QCD from quark, gluon, and meson correlators

Mario Mitter

Karl-Franzens-Universität Graz

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fQCD collaboration - QCD (phase diagram) with FRG:

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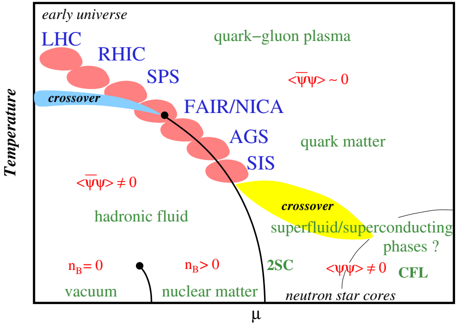

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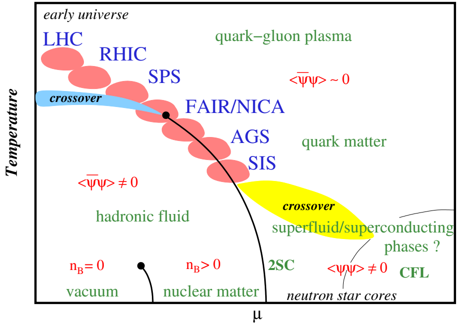

large part of this effort: vacuum YM-theory and QCD

QCD from the effective action

$$\Gamma[\Phi] = \sum_n \int_{\{p_i\}} \Gamma^{(n)}_{\Phi_1 \cdots \Phi_n}(p_1, \ldots, p_{n-1}) \Phi^1(p_1) \cdots \Phi^n(-p_1 - \cdots - p_{n-1})$$

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    e.g. [Roberts, Williams, '94], [Alkofer, Smekal, '00], [Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, '16]
  - form factors: photon-particle correlators

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  - thermodynamic quantities: $\Gamma[\Phi] \propto$ grand potential
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  - further quantities: $\Gamma[\Phi] \propto$ eff. potential, propagators, 't Hooft determinant
    - chiral condensate(s)/$\langle\sigma\rangle$

      e.g. [Schaefer, Wambach '04], [Fischer, Luecker, Mueller '11], [MM, Schaefer, '13]
    - (dressed) Polyakov loop

      e.g. [Braun, Gies, Pawlowski, '07], [Fischer, '09], [Braun, Haas, Marhauser, Pawlowski, '09]
    - axial anomaly e.g.[Grahl, Rischke, '13], [MM, Schaefer, '13], [Fejos, '15], [Heller, MM, '15]
    - spectral functions e.g. [Tripolt, Strodthoff, Smekal, Wambach, '14]

Correlation functions with the FRG

- $S[\Phi] = \Gamma_\Lambda[\Phi]$: use only perturbative QCD input
  - $\alpha_S(\Lambda = \mathcal{O}(10)\text{ GeV})$
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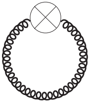
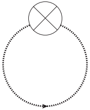
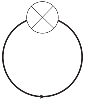
$$\partial_k \Gamma_k[A, \bar{c}, c, \bar{q}, q] = \frac{1}{2}$$  $-$  $-$ 

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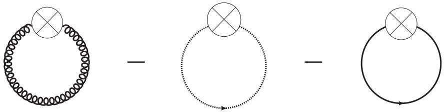

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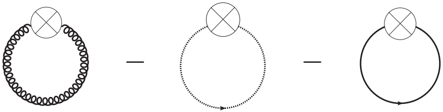

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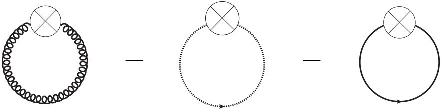


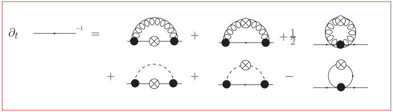
$\Rightarrow$ full quantum effective action

