TESTING THE COSMOLOGICAL PRINCIPLE

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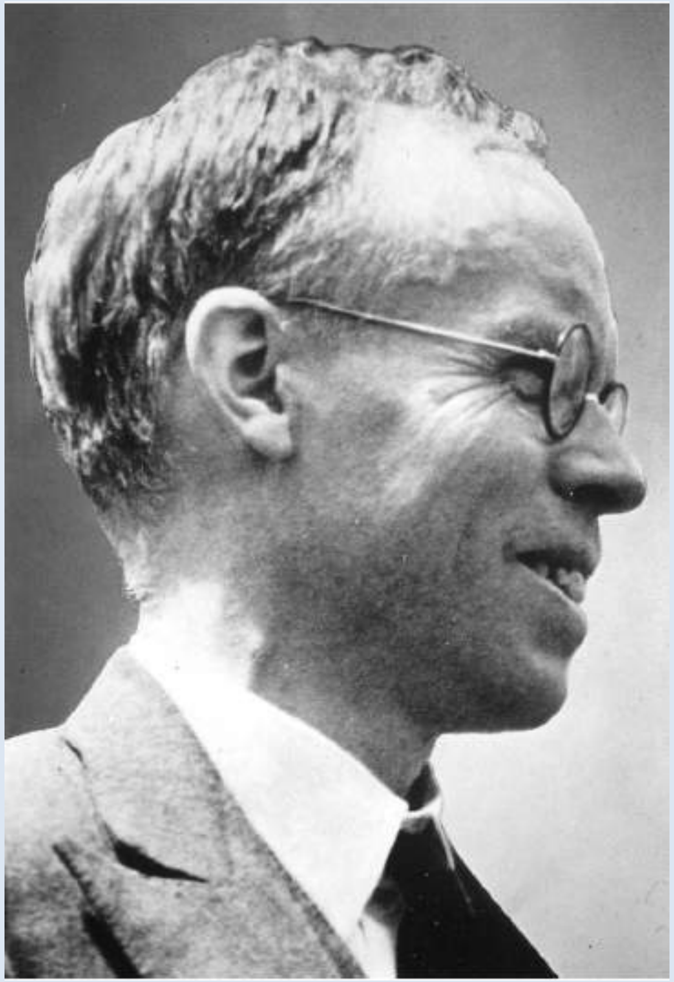
None of us can understand why there is a Universe at all, why anything should exist; that is the ultimate question. But while we cannot answer this question, we can at least make progress with the next simpler one, of what the Universe as a whole is like.

Dennis Sciama (1978)

Particle, Astroparticle & Cosmology Symposium, Tallinn, 18-22 June 2018

*… the Universe must appear to be the same to all observers wherever they are. This ‘**cosmological principle**’ …*



Edward Arthur Milne

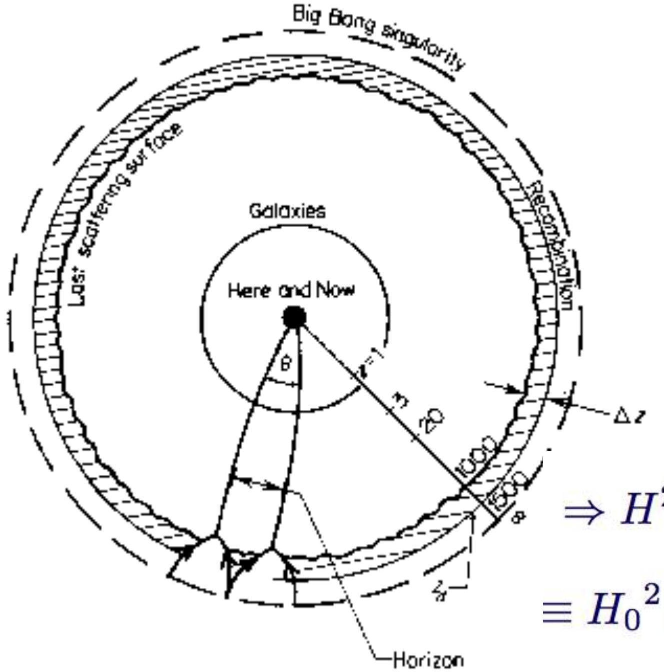
Rouse Ball Professor of Mathematics & Fellow of Wadham College, Oxford, 1929-50

Standard cosmological model

Universe is isotropic + Universe is homogeneous (when averaged on large scales)
⇒ Maximally-symmetric space-time and ideal fluid energy-momentum tensor

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu$$
$$= a^2(\eta)\left[d\eta^2 - d\bar{x}^2\right]$$
$$a^2(\eta) d\eta^2 \equiv dt^2$$

Robertson-Walker



$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \lambda g_{\mu\nu}$$
$$= 8\pi G_{\mathrm{N}} T_{\mu\nu}$$

Einstein

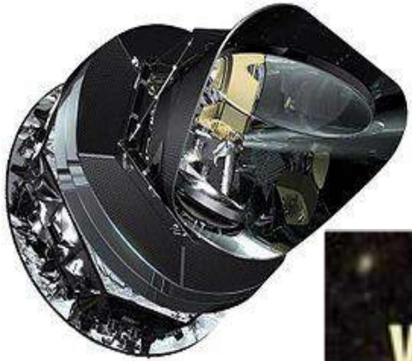
$$\Rightarrow H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_{\mathrm{N}} \rho_{\mathrm{m}}}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$
$$\equiv {H_0}^2 \left[\Omega_{\mathrm{m}}(1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda\right]$$

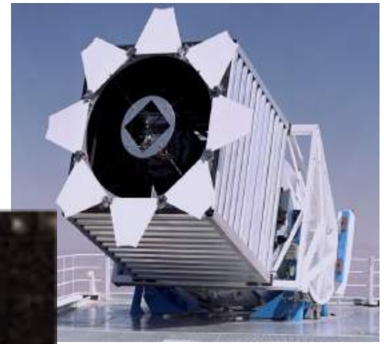
So the **Friedmann-Lemaitre** equ**ation** ⇒ Cosmic sum rule: $\mathbf{\Omega_{matter} + \Omega_{curvature} + \Omega_\Lambda = 1}$

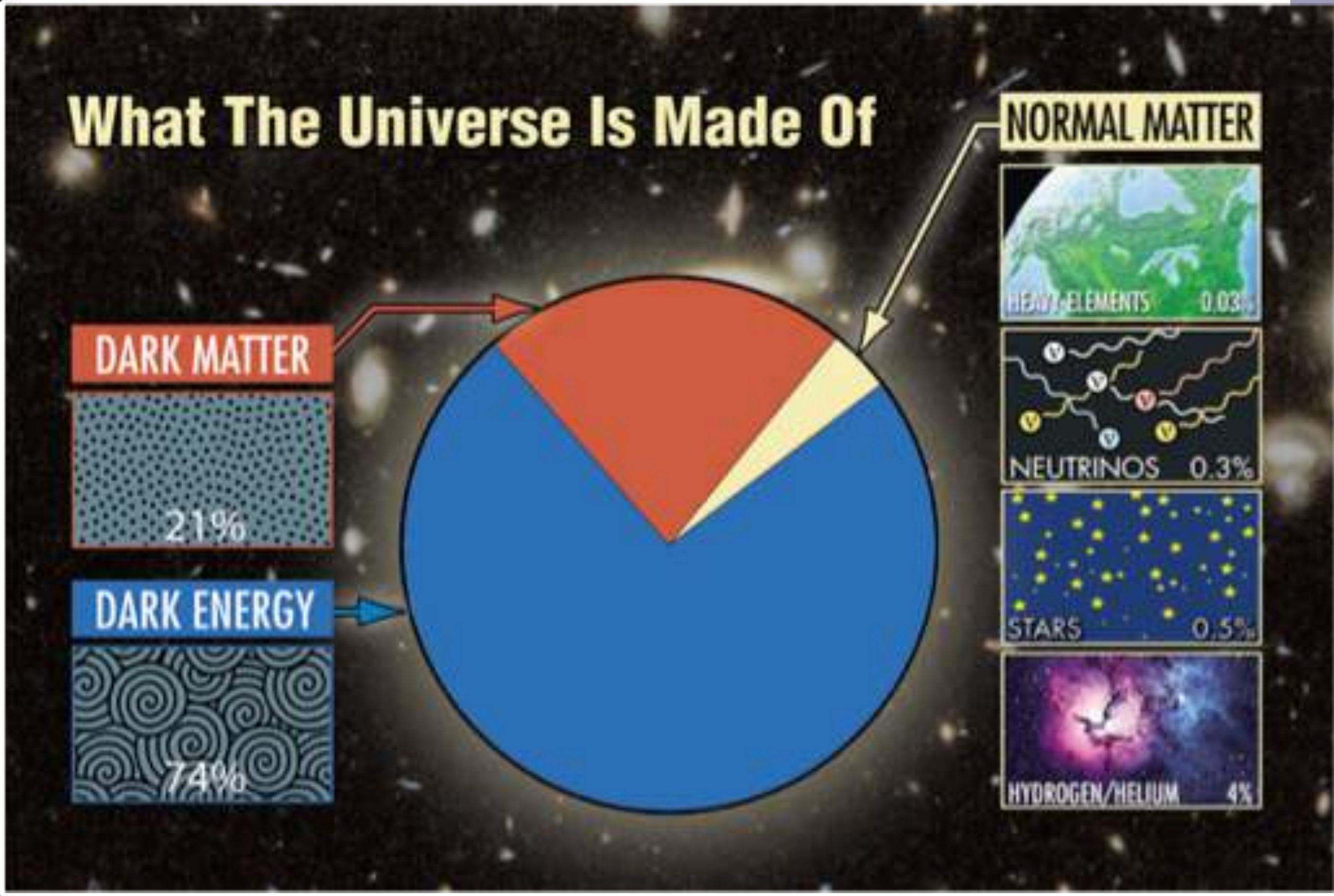
We observe ~zero curvature (CMB fluctuations) + insufficient matter to make up critical density

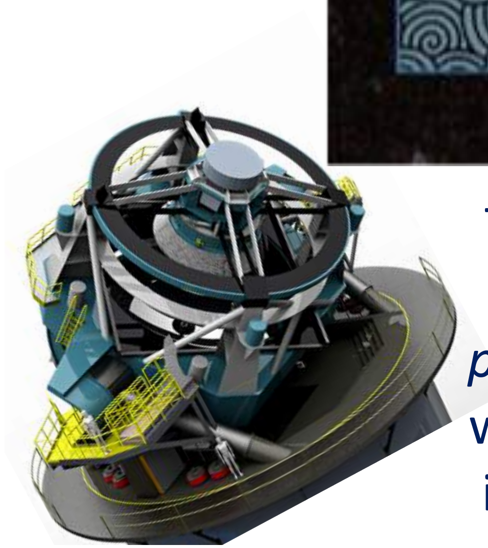
➙ **Universe is dominated by dark energy** with: $\mathbf{\Omega_\Lambda = 1 - \Omega_m - \Omega_k \sim 0.7} \Rightarrow \Lambda \sim 2{H_0}^2$

> Since 1998 (Riess *et al.*[1], Perlmutter *et al.*[2]), surveys of cosmologically distant Type Ia supernovae (SNe Ia) have indicated an acceleration of the expansion of the Universe, distant SNe Ia being dimmer that expected in a decelerating Universe. With the assumption that the Universe can be described on average as isotropic and homogeneous, this acceleration implies either the existence of a fluid with negative pressure usually called "Dark Energy", a constant in the equations of general relativity or modifications of gravity on cosmological scales.

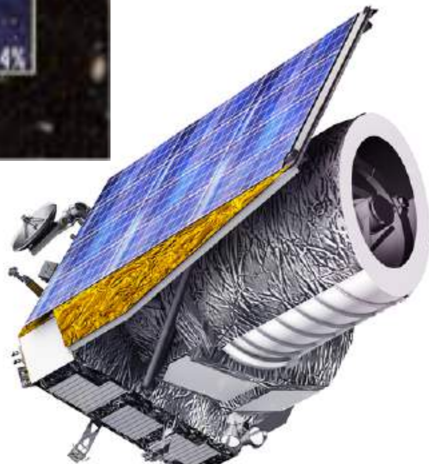




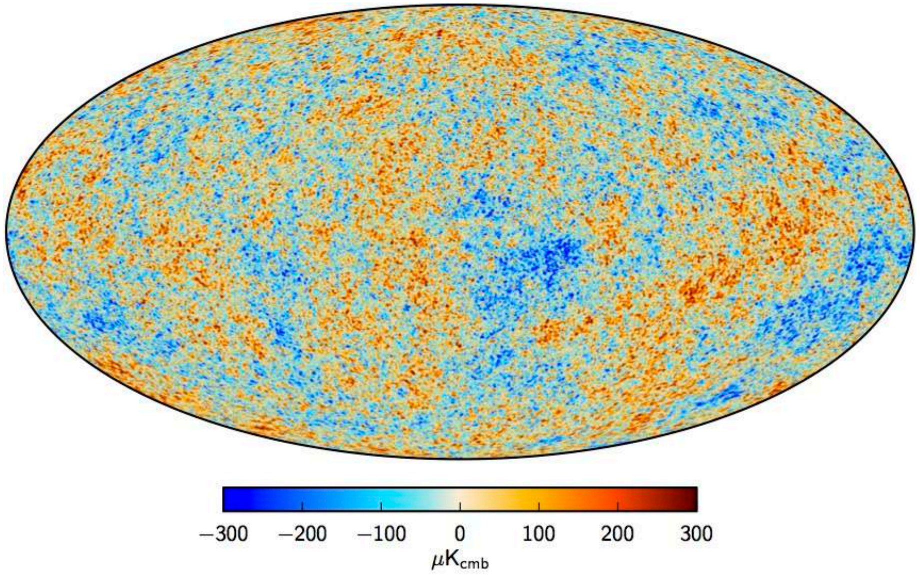


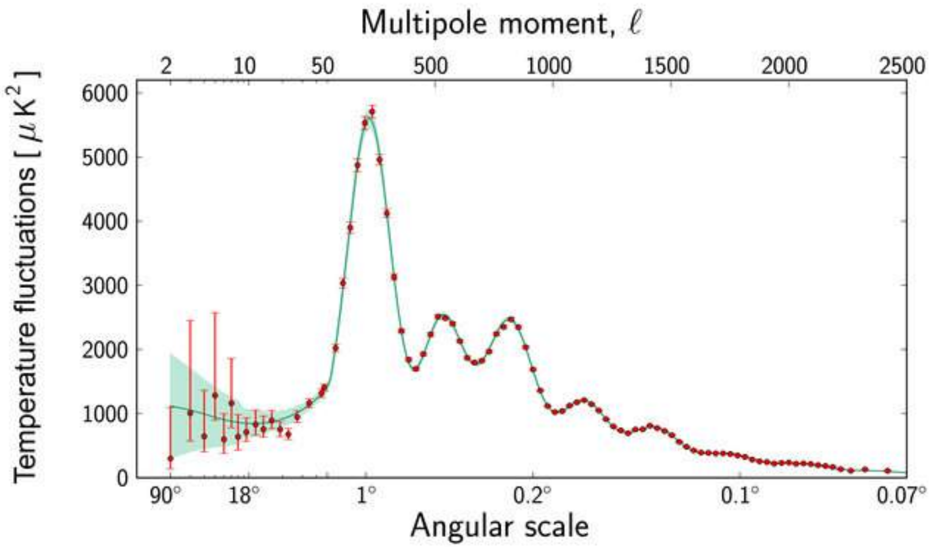


There has been substantial investment in major satellites and telescopes to *measure the parameters* of the 'standard cosmological model' with increasing 'precision'… but surprisingly little interest in ***testing its foundational assumptions***



