Analytic approach to πK scattering and strange resonances.

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Based on:

J.R.Peláez, A.Rodas and J.Ruiz de Elvira, Strange resonance poles from $K\pi$ scattering below 1.8GeV, Eur.Phys.J.C **77**, no. 2, 91 (2017) [arXiv:1612.07966 [hep-ph]],
J.R.Peláez and A.Rodas, Pion-kaon scattering amplitude constrained with forward dispersion relations up to 1.6GeV, Phys.Rev.D **93**, no. 7, 074025 (2016) [arXiv:1602.08404 [hep-ph]].

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Motivation

- πK scattering appears as final state in many hadronic processes.
- Good description in many previous works with Unitarized Chiral Perturbation Theory: Oller,Oset (1999).Dobado,Peláez (1997). Oller,Oset,Peláez (1999). Jamin,Oller,Pich (2000). Gomez Nicola,Peláez (2002).
- Best determination Roy-Steiner analysis: Büttiker,Descotes-Genon,Moussallam (2004).
- $K_0^*(800)/\kappa$ appears in these works and in other papers.
- However $K_0^*(800)/\kappa$ still needs confirmation according to PDG.
- Relevant to complete the lightest scalar nonet and rule out σ-glueball interpretation.
- We have been encouraged to perform a similar analysis for the $K_0^*(800)/\kappa$ as done for the $f_0(500)/\sigma$ by our group.
- Experimental groups ask for simple but consistent parametrizations to be used at LHCb.

Introduction

- Steps:
  1. Simple fits with unitarity and analyticity. There is no dynamical input.
  2. Check of the Forward Dispersion Relations (FDR).
  3. Impose FDR to the fits.
  4. Important to obtain the correct parameter of the poles.
- Data obtained from LASS experiments (Aston et al., Estabrooks et al.).
- First step in a long term project.

Forward dispersion relations

- We form symmetric or antisymmetric amplitudes under $s \leftrightarrow u$ exchange.

$$T^{+} = \frac{1}{3}T^{1/2} + \frac{2}{3}T^{3/2},$$
$$T^{-} = \frac{1}{3}T^{1/2} - \frac{1}{3}T^{3/2}. \quad (1)$$

- T^{I} is the amplitude of defined isospin I.

Forward dispersion relations

- We will take for our analysis $t = 0$, they are called FDR.
- For the symmetric $s \leftrightarrow u$ amplitude one subtraction is needed

$$\mathrm{Re}\, T^+(s) = T^+(s_{th}) + \frac{(s - s_{th})}{\pi} \tag{2}$$

$$P \int_{s_{th}}^{\infty} ds' \left[\frac{\mathrm{Im}\, T^+(s')}{(s' - s)(s' - s_{th})} - \frac{\mathrm{Im}\, T^+(s')}{(s' + s - 2\Sigma_{\pi K})(s' + s_{th} - 2\Sigma_{\pi K})} \right], \tag{3}$$

- For the antisymmetric amplitude no subtraction is needed

$$\mathrm{Re}\, T^-(s) = \frac{(2s - 2\Sigma_{\pi K})}{\pi} P \int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im}\, T^-(s')}{(s' - s)(s' + s - 2\Sigma_{\pi K})}. \tag{4}$$

Unconstrained Fits (UFD):Elastic region

- We use the unitary functional form for the partial waves

$$t_l^I(s) = \frac{1}{\sigma(s)} \frac{1}{cot\delta_l^I(s) - i} \tag{5}$$

- Where

$$cot\delta_l^I(s) = \frac{\sqrt{s}}{2q^{2l+1}} \sum B_n \omega(s)^n \tag{6}$$

- with $\omega(s) = \frac{\sqrt{y(s)} - \alpha\sqrt{y(s0) - y(s)}}{\sqrt{y(s)} + \alpha\sqrt{y(s0) - y(s)}}$ as our new variable (conformal mapping).
- Here $y(s) = (\frac{s-su}{s+su})^2$ defines the circular cut on the next figure.
- ω used to maximize the analyticity domain.