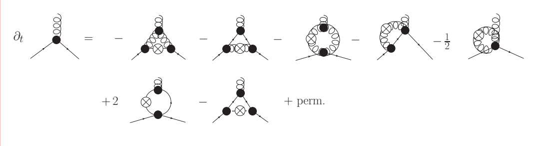
- gauge-fixed approach (Landau gauge): ghosts appear
- functional derivatives $\Rightarrow$ equations for correlators
- aim for "apparent convergence" of $\Gamma[\Phi] = \lim_{k\to 0} \Gamma_k[\Phi]$

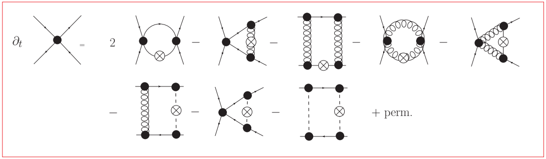
Some representative equations

[MM, Strodthoff, Pawlowski, 2014],

[Cyrol, Fister, MM, Strodthoff, Pawlowski, 2016]







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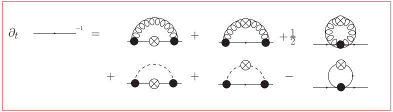

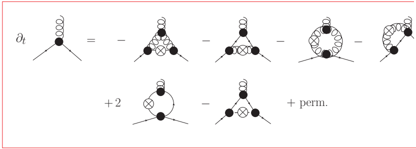

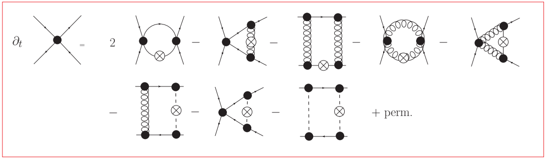

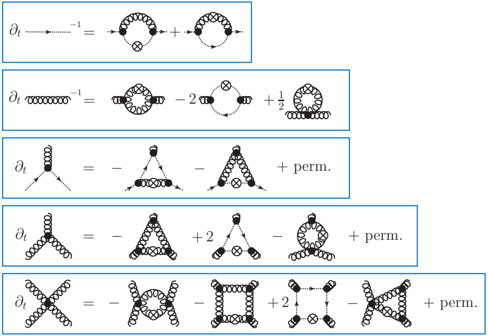

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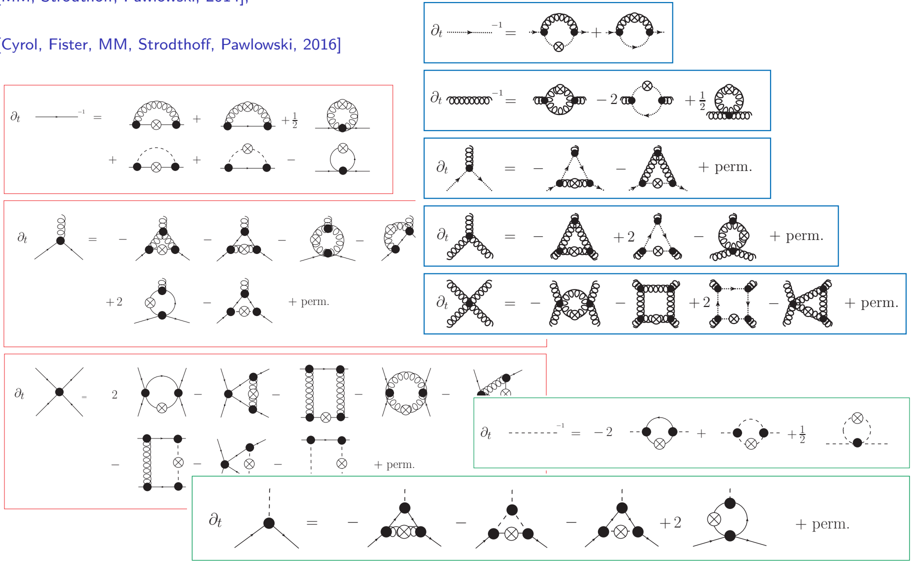

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- outlook: [Cyrol, MM, Strodthoff, Pawlowski, in preparation]
    - unquenching
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- use results from lattice QCD to gauge truncation