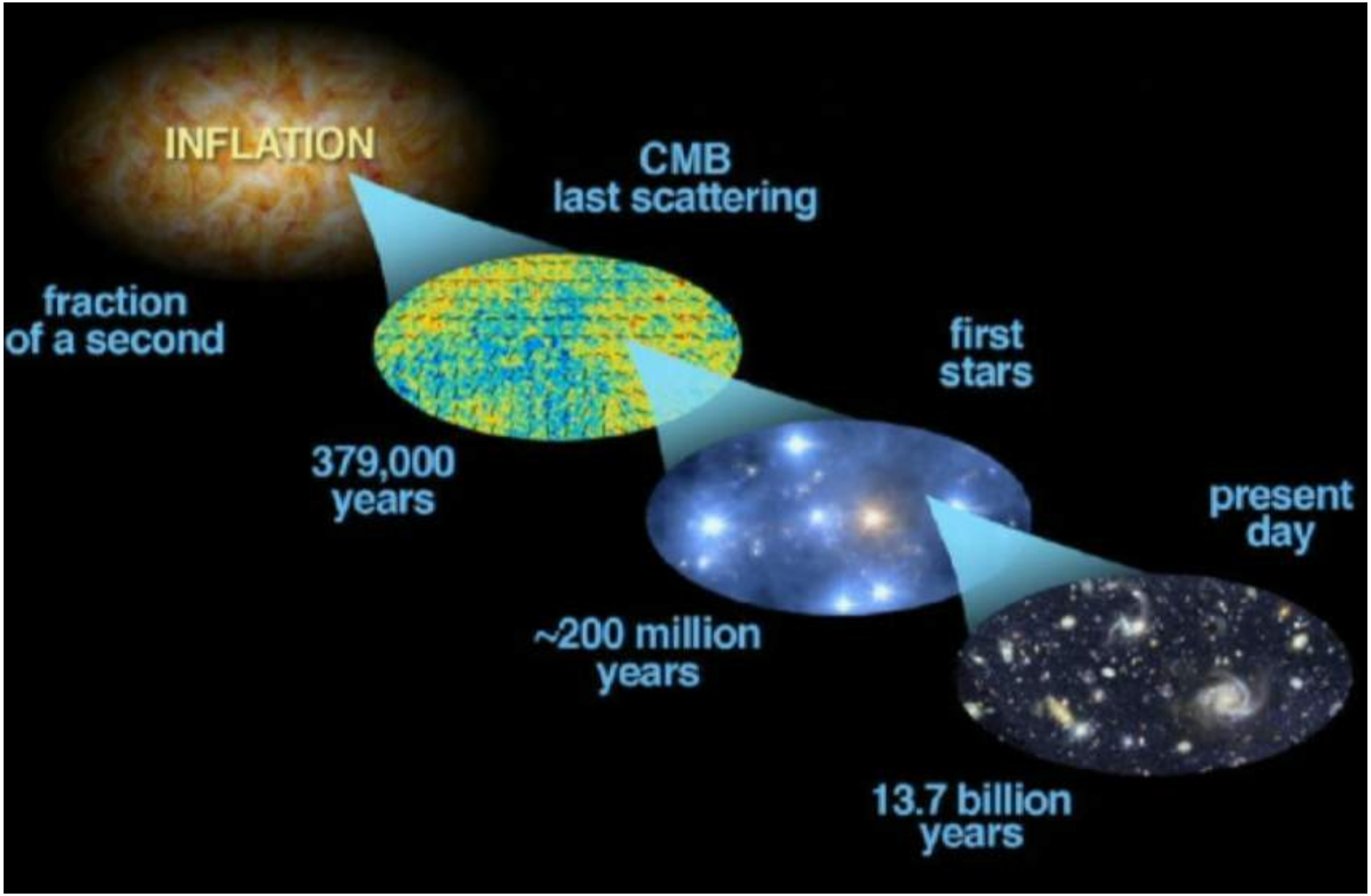
"Data from the Planck satellite show the universe to be highly isotropic" (Wikipedia)





We observe a statistically isotropic *Gaussian* random field of small temperature fluctuations (fully quantified by the 2-point correlations ➛ angular power spectrum)

Standard model of structure formation

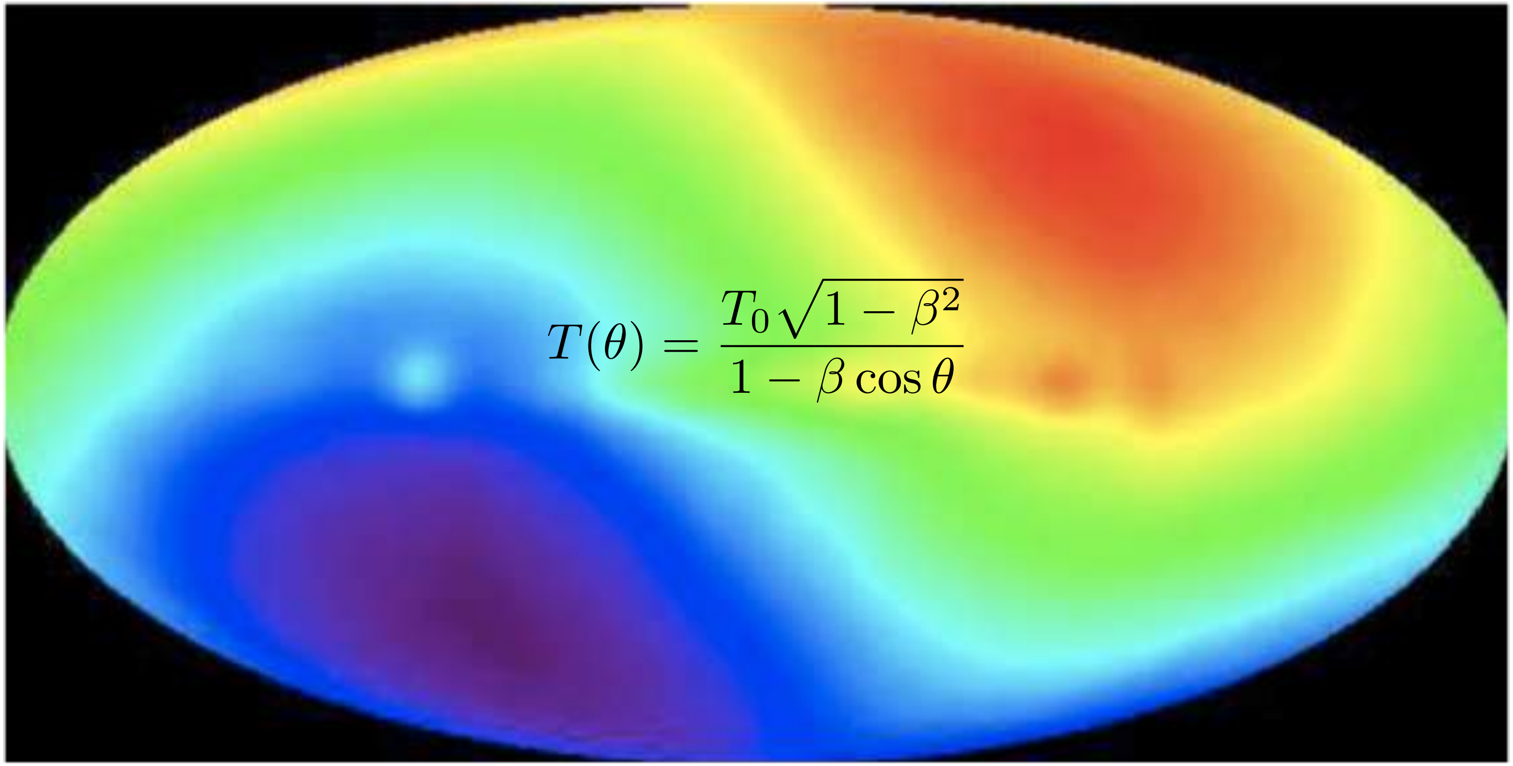


The tiny **CMB temperature fluctuations** are understood as due to **scalar density perturbations** with an ~scale-invariant spectrum which were generated during an early phase of inflationary expansion … these perturbations have subsequently grown into the **large-scale structure** of galaxies observed today through **gravitational instability** in a sea of **dark matter**

But the CMB sky is in fact *very* anisotropic

There is a ~100 times *bigger* signal than the fluctuations in the form of a dipole with

$$\Delta T/T \sim 10^{-3}$$



$$T(\theta) = \frac{T_0\sqrt{1-\beta^2}}{1-\beta\cos\theta}$$

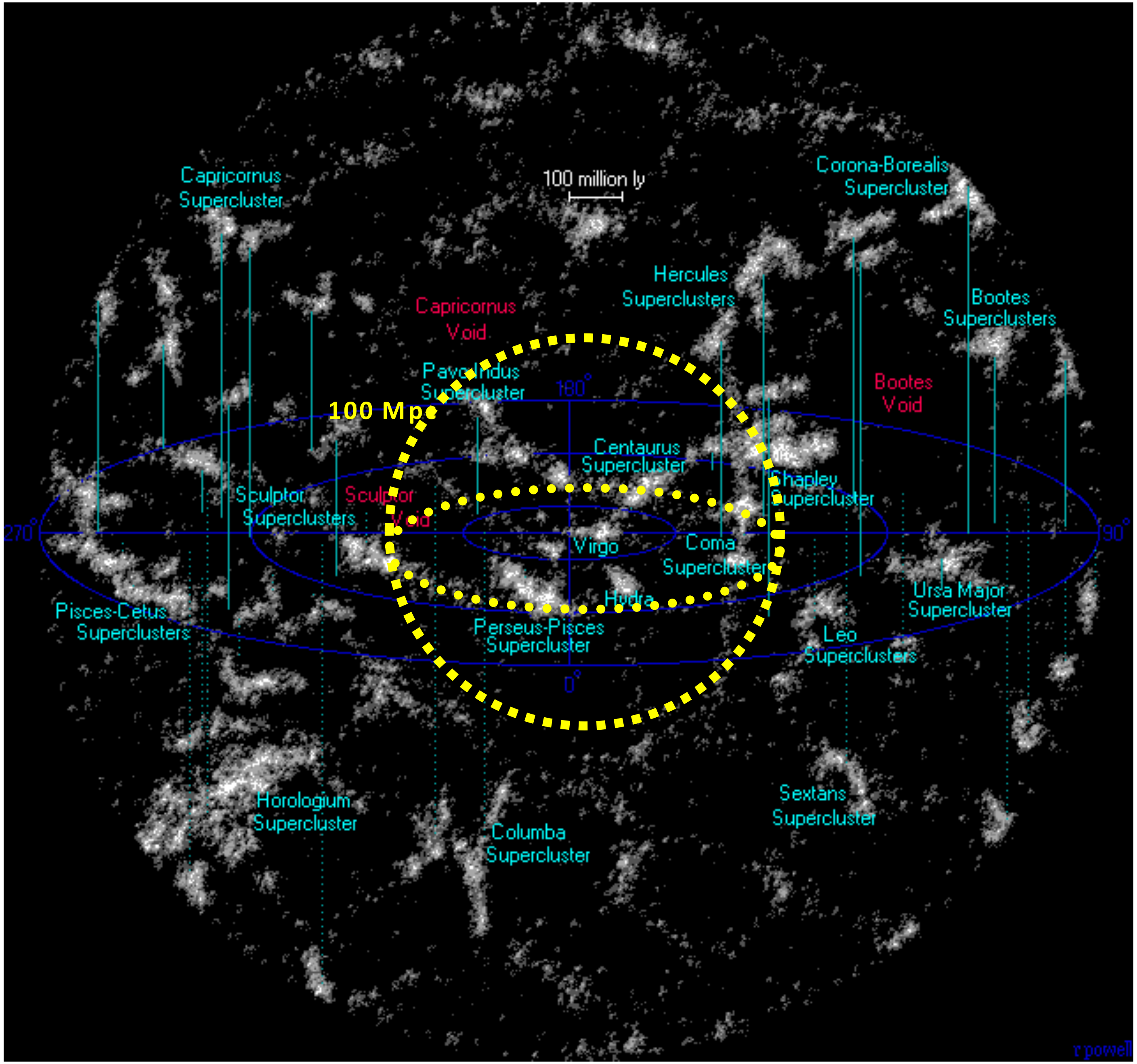
Stewart & Sciama 1967, Peebles & Wilkinson 1968

This is *interpreted* as due to our motion at 368 km/s wrt the frame in which the CMB is truly isotropic ⇒ motion of the Local Group at 627 km/s towards l=263.85°, b=48.25°

This motion is presumed to be due to ***local inhomogeneity*** in the matter distribution

Its scale – *beyond* which we converge to the CMB frame – is supposedly of $\mathcal{O}(100)$ Mpc (Counts of galaxies in SDSS & WiggleZ surveys are said to scale as ~r^3 on larger scales)

THIS IS WHAT OUR UNIVERSE *ACTUALLY* LOOKS LIKE (OUT TO ~300 MPC)



Our motion is towards the Shapley supercluster, supposedly due to a 'Great Attractor' beyond …

Theory of peculiar velocity fields

In linear perturbation theory, the growth of the density contrast $\delta(x) = [\rho(x) - \bar{\rho}]/\bar{\rho}$ as a function of commoving coordinates and time is governed by:

$$\frac{\partial^2 \delta}{\partial t^2} + 2H(t)\frac{\partial \delta}{\partial t} = 4\pi G_{\mathrm{N}}\bar{\rho}\delta$$

We are interested in the 'growing mode' solution – the density contrast grows self-similarly and so does the perturbation potential and its gradient … so the direction of the acceleration (and its integral – the peculiar velocity) remains *unchanged*.

