Heavy and heavy-light mesons and the Lorentz structure of the quark-antiquark kernel

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Excited QCD, Sintra, May 8, 2017

Motivation

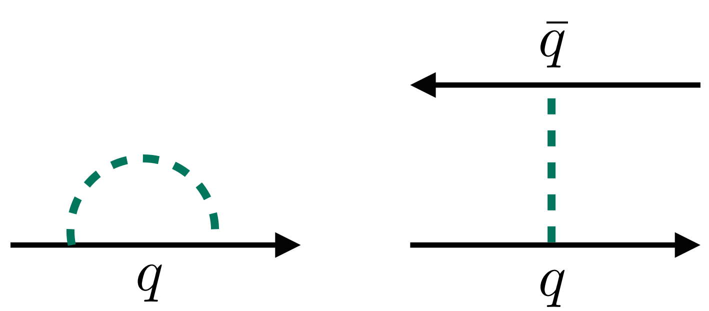
- Intense experimental activity to explore meson structure at LHC, BaBaR, Belle, CLEO and soon at GlueX (Jlab) and PANDA (GSI)
- Search for exotic mesons (hybrids, glueballs, … maybe $qq̄$?)
- Need to understand also "conventional" $q\bar{q}$-mesons in more detail
- Study production mechanisms, transition form factors (also important for hadronic contributions to light-by-light scattering)

Theory: a huge amount of work has already been done on meson structure (LQCD, BS/DSE, constrained dynamics two-body Dirac equation, relativized Schrödinger equation, …)

Guiding principles of our approach (CST - Covariant Spectator Theory):

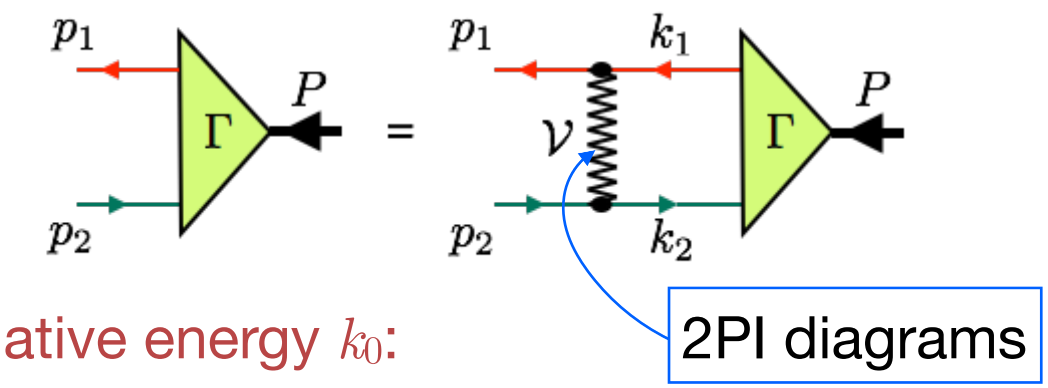
- Find $q\bar{q}$ interaction that can be used in all mesons (unified model)
- Must be relativistic (relativity necessary with light quarks), and reduce to linear+Coulomb in the nonrelativistic limit
- Manifest covariance: strongly constrains spin-dependence of interactions
- Learn about the Lorentz structure of the confining interaction
- Quark masses are dynamic: self-interaction should be consistent with $q\bar{q}$ interaction

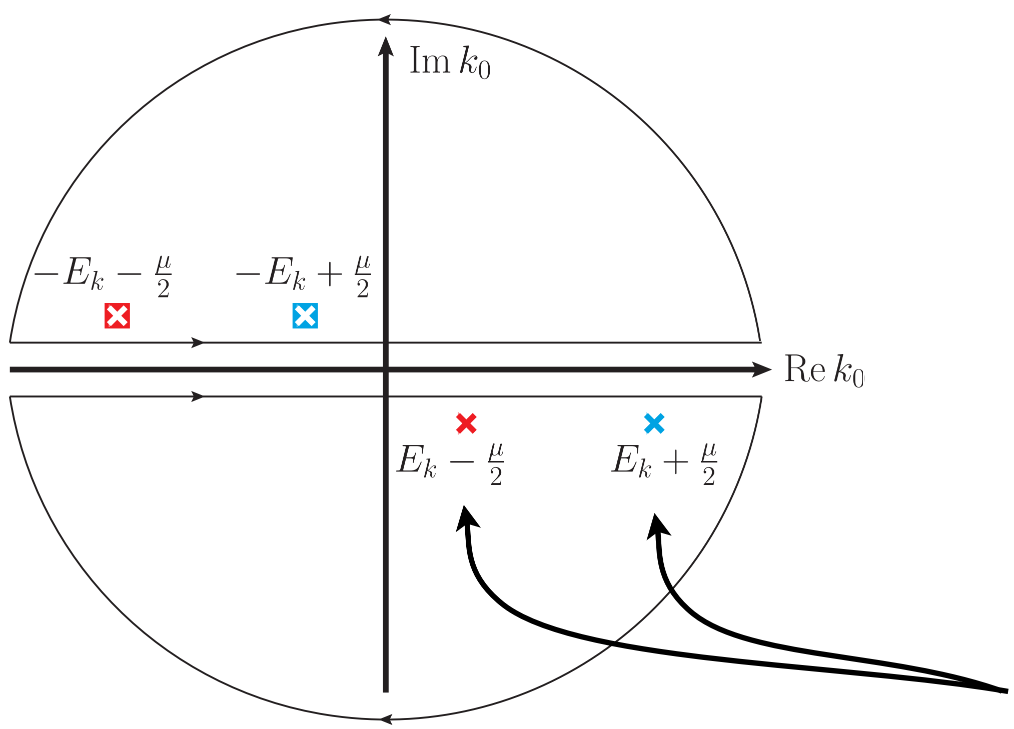
Huge mass variation:
from pions (~0.14 GeV)
to bottomonium (> 10 GeV)



CST equation for two-body bound states

Bethe-Salpeter equation for $q\bar{q}$ bound-state with mass μ





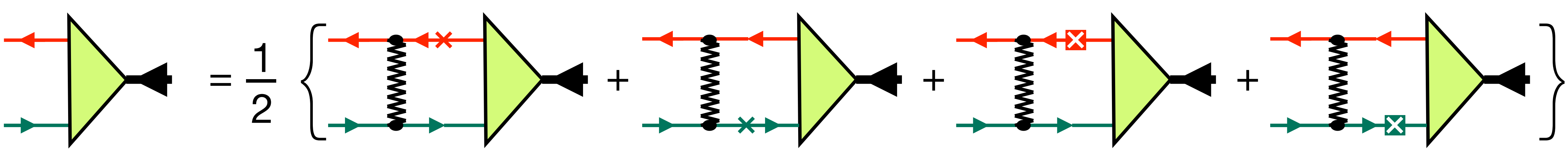
Integration over relative energy k_0:

- Keep only pole contributions from constituent particle propagators
- Poles from particle exchanges appear in higher-order kernels (usually neglected — tend to cancel)
- Reduction to 3D loop integrations, but covariant
- Correct one-body limit

If bound-state mass μ is small:
both poles are close together (both important)

Symmetrize pole contributions from both half planes: charge conjugation symmetry

BS vertex (approx.) CST vertices



Once the four CST vertices (with one quark on-shell) are all known, one can use this equation to get the vertex function for other momenta (also Euclidean).