Unconstrained Fits (UFD):Elastic region

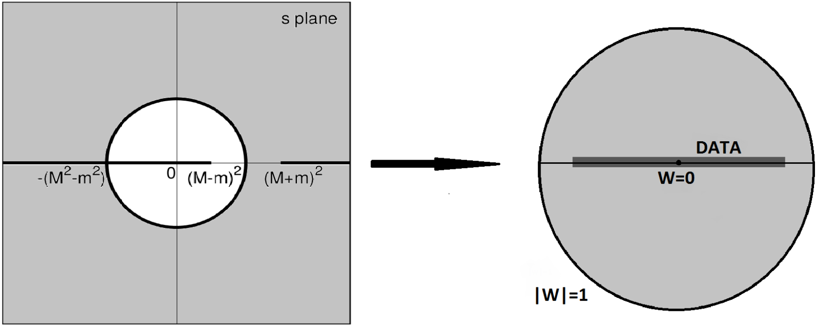

Figure: Structure of the PW.

- α is used to center the point of energy s_c for the expansion.

Unconstrained Fits (UFD):Inelastic region

- In the inelastic region $t_l^I = \frac{\eta_l^I(s)e^{2i\delta_l^I(s)}-1}{2i} = |t_l^I|e^{i\phi_l^I}$.
- We use complex rational functions that near each resonance look like BW.
- We impose matching conditions on the inelastic ηk threshold.
- We use up to $F^{1/2}$ which is very small and neglect $G^{1/2}$ in the studied energy region.
- Although we use for our analysis the $P^{3/2}, D^{3/2}$ and the $F^{1/2}$ their contribution is small. Not shown here.

FDR check of UFD

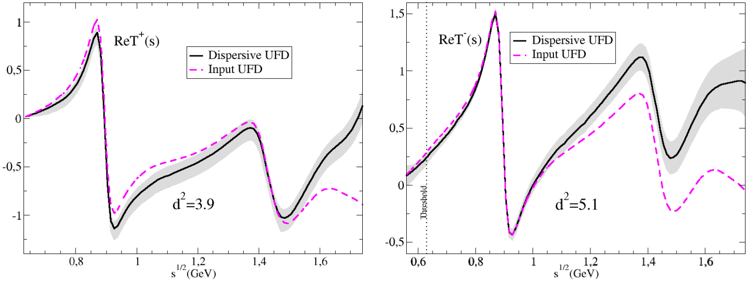

Figure: FDR unconstrained, symmetric and antisymmetric. Clear difference between the input and output

- Symmetric incompatibilities caused by the $S^{1/2}$ and the $S^{3/2}$ PW.
- Antisymmetric deviations due to Regge contribution.
- Room for improvement $\rightarrow$ Constrained fits.
- Above $1.8 GeV$ the discrepancies are too big to impose FDR.

Constrained fits to data (CFD)

- The change in the symmetric amplitude around $1 - 1.2GeV$ is caused by the change of the $S^{3/2}$-wave. The Regge contribution in this region is small.
- The huge change of the antisymmetric one is caused by the Regge πK factorization constant.

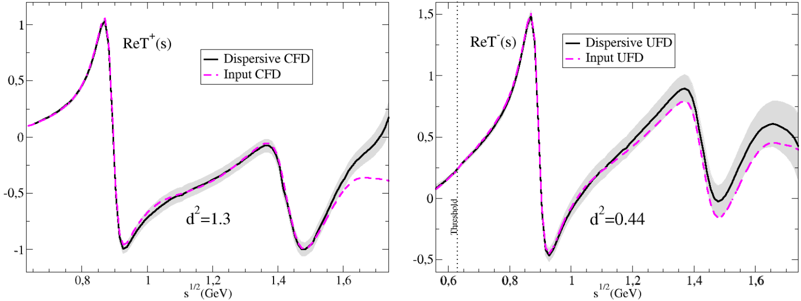

Figure: FDR constrained, symmetric and antisymmetric. Fairly compatible up to 1.6 GeV