The peculiar velocity field is related to the density contrast as:

$$v(\mathbf{x}) = \frac{2}{3H_0}\int \mathrm{d}^3 y \frac{\mathbf{x}-\mathbf{y}}{|\mathbf{x}-\mathbf{y}|^3}\delta(\mathbf{y}),$$

So the peculiar Hubble flow, $\boldsymbol{\delta}H(\boldsymbol{x}) = H_{\mathrm{L}}(\boldsymbol{x}) - H_0$ ($\Rightarrow$ trace of the shear tensor), is:

$$\delta H(\mathbf{x}) = \int \mathrm{d}^3\mathbf{y}\ \mathbf{v}(\mathbf{y}) \cdot \frac{\mathbf{x}-\mathbf{y}}{|\mathbf{x}-\mathbf{y}|^2} W(\mathbf{x}-\mathbf{y}),$$

where $H_{\mathrm{L}}(\mathbf{x})$ is the **local** value of the Hubble parameter and $W(\mathbf{x}-\mathbf{y})$ is the 'window function' (e.g. $\boldsymbol{\theta}(R - |\mathbf{x}-\mathbf{y}|)\ (4\pi R^3/3)^{-1}$ for a volume-limited survey, out to distance R)

Theory of peculiar velocity fields (cont.)

Rewrite in terms of the Fourier transform $\delta(\mathbf{k}) \equiv (2\pi)^{3/2} \int \mathrm{d}^3x \; \delta(\mathbf{x}) \mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}}$:

$$\frac{\delta H}{H_0} = \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}} \delta(k) \mathcal{W}_H(kR) \mathrm{e}^{ik.x}, \; \mathcal{W}_H(x) = \frac{3}{x^3}\left(\sin x - \int_o^x \mathrm{d}y \frac{\sin y}{y}\right)$$

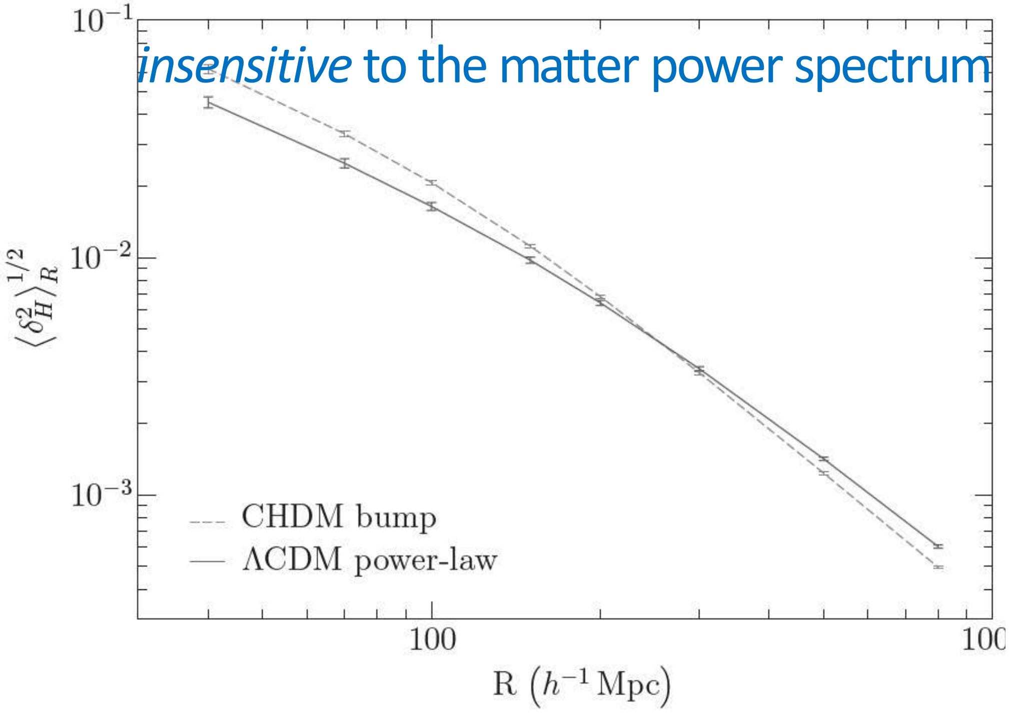
Window function

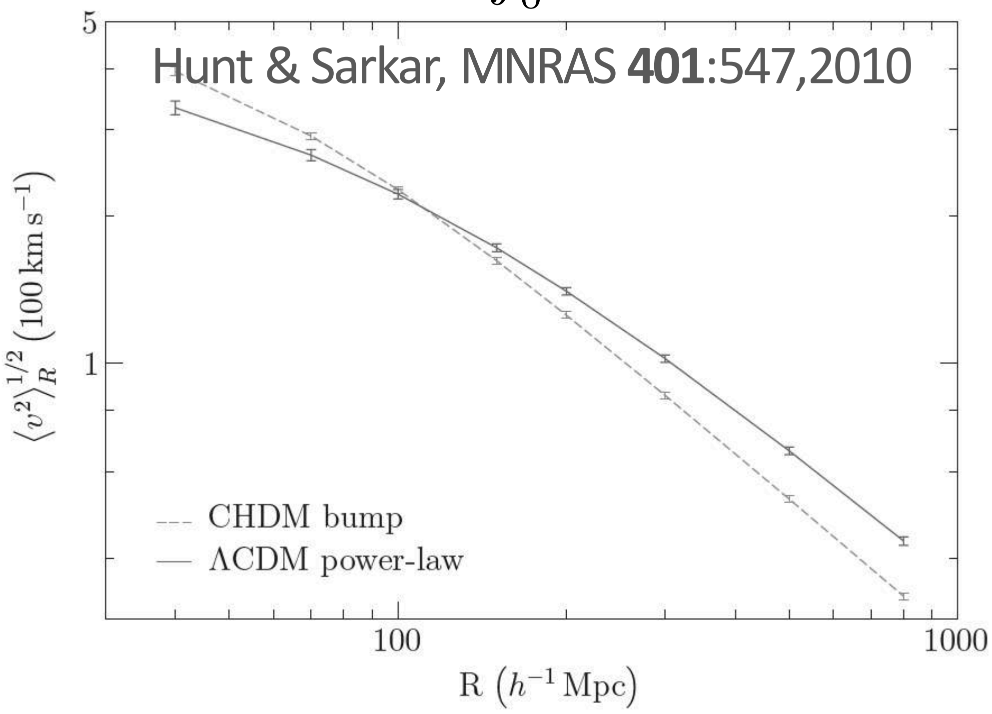
Then the RMS fluctuation in the local Hubble constant $\delta_H \equiv \langle (\delta H / H_0)^2 \rangle^{1/2}$ is:

$$\delta_H^2 = \frac{f^2}{2\pi^2} \int_0^\infty k^2 \mathrm{d}k \; P(k) \mathcal{W}^2(kR), P(k) \equiv |\delta(k)^2|, f \simeq \Omega_{\mathrm{m}}^{4/7} + \frac{\Omega_\Lambda}{70}(1 + \frac{\Omega_{\mathrm{m}}}{2})$$

Growth rate

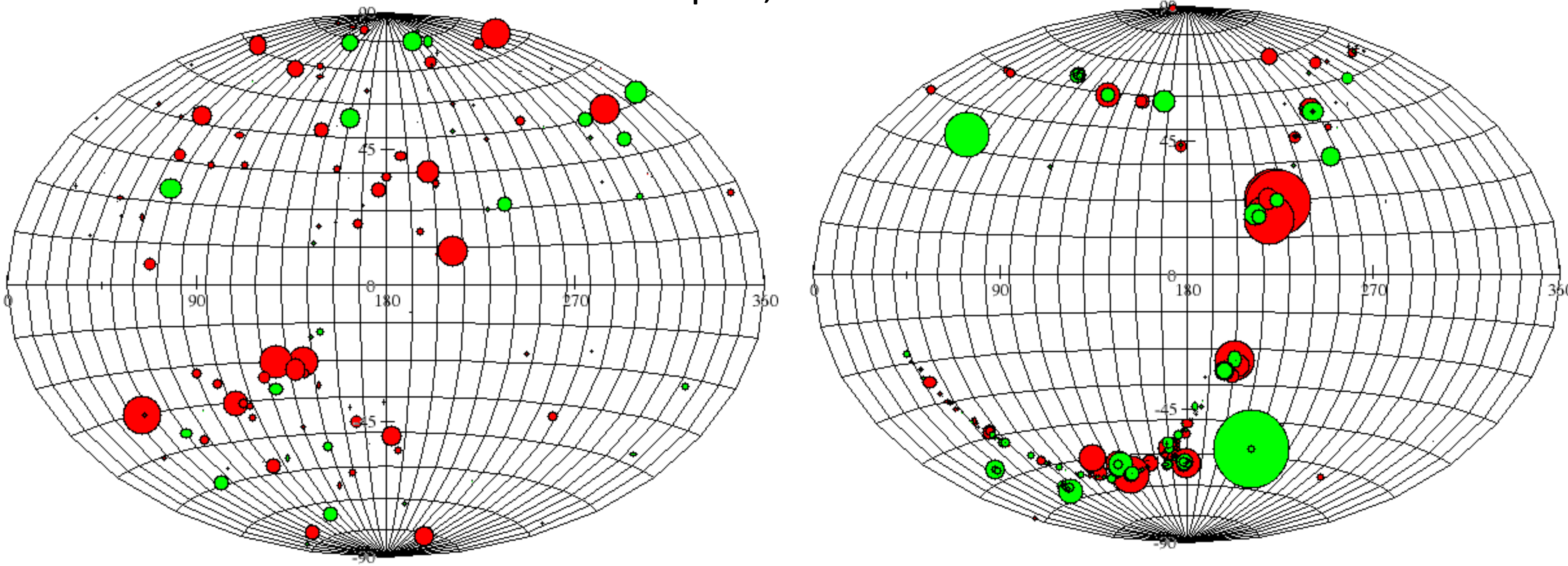
Similarly the variance of the peculiar velocity is: $\langle v^2 \rangle_R = \frac{f^2 H_0^2}{2\pi^2} \int_0^\infty \mathrm{d}k P(k) \mathcal{W}^2(kR)$





Union 2 compilation of 557 SNe Ia

Aitoff-Hammer plot, Galactic coordinates



Left panel: The red spots represent the data points for $z < 0.06$ with distance moduli $\mu_{\rm data}$ bigger than the values $\mu_{\rm CDM}$ predicted by ΛCDM, and the green spots are those with $\mu_{\rm data}$ less than $\mu_{\rm CDM}$; the spot size is a relative measure of the discrepancy. A dipole anisotropy is visible around the direction $b = -30^\circ$, $l = 96^\circ$ (red points) and its opposite direction $b = 30^\circ$, $l = 276^\circ$ (small green points), which is the direction of the CMB dipole. **Right panel**: Same plot for $z > 0.06$

Colin, Mohayaee, Sarkar & Shafieloo, MNRAS **414**:264,2011

Use this to do *tomography* of the local Hubble flow by asking if the supernovae are at the expected distances: any residuals ⇒ 'peculiar velocity' flow in local universe

Method of Residuals and Smoothing

$$q_i(z_i, \theta_i, \phi_i) = \frac{\mu_i(z_i, \theta_i, \phi_i) - \tilde{\mu}_i(z_i, \theta_i, \phi_i)}{\sigma_i(z_i, \theta_i, \phi_i)}$$

Calculation of Residuals

$$Q(\theta, \phi) = \sum_{i=1}^{N} q_i(z_i, \theta_i, \phi_i) W(\theta, \phi, \theta_i, \phi_i)$$

2D smoothing on unit sphere

$$W(\theta, \phi, \theta_i, \phi_i) = \frac{1}{\sqrt{2\pi}\,\delta} \exp\left[-\frac{L(\theta, \phi, \theta_i, \phi_i)^2}{2\delta^2}\right]$$

Window function

$$L(\theta, \phi, \theta_i, \phi_i) = 2 \arcsin \frac{R}{2}, \quad R = \Big([\sin(\theta_i)\cos(\phi_i) - \sin(\theta)\cos(\phi)]^2 + [\sin(\theta_i)\sin(\phi_i) - \sin(\theta)\sin(\phi)]^2 + [\cos(\theta_i) - \cos(\theta)]^2\Big)^{1/2}$$