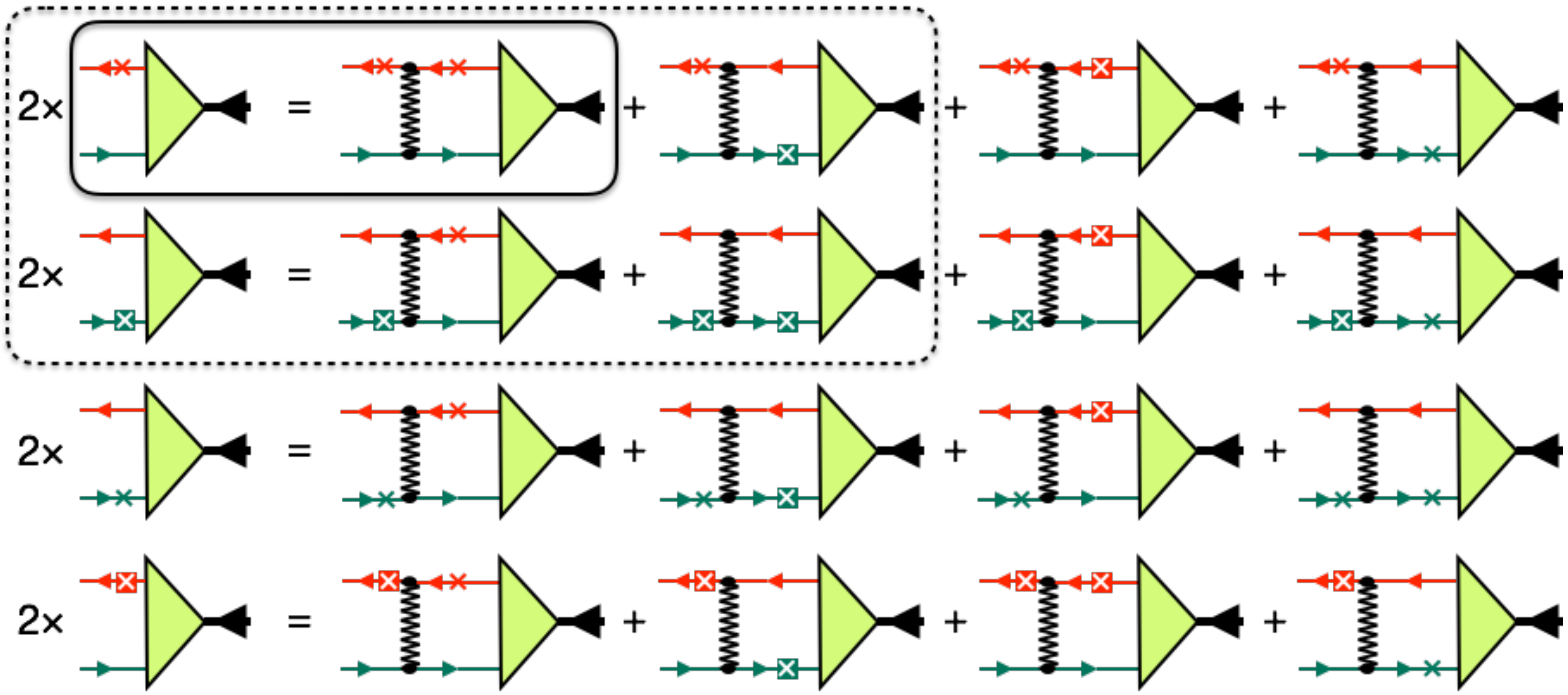
CST equations

Closed set of equations when external legs are systematically placed on-shell

1CSE

2CSE

4CSE



Solutions: bound state masses μ and corresponding vertex functions Γ

One-channel spectator equation (1CSE):

- Particularly appropriate for unequal masses
- Numerical solutions easier (fewer singularities)
- But not charge-conjugation symmetric

Two-channel spectator equation (2CSE):

- Restores charge-conjugation symmetry
- Additional singularities in the kernel

Four-channel spectator equation (4CSE):

- Necessary for light bound states (pion!)

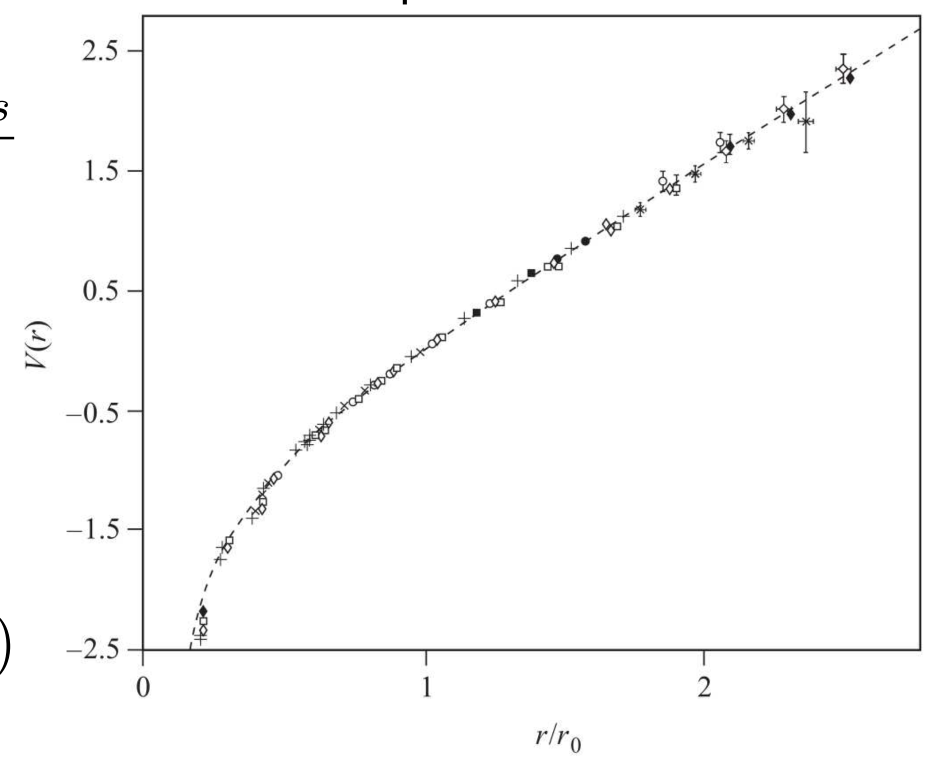
All have smooth one-body limit (Dirac equation) and nonrelativistic limit (Schrödinger equation).

Confining potential in momentum space

Phenomenological $q\bar{q}$ kernel

Inspired by Cornell potential: $V(r) = \sigma r - C - \frac{\alpha_s}{r}$

Static QCD potential from the lattice



Allton et al, UKQCD Collab., PRD **65**, 054502 (2002)

NR linear potential in momentum space:

Fourier transform of screened potential

Usually: $\sigma r = \lim_{\epsilon \to 0} \sigma \frac{\partial^2}{\partial \epsilon^2} \frac{e^{-\epsilon r}}{r}$

But simpler: $\sigma r = \lim_{\epsilon \to 0} -\frac{\sigma}{\epsilon} \left(e^{-\epsilon r} - 1 \right) \equiv \tilde{V}_A(r) - \tilde{V}_A(0)$

FT: $V_L(\mathbf{q}) = V_A(\mathbf{q}) - (2\pi)^3 \delta(\mathbf{q}) \int \frac{d^3 q'}{(2\pi)^3} V_A(\mathbf{q}')$

with $\boxed{V_A(\mathbf{q}) = -\frac{8\pi\sigma}{\mathbf{q}^4}}$

Leitão, Stadler, Peña, Biernat, PRD **90**, 096003 (2014)
Gross, Milana, PRD **43**, 2401 (1991)
Savkli, Gross, PRC **63**, 035208 (2001)

$$\langle V_L \phi \rangle (\mathbf{p}) = \int \frac{d^3 k}{(2\pi)^3} V_L(\mathbf{p} - \mathbf{k}) \phi(\mathbf{k}) = -8\pi\sigma \int \frac{d^3 k}{(2\pi)^3} \frac{\phi(\mathbf{k}) - \phi(\mathbf{p})}{(\mathbf{p} - \mathbf{k})^4}$$

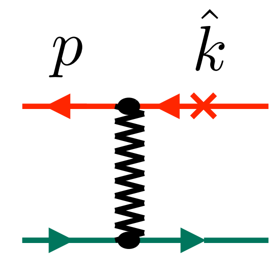
highly singular

automatic subtraction

only a Cauchy principal value singularity remains

Covariant confining kernel in CST

Covariant generalization: $\mathbf{q}^2 \to -q^2$



initial state:
either quark or
antiquark onshell

This leads to a kernel that acts like

$$\langle V_L \phi\rangle(p) = \int \frac{d^3k}{(2\pi)^3}\frac{m}{E_k} V_L(p,\hat{k})\phi(\hat{k}) = -8\pi\sigma \int \frac{d^3k}{(2\pi)^3}\frac{m}{E_k}\frac{\phi(\hat{k})-\phi(\hat{p}_R)}{(p-\hat{k})^4}$$

any regular function

$$\hat{k} = (E_k, \mathbf{k})$$

$$\hat{p}_R = (E_{p_R}, \mathbf{p}_R)$$

$\mathbf{p}_R = \mathbf{p}_R(p_0, \mathbf{p})$ value of $\mathbf{k}$ at which kernel becomes singular

Properties:

- Subtraction regularizes kernel to Cauchy principal value
- Nonrelativistic limit → linear potential
- Satisfies the condition

Shorthand

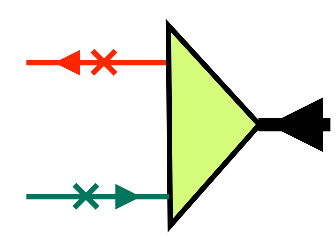
$$\int \frac{d^3k}{(2\pi)^3}\frac{m}{E_k} \to \int_k$$

$$\boxed{\langle V_L\rangle = \int_k V_L(p,\hat{k}) = 0}$$

← corresponds to $\tilde{V}_L^{\rm nr}(r=0) = 0$

But does it still confine?