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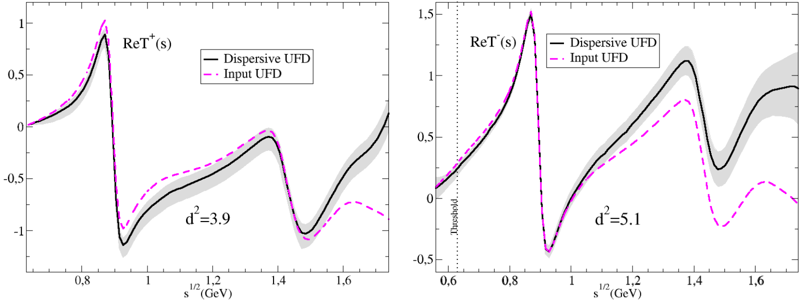


Figure: FDR unconstrained, symmetric and antisymmetric. It is clear the huge difference between the input and output

- We study the FDR up to $1.7 - 1.8 GeV$.
- We define a χ_1^2 between the input and output, with a weight using the degrees of freedom of the amplitudes.
- There is a χ_2^2 between the UFD parameters and the new ones.
- After the minimization of the total function we obtain

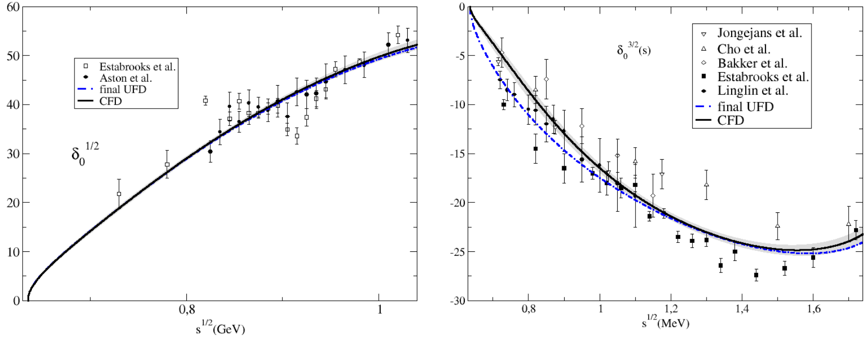


Figure: $S^{1/2}$ and $S^{3/2}$ phase shifts.

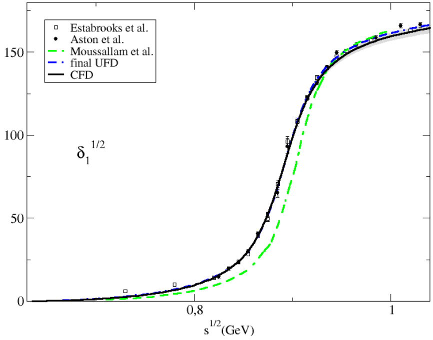
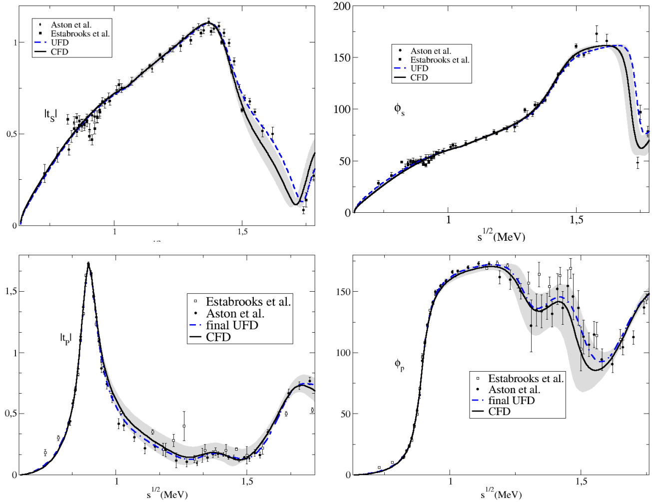


Figure: $P^{1/2}$ phase shifts.