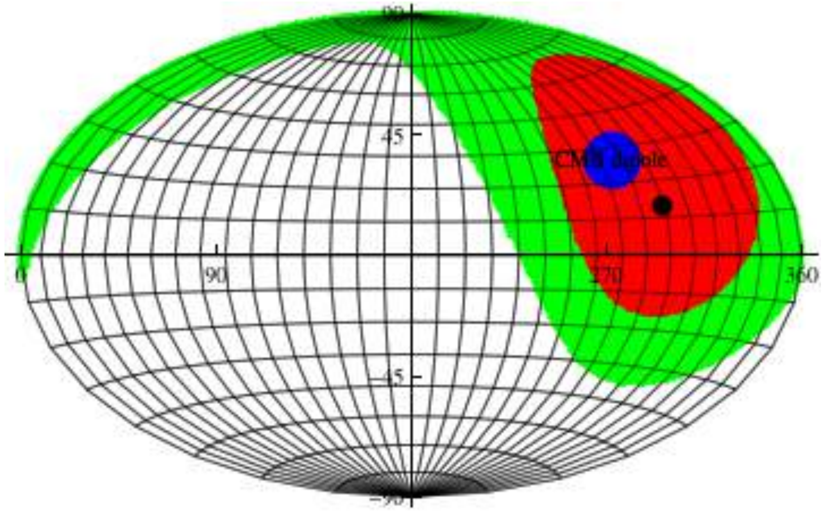
$$\Delta Q_{\mathrm{data}} = Q(\theta_{\max}, \phi_{\max}) - Q(\theta_{\min}, \phi_{\min})$$

Statistical measure

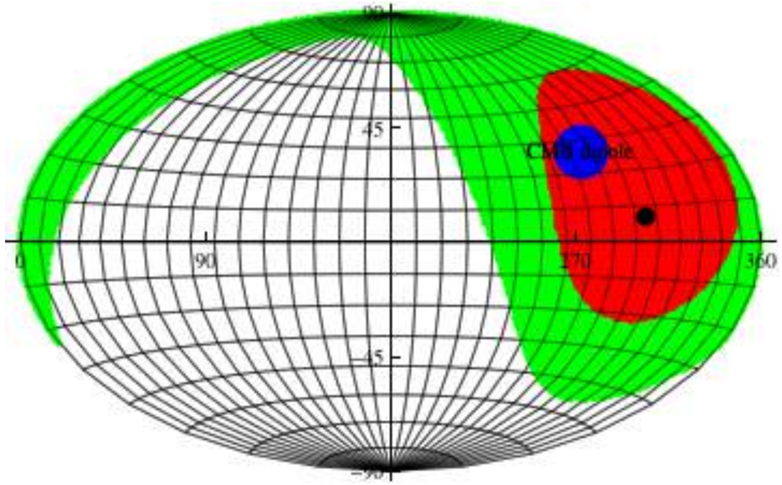
Calculate for the data (as well as for Monte Carlo simulations of isotropic distribution, in order to obtain p-value), using a ratio method to *minimise* systematic uncertainties

There *is* a dipole in the SNe Ia velocity field *towards* the CMB Dipole

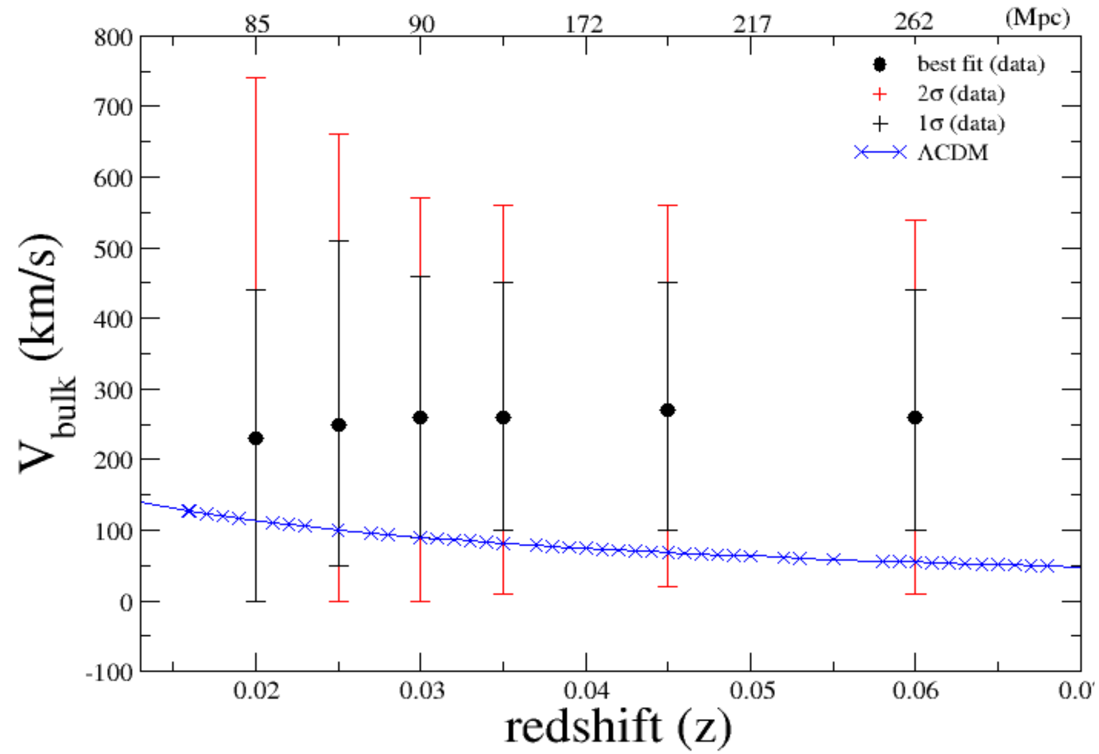
$0.015 < z < 0.045$, $v = 270$ km/s, $l = 291$, $b = 15$



$0.015 < z < 0.06$, $v = 260$ km/s, $l = 298$, $b = 8$



Colin et al, MNRAS **414**:264,2011



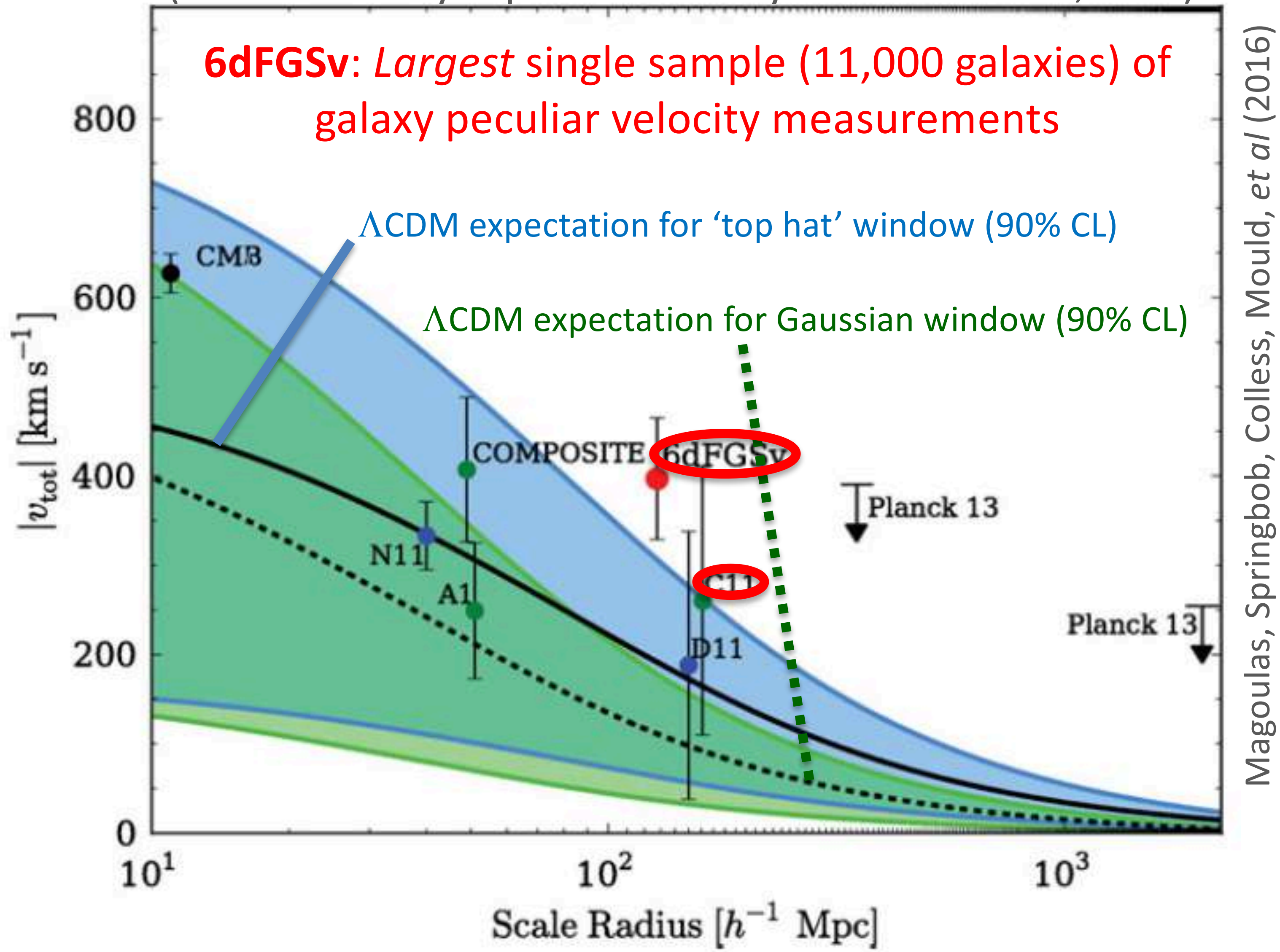
This is ~1.5σ higher than expected for the standard ΛCDM model and extends *beyond* Shapley supercluster (260 Mpc)

Consistent with Watkins *et al* (2009) who found a bulk flow of 416±78 km/s towards $b = 60 \pm 6^0$, $l = 282 \pm 11^0$ extending up to ~100 h^{-1} Mpc)

There is *no* convergence to CMB frame, well beyond 'scale of homogeneity'!

OUR RESULT HAS BEEN *CONFIRMED* BY THE 6-DEGREE FIELD GALAXY SURVEY

(also the Nearby Supernova Factory data: Feind *et al*, 2013)



We find that in the 'Dark Sky' ΛCDM simulations, *less than 1%* of Milky Way–like observers experience a bulk flow as large as is observed, extending out as far as is seen

Rameez, Mohayaee, Sarkar & Colin, MNRAS **477**:1722,2018

A moving observer → *kinematic* dipole

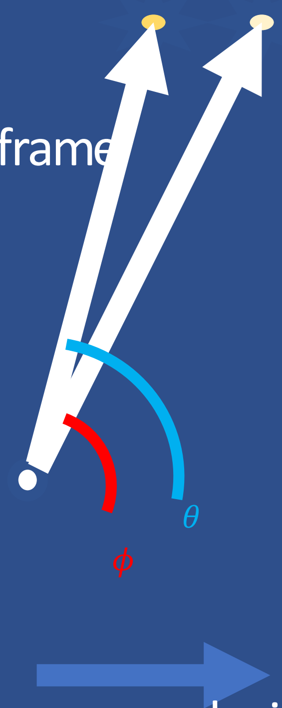
$$\sigma(\theta)_{obs} = \sigma_{rest}[1 + [2 + x(1+\alpha)]\frac{v}{c}\cos(\theta)]$$

Aberration

Rest frame

Moving frame

$$\tan\phi = \frac{\sin\theta}{\gamma * cos\theta \; - \frac{v}{c}}$$

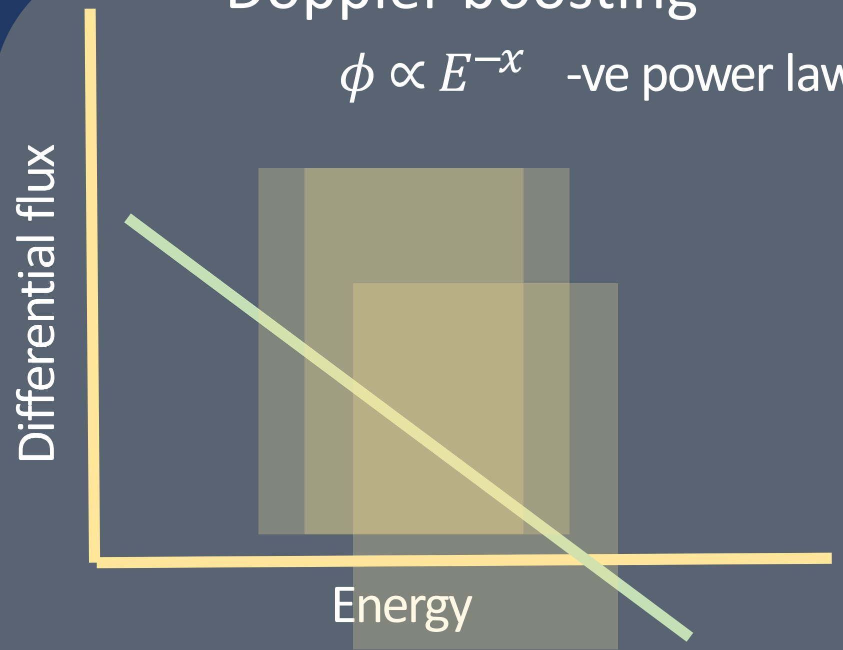


Observer, velocity *v*

+

Doppler boosting

$\phi \propto E^{-x}$ -ve power law

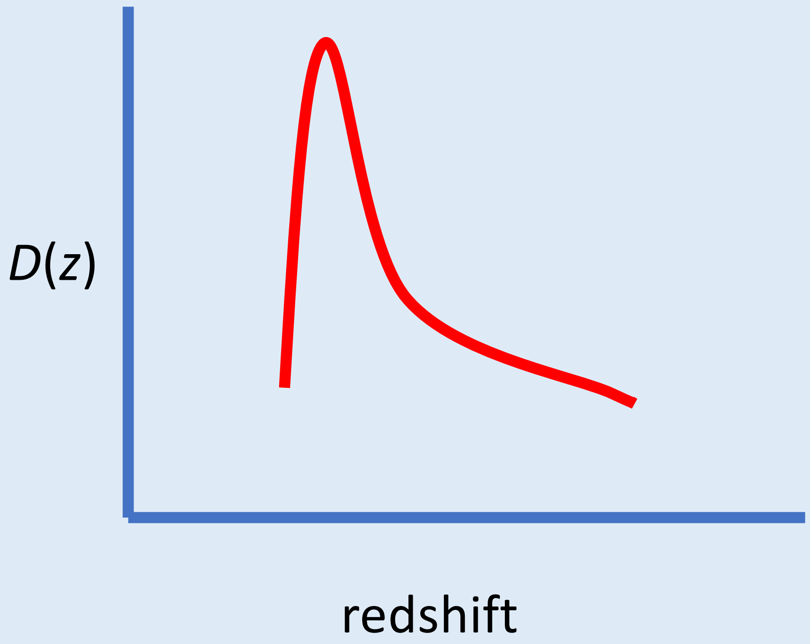


Flux limited catalog → more sources in direction of motion

Ellis & Baldwin (1984)