Yes: the vertex function vanishes if both quarks are on-shell!



More details: Savkli, Gross, PRC **63**, 035208 (2001)

The covariant kernel

Our kernel: $\mathcal{V}(p,k;P) = \frac{3}{4}\mathbf{F}_1 \cdot \mathbf{F}_2 \sum_K V_K(p,k;P)\,\Theta_1^{K(\mu)} \otimes \Theta^K_{2(\mu)}$

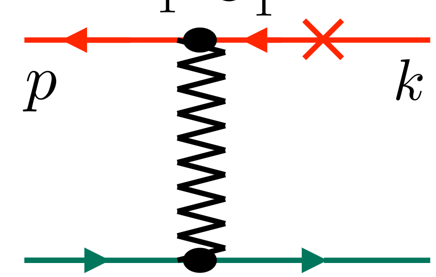
$F_a = \frac{1}{2}\lambda_a$ color SU(3) generators

1 for $q\bar{q}$ color singlets

momentum dependence

Dirac structure

$\Theta_i^{K(\mu)} = \mathbf{1}_i, \gamma_i^5, \gamma_i^\mu$



- Confining interaction: Lorentz (scalar + pseudoscalar) mixed with vector
  Coupling strength σ, mixing parameter y

  $y = 0$ pure S+PS

  $y = 1$ pure V

for correct nonrelativistic limit

$$\mathcal{V}_{\rm L}(p,k;P) = \left[(1-y)\left(\mathbf{1}_1 \otimes \mathbf{1}_2 + \gamma_1^5 \otimes \gamma_2^5\right) - y\,\gamma_1^\mu \otimes \gamma_{\mu 2}\right] V_{\rm L}(p,k;P)$$

equal weight (constraint from chiral symmetry)

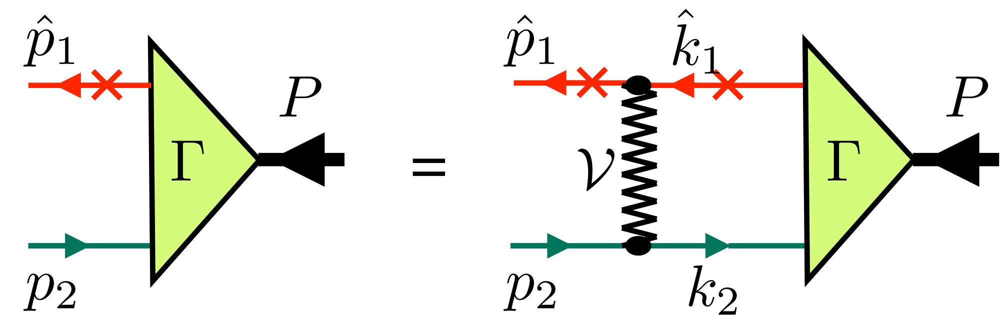
→ E.P. Biernat et al., PRD 90, 096008 (2014)

- One-gluon exchange with constant coupling strength α_s
  \+ Constant interaction (in r-space) with strength C

  } Lorentz vector

$$\mathcal{V}_{\rm OGE+C}(p,k;P) = -\gamma_1^\mu \otimes \gamma_{\mu 2}\left[V_{\rm OGE}(p,k;P) + V_{\rm C}(p,k;P)\right]$$

The One-Channel Spectator Equation (1CSE)



We solve the 1CSE for heavy and heavy-light systems

- Should work well for bound states with at least one heavy quark
- Easier to solve numerically than 2CSE or 4CSE
- C-parity splitting small in heavy quarkonia
- For now with constant constituent quark masses (quark self-energies will be included later)

$$\Gamma(\hat p_1, p_2) = -\int \frac{d^3k}{(2\pi)^3}\frac{m_1}{E_{1k}}\sum_K V_K(\hat p_1, \hat k_1)\Theta_1^{K(\mu)}\frac{m_1+\hat{\not k}_1}{2m_1}\Gamma(\hat k_1, k_2)\frac{m_2+\not k_2}{m_2^2-k_2^2-i\epsilon}\Theta^K_{2(\mu)}$$

$$E_{ik} = \sqrt{m_i^2+\mathbf{k}^2}$$

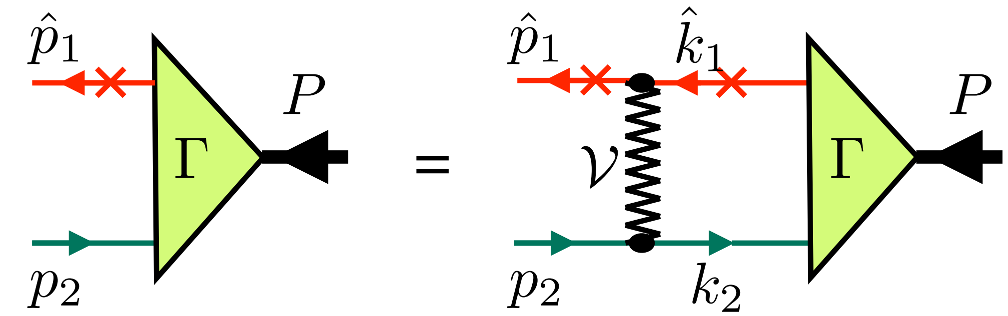
- Momentum-dependence of kernels is also simpler

$$V_{\rm L}(\hat p_1, \hat k_1) = -8\sigma\pi\left[\frac{1}{(\hat p_1-\hat k_1)^4} - \frac{E_{p_1}}{m_1}(2\pi)^3\delta^3(\mathbf{p}_1-\mathbf{k}_1)\int\frac{d^3k_1'}{(2\pi)^3}\frac{m_1}{E_{k_1'}}\frac{1}{(\hat p_1-\hat k_1')^4}\right]$$

$$V_{\rm OGE}(\hat p_1, \hat k_1) = -\frac{4\pi\alpha_s}{(\hat p_1-\hat k_1)^2} \qquad V_{\rm C}(\hat p_1, \hat k_1) = (2\pi)^3\frac{E_{k_1}}{m_1}C\delta^3(\mathbf{p}_1-\mathbf{k}_1)$$

- Linear and OGE kernels need to be regularized
  We chose Pauli-Villars regularizations with parameter $\Lambda = 2m_1$

Numerical solution of the 1CSE



- Work in rest frame of the bound state $P = (\mu, \mathbf{0})$

- Use ρ-spin decomposition of the propagator

$$\frac{m_2 + \not{k}_2}{m_2^2 - k_2^2 - i\epsilon} = \frac{m_2}{E_{2k}} \sum_{\rho,\lambda_2} \rho \frac{u_2^\rho(\mathbf{k},\lambda_2)\bar{u}_2^\rho(\mathbf{k},\lambda_2)}{E_{2k} - \rho k_{20} - i\epsilon}$$

$u^+(\mathbf{k},\lambda) \equiv u(\mathbf{k},\lambda)$
$u^-(\mathbf{k},\lambda) \equiv v(-\mathbf{k},\lambda)$
ρ-spinors with helicity λ

$$E_{ik} = \sqrt{m_i^2 + \mathbf{k}^2}$$

- Project 1CSE onto ρ-spin helicity channels

$$\Gamma^{+\rho'}_{\lambda\lambda'}(p) \equiv \bar{u}_1^+(\mathbf{p},\lambda)\Gamma(p)u_2^{\rho'}(\mathbf{p},\lambda')$$

- Define relativistic “wave functions”

$$\Psi^{+\rho}_{\lambda\lambda'}(p) \equiv \sqrt{\frac{m_1 m_2}{E_{1p}E_{2p}}} \frac{\rho}{E_{2p} - \rho(E_{1p} - \mu)} \Gamma^{+\rho}_{\lambda\lambda'}(p)$$