- If the K*(892) is to be well described the phase shift must be lower at $1GeV$.
- Paris group → solve Roy equations. They use the data at $0.935GeV$ as the matching point.

Constrained Fits (UFD):Inelastic region

- Almost unchanged below 1.5 GeV.
- Changes above that point, the CFD solution starts to be incompatible with the UFD.



- Now we can obtain the threshold parameters for the most important partial waves.

Table: Scattering lengths.

SL	UFD	CFD	Roy-Steiner result
$m_\pi a_0^{1/2}$	0.222±0.014	0.218±0.014	0.224± 0.022
$m_\pi a_0^{3/2}$	-0.101±0.03	-0.054±0.014	-0.0448± 0.0077
$m_\pi^3 a_1^{1/2}$	0.031±0.008	0.024±0.005	0.019± 0.001

- Dirac collaboration measure the difference between the scalar partial waves.

$$\frac{1}{3}\left(a_0^{1/2} - a_0^{3/2}\right) = 0.11^{+0.09}_{-0.04}\, m_\pi^{-1}, \quad \text{(DIRAC)} \tag{7}$$

- Our results are compatible, although we obtain much smaller errors.

$$\frac{1}{3}\left(a_0^{1/2} - a_0^{3/2}\right) = 0.091^{+0.006}_{-0.005}\, m_\pi^{-1}. \quad \text{(CFD)} \tag{8}$$

Pole parameters

- We could use the parameterizations to calculate the poles
- For the kappa resonance $K_0^*(800)$ we obtain

Table: $K_0^*(800)$ parameters.

Group	Mass	Width
UFD	673 ± 19	674 ± 24
CFD	680 ± 19	667 ± 23
Moussallam et al.	658 ± 13	557 ± 24
D.Bugg	663 ± 34	658 ± 44
Zheng,Zhou	694 ± 53	606 ± 59

- The values of the masses are compatible. Our width is not compatible with Moussallam's result. However the values are obtained using parametrizations (model dependent).

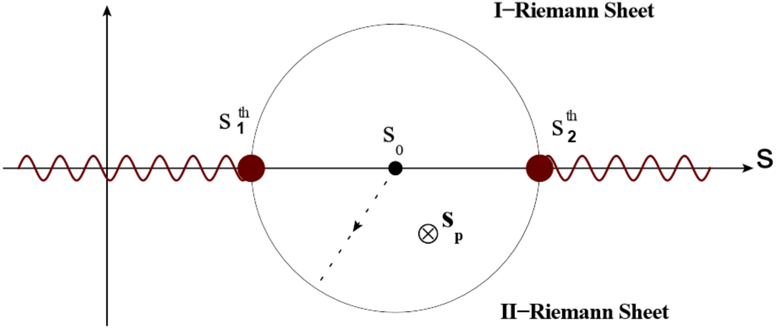
Padé approximants and Pole parameters

- We use this Padé approximants to calculate the parameters of the strange resonances.
- It is a model independent calculation.
- Based on its analytic properties, for example, when searching one pole the approximants read

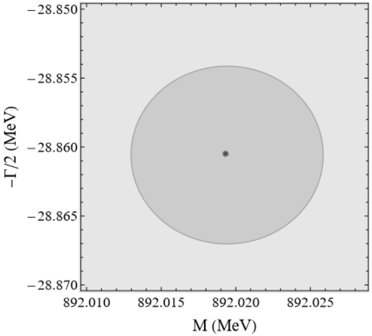
$$P_1^N(s,s_0) = \sum_{k=0}^{N-1} a_K (s-s_0)^k + \frac{a_N (s-s_0)^N}{1 - \frac{a_{N+1}}{a_N}(s-s_0)} \quad (9)$$