Pure YM-theory

[Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]

- truncation: momentum dependent dressing functions for all classical tensors
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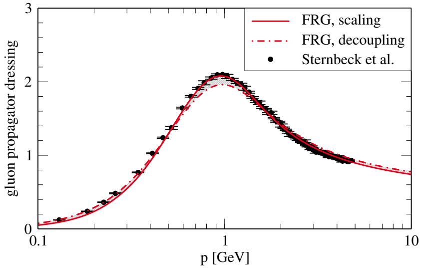
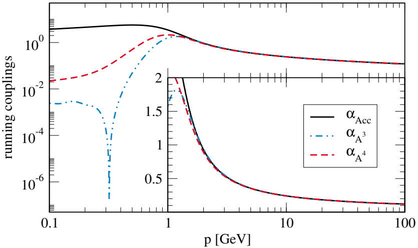
- hardest part of solution: fulfilling the modified STI ($\Rightarrow$ scaling solution)

- $\Gamma^{(2)}_{AA}(p) \propto Z_A(p)\, p^2 \,\left(\delta^{\mu\nu} - p^\mu p^\nu / p^2\right)$
- IR-suppression $\Leftrightarrow$ “confinement”
- smooth transition to perturbation theory

- running couplings
- degeneracy at large p due to STI
- test of truncation





lattice data: A. Sternbeck, E. M. Ilgenfritz, M. Muller-Preussker, A. Schiller, and I. L. Bogolubsky, PoS LAT2006, 076.

Chiral symmetry breaking

- χSB $\Leftrightarrow$ resonance in 4-Fermi interaction λ (pion pole):

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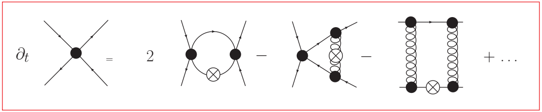
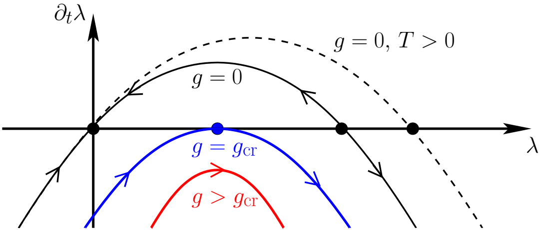
- β-function of momentum independent 4-Fermi interaction:

$$\partial_t \lambda = 2\lambda + a\,\lambda^2 + b\,\lambda\,\alpha + c\,\alpha^2, \quad b > 0,\ a, c \leq 0$$

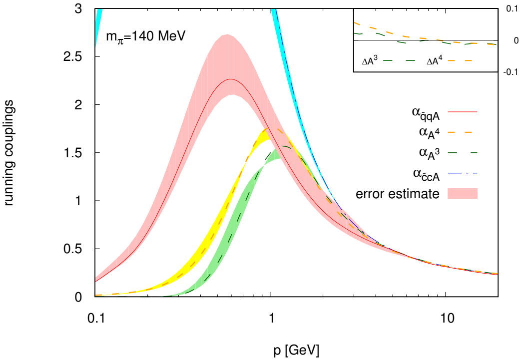




[Braun, 2011]

(transverse) running couplings

[Cyrol, MM, Pawlowski, Strodthoff, in preparation]

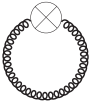
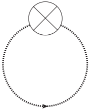
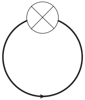
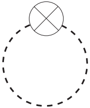


- agreement in perturbative regime required by Slavnov-Taylor identities
- non-degenerate in nonperturbative regime: reflects gluon mass gap
- $\alpha_{\bar{q}Aq} > \alpha_{cr}$: necessary for chiral symmetry breaking
- area above α_{cr} very sensitive to errors

  $\Rightarrow$ use STI in perturbative regime

4-Fermi vertex via dynamical hadronization [Gies, Wetterich, 2002]