The 1CSE becomes a generalized linear EV problem for the mass eigenvalues *μ*

- Switch to basis of eigenstates of total orbital angular momentum *L* and of total spin *S* (not necessary, but useful for spectroscopic identification of solutions)

- Expand wave functions in a basis of B-splines (modified for correct asymptotic behavior) and solve eigenvalue problem → expansion coefficients and mass eigenvalues

Global fits with fixed quark masses and y=0

First step: we perform global fits to the heavy + heavy-light meson spectrum

Adjustable model parameters: $\sigma \quad \alpha_s \quad C$

Model parameters not adjusted in the fits:

Constituent quark masses (in GeV) m_b=4.892, m_c=1.600, m_s=0.448, m_q=0.346

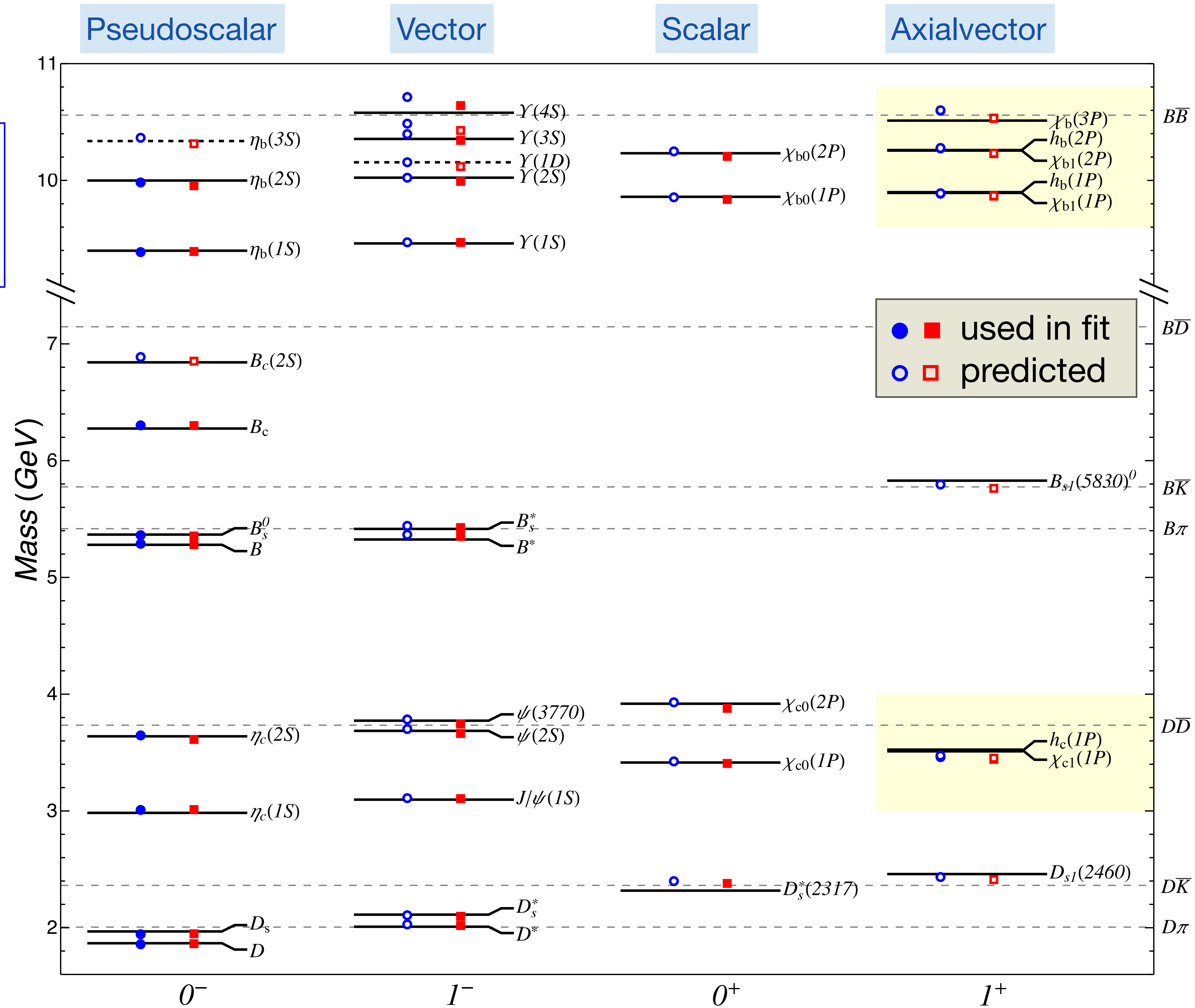
Scalar + pseudoscalar confinement $y = 0$

"q" means "light quark"
m_u=m_d=m_q

- Model P1: fitted to 9 pseudoscalar meson masses only
- Model PSV1: fitted to 25 pseudoscalar, vector, and scalar meson masses

Global fits with fixed quark masses and scalar confinement (y=0)

Blue: model P1 fitted to 9 0^-(P) only



Red: model PSV2 fitted to 25 P+S+V

Global fits with fixed quark masses and y=0

The results of the two fits are remarkably similar!

Model	σ [GeV^2]	α_s	C [GeV]
P1	0.2493	0.3643	0.3491
PSV1	0.2247	0.3614	0.3377

rms differences to experimental masses:

Model	Δ_{rms} [GeV]
P1	0.036
PSV1	0.030

- Kernel parameters are already well determined through pseudoscalar states ($J^P = 0^-$)

Almost 100% L=0, S=0
(S-wave, spin singlet)

$$\langle 0^- | \mathbf{L} \cdot \mathbf{S} | 0^- \rangle = 0$$ Spin-orbit force vanishes

$$\langle 0^- | S_{12} | 0^- \rangle = 0$$ Tensor force vanishes

$$\langle 0^- | \mathbf{S}_1 \cdot \mathbf{S}_2 | 0^- \rangle = -3/4$$ Spin-spin force acts in singlet only

- Good test for a covariant kernel:

Pseudoscalar states do not constrain spin-orbit and tensor forces, and cannot separate spin-spin from central force.

But they should be determined through covariance.

Model P1 indeed predicts spin-dependent forces correctly!

Published in: Leitão, Stadler, Peña, Biernat, Phys. Lett. B **764** (2017) 38

Fits with variable quark masses and confinement (S+PS)-V mixing *y*

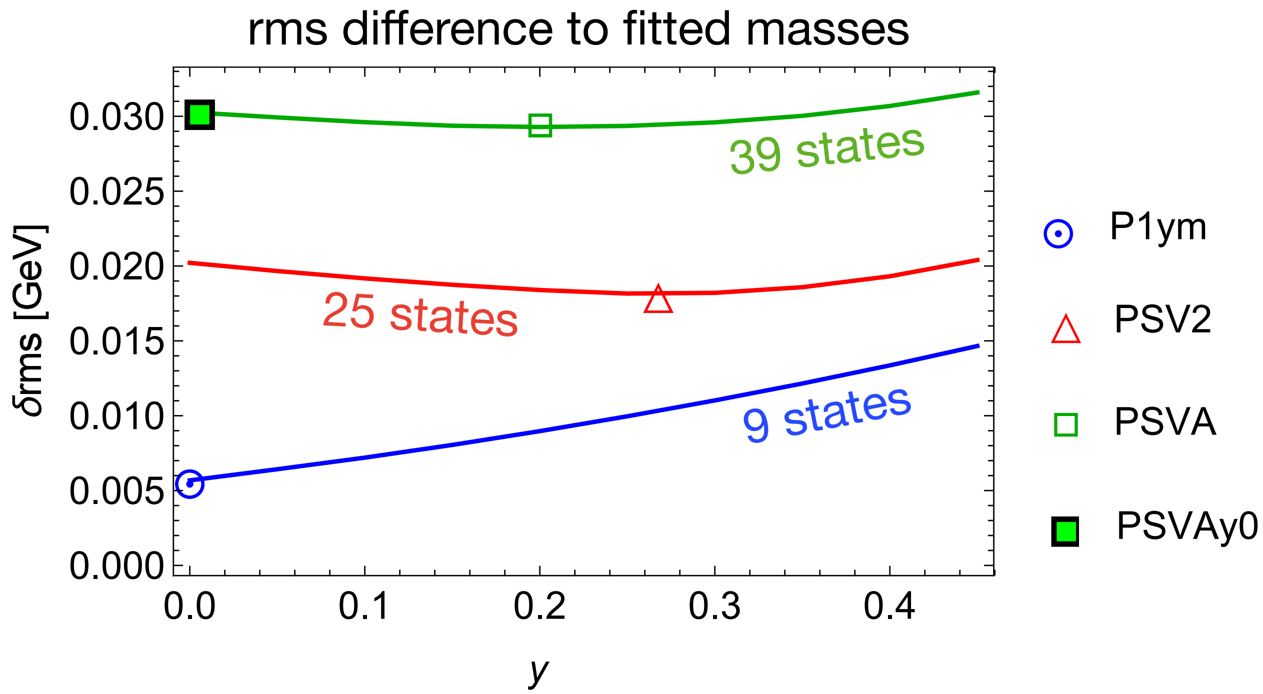
In a new series of fits we treat quark masses and mixing parameter *y* as adjustable parameters.