DIPOLES IN A CATALOGUE OF GALAXIES

All-sky catalogue with N sources with redshift distribution $D(z)$ from a directionally unbiased survey



$$\vec{\delta} = \vec{\mathcal{K}}\,(\vec{v}_{obs}, x, \alpha) + \vec{\mathcal{R}}\,(N) + \vec{\mathcal{S}}\,(D(z))$$

$\vec{\mathcal{K}} \rightarrow$ The kinematic dipole: *independent* of source distance, but depends on source spectrum, source flux function, observer velocity

$\vec{\mathcal{R}} \rightarrow$ The random dipole: $\propto 1/\sqrt{N}$ isotropically distributed

$\vec{\mathcal{S}} \rightarrow$ The dipole component of an actual anisotropy in the distribution of sources in the cosmic rest frame (significant for shallow surveys)

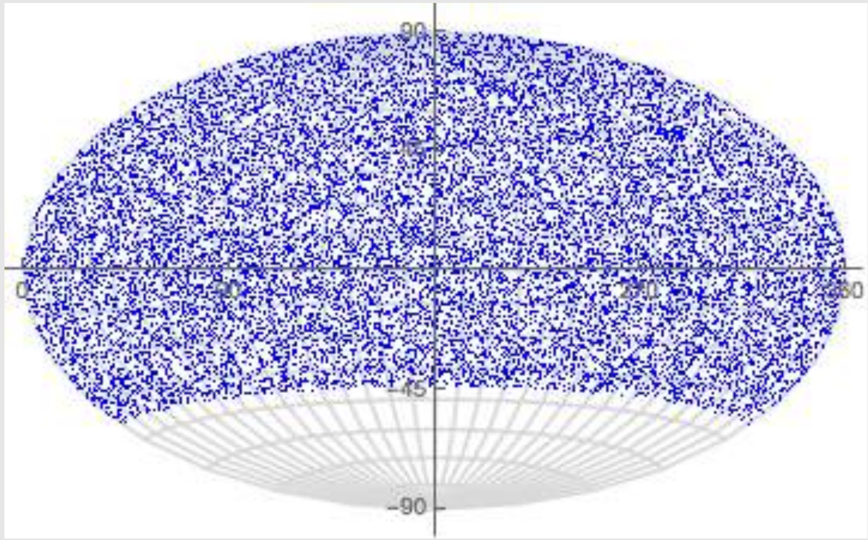
Radio sources: NVSS + SUMSS, 600,000 galaxies $z \sim 1$, $\vec{\mathcal{S}}\,(D(z)) \rightarrow 0$

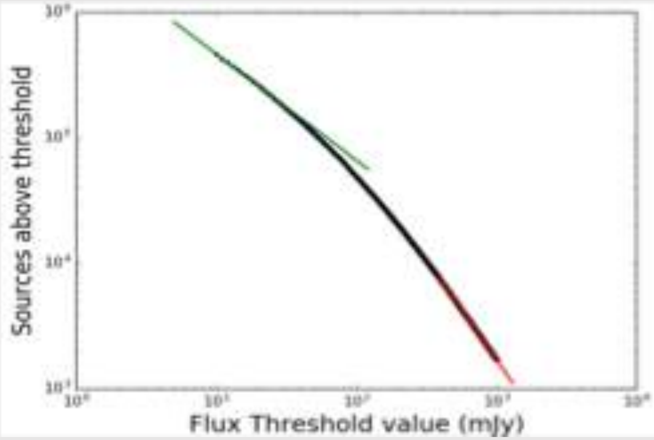
Colin, Mohayaee, Rameez & Sarkar, MNRAS **471**:1045,2017

Wide Field Infrared Survey Explorer, 2,400,000 galaxies, $z \sim 0.14$, $\vec{\mathcal{S}}\,(D(z))$ significant

Rameez, Mohayaee, Sarkar & Colin MNRAS **477**:1722,2018

The NRAO VLA Sky Survey (NVSS)

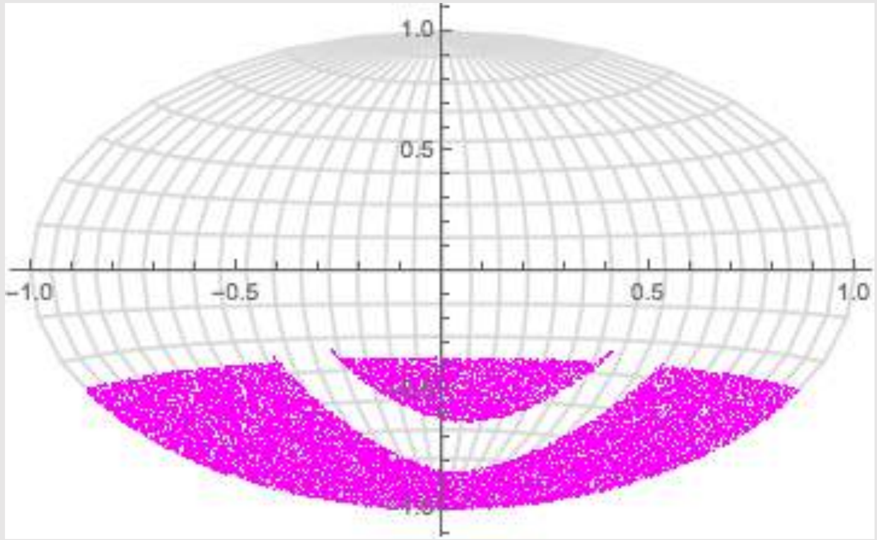


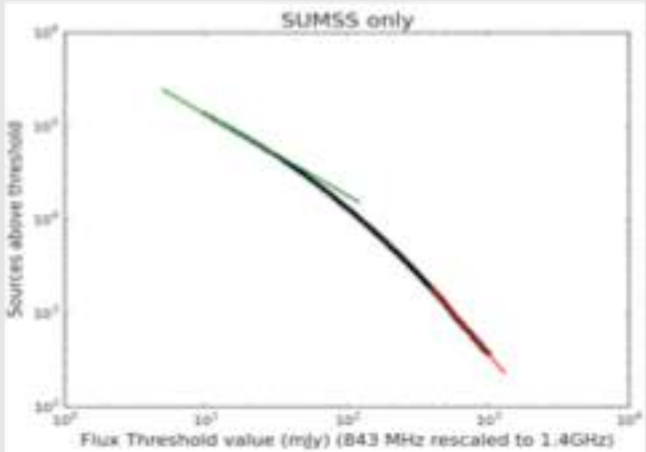


1.4 GHz survey (down to Dec = -40.4°)
National Radio Astronomy Observatory

1,773,488 sources >2.5 mJy
(complete above 10 mJy)
Most are believed to be at $z \gtrsim 1$

Sydney University Molonglo Sky Survey (SUMSS)

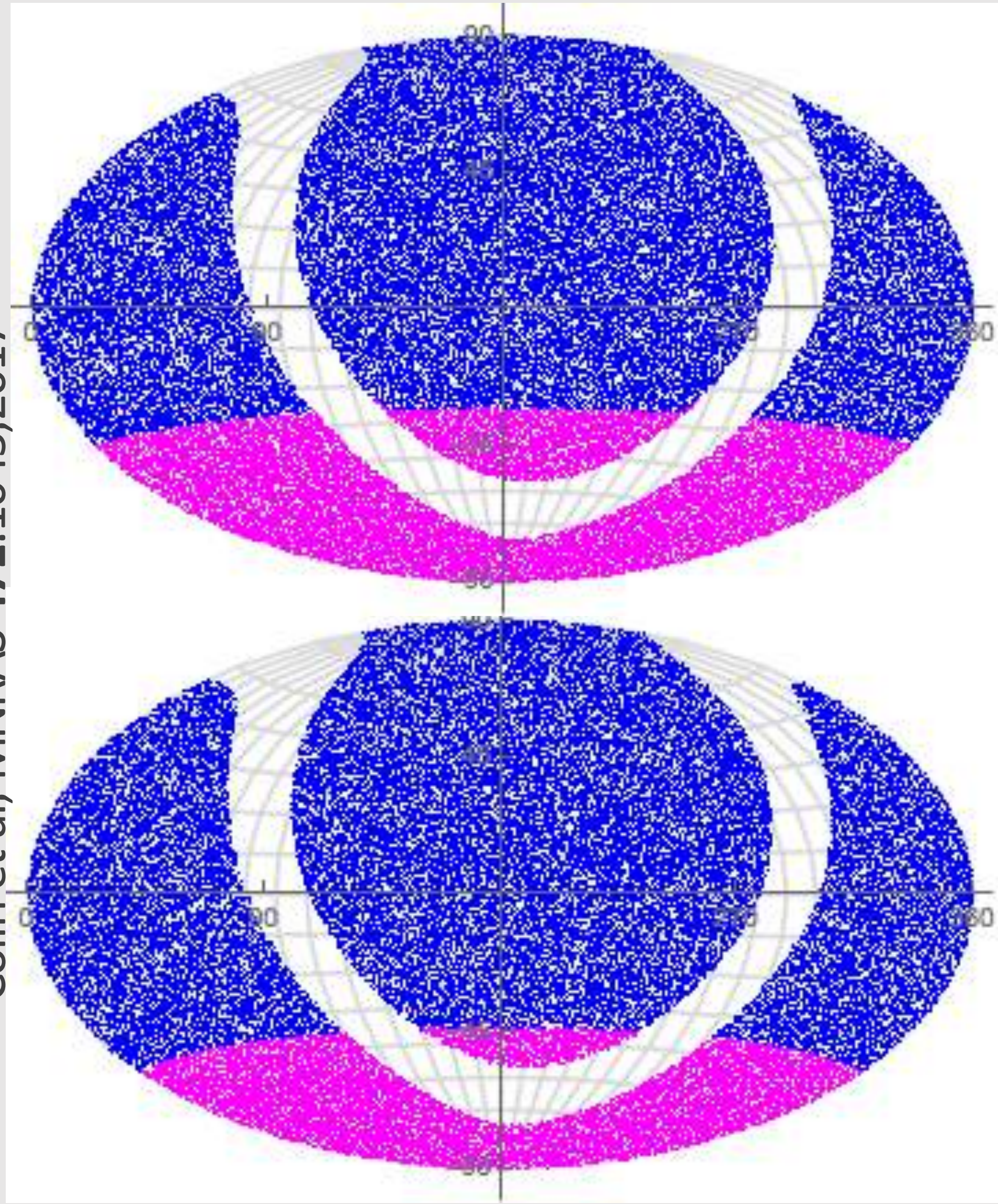




843 MHz survey (Dec < -30.0°)
Molonglo Observatory Synthesis telescope

211,050 sources (with similar sensitivity and resolution to NVSS catalogue)
... Similar expected redshift distribution

The NVSUMSS-Combined All Sky catalog

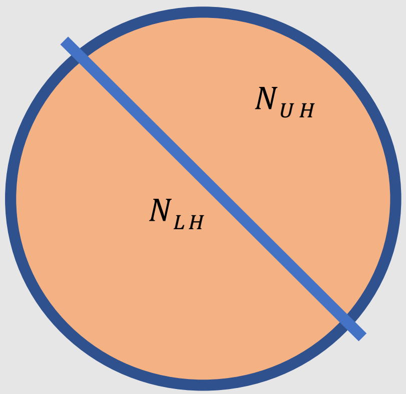


Colin et al, MNRAS **471**:1045,2017

- Rescale SUMSS fluxes by $(843/1400)^{-0.75} \sim 1.46$ to match with NVSS (within ~1%)
- Remove Galactic Plane at $\pm 10^{o}$ (also super-galactic plane)
- Remove NVSS sources below, and SUMSS sources above, dec -30 (or -40)
- Apply common threshold flux cut to both samples
- Remove *any* nearby sources (common with 2MRS & LRS)

Estimators for the Dipole

$$\vec{D}_H = \hat{z} * \frac{N_{UH} - N_{LH}}{N_{UH} + N_{LH}}$$



- Vary the direction of the hemispheres until maximum asymmetry is observed

- Easy visualisation

- But high bias (find by Monte Carlo)

$$\vec{D}_C = \frac{1}{N}\sum_{i=1}^{N} \hat{r}_i$$

- Add up the unit vectors corresponding to directions in the sky for every source

- Relatively low bias and statistical error $1/\sqrt{N}$

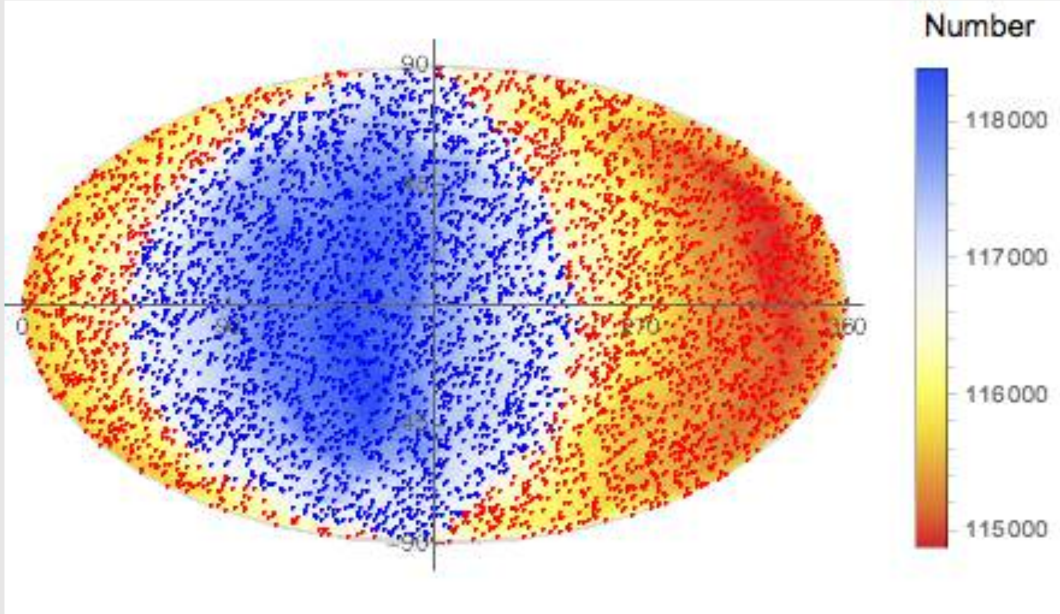
$$\vec{D}_C = \frac{\hat{z}}{N}\int_{\phi=0}^{\phi=2\pi}\int_{\theta=0}^{\theta=\pi} \sigma(\theta)\cos\theta\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