- change of variables: particular 4-Fermi channels → meson exchange
- efficient inclusion of pole structure ⇒ no spurious singularities
- calculation of low-energy model parameters from QCD

$\partial_k \Gamma_k = \frac{1}{2}$  $-$  $-$  $+ \frac{1}{2}$ 

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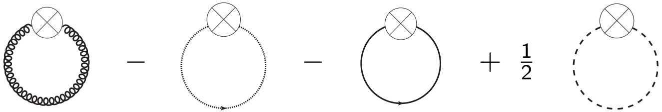
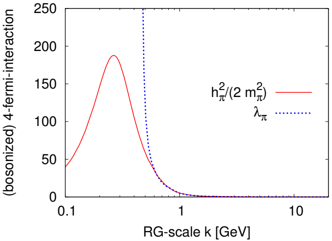
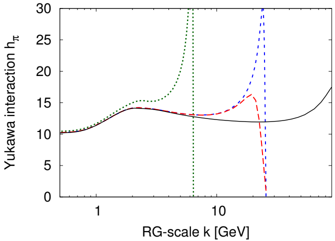
$\partial_k \Gamma_k = \frac{1}{2}$ 



[MM, Strodthoff, Pawlowski, 2014]



[Braun, Fister, Haas, Pawlowski, Rennecke, 2014]

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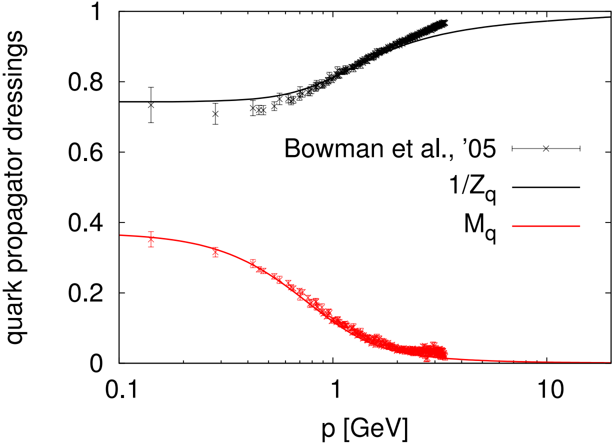
Quenched QCD: Quark propagator

[MM, Pawlowski, Strodthoff, 2014]

- $\Gamma^{(2)}_{\bar{q}q}(p) \propto Z_q(p)\,(\not{p} + M(p))$



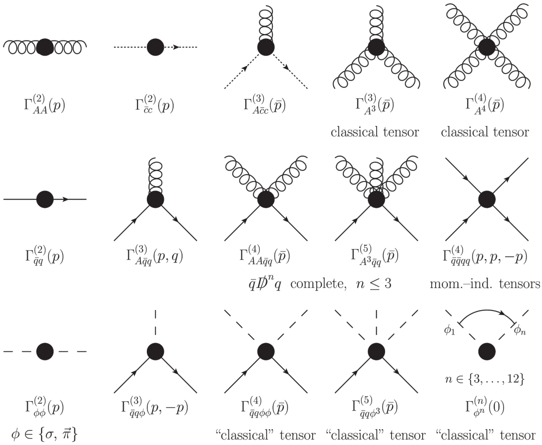
- FRG vs. lattice: bare mass, quenched, scale set via gluon propagator
- not sufficient: need convergence of unquenched QCD at $T, \mu \neq 0$

lattice data: Bowman, Heller, Leinweber, Parappilly, Williams, Zhang , Phys. Rev. D71, 054507 (2005).