Model	σ [GeV^2]	α_s	C [GeV]	y	m_b [GeV]	m_c [GeV]	m_s [GeV]	m_q [GeV]	$n_{\text{statesfit}}$	δ_{rms} [GeV]	Δ_{rms} [GeV]
P1	0.2493	0.3643	0.3491	0.0000**	4.892**	1.600**	0.4478**	0.3455**	9	0.017	0.037
P1ym○	0.2235	0.3941	0.0591	0.0000	4.768	1.398	0.2547	0.1230	9	0.006	0.041
PSV1	0.2247	0.3614	0.3377	0.0000**	4.892**	1.600**	0.4478**	0.3455**	25	0.028	0.036
PSV1ym	0.1893	0.4126	0.1085	0.2537	4.825	1.470	0.2349	0.1000	25	0.022	0.033
PSV2△	0.2017	0.4013	0.1311	0.2677	4.822	1.464	0.2365	0.1000	24	0.018	0.033
PSVA□	0.2022	0.4129	0.2145	0.2002	4.875	1.553	0.3679	0.2493	39	0.030	0.030
PSVAy0■	0.2058	0.4172	0.2821	0.0000**	4.917	1.624	0.4616	0.3514	39	0.031	0.031

include AV states in fit

**parameter held fixed during fit

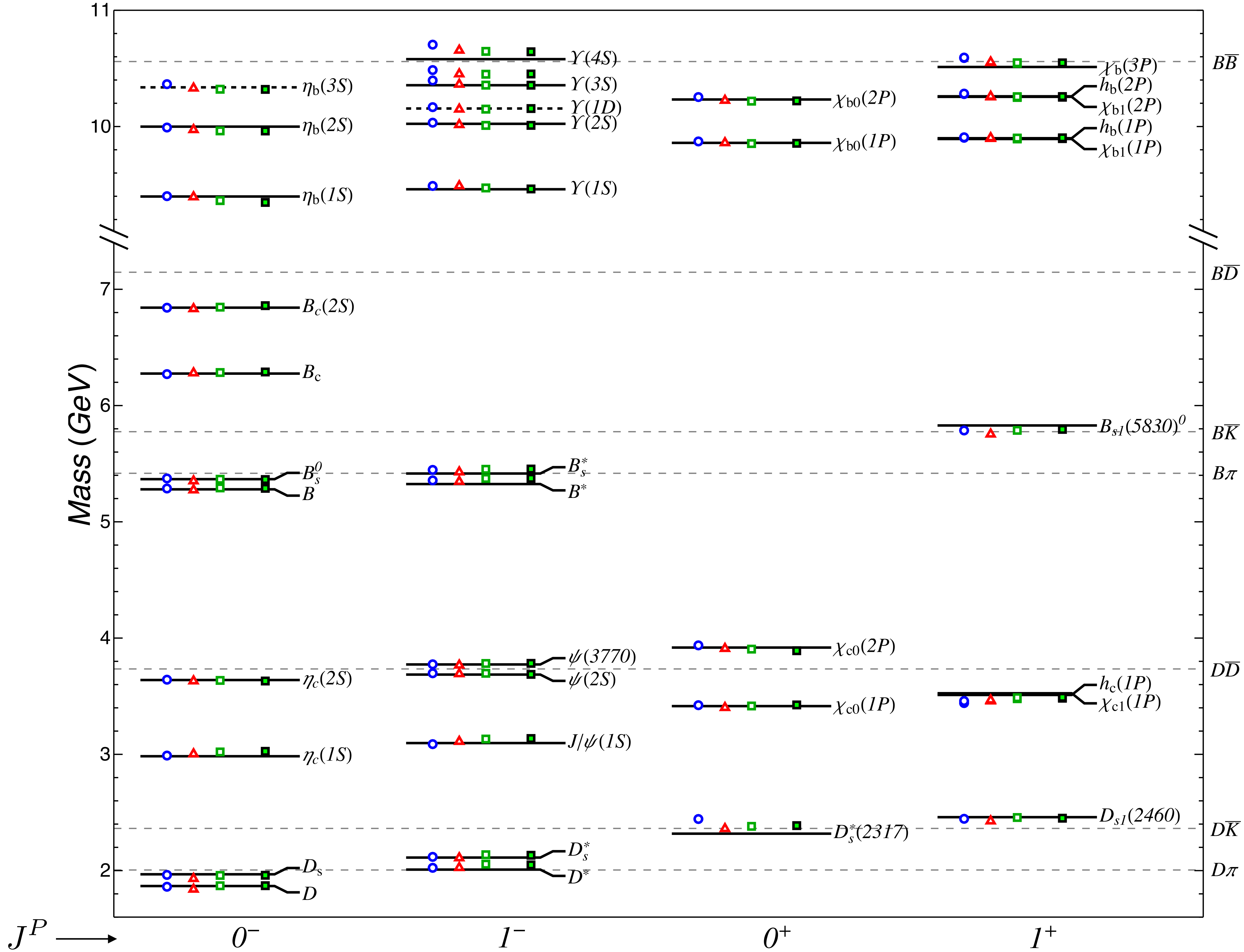
y held fixed, other parameters refitted



- Quality of fits not much improved
- Best model PSVA has y=0.20, but minimum is very shallow

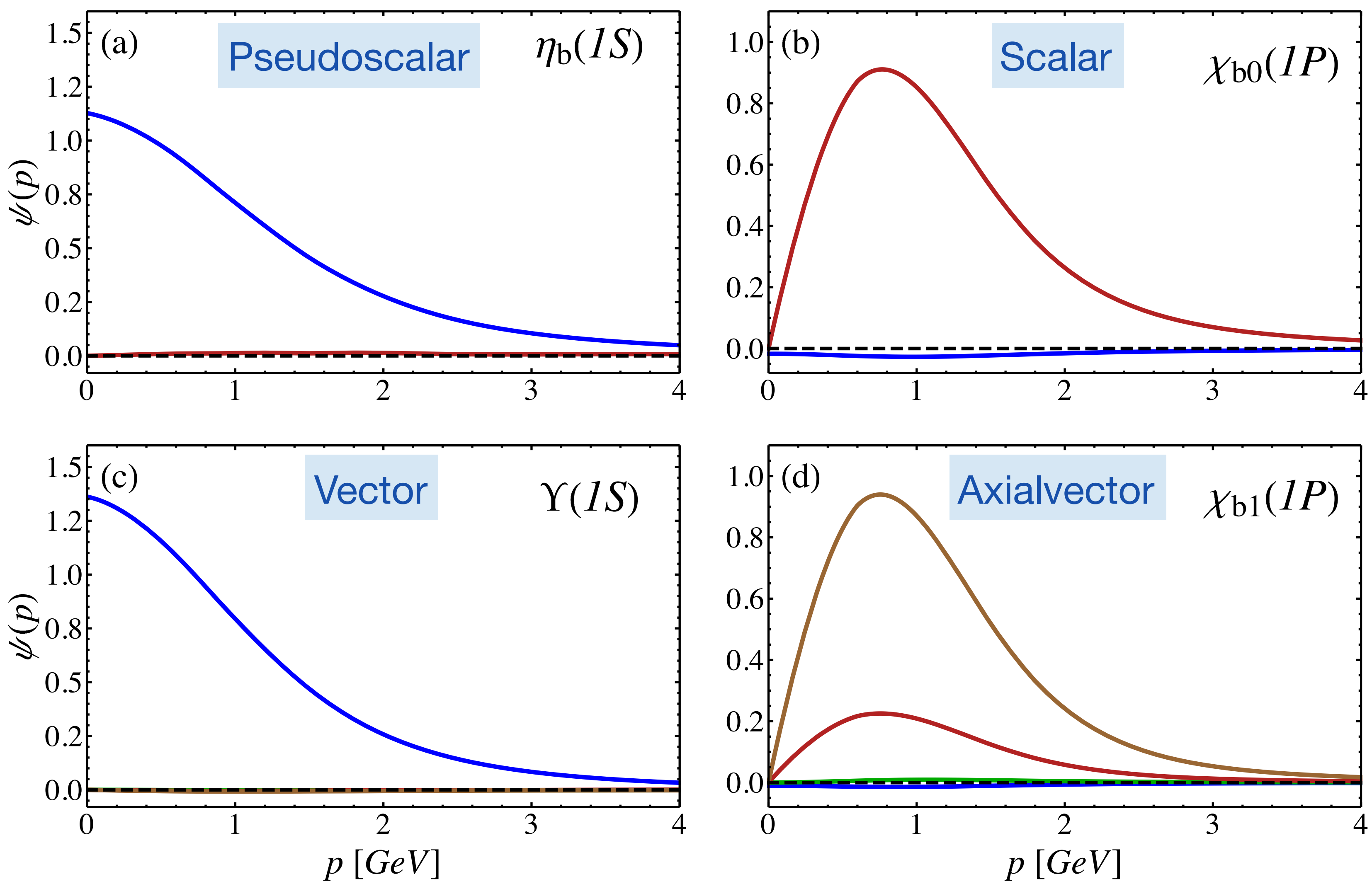
y and quark masses are not much constrained by mass spectrum.