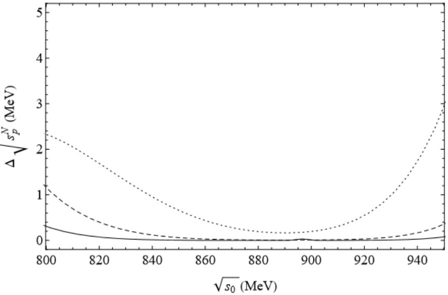
- With a pole located at $s_p = s_0 + \frac{a_N}{a_{N+1}}$. Where $a_n = F^{(n)}(s_0)/n!$.
- We always truncate the sequence when the difference between the poles is smaller than the experimental error.

- The method is suitable for the calculation of both elastic and inelastic resonances.
- The Padé sequence gives us the continuation to the continuous Riemann Sheet.
- We take care of the calculation of the errors. Apart from the experimental and systematic errors of each parameterization we also include different fits.



- For every fit we search the s_0 thats gives the minimum difference for the truncation of the sequence.
- We stop at a N ($N+1$ derivatives) where the systematic uncertainty is smaller than the statistical one (usually $N=4$ is enough).
- Run a montecarlo for every fit to calculate the parameters an errors of every resonance.

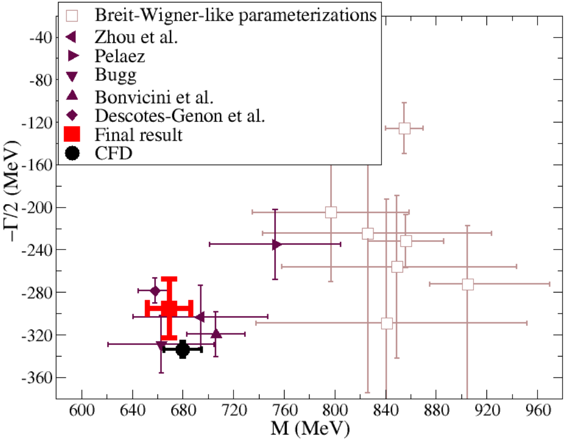




Elastic resonances

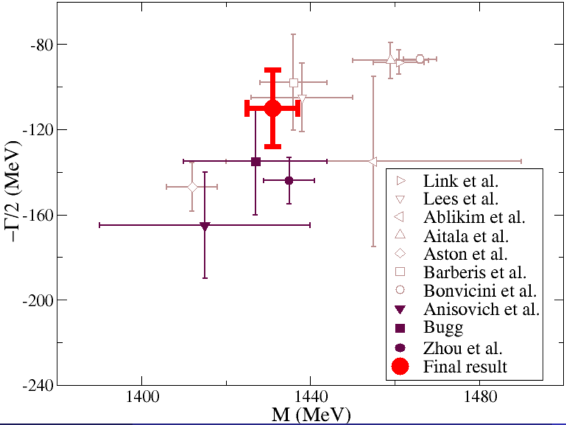
- For the $K_0^*(800)$ resonance we obtain

$$\begin{aligned} \sqrt{s_p} &= (670 \pm 18) - i(295 \pm 28)MeV \\ \sqrt{s_p} &= (682 \pm 29) - i(274 \pm 12)MeV(PDG) \end{aligned} \qquad (10)$$