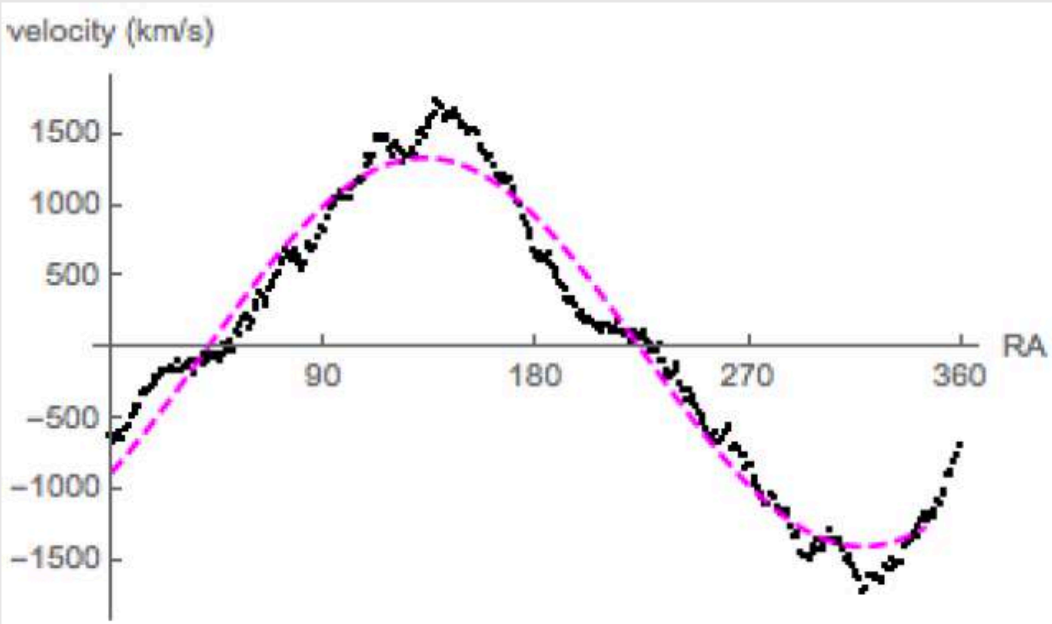
$$\vec{D}_H = \frac{\hat{z}}{N}\int_{\phi=0}^{\phi=2\pi}\int_{\theta=0}^{\theta=\pi} \sigma(\theta)\frac{|\cos\theta|}{\cos\theta}\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

Rubart & Schwarz (2013)

Our Peculiar velocity wrt radio sources

Colin et al, MNRAS **471**:1045,2017





Velocity ~ 1355 ± 174 km/s
(with the 3D linear estimator)

Direction within 10° of CMB dipole (but **4 times *faster***)!

Statistical significance: 99.75%
⇒ 2.81**σ** (by Monte Carlo)

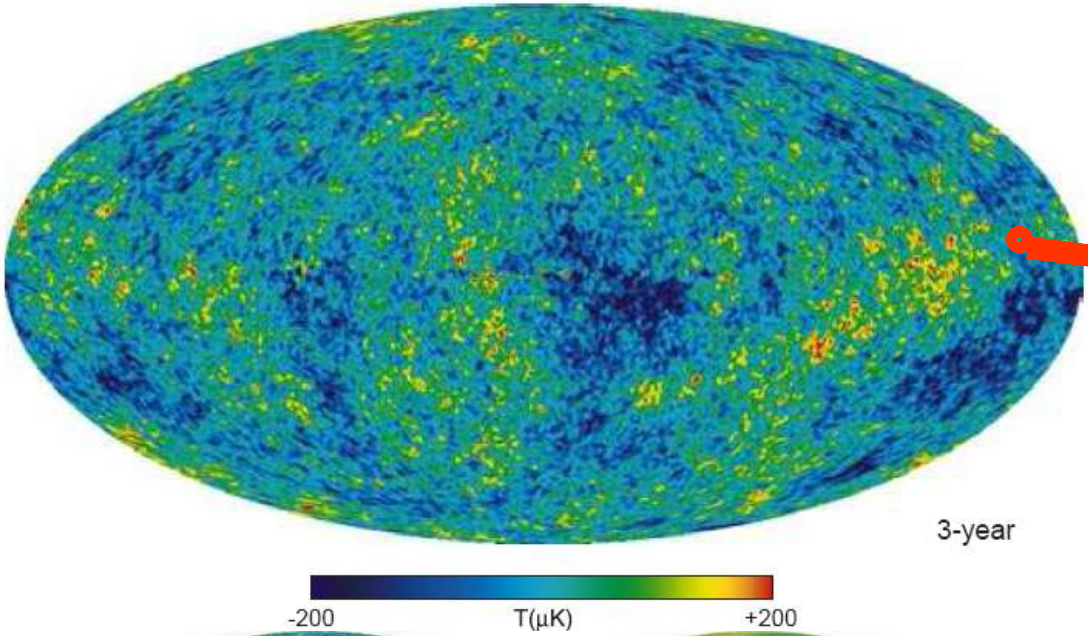
Confirms claim by Singal (2011) which was criticized subsequently
(Gibelyou & Huterer 2012, Rubart & Schwarz 2013, Nusser & Tiwari 2015)

We have addressed *all* the concerns but this strange anomaly remains!

Look forward to data from SKA

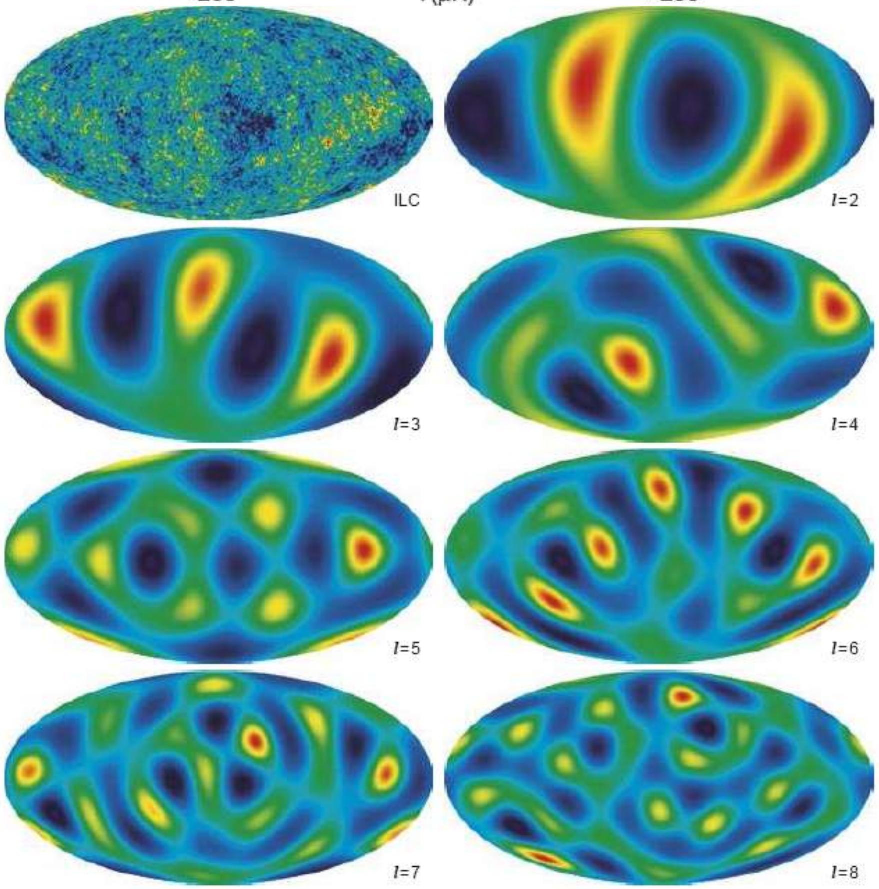
Coherent oscillations in photon-baryon plasma, excited by density erturbations on *super*-horizon scales …

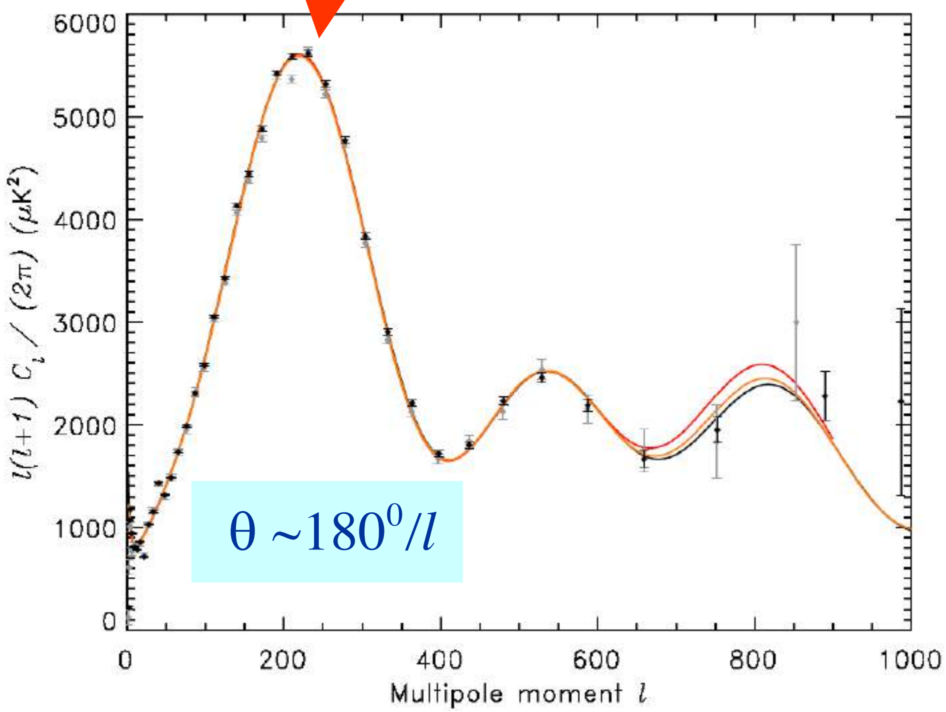
(Hubble radius at t_{rec})



$$\Delta T(\mathbf{n}) = \sum a_{lm} Y_{lm}(\mathbf{n})$$

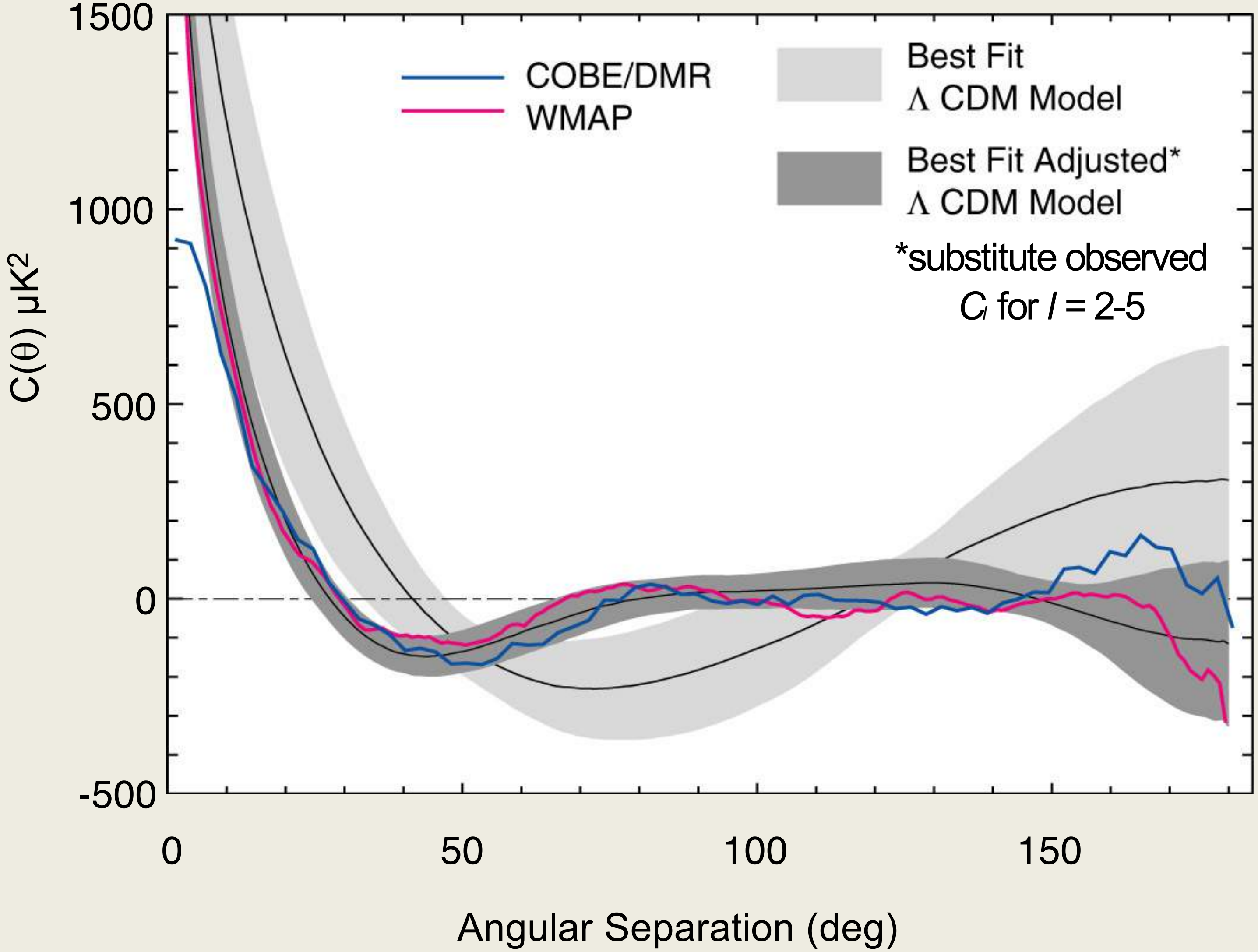
$$C_l \equiv \frac{1}{2l+1} \sum |a_{lm}|^2$$