Mass spectra of heavy and heavy-light mesons



Bottomonium ground-state wave functions

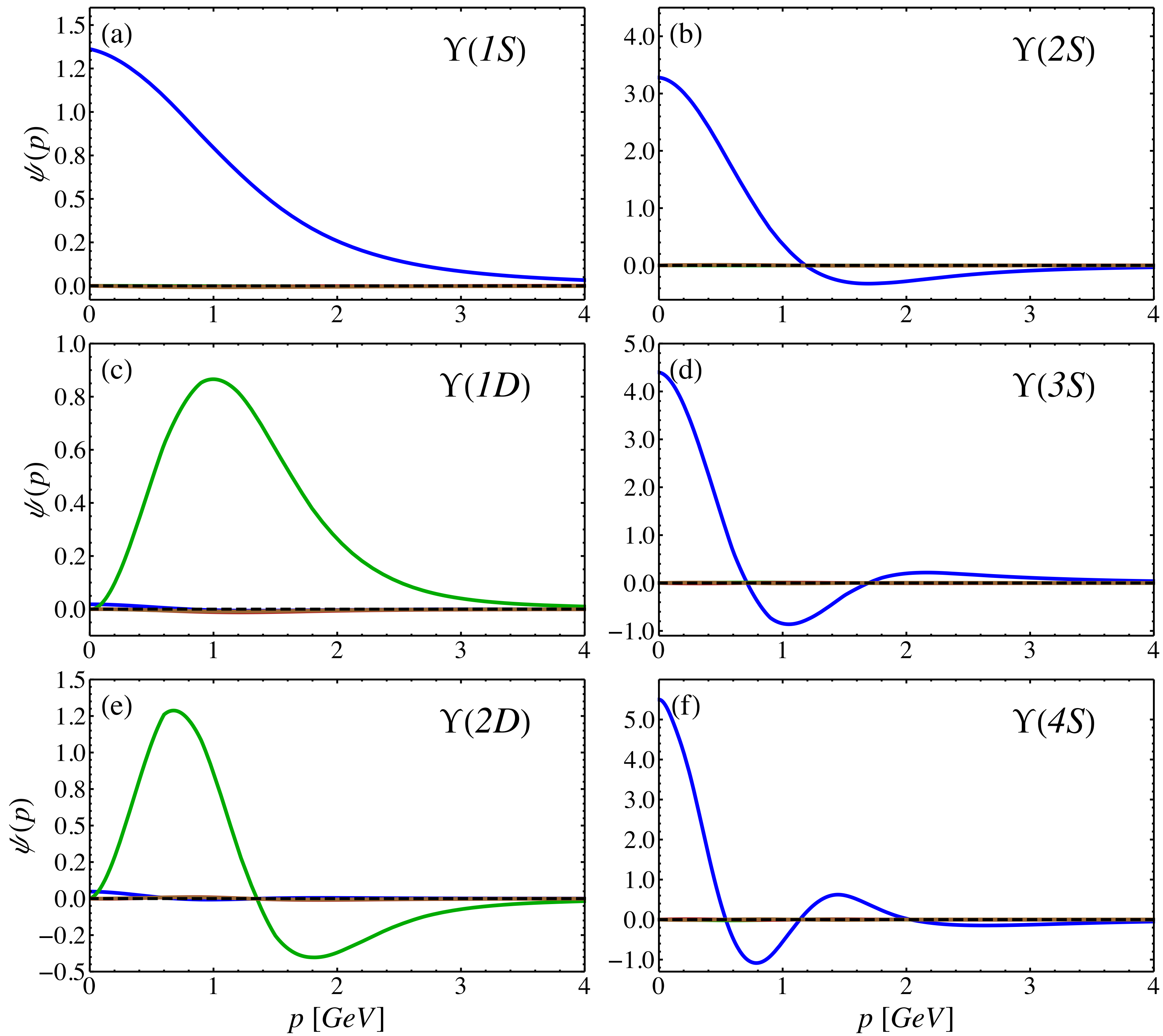
Model PSVA

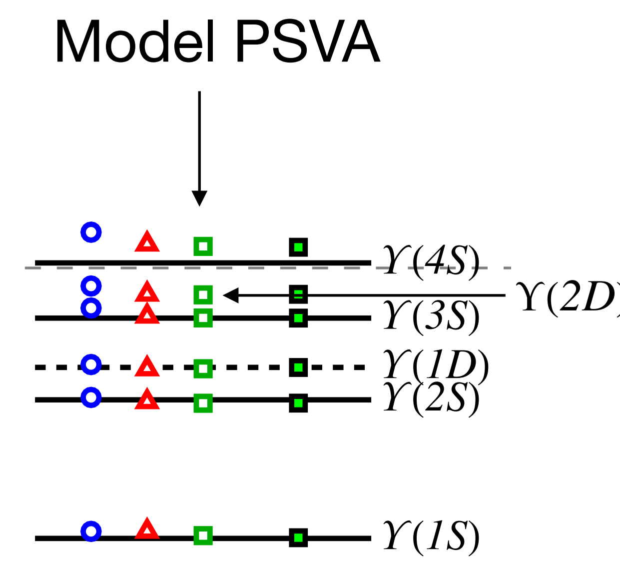


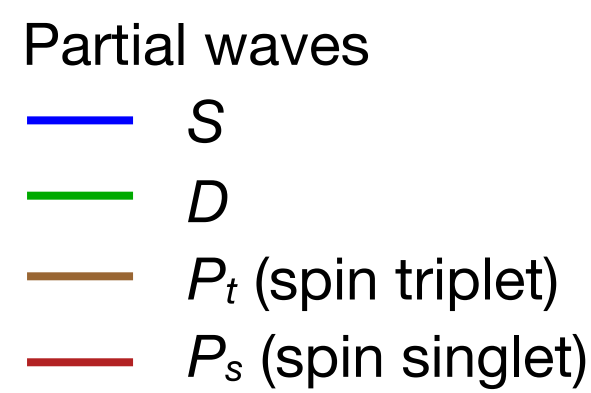
Partial waves
- S
- P

Partial waves
- S
- D
- P_t (spin triplet)
- P_s (spin singlet)