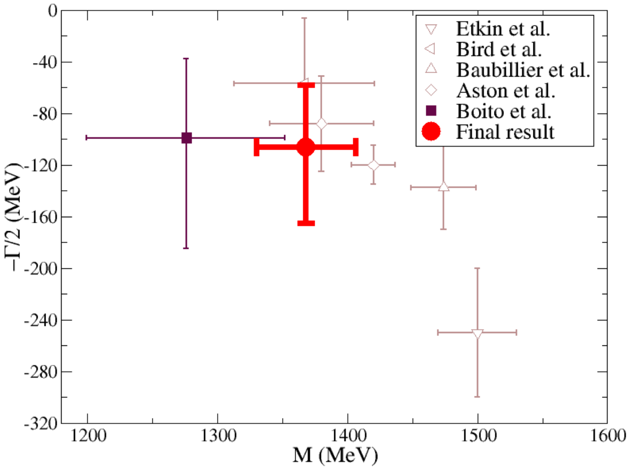
- For the $K_0^*(1430)$ we find

$$\begin{aligned}\sqrt{s_p} &= (1431 \pm 6) - i(110 \pm 19) MeV \\ \sqrt{s_p} &= (1425 \pm 50) - i(135 \pm 40) MeV (PDG) \qquad (11)\end{aligned}$$



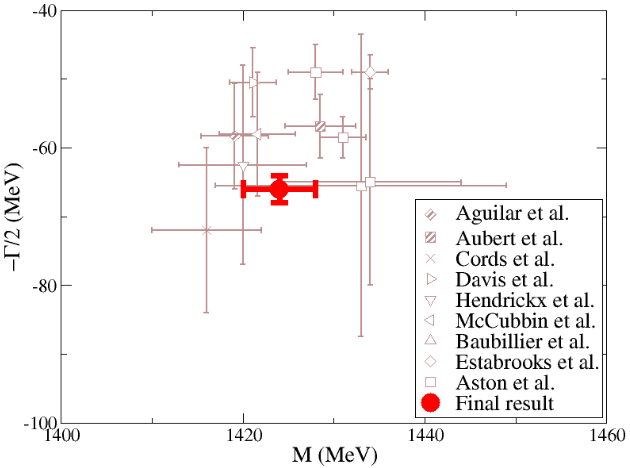
- For the $K_1^*(1410)$ we find

$$\sqrt{s_p} = (1368 \pm 38) - i(106^{+48}_{-59})MeV$$
$$\sqrt{s_p} = (1414 \pm 15) - i(116 \pm 10)MeV(PDG) \qquad (12)$$



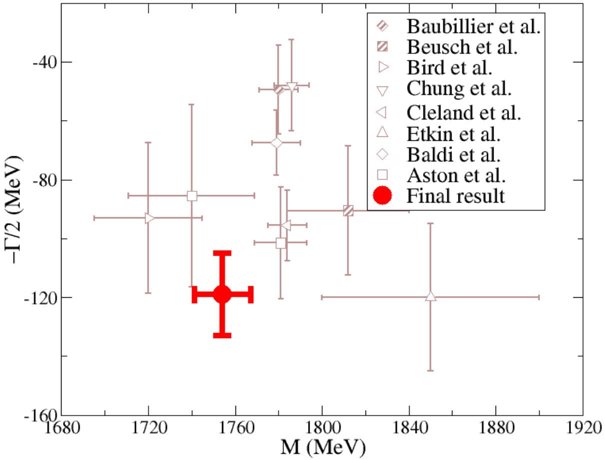
- For the $K_2^*(1430)$ we find

$$\begin{aligned}\sqrt{s_p} &= (1424 \pm 4) - i(66 \pm 2)MeV \\ \sqrt{s_p} &= (1432.4 \pm 1.3) - i(55 \pm 3)MeV(PDG) \qquad (13)\end{aligned}$$



- For the $K_3^*(1780)$ we find

$$\begin{aligned} \sqrt{s_p} &= (1754 \pm 13) - i(119 \pm 14) MeV \\ \sqrt{s_p} &= (1776 \pm 7) - i(80 \pm 11) MeV (PDG) \end{aligned} \tag{14}$$



Conclusions

- The data sets are not compatible with Forward Dispersion Relations.
- We provide simple constrained data fits compatible with Forward Dispersion Relations below 1.7 GeV.
- Resonance parameters obtained from model-independent analytic approach including systematic uncertainties
  - κ pole confirmed. Parameters compatible with PDG and Roy-Steiner equations.
  - Inelastic resonances are compatible with the values listed in the PDG.

Thank you for your attention!