The lack of power on large angular scales is most striking, although it is claimed to be *not* unlikely taking cosmic variance and foreground subtraction uncertainties into account

➛ chance probability of ~1%



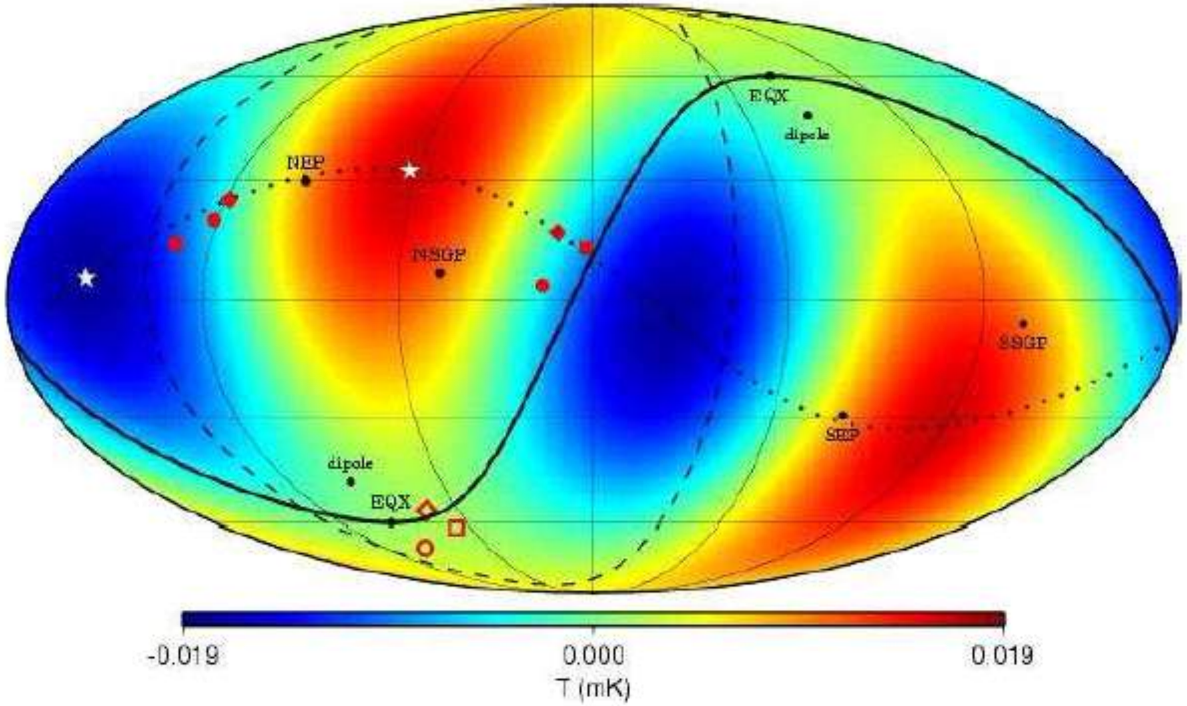
$$C(\theta) = \left\langle T(n_i)T(n_j)\right\rangle$$

$$n_i \cdot n_j = \cos\theta$$

$$S = \int_{60^\circ}^{180^\circ} C(\theta)^2 d\theta$$

A posteriori likelihood of observed *S* is only (0.15 - 0.3) %

Moreover there is an unexpected alignment of low multipoles, a ‘cold spot’, and an asymmetry between the North and South ecliptic hemispheres



Curious alignment of quadrupole and octupole (along the ecliptic)

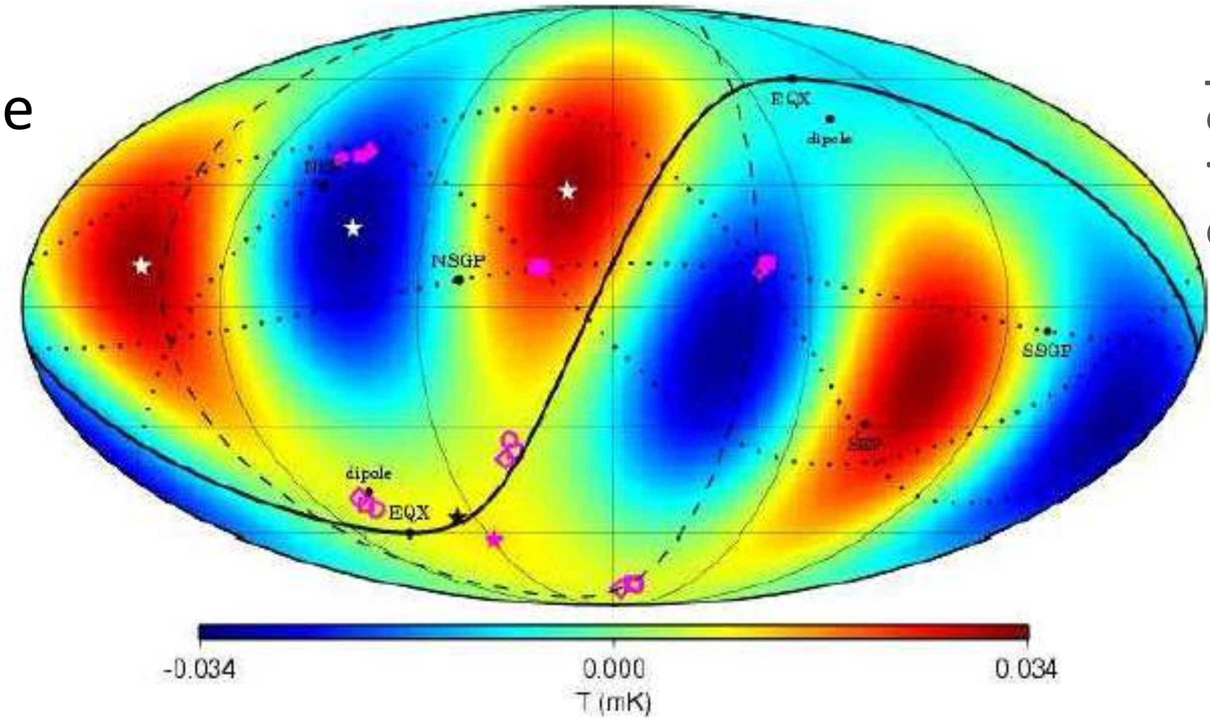
Power concentrated in plane tilted by ~30° from the Galactic plane ($m = \pm\, l$ in suitable coord. system)

Copi, Schwarz, Starkman (2004)

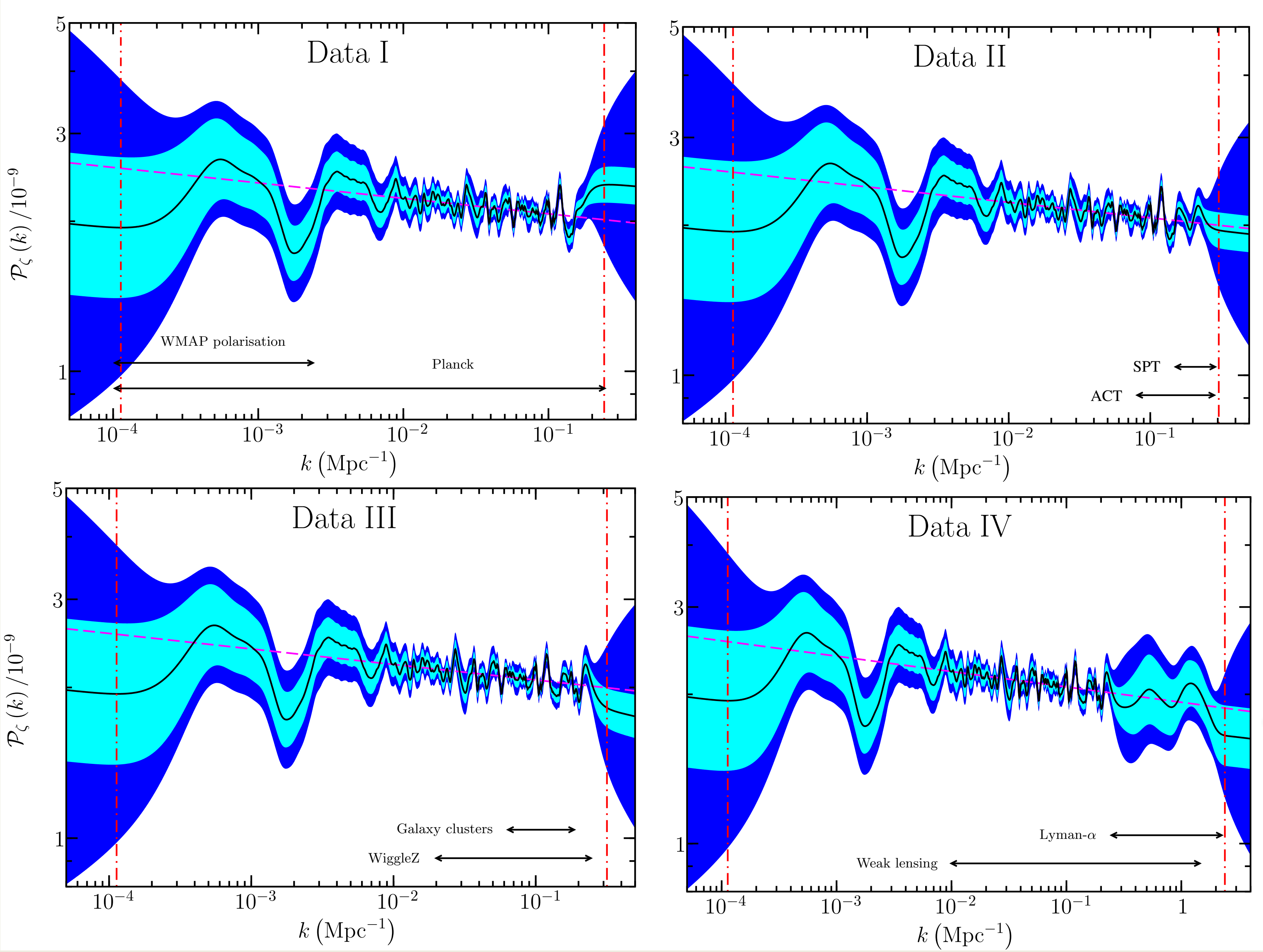
Probability of low quadrupole + alignment + "planarity":

$\sim$ **4 x 10^{-5}**

Tegmark *et al* (2003, 2004)



The primordial spectrum of perturbations can be deconvoluted from CMB & LSS data *non*-parametrically, using 'Tikhonov regularisation' (Hunt & Sarkar, JCAP **12**:052,2015)



This is the *assumed* power-law spectrum which is supposedly the best-fit to data: $n_s = 0.969$

Comparison with Monte Carlo simulations shows ~2σ deviations from a power-law spectrum

Reconstruction of a direction-dependent primordial power spectrum from Planck CMB data
Durakovic, Hunt, Mukherjee, Sarkar & Souradeep, JCAP **02**:012,2018

We can also consider a **direction-dependent** component of the power spectrum of the CMB fluctuations, which is also allowed to vary with the scale (wave number):

$$\mathcal{P}(\mathbf{k}) = \mathcal{P}(k) + \sqrt{4\pi} \sum_{LM} g_{LM}(k) Y_{LM}\left(\hat{\mathbf{k}}\right)$$

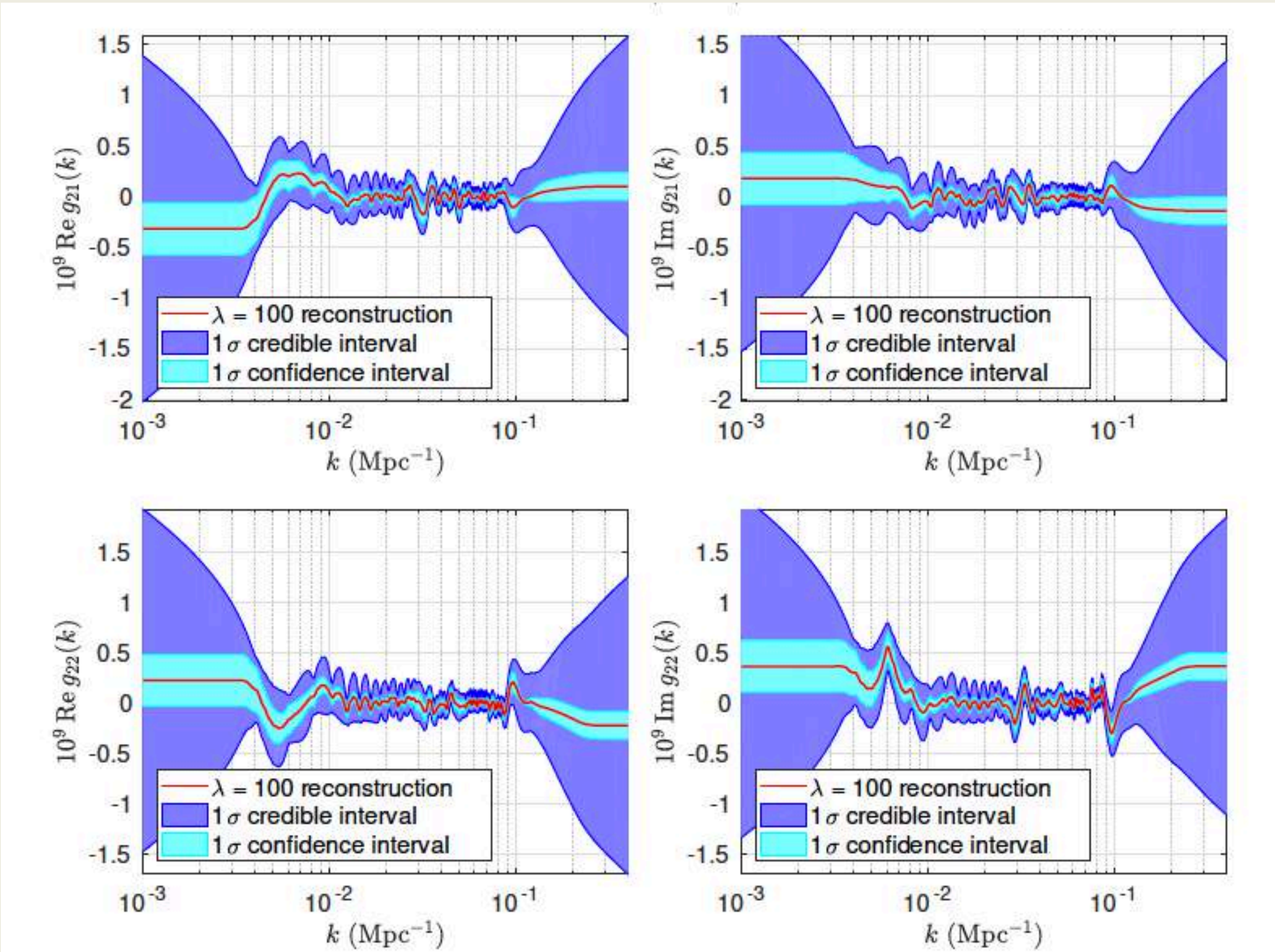
... and focus on the **quadrupole** modulation (NB: Density field is *real*, hence symmetry requires *L* to be *even* – see Hajian & Souradeep 2005, Pullen & Kamionkowski 2007)