Radial excitations in vector bottomonium

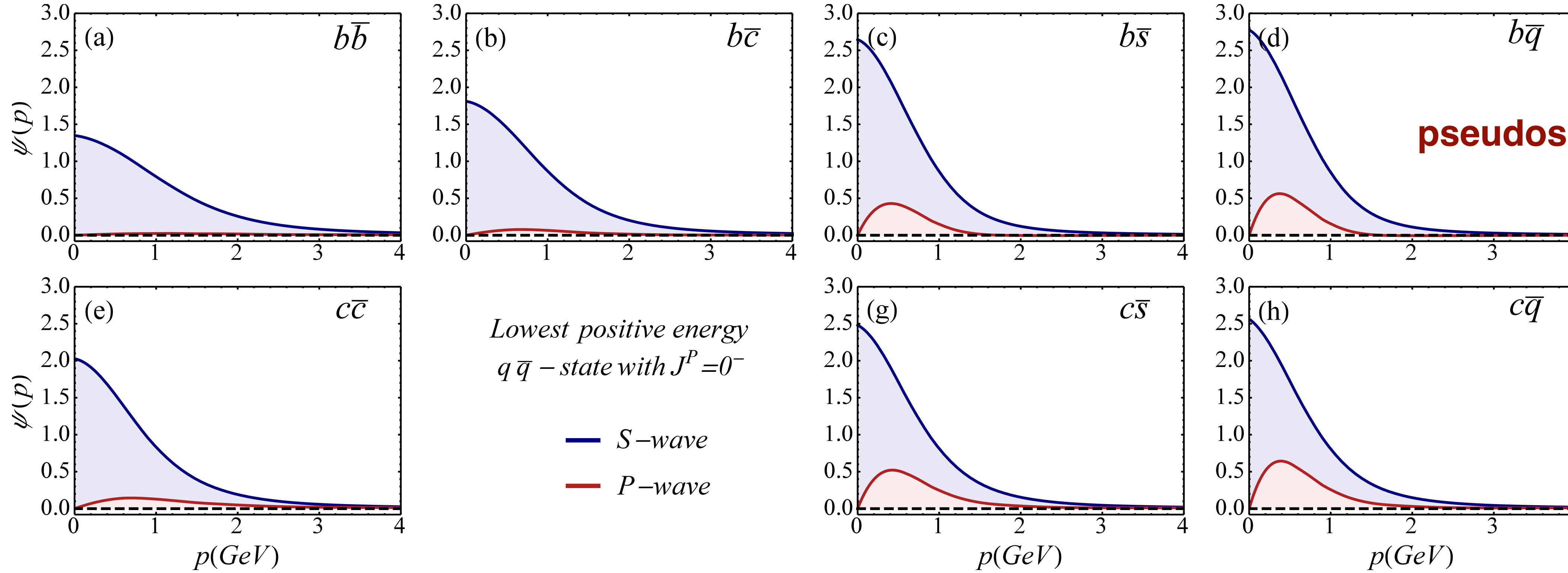




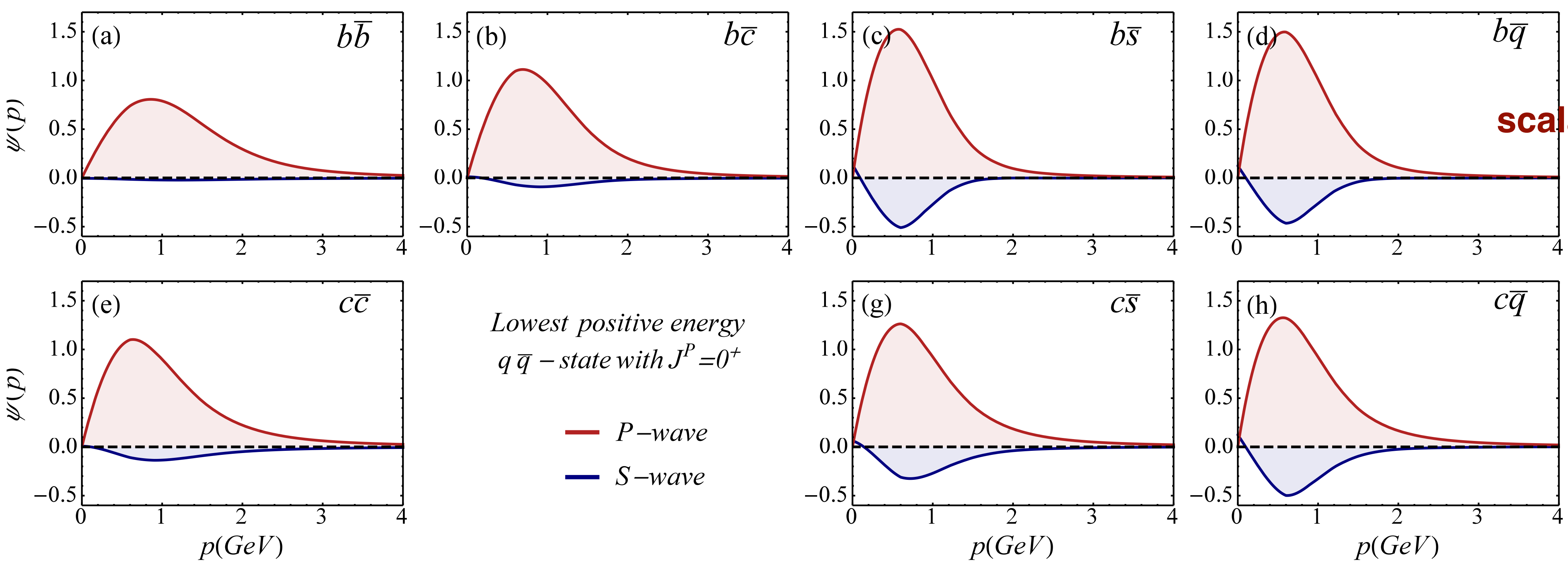


Importance of relativistic components

Ground-state wave functions of model PSVA.