Outlook: unquenching, $N_f = 2$

[Cyrol, MM, Pawlowski, Strodthoff, in prep.]

extended truncation:



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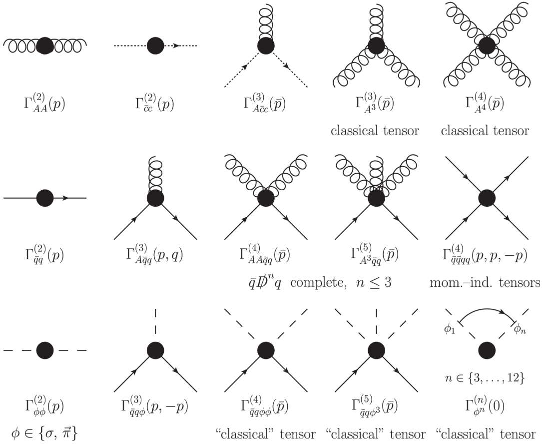

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$\Rightarrow$ BRST-invariant operators, e.g. $\bar{\psi}\not{D}^n\psi$

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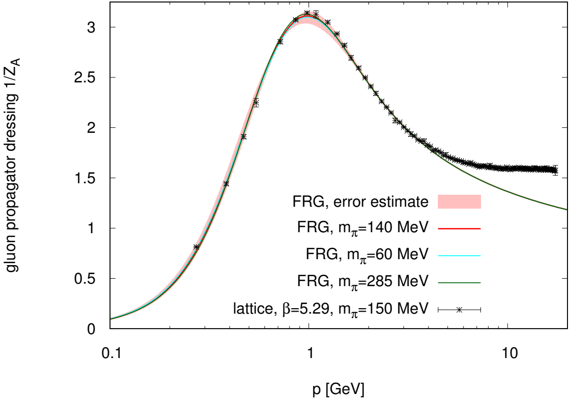

- infrared suppression ⇔ "confinement"
- insensitive to pion mass
- transition to perturbation theory
- scaling solution

lattice data: A. Sternbeck, K. Maltman, M. Müller-Preussker, L. von Smekal, PoS LATTICE2012 (2012) 243.

Outlook: quark propagator

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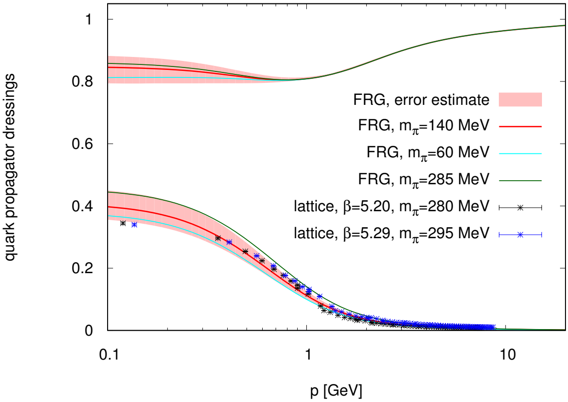

- wave function renormalisation Z_q
- sensitive to $\bar{q}qA$-interaction, relative scales
- mass function M_q
- lattice Z_q?

lattice data: O. Oliveira, A. Kzlersu, P. J. Silva, J.-I. Skullerud, A. Sternbeck, A. G. Williams, arXiv:1605.09632 [hep-lat].

Outlook: Quark-gluon interactions [Cyrol, MM, Pawlowski, Strodthoff, in prep.]

- quark-gluon interaction most crucial for chiral symmetry breaking
- transverse tensor basis (8 tensors), e.g. $\gamma^\mu,\ \mathrm{i}\left(\not{p}+\not{q}\right)\gamma^\mu,\ \frac{1}{2}\left[\not{p},\not{q}\right]\gamma^\mu$
- $\lambda^{(i)}(p,q) \to \lambda^{(i)}(p^2,q^2,p\cdot q)$

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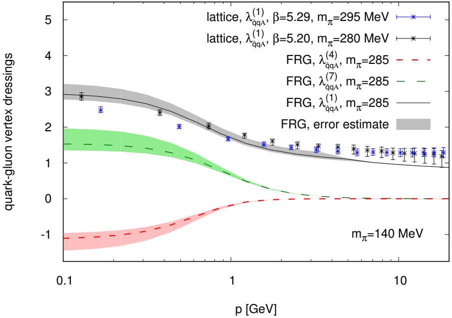

