W. Eshraim, S. Janowski, F.G., D. Rischke, **Phys.Rev. D87 (2013) 054036**. [arxiv: 1208.6474](https://arxiv.org/abs/1208.6474) .

$M_G = 2.6$ GeV from lattice as been used as an input.



X(2370) found at BESIII is a possible candidate.

The pseudoscalar glueball from the eLSM/2



Coupling to baryons

$$\frac{\Gamma_{\tilde{G} \rightarrow NN}}{\Gamma_{\tilde{G} \rightarrow N^* N + \text{h.c.}}} \simeq 1.96$$

$$p + \bar{p} \rightarrow p + \bar{p}(1535) + \text{h.c.}$$

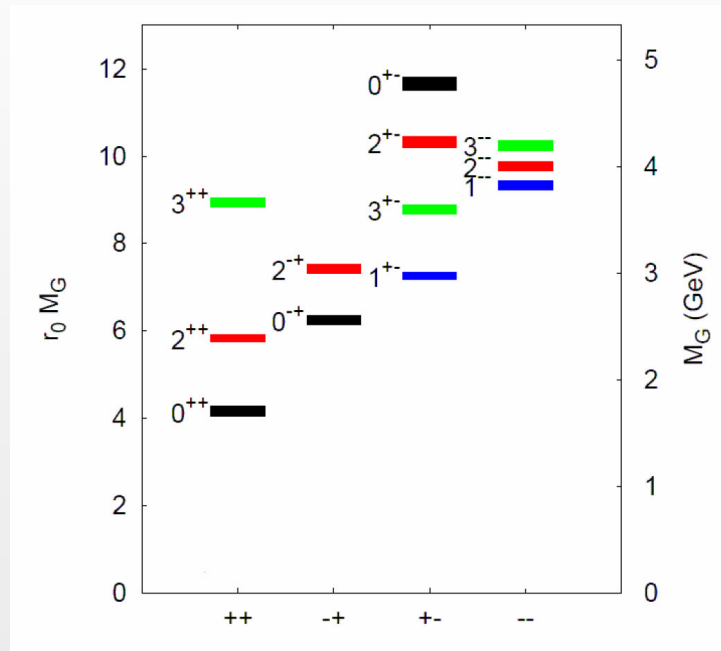
W. Eschraim, S. Janowski, K. Neuschwander, A. Peters, F.G., **Acta Phys. Pol. B**, Proc. Suppl. 5/4, **arxiv: 1209.3976**

Heavier glueballs: vector and pseudotensor



vector, pseudotensor,... glueballs.

Oddballs.



My personal plan: make predictions for decays of various glueballs (9 to do)

Vector glueball

From arXiv:1607.03640 [hep-ph], to appear in PRD,

Decays of the vector glueball by F.G, J. Sammet, S. Janowski

We predict that the vector glueball decays mostly into:

$$\mathcal{O} \rightarrow b_1 \pi \rightarrow \omega \pi \pi$$

$$\mathcal{O} \rightarrow \omega \pi \pi$$

$$\mathcal{O} \rightarrow \pi K K^*(892)$$

The lattice mass of 3.8 GeV has been used; Tables of ratios are in the preprint.

Fields: new entry



Nonet	L	S	J^{PC}	Current	Assignment
P	0	0	0^{-+}	$P_{ij} = \frac{1}{\sqrt{2}} \bar{q}_j i \gamma^5 q_i$	π, K, η, η'
S	1	1	0^{++}	$S_{ij} = \frac{1}{\sqrt{2}} \bar{q}_j q_i$	$a_0(1450), K_0^*(1430), f_0(1370), f_0(1510)$
V^μ	0	1	1^{--}	$V_{ij}^\mu = \frac{1}{\sqrt{2}} \bar{q}_j \gamma^\mu q_i$	$\rho(770), K^*(892), \omega(785), \phi(1024)$
A^μ	1	1	1^{++}	$A_{ij}^\mu = \frac{1}{\sqrt{2}} \bar{q}_j \gamma^5 \gamma^\mu q_i$	$a_1(1230), K_{1,A}, f_1(1285), f_1(1420)$
B^μ	1	0	1^{+-}	$B_{ij}^\mu = \frac{1}{\sqrt{2}} \bar{q}_j \gamma^5 \overleftrightarrow{\partial}^\mu q_i$	$b_1(1230), K_{1,B}, h_1(1170), h_1(1380)$
E_{ang}^μ	2	1	1^{--}	$E_{\text{ang},ij}^\mu = \frac{1}{\sqrt{2}} \bar{q}_j i \overleftrightarrow{\partial}^\mu q_i$	$\rho(1700), K^*(1680), \omega(1650), \phi(???)$



Chiral multiplet	Current	$U_R(3) \times U_L(3)$	P	C
$\Phi = S + iP$	$\sqrt{2} \bar{q}_{R,j} q_{L,i}$	$U_L \Phi U_R^\dagger$	Φ^\dagger	Φ^t
$R^\mu = V^\mu - A^\mu$	$\sqrt{2} \bar{q}_{R,j} \gamma^\mu q_{R,i}$	$U_R R^\mu U_R^\dagger$	L_μ	$L^{t\mu}$
$L^\mu = V^\mu + A^\mu$	$\sqrt{2} \bar{q}_{L,j} \gamma^\mu q_{L,i}$	$U_L R^\mu U_L^\dagger$	R_μ	$R^{t\mu}$
$\tilde{\Phi}^\mu = E_{\text{ang}}^\mu - iB^\mu$	$\sqrt{2} \bar{q}_{R,j} i \overleftrightarrow{\partial}^\mu q_{L,i}$	$U_L \tilde{\Phi}^\mu U_R^\dagger$	$\tilde{\Phi}^{\dagger\mu}$	$-\tilde{\Phi}^{t\mu}$

ArXiv: 1607.03640

Vector glueball into pseudoscalar-pseudovector

$$\mathcal{L}_1 = \lambda_{\mathcal{O},1} G \mathcal{O}_\mu \text{Tr} [\Phi^\dagger \tilde{\Phi}^\mu + \tilde{\Phi}^{\mu\dagger} \Phi]$$

Quantity	Value
$\frac{\mathcal{O} \rightarrow \eta h_1(1170)}{\mathcal{O} \rightarrow b_1 \pi}$	0.17
$\frac{\mathcal{O} \rightarrow \eta h_1(1380)}{\mathcal{O} \rightarrow b_1 \pi}$	0.11
$\frac{\mathcal{O} \rightarrow \eta' h_1(1170)}{\mathcal{O} \rightarrow b_1 \pi}$	0.15
$\frac{\mathcal{O} \rightarrow \eta' h_1(1380)}{\mathcal{O} \rightarrow b_1 \pi}$	0.10
$\frac{\mathcal{O} \rightarrow K K_1(1270)}{\mathcal{O} \rightarrow b_1 \pi}$	0.75
$\frac{\mathcal{O} \rightarrow K K_1(1400)}{\mathcal{O} \rightarrow b_1 \pi}$	0.30
$\frac{\mathcal{O} \rightarrow K_0^*(1430) K^*(1680)}{\mathcal{O} \rightarrow b_1 \pi}$	0.20
$\frac{\mathcal{O} \rightarrow a_0(1450) \rho(1700)}{\mathcal{O} \rightarrow b_1 \pi}$	0.14
$\frac{\mathcal{O} \rightarrow f_0(1370) \omega(1650)}{\mathcal{O} \rightarrow b_1 \pi}$	0.034

ArXiv: 1607.03640

$$\mathcal{O} \rightarrow b_1 \pi \rightarrow \omega \pi \pi$$

Francesco Giacosa

Vector glueball into VPP

$$\mathcal{L}_2 = \lambda_{\mathcal{O},2} \mathcal{O}_\mu \text{Tr} [L^\mu \Phi \Phi^\dagger + R^\mu \Phi^\dagger \Phi]$$

Quantity	Value
$\frac{\mathcal{O} \rightarrow KK\rho}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.50
$\frac{\mathcal{O} \rightarrow KK\omega}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.17
$\frac{\mathcal{O} \rightarrow KK\phi}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.21
$\frac{\mathcal{O} \rightarrow \pi K K^* (892)}{\mathcal{O} \rightarrow \omega\pi\pi}$	1.2
$\frac{\mathcal{O} \rightarrow \eta\eta\omega}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.064
$\frac{\mathcal{O} \rightarrow \eta\eta'\omega}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.019
$\frac{\mathcal{O} \rightarrow \eta'\eta'\omega}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.019
$\frac{\mathcal{O} \rightarrow \eta\eta\phi}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.039
$\frac{\mathcal{O} \rightarrow \eta\eta'\phi}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.011
$\frac{\mathcal{O} \rightarrow \eta'\eta'\phi}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.011
$\frac{\mathcal{O} \rightarrow a_0(1450)a_0(1450)\omega}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.00029

Quantity	Value
$\frac{\mathcal{O} \rightarrow a_0(1450)\rho}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.47
$\frac{\mathcal{O} \rightarrow f_0(1370)\omega}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.15
$\frac{\mathcal{O} \rightarrow K_0^*(1430)K^*(892)}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.30
$\frac{\mathcal{O} \rightarrow KK}{\mathcal{O} \rightarrow \omega\pi\pi}$	0.018

ArXiv: 1607.03640

$$\mathcal{O} \rightarrow \omega\pi\pi \quad \mathcal{O} \rightarrow \pi K K^* (892)$$

Vector glueball into VP

$$\mathcal{L}_3 = \alpha \epsilon_{\mu\nu\rho\sigma} \partial^\rho \mathcal{O}^\sigma \text{Tr} [L^\mu \Phi R^\nu \Phi^\dagger]$$

Quantity	Value
$\frac{\mathcal{O} \rightarrow KK^*(892)}{\mathcal{O} \rightarrow \rho\pi}$	1.3
$\frac{\mathcal{O} \rightarrow \eta\omega}{\mathcal{O} \rightarrow \rho\pi}$	0.16
$\frac{\mathcal{O} \rightarrow \eta'\omega}{\mathcal{O} \rightarrow \rho\pi}$	0.13
$\frac{\mathcal{O} \rightarrow \eta\phi}{\mathcal{O} \rightarrow \rho\pi}$	0.21
$\frac{\mathcal{O} \rightarrow \eta'\phi}{\mathcal{O} \rightarrow \rho\pi}$	0.18
$\frac{\mathcal{O} \rightarrow \rho a_1(1230)}{\mathcal{O} \rightarrow \rho\pi}$	1.8
$\frac{\mathcal{O} \rightarrow \omega f_1(1285)}{\mathcal{O} \rightarrow \rho\pi}$	0.55
$\frac{\mathcal{O} \rightarrow \omega f_1(1420)}{\mathcal{O} \rightarrow \rho\pi}$	0.82

ArXiv: 1607.03640

dominant decay modes  $\rho\pi$, $KK^*(892)$, and $\rho a_1(1230)$

Pseudotensor glueball



Eur. Phys. J. A (2016) 52: 356
DOI 10.1140/epja/i2016-16356-x

THE EUROPEAN
PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Phenomenology of pseudotensor mesons and the pseudotensor glueball

“ $G_2(3040)$ ”	
Branching ratio	Theory
$\Gamma^{\text{th}}(a_2(1320) \pi) / \Gamma^{\text{th}}(K_2^*(1430) K + c.c.)$	0.9
$\Gamma^{\text{th}}(a_2(1320) \pi) / \Gamma^{\text{th}}(f_2(1270) \eta)$	6.0
$\Gamma^{\text{th}}(a_2(1320) \pi) / \Gamma^{\text{th}}(f_2(1270) \eta'(958))$	8.5
$\Gamma^{\text{th}}(a_2(1320) \pi) / \Gamma^{\text{th}}(f_2'(1525) \eta)$	9.0
$\Gamma^{\text{th}}(a_2(1320) \pi) / \Gamma^{\text{th}}(f_2'(1525) \eta'(958))$	11.0

Table 8: Theoretical branching ratios for the pseudotensor glueball “ $G_2(3040)$ ”.

ArXiv: 1608.8777

Still hypothetical states. Need of experiment (PANDA in first place).

Pseudotensor meson: surprising large mixing?



Abstract. We study the decays of the pseudotensor mesons ($\pi_2(1670)$, $K_2(1770)$, $\eta_2(1645)$, $\eta_2(1870)$) interpreted as the ground-state nonet of 1^1D_2 $\bar{q}q$ states using interaction Lagrangians which couple them to pseudoscalar, vector, and tensor mesons. While the decays of $\pi_2(1670)$ and $K_2(1770)$ can be well described, the decays of the isoscalar states $\eta_2(1645)$ and $\eta_2(1870)$ can be brought in agreement with the present experimental data only if the mixing angle between nonstrange and strange states is surprisingly large (about -42° , similar to the mixing in the pseudoscalar sector, in which the chiral anomaly is active). Such a large mixing angle is however at odd with all other conventional quark-antiquark nonets:

ArXiv: 1608.8777

$$\begin{pmatrix} \eta_2(1645) \\ \eta_2(1870) \end{pmatrix} = \begin{pmatrix} \cos \theta_{PT} & \sin \theta_{PT} \\ -\sin \theta_{PT} & \cos \theta_{PT} \end{pmatrix} \begin{pmatrix} \eta_{2,N} = \frac{\bar{u}u + \bar{d}d}{\sqrt{2}} \\ \eta_{2,S} = \bar{s}s \end{pmatrix} \quad \theta_{PT} \simeq -42^\circ$$

Concluding remarks



- We did not find glueballs yet.
- We have some good candidates for the scalar sector and less good candidates for the tensor and pseudoscalar sector.
- Vector and pseudotensor glueballs: predictions
- Are (at least some) glueballs narrow enough? This is a crucial question. Large- N_c tells so. Help from Lattice would be very welcome!
- Other glueballs can be studied; oddballs and glueballs with $J=3$ are on my to-do list
- Ongoing and future experiments:
BESIII
COMPASS@CERN - LHCb@CERN
CLAS12@JLAB, GLUEX@JLAB
BELLE2
RHIC/STAR
PANDA@FAIR is very well suited for the search of glueballs.

Concluding remarks



The search for glueballs is an active field.
Expected other 1000 publications in the next 20 years.
Hopefully with conclusive results!

Thank You

Back-up slides

Trace anomaly: the emergence of a dimension



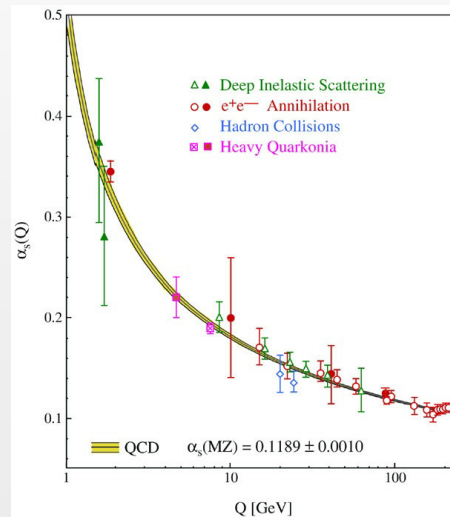
Chiral limit: $m_i = 0$

$$x^\mu \rightarrow x'^\mu = \lambda^{-1} x^\mu$$

is a classical symmetry, which is how broken by quantum fluctuations (trace anomaly)

Dimensional transmutation $\Lambda_{YM} \approx 250 \text{ MeV}$

$$\alpha_s(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



In Yang-Mills (QCD without quarks) it is:

Gluon condensate: $\langle G_{\mu\nu}^a G^{a,\mu\nu} \rangle \neq 0$

$$\partial_\mu J^\mu = T_\mu^\mu = \frac{\beta(g)}{4g} G_{\mu\nu}^a G^{a,\mu\nu} \neq 0$$

The trace anomaly: addendum

Emergence of a low-energy scale: $\Lambda_{\text{YM}} \approx 250 \text{ MeV}$

Gluon condensate: $\langle G_{\mu\nu}^a G^{a,\mu\nu} \rangle \neq 0$

„Effective“ gluon „mass“ (better: ‚dressing‘ of the propagator)

Analytic Structure of the Landau-Gauge Gluon Propagator
Stefan Strauss, Christian S. Fischer, and Christian Kellermann
Phys. Rev. Lett. **109**, 252001 – Published 19 December 2012

„Moreover, the gluon is certainly not a massive particle in the usual sense. Within the present accuracy we find $600 \text{ MeV} < m_g < 700 \text{ MeV}$.“

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}$$

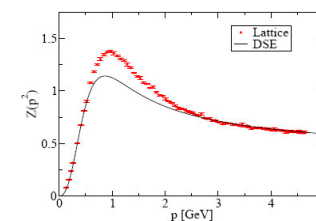
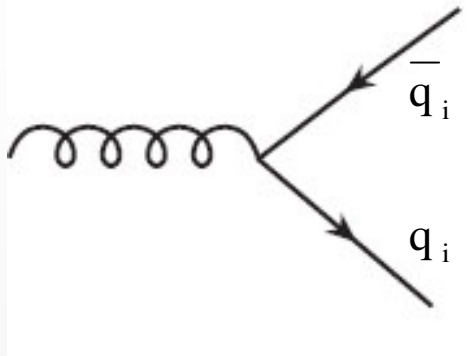


FIG. 2: Results for the gluon dressing function $Z(p^2)$ from lattice calculations [12] compared to the result from DSEs [9].

Flavor symmetry

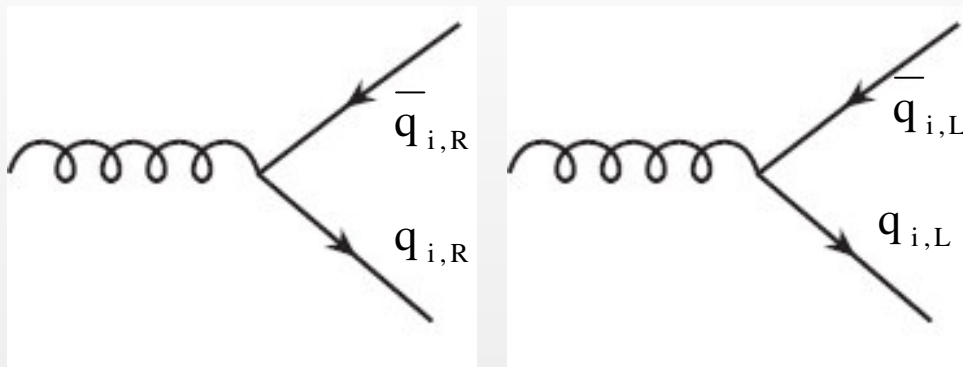
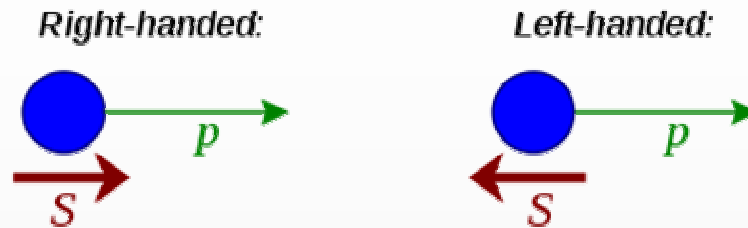


Gluon-quark-antiquark vertex.

The gluon is democratic! It couples to each flavor with the same strength.

$$q_i \rightarrow U_{ij} q_j \quad U \in U(3)_V \rightarrow U^\dagger U = 1$$

Chiral symmetry



$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1 + \gamma^5) q_i$$

$$q_{i,L} = \frac{1}{2}(1 - \gamma^5) q_i$$

$$q_i = q_{i,R} + q_{i,L} \rightarrow U_{ij}^R q_{j,R} + U_{ij}^L q_{j,L}$$

Even more: the gluon couples with the same strength to quarks with different chiralities.

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

In the chiral limit ($m_i=0$) chiral symmetry is exact.

Spontaneous breaking of chiral symmetry

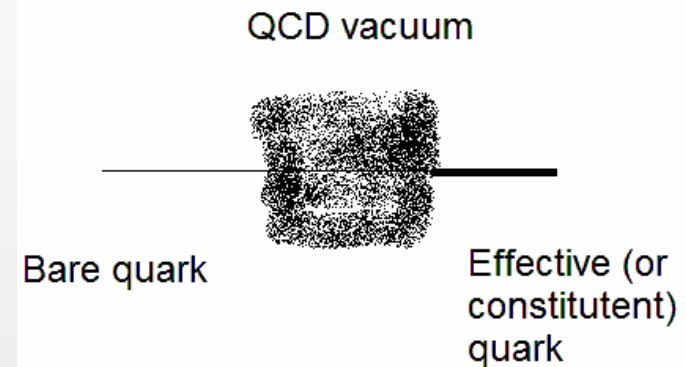
$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

$$\text{SSB: } SU(3)_R \times SU(3)_L \rightarrow SU(3)_{V=R+L}$$

Chiral symmetry \rightarrow Flavor symmetry

$$\langle \bar{q}_i q_i \rangle = \langle \bar{q}_{i,R} q_{i,L} + \bar{q}_{i,L} q_{i,R} \rangle \neq 0$$

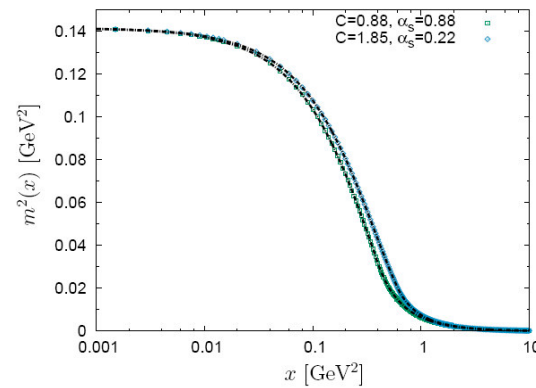
$$m \approx m_u \approx m_d \approx 5 \text{ MeV} \rightarrow m^* \approx 300 \text{ MeV}$$



$$m_{\rho\text{-meson}} \approx 2m^*$$

$$m_{\text{proton}} \approx 3m^*$$

See also:
All-order equation of the effective gluon mass
D. Binosi, D. Ibañez, and J. Papavassiliou
Phys. Rev. D **86**, 085033 (2012)



Many other works:

A Dynamical gluon mass solution in a coupled system of the Schwinger-Dyson equations

[A.C. Aguilar, A.A. Natale](#)

Published in **JHEP 0408 (2004) 057**

Off diagonal gluon mass generation and infrared Abelian dominance in the maximally Abelian gauge in lattice QCD

[Kazuhisa Amemiya, Hideo Suganuma](#)

Published in **Phys.Rev. D60 (1999) 114509**

Meson



Definition(s):

- 1) A meson is a strongly interacting particle with integer spin.
- 2) A meson is a strongly interacting particle with zero baryon number.

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Quark-antiquark states: the large- N_c limit



As Isgur-Godfrey have shown, the quark model works.
Theoretical justification relies on the large- N_c expansion.

G. 't Hooft, Nucl. Phys. B **72** (1974) 461

For comprehensive reviews on N_c :

Baryons in the $1/n$ Expansion

Edward Witten

Published in **Nucl.Phys. B160 (1979) 57**

S. R. Coleman, Lectures given at 17th International School of Subnuclear Physics: Pointlike Structures Inside and Outside Hadrons 31 Jul - 10 Aug 1979. Erice, Italy. SLAC-PUB-2484.

R. F. Lebed, Czech. J. Phys. **49** (1999) 1273 [nucl-th/9810080].

$$|\rho^+\rangle \propto |u\bar{d}\rangle + \frac{1}{N_c} (|\pi^+\pi^0\rangle + \dots)$$

where

$$|u\bar{d}\rangle = |\text{valence } u + \text{valence } \bar{d} + \text{gluons}\rangle$$

Mesons beyond q - q bar: the first term in the first expansion is of non-quarkonium type

Other approaches/1



QCD Sum rules

H. G. Dosch and S. Narison, Nucl. Phys. Proc. Suppl. 121 (2003) 114 [arXiv:hep-ph/0208271].

H. Forkel, Phys. Rev. D 71, 054008 (2005) [arXiv:hep-ph/0312049].

$$M_{0++} \simeq 1.3 \text{ GeV}$$

$$M_{2++} \simeq 2.0 \text{ GeV}$$

$$M_{0-+} \simeq 2.2 \text{ GeV}$$

Hamiltonian QCD

A.P. Szczepaniak, E.S. Swanson, Phys. Lett. B577 (2003) 61. hep-ph/0308268.

$$M_{0++} \simeq 1.9 \text{ GeV}$$

$$M_{2++} \simeq 2.4 \text{ GeV}$$

$$M_{0-+} \simeq 2.2 \text{ GeV}$$

Masses: summary



Agreement of different methods.

Lightest state: a scalar glueball between 1- 2 GeV

Next: tensor and pseudoscalar glueballs.

Glueballs with exotic quantum numbers (oddballs) are also predicted to exist.

Ellis-Lanik warning (1984)



IS SCALAR GLUONIUM OBSERVABLE?

John ELLIS
CERN, Geneva, Switzerland

and

Jozef LÁNIK
JINR, Dubna, USSR

Received 26 October 1984

We discuss couplings of scalar gluonium σ on the basis of the low energy theorems of broken chiral symmetry and the anomalous trace of the energy-momentum tensor, implemented using a phenomenological lagrangian. Taking the ITEP value of the gluon condensate as input, we find $\Gamma(\sigma \rightarrow \pi\pi) \simeq 0.6 \text{ GeV} \times (m_\sigma/1 \text{ GeV})^5$ and $\Gamma(\sigma \rightarrow \gamma\gamma) \simeq 90 \text{ eV} \times (m_\sigma/1 \text{ GeV})^5$, while m_σ is undetermined. These results suggest that if the scalar gluonium mass is above 1 GeV, it is probably unobservably wide, while production in $\gamma\gamma$ collisions is probably too small to be detectable if $m_\sigma < 1.5 \text{ GeV}$. We comment on the observability of $J/\psi \rightarrow \sigma + \gamma$ and on the relevance of our results to other gluonia.

Physics Letters 150 B, 1984

Dilaton Lagrangian which mimics the trace anomaly: very large glueball is found.
Decay into pion reads:

$$\Gamma = 0.6(M_G/1 \text{ GeV})^5 \text{ GeV}$$

For a glueball of about 1.5 GeV in mass, one gets a width of about 4.5 GeV!

Disagreement with the large- N_c expectation

M. Migdal and Shifman, Phys. Lett. 114B, 445 (1982)
J. Schechter et al, Phys. Rev. D 24, 2545 (1981)
Francesco Giacosa

Other approaches/2



- Flux-tube models

N. Isgur, J.E. Paton,
Phys. Lett. B124 (1983) 247

$$M_{0^{++}} \simeq 1.5 \text{ GeV}$$

$$M_{2^{++}} \simeq 2.8 \text{ GeV}$$

$$M_{0^{-+}} \simeq 2.8 \text{ GeV}$$

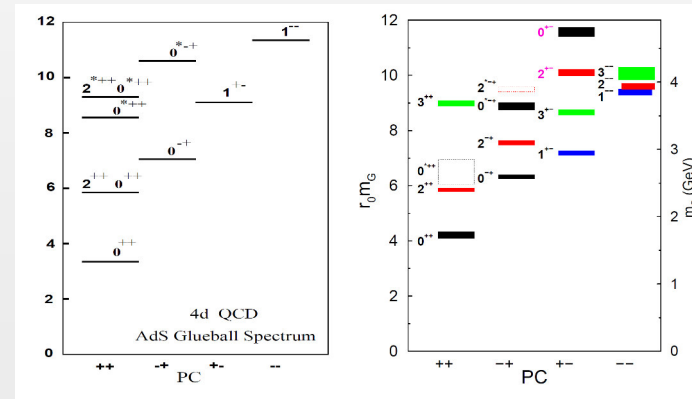
- Bethe-Salpeter approach

Sanchis-Alepuz et al,
Phys. Rev. D 92, 034001 (2015)

$$M_{0^{++}} \simeq 1.6 \text{ GeV}$$

$$M_{0^{-+}} \simeq 4.5 \text{ GeV}$$

- ADS/QCD



R. C. Brower, S. D. Mathur and C. I. Tan, *Nucl. Phys. B* **587** (2000) 249 [arXiv:hep-th/0003115]

The light scalar mesons



$a_0(980)$ $k(800)$ $f_0(980)$ $f_0(500)$

$$J^{PC} = 0^{++}$$

We saw that they are not quark-antiquark states:
(in our approach this is achieved by exclusion,
but this result is common to many others in low-energy QCD)!!!

Close-Amsler (1995 and 1996)



C. Amsler and F.E. Close Phys. Lett. **B353** (1995) 385

C. Amsler and F.E. Close Phys. Rev. **D53**, 295 (1996)

Is $f_0(1500)$ a scalar glueball?

Claude Amsler*

Physik-Institut, Universität Zürich, CH-8057 Zürich, Switzerland

Frank E. Close†

Particle Theory, Rutherford-Appleton Laboratory, Chilton, Didcot OX11 0QX, United Kingdom
(Received 24 July 1995)

Following the discovery of two new scalar mesons $f_0(1370)$ and $f_0(1500)$ at the Low Energy Antiproton Ring at CERN, we argue that the observed properties of this pair are incompatible with them both being $Q\bar{Q}$ mesons. We show instead that $f_0(1500)$ is compatible with the ground state glueball expected around 1500 MeV mixed with the nearby states of the $0^{++}Q\bar{Q}$ nonet. Tests of this hypothesis include the prediction of a further scalar state $f'_0(1500-1800)$ which couples strongly to $K\bar{K}$, $\eta\eta$, and $\eta\eta'$. Signatures for a possible tensor glueball at ~ 2 GeV are also considered.

Decuplet: nonet+glueball.
3P0 model for decay.
Flavour symmetry.
Fit to data on decays.

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} -0.91 & 0.07 & 0.40 \\ -0.41 & 0.35 & -0.84 \\ 0.09 & 0.93 & 0.36 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ \bar{s}s \\ \text{Glueball: } gg \end{pmatrix}$$

Lee and Weingarten (1999)



PHYSICAL REVIEW D, VOLUME 61, 014015

Scalar quarkonium masses and mixing with the lightest scalar glueball

W. Lee* and D. Weingarten

IBM Research, P.O. Box 218, Yorktown Heights, New York 10598

(Received 18 August 1999; published 10 December 1999)

We evaluate the continuum limit of the valence (quenched) approximation to the mass of the lightest scalar quarkonium state, for a range of different quark masses, and to the mixing energy between these states and the lightest scalar glueball. Our results support the interpretation of $f_0(1710)$ as composed mainly of the lightest scalar glueball.

One of the first lattice studies.
Coupling of the glueball to
pions and kaons (via mixing)

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.82 & 0.29 & -0.49 \\ -0.40 & 0.91 & -0.13 \\ 0.41 & 0.30 & 0.85 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ \bar{s}s \\ \text{Glueball: } gg \end{pmatrix}$$

My PhD time (2005)



PHYSICAL REVIEW D **72**, 094006 (2005)

Scalar nonet quarkonia and the scalar glueball: Mixing and decays in an effective chiral approach

F. Giacosa, Th. Gutsche, V. E. Lyubovitskij, and Amand Faessler

Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany
(Received 22 September 2005; published 9 November 2005)

We study the strong and electromagnetic decay properties of scalar mesons above 1 GeV within a chiral approach. The scalar-isoscalar states are treated as mixed states of quarkonia and glueball configurations. A fit to the experimental mass and decay rates listed by the Particle Data Group is performed to extract phenomenological constraints on the nature of the scalar resonances and to the issue of the glueball decays. A comparison to other experimental results and to other theoretical approaches in the scalar meson sector is discussed.

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger + \chi_+ \rangle + \frac{1}{2} \langle D_\mu S D^\mu S - M_S^2 S^2 \rangle \\ & + \frac{1}{2} \langle \partial_\mu G \partial^\mu G - M_G^2 G^2 \rangle + c_d^s \langle S u_\mu u^\mu \rangle \\ & + c_m^s \langle S \chi_+ \rangle + \frac{c_d^s}{\sqrt{3}} G \langle u_\mu u^\mu \rangle + \frac{c_m^s}{\sqrt{3}} G \langle \chi_+ \rangle \\ & + c_e^s \langle S F_{\mu\nu}^+ F^{+\mu\nu} \rangle + \frac{c_e^s}{\sqrt{3}} G \langle F_{\mu\nu}^+ F^{+\mu\nu} \rangle + \mathcal{L}_{\text{mix}}^P \end{aligned}$$

Starting point: Lagrangian (with both derivative and non-derivative terms is in agreement with ChPt; nonlinear real. of chiral symmetry)

Flavour symmetry is crucial.

Fit to all available PDG data.

$$\begin{pmatrix} \mathbf{f}_0(1370) \\ \mathbf{f}_0(1500) \\ \mathbf{f}_0(1710) \end{pmatrix} = \begin{pmatrix} 0.86 & 0.24 & 0.45 \\ -0.45 & -0.06 & 0.89 \\ -0.24 & 0.97 & -0.06 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ \bar{s}s \\ \text{Glueball: } gg \end{pmatrix}$$

My PhD time (2005)/2



But we found also an alternative solution:

$$\begin{pmatrix} \mathbf{f}_0(1370) \\ \mathbf{f}_0(1500) \\ \mathbf{f}_0(1710) \end{pmatrix} = \begin{pmatrix} 0.81 & 0.19 & 0.54 \\ -0.49 & 0.72 & 0.49 \\ -0.30 & 0.67 & -0.68 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ \bar{s}s \\ \text{Glueball: } gg \end{pmatrix}$$

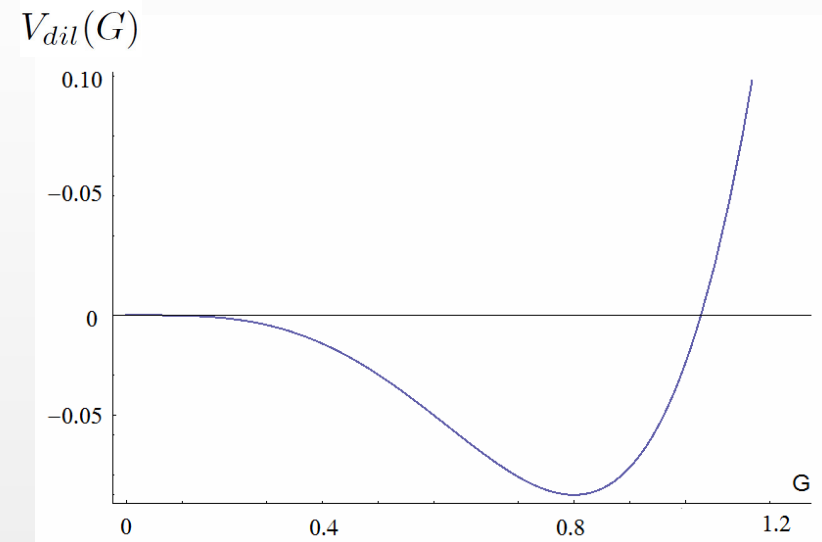
Scalar glueball: often described as a dilaton

At the hadronic level, we describe these properties as:

$$G^4 \sim G_{\mu\nu}^a G^{a,\mu\nu}$$

$$\mathcal{L}_{dil} = \frac{1}{2} (\partial_\mu G)^2 - V_{dil}(G)$$

$$V_{dil}(G) = \frac{1}{4} \frac{m_G^2}{\Lambda_G^2} \left[G^4 \ln \left(\frac{G}{\Lambda_G} \right) - \frac{G^4}{4} \right]$$



Λ_G **dimensionful param that breaks dilatation inv!**

$$\langle G \rangle = G_0 = \Lambda_G \propto \Lambda_{YM}$$

$$\partial_\mu J^\mu = T_\mu^\mu = -\frac{1}{4} \frac{m_G^2}{\Lambda_G^2} G^4$$

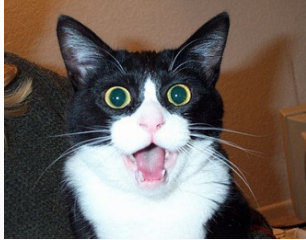
In Yang-Mills (QCD wo quarks) it is:

$$\partial_\mu J^\mu = T_\mu^\mu = \frac{\beta(g)}{4g} G_{\mu\nu}^a G^{a,\mu\nu} \neq 0$$

J. Schechter et al,
Phys. Rev. D 24, 2545 (1981)

M. Migdal and Shifman,
Phys. Lett. 114B, 445 (1982)

Model of QCD – eLSM with scalar Glueball



$$\begin{aligned}
 \mathcal{L} = & \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda^2} \left(G^4 \ln \left| \frac{G}{\Lambda} \right| - \frac{G^4}{4} \right) + \text{Tr} [(D^\mu \Phi)^\dagger (D_\mu \Phi)] \\
 & - m_0^2 \left(\frac{G}{G_0} \right)^2 \text{Tr} [\Phi^\dagger \Phi] - \lambda_1 (\text{Tr} [\Phi^\dagger \Phi])^2 - \lambda_2 \text{Tr} [(\Phi^\dagger \Phi)^2] \\
 & + \left(\frac{G}{G_0} \right)^2 \text{Tr} \left[\left(\frac{m_1^2}{2} + \Delta \right) ((L^\mu)^2 + (R^\mu)^2) \right] \\
 & - \frac{1}{4} \text{Tr} [(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \text{Tr} [H (\Phi^\dagger + \Phi)] \\
 & + c_1 [\det(\Phi) - \det(\Phi^\dagger)]^2 + \frac{h_1}{2} \text{Tr}[\Phi^\dagger \Phi] \text{Tr}[L_\mu L^\mu + R_\mu R^\mu] \\
 & + h_2 \text{Tr}[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] + 2h_3 \text{Tr}[\Phi R_\mu \Phi^\dagger L^\mu]
 \end{aligned}$$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{*+} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{*0} + iK^0 \\ K_0^{*-} + iK^- & \bar{K}_0^{*0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

$$L^\mu, R^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N \pm \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \pm a_1^0}{\sqrt{2}} & \rho^+ \pm a_1^+ & K^{*+} \pm K_1^+ \\ \rho^- \pm a_1^- & \frac{\omega_N \mp \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \mp a_1^0}{\sqrt{2}} & K^{*0} \pm K_1^0 \\ K^{*-} \pm K_1^- & \bar{K}^{*0} \pm i\bar{K}_1^0 & \omega_S \pm f_{1S} \end{pmatrix}$$

S. Janowski, D. Parganlija, F. Giacosa, D. H. Rischke, **Phys. Rev. D84, 054007 (2011)**
D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa, D. H. Rischke, **Phys.Rev. D87 (2013) 014011**

Results of the fit/1 (11 parameters, 21 exp. quantities)

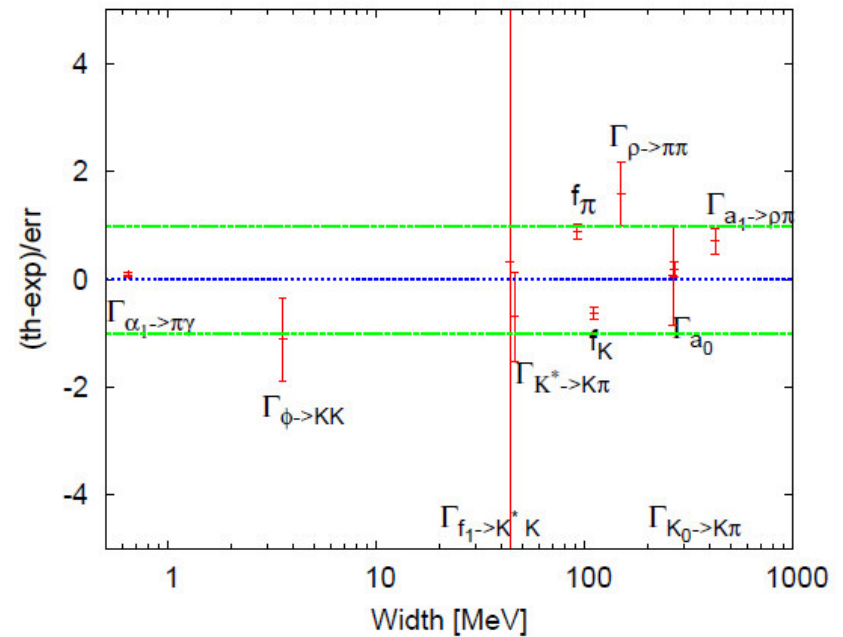
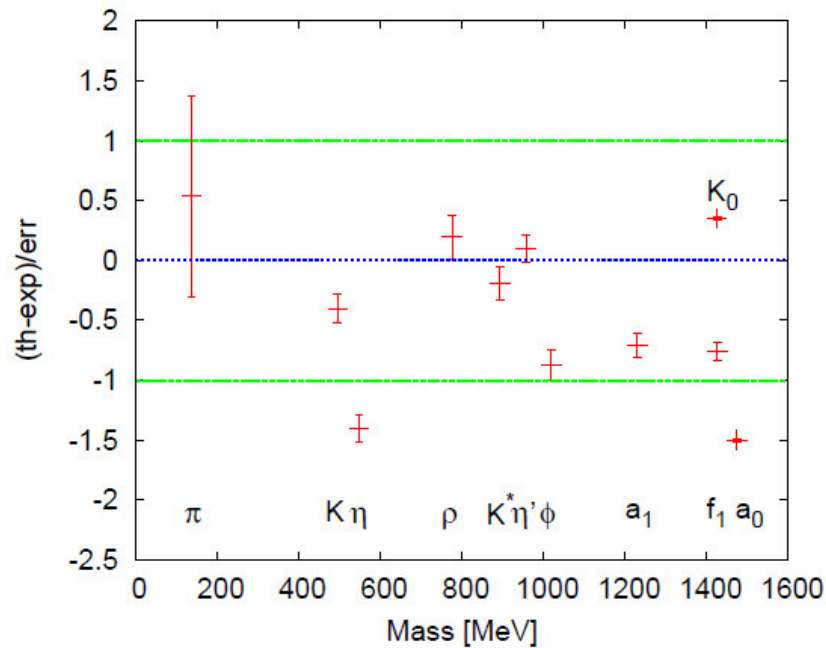
Error from PDG or 5% of exp.
Scalar-isoscalar sector not
included.

$$\chi_{red}^2 = 1.2$$

Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
m_{K^*}	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^*}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho \rightarrow \pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^* \rightarrow K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi \rightarrow \bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1 \rightarrow \rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1 \rightarrow \pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420) \rightarrow K^*K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^* \rightarrow K\pi}$	285 ± 12	270 ± 80

arXiv:1208.0585

Results of the fit/2 (11 parameters, 21 exp. quantities)



arXiv:1208.0585

Overall phenomenology is good (many more quantities can be calculated)

Scalar mesons $a_0(1450)$ and $K_0(1430)$ above 1 GeV and are quark-antiquark states.

Importance of the (axial-)vector mesons.

Large- N_c : except from G, all states scale as quark-antiquark states.

eLSM and scalar glueballs: details



$$\sigma_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2} \cong f_0(1370) \quad \sigma_S \equiv \bar{s}s \cong f_0(1500) \quad G \equiv gg \cong f_0(1710)$$

$\chi^2/\text{d.o.f.} = 0.35$

Parameter	Value
Λ_{dil}	3297 [MeV]
m_G	1525 [MeV]
λ_1	6.25
h_1	-3.22
ϵ_S	0.421×10^6 [MeV ²]

$\Lambda_{dil} \rightarrow C \approx 1.8 \text{ GeV}$

Quantity	Fit [MeV]	Exp. [MeV]
$M_{f_0(1370)}$	1444	1200-1500
$M_{f_0(1500)}$	1534	1505 ± 6
$M_{f_0(1710)}$	1750	1720 ± 6
$f_0(1370) \rightarrow \pi\pi$	423.6	-
$f_0(1500) \rightarrow \pi\pi$	39.2	38.04 ± 4.95
$f_0(1500) \rightarrow K\bar{K}$	9.1	9.37 ± 1.69
$f_0(1710) \rightarrow \pi\pi$	28.3	29.3 ± 6.5
$f_0(1710) \rightarrow K\bar{K}$	73.4	71.4 ± 29.1

$$\sigma_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2} \cong f_0(1370) \quad \sigma_S \equiv \bar{s}s \cong f_0(1500) \quad G \equiv gg \cong f_0(1710)$$

Decay Channel	Our Value [MeV]	Experiment [MeV]
$f_0(1370) \rightarrow KK$	117.5	–
$f_0(1370) \rightarrow \eta\eta$	43.3	–
$f_0(1370) \rightarrow \rho\rho \rightarrow 4\pi$	13.8	–
$f_0(1500) \rightarrow \eta\eta$	4.7	5.56 ± 1.34
$f_0(1500) \rightarrow \rho\rho \rightarrow 4\pi$	0.2	$> 54.0 \pm 7.1$
$f_0(1710) \rightarrow \eta\eta$	57.9	34.3 ± 17.6
$f_0(1710) \rightarrow \rho\rho \rightarrow 4\pi$	0.5	–

Gluonium content in η'



Eur. Phys. J. C (2010) 68: 619–681
DOI 10.1140/epjc/s10052-010-1351-1

THE EUROPEAN
PHYSICAL JOURNAL C

Review

Physics with the KLOE-2 experiment at the upgraded DAΦNE

In general, one has a 3-body mixing.
Chiral anomaly important.
However, the mixing should be suppressed
by the large glueball mass (2.6 GeV).

Results not conclusive yet...

$$|q\bar{q}\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle) \quad |s\bar{s}\rangle$$

$$|\eta'\rangle = \cos\psi_G \sin\psi_P |q\bar{q}\rangle + \cos\psi_G \cos\psi_P |s\bar{s}\rangle + \sin\psi_G |GG\rangle,$$

$$|\eta\rangle = \cos\psi_P |q\bar{q}\rangle - \sin\psi_P |s\bar{s}\rangle,$$

$$Z_G^2 = \sin^2\psi_G$$

Table 12 Fit to the gluonium content in η' assuming 1% error on the η' branching fractions. The $\eta' \rightarrow \gamma\gamma/\pi^0 \rightarrow \gamma\gamma$ constraint is used (not used) in the left (right) column

	with $\eta' \rightarrow \gamma\gamma/\pi^0 \rightarrow \gamma\gamma$	without $\eta' \rightarrow \gamma\gamma/\pi^0 \rightarrow \gamma\gamma$
Z_G^2	0.11 ± 0.03	0.11 ± 0.04



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On the gluon content of the η and η' mesons

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ABSTRACT: A phenomenological analysis of radiative $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ decays is performed with the purpose of determining the gluonic content of the η and η' wave functions. Our results show that within our model there is no evidence for a gluonium contribution in the η , $Z_\eta^2 = 0.00 \pm 0.12$, or the η' , $Z_{\eta'}^2 = 0.04 \pm 0.09$. In terms of a mixing angle description this corresponds to $\phi_P = (41.4 \pm 1.3)^\circ$ and $|\phi_{\eta'G}| = (12 \pm 13)^\circ$. In addition, the η - η' mixing angle is found to be $\phi_P = (41.5 \pm 1.2)^\circ$ if we don't allow for a gluonium component.

Situation not clarified yet.

According to Lattice, the pseudoscalar glueball has a mass of about 2.6 GeV.

Mixing should be suppressed.

Glueball decays: blindness

Flavour blindness (widely used for the scalar glueball):

$$\left| \frac{A_{G \rightarrow \pi\pi}}{A_{G \rightarrow KK}} \right|^2 = \frac{3}{4} \quad \left| \frac{A_{G \rightarrow \eta\eta}}{A_{G \rightarrow KK}} \right|^2 = \frac{3}{4} \quad A_{G \rightarrow \eta\eta'} = 0$$

Even more: chiral blindness (could be relevant for heavier glueballs):

$$\left| \frac{A_{G \rightarrow \rho\rho}}{A_{G \rightarrow a_1(1230) a_1(1230)}} \right|^2 = 1$$

Glueball decay: Large- N_c



Glueball masses are N_c -independent for large- N_c
(just a conventional quark-antiquark mesons)

$$M_G \propto N_c^0$$

Decay amplitude of a glueball into (conventional) mesons scales as:

$$A_{G \rightarrow M_1 M_2} \propto N_c^{-1} \quad \Gamma_{G \rightarrow M_1 M_2} \propto N_c^{-2}$$

Recall that for a conventional quark-antiquark state:

$$A_{M \rightarrow M_1 M_2} \propto N_c^{-1/2} \quad \Gamma_{M \rightarrow M_1 M_2} \propto N_c^{-1}$$

Glueball-quarkonium: mixing and large- N_c

The picture is complicated by mixing.
A glueball with conventional quantum numbers mix with nearby quark-antiquark states.

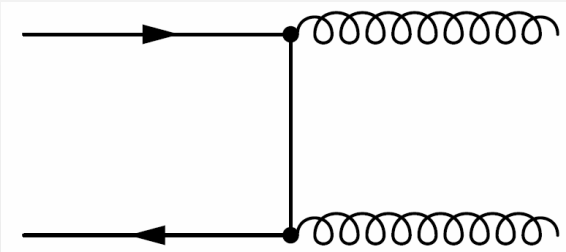
Mixing suppressed in large- N_c but phenom. relevant

$$A_{G-M} \propto N_c^{-1/2}$$

For comparison:

$$A_{\sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) - \bar{s}s} \propto N_c^{-1}$$

$$|\bar{q}q\rangle \longleftrightarrow |gg\rangle$$



Estimate of the glueball's width/1

OZI-dominant decays of conventional quark-antiquark mesons:

$$\Gamma_{\rho \rightarrow \pi\pi} \propto \frac{1}{N_c} \text{ and } \Gamma_{\rho \rightarrow \pi\pi}^{\text{exp}} = 148 \text{ MeV.}$$

$$\Gamma_{f_2(1270) \rightarrow \pi\pi} \propto \frac{1}{N_c} \text{ and } \Gamma_{f_2(1270) \rightarrow \pi\pi}^{\text{exp}} = 157 \text{ MeV}$$

$$\Gamma_{f_2'(1525) \rightarrow KK} \propto \frac{1}{N_c} \text{ and } \Gamma_{f_2'(1525) \rightarrow KK}^{\text{exp}} = 65 \text{ MeV}$$

$$\Gamma_{\bar{q}q \rightarrow MM}^{\text{OZI-allowed}} \propto \frac{1}{N_c} \text{ and } \Gamma_{\bar{q}q \rightarrow MM}^{\text{OZI-allowed}} \sim 100 \text{ MeV.}$$

Estimate of the glueball's width/2

OZI-suppressed decays of conventional quark-antiquark mesons:

$$\Gamma_{f_2'(1525) \rightarrow \pi\pi} \propto \frac{1}{N_c^3} \text{ and } \Gamma_{f_2'(1525) \rightarrow \pi\pi}^{\text{exp}} = 0.6 \text{ MeV}$$

$$\Gamma_{j/\psi \rightarrow \text{hadrons}} \propto \frac{1}{N_c^3} \text{ and } \Gamma_{j/\psi \rightarrow \text{hadrons}}^{\text{exp}} = 0.08 \text{ MeV}$$

$$\Gamma_{\bar{q}q \rightarrow MM}^{\text{OZI-suppressed}} \propto \frac{1}{N_c^3} \text{ and } \Gamma_{\bar{q}q \rightarrow MM}^{\text{OZI-suppressed}} \lesssim 1 \text{ MeV}$$

Hence, for glueballs we *guess*:

$$\Gamma_{G \rightarrow MM} \propto \frac{1}{N_c^2},$$

$$1 \text{ MeV} \sim \Gamma_{\bar{q}q \rightarrow MM}^{\text{OZI-suppressed}} \propto \frac{1}{N_c^3} < \Gamma_{G \rightarrow MM} \propto \frac{1}{N_c^2} < \frac{1}{N_c} \propto \Gamma_{\bar{q}q \rightarrow MM}^{\text{OZI-allowed}} \sim 100 \text{ MeV}.$$

$$\Gamma_{G \rightarrow MM} \sim 10 \text{ MeV}.$$

The PANDA experiment