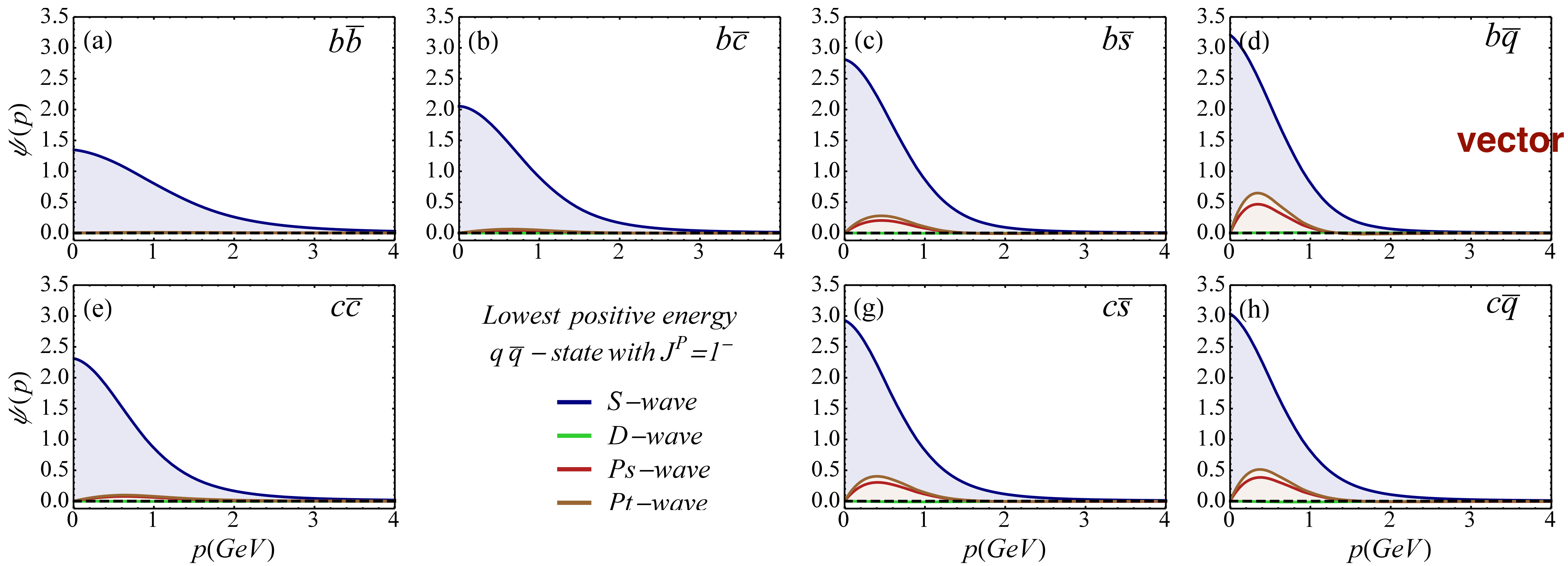
pseudoscalar

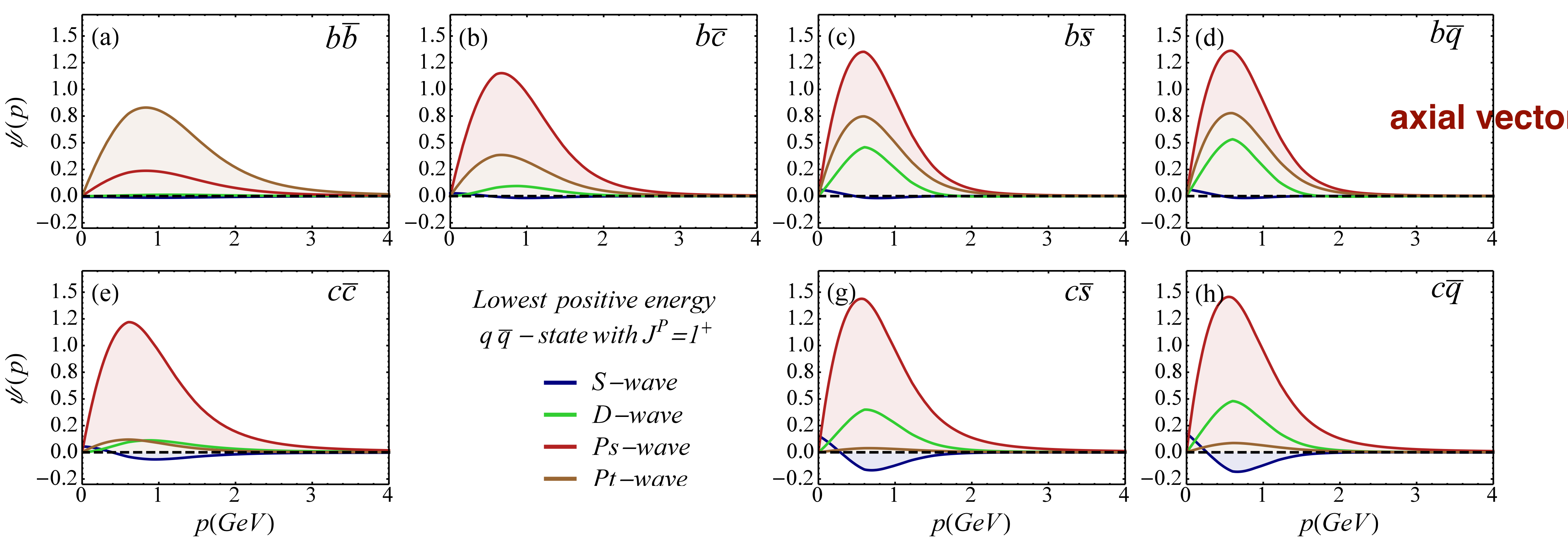


scalar

Importance of relativistic components

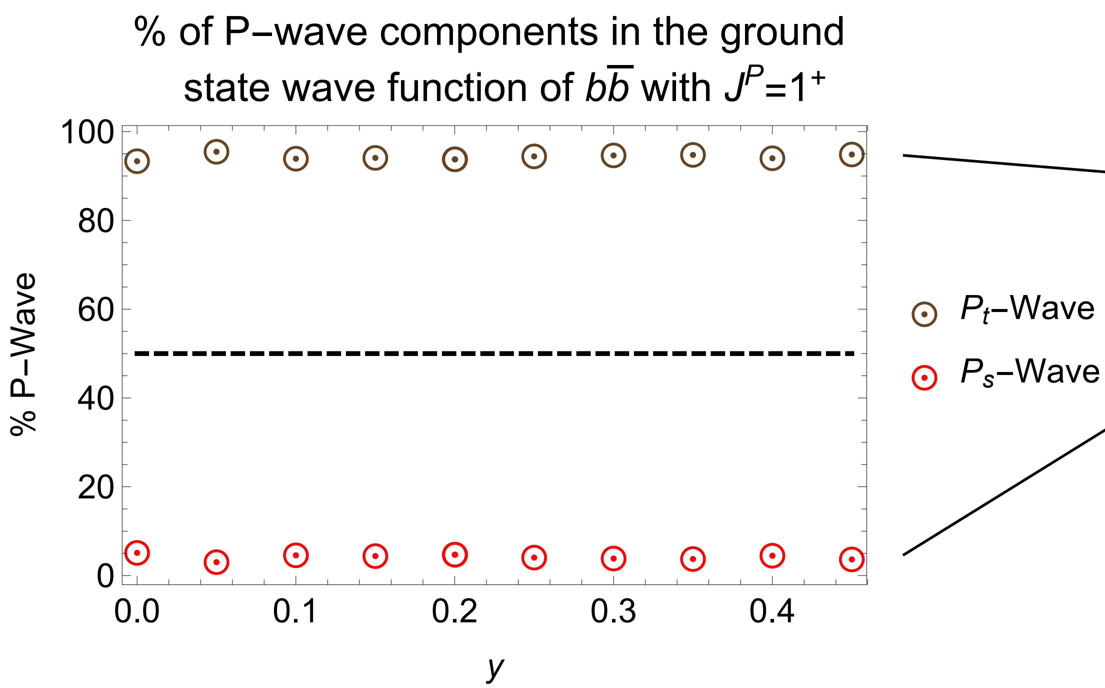
Ground-state wave functions of model PSVA.





Sensitivity to *y*

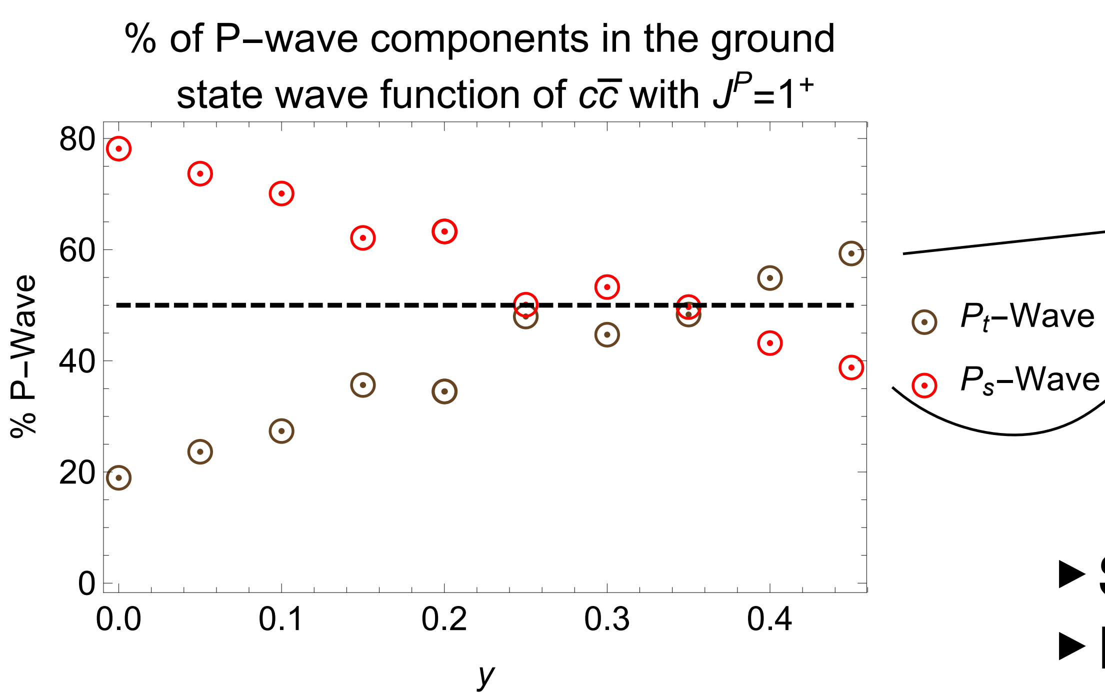
Axial-vector ground-state wave functions of model PSVA.



Bottomonium

L=1, S=1 χ_{b1} (1^{++})

L=1, S=0 h_{b1} (1^{+-})



Charmonium

L=1, S=1 χ_{c1} (1^{++})

L=1, S=0 h_{c1} (1^{+-})

Triplet becomes ground-state only for larger *y*

- Sign for interesting sensitivity to y in details
- Need C-symmetric equation to confirm

Outlook

The results so far are very encouraging, but much work remains to be done:

- Calculation of tensor mesons (spin ≥ 2)
- Inclusion of running quark-gluon coupling
- Implement charge-conjugation symmetry
- Study more constraints on Lorentz structure of confining interaction
- Extension of current model to the light-quark sector
- Calculation of self-consistent dynamical quark masses
- Calculation of meson decay properties
- Calculation of consistent photon-quark current, and then e.m. form factors
- Calculation of parton distribution functions
- Calculate exotic mesons (quark-antiquark states with exotic J^{PC})