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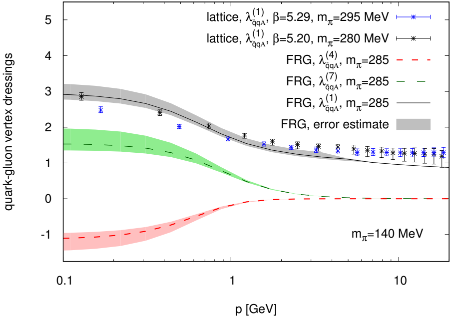

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- chirally symmetric tensors from operator $\bar{q}\not{D}^3 q$ worsen result
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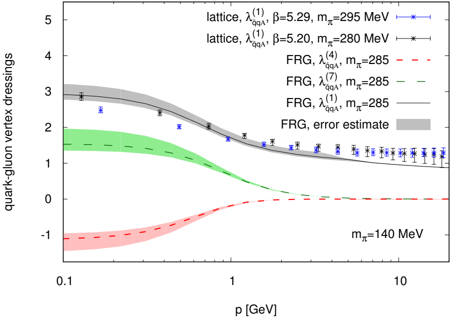
Outlook: Quark-gluon interactions [Cyrol, MM, Pawlowski, Strodthoff, in prep.]

- quark-gluon interaction most crucial for chiral symmetry breaking
- transverse tensor basis (8 tensors), e.g. γ^μ, $\mathrm{i}\left(\not{p}+\not{q}\right)\gamma^\mu$, $\frac{1}{2}\left[\not{p},\not{q}\right]\gamma^\mu$
- $\lambda^{(i)}(p,q) \to \lambda^{(i)}(p^2,q^2,p\cdot q)$



- 3 leading tensors:
  - classical tensor: constrained by STI at large momenta
  - chirally symmetric
  - break chiral symmetry
- systematic lattice error?

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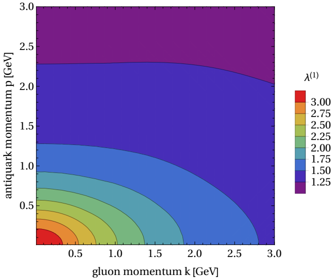
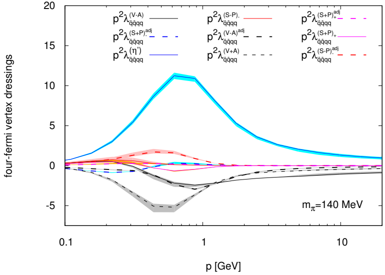
$\Rightarrow$ expansion in BRST-invariant operators improves convergence?

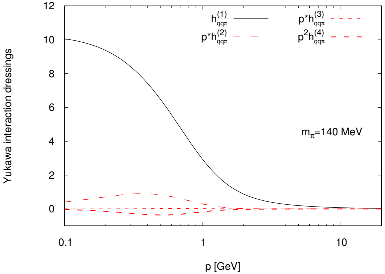
lattice data: O. Oliveira, A. Kzlersu, P. J. Silva, J.-I. Skullerud, A. Sternbeck, A. G. Williams, arXiv:1605.09632 [hep-lat].

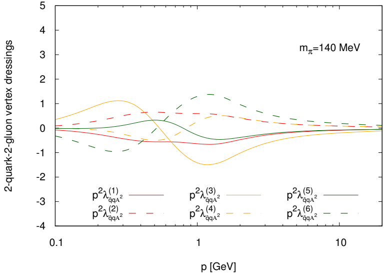
Outlook: more correlators

[Cyrol, MM, Pawlowski, Strodthoff, in prep.]