$$\mathcal{P}(\mathbf{k}) = \mathcal{P}(k) + \sqrt{4\pi} \sum_{M=-2}^{2} g_{2M}(k) Y_{2M}\left(\hat{\mathbf{k}}\right)$$

We compute these '**bipolar spherical harmonics**' for the Planck DR2-2015 SMICA map, and estimate the noise covariance from *Planck Full Focal Plane 9* simulations

Previous work by: Groeneboom & Eriksen (2009), Kim & Komatsu (2013); Theoretical models by: Ford (1989), Chibisov (1989), Ackerman *et al* (2007), Pitrou *et al* (2008), Himmetoglu *et al* (2009), Watanabe *et al* (2009), Bartolo *et al* (2013, 2018), ...

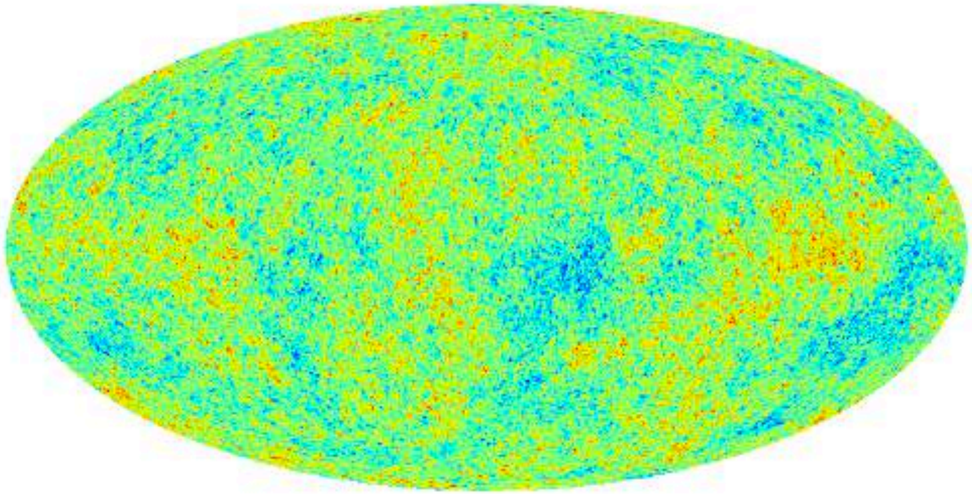
When a *constant* **quadrupolar modulation** is fitted to Planck data in the range $0.005 \leq k/\mathrm{Mpc}^{-1} \leq 0.008$, its **preferred directions** are found to be ***related*** to the **cosmic hemispherical asymmetry**, and the **CMB dipole**

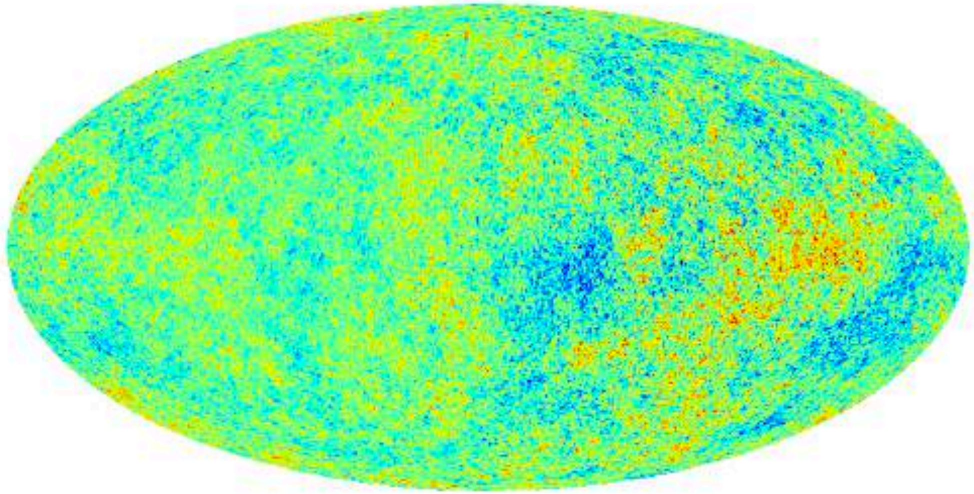


Durakovic *et al*, JCAP **02**:012,2018

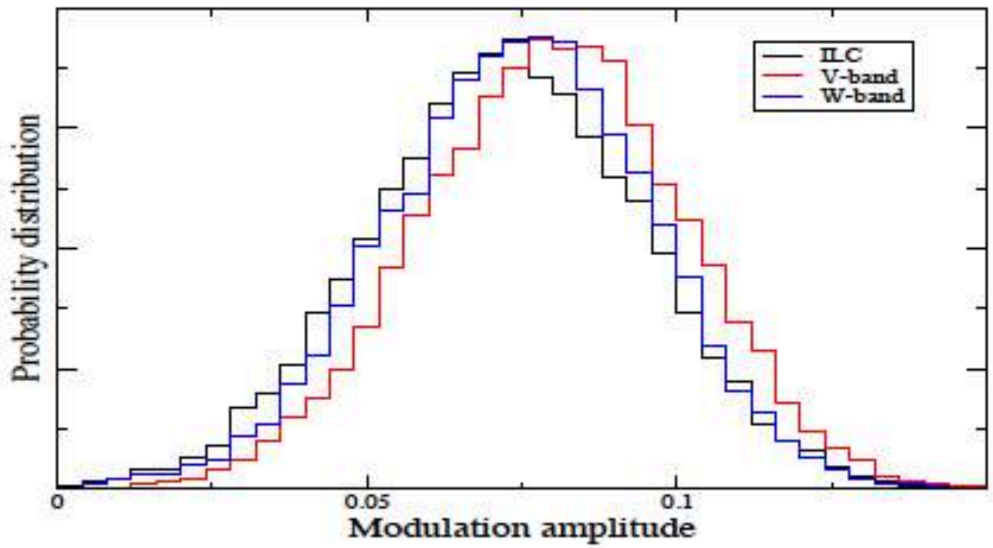
The significance is 2.1σ with a test statistic sensitive only to the amplitude of the modulation … but with a statistic sensitive also to the direction, it rises to 6.9σ!

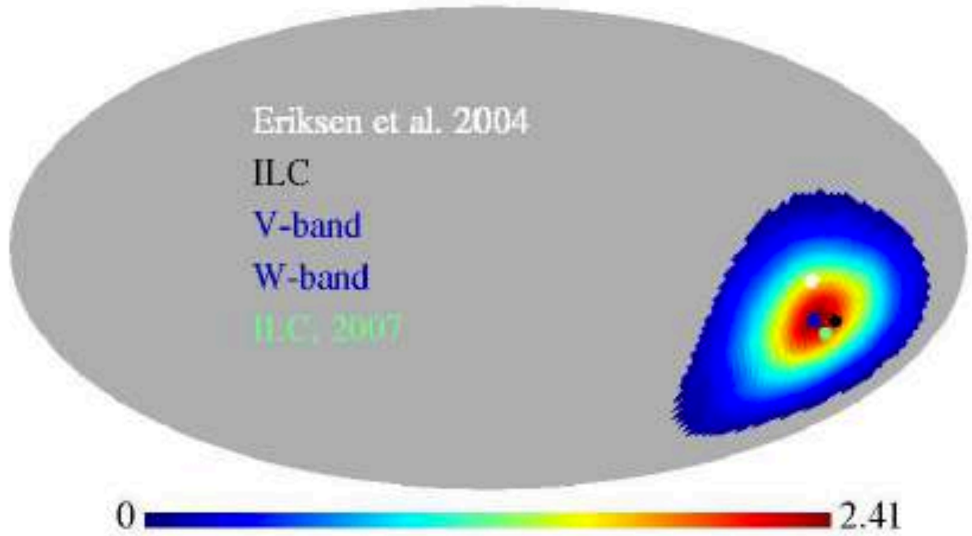
Eriksen *et al* (2004) found that the CMB fluctuations are stronger in one hemisphere of the sky than in the other (@3σ) … as if the perturbations are modulated by a *dipole*



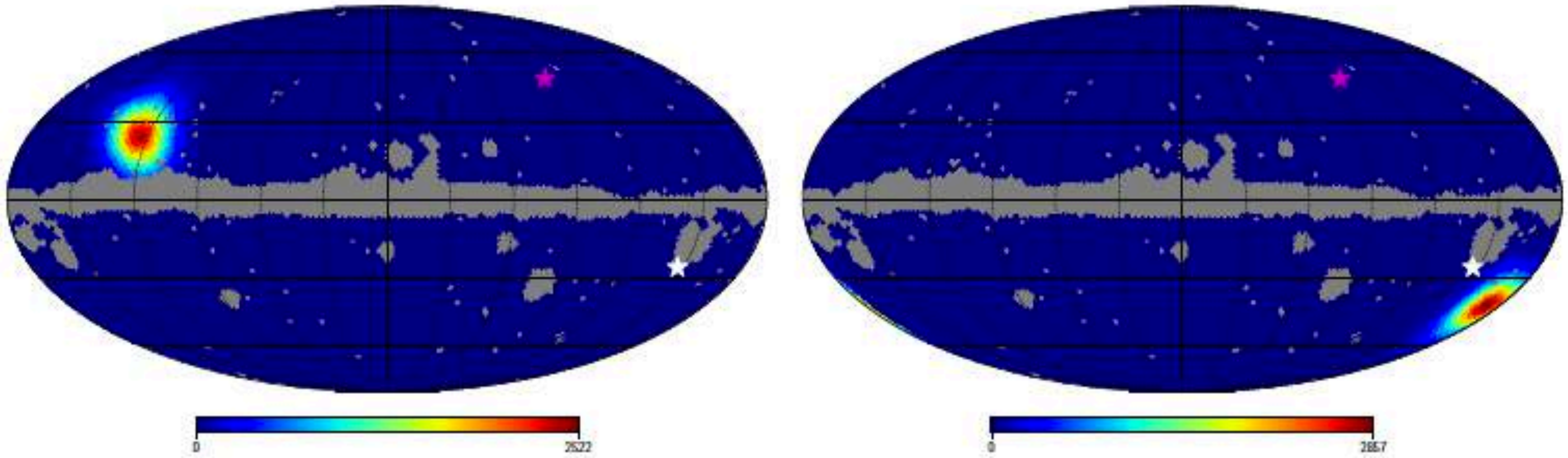


$$T_{\rm iso}(\hat{\mathbf{n}}) \qquad\qquad T_{\rm iso}(\hat{\mathbf{n}})(1 + A\hat{\mathbf{p}}\cdot\hat{\mathbf{n}})$$





Alignments on the sky : What does this imply for inflation?



Hot ($10^9 g_A = 0.76 \pm 0.22$) quadrupole modulation (left panel), and cold ($10^9 g_A = -0.82 \pm 0.21$) modulation (right panel). The magenta and white stars indicate the direction of the CMB dipole and of the hemispherical asymmetry respectively.

Durakovic *et al*, JCAP **02**:012,2018

For $k = 0.005$-$0.008\,\mathrm{Mpc}^{-1}$:		Angular distances to:	
Amp. $10^9 g_A$	Direction (l, b)	CMB dipole $(264°, 48°)$	Hemisph. asym. $(213°, -26°)$
0.76 ± 0.22	$(128^{\circ}{}^{+14}_{-14}, 25^{\circ}{}^{+11}_{-9})$	97°	97°
-0.82 ± 0.21	$(191^{\circ}{}^{+15}_{-14}, -41^{\circ}{}^{+10}_{-11})$	110°	24°

Conclusions

- The distribution of directions of radio galaxies in NVSS and SUMSS is at 2.8$\boldsymbol{\sigma}$ tension with the kinematic interpretation of the CMB dipole as being due to our motion at 369 km/s w.r.t the CMB rest frame
- There is a scale-*dependent* quadrupolar modulation of CMB anisotropy … the direction is ~orthogonal to the CMB dipole but ~aligned with that of the 'hemispherical asymmetry'

Could all this be an indication of new horizon-scale physics,
e.g. pre-inflationary relics lurking just *outside* our present horizon?
(Gunn 1988, Paczynski & Piran 1990, Turner 1991: `A tilted universe')

The 'standard' assumptions of *exact* isotropy and homogeneity are *questionable* – data from forthcoming surveys (