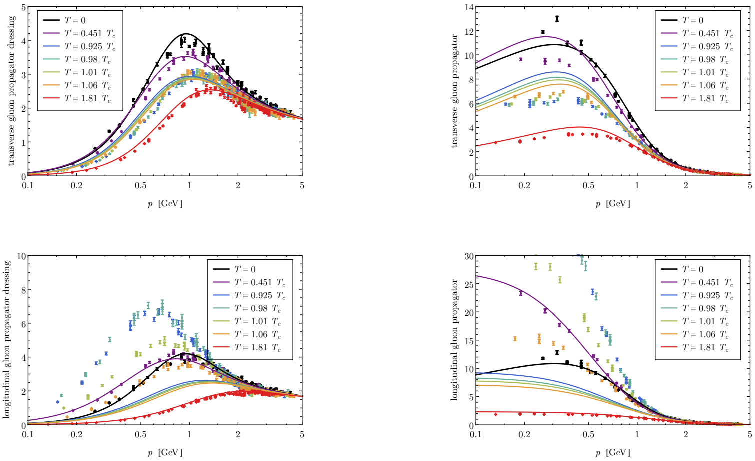




Outlook: YM correlators I at $T \neq 0$ [Cyrol, MM, Pawlowski, Strodthoff, in prep.]

full splitting of classical tensors into electric and magnetic parts

Zeroth mode correlation functions

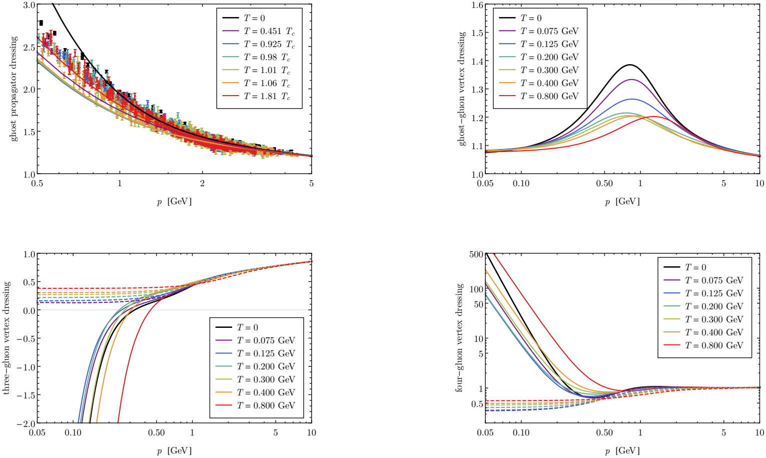


lattice data: A. Maas, J. M. Pawlowski, L. von Smekal, D. Spielmann, Phys.Rev. D85 (2012) 034037.

Outlook: YM correlators II at $T \neq 0$ [Cyrol, MM, Pawlowski, Strodthoff, in prep.]

full splitting of classical tensors into electric and magnetic parts

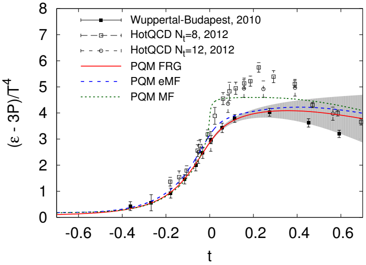
Zeroth mode correlation functions (solid lines: magnetic dressings; dashed lines: dressings with two electric legs)



lattice data: A. Maas, J. M. Pawlowski, L. von Smekal, D. Spielmann, Phys.Rev. D85 (2012) 034037.

"QCD-enhanced" models

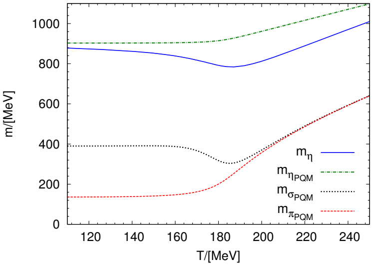
- equation of state
- $N_f = 2 + 1$ PQM model with FRG
- unquenched Polyakov-loop potential from [Braun, Haas, Marhauser, Pawlowski, '11]



[Herbst, MM, Pawlowski, Schaefer, Stiele, '13]

[Haas, Stiele, Braun, Pawlowski, Schaffner-Bielich, '13]

- η'-meson screening mass
- $N_f = 2$ PQM model, extended MF
- running 't Hooft determinant from [MM, Pawlowski, Strodthoff, '14]



[Heller, MM, '15]

Status and Outlook

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- further applications:
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  - other strongly-interacting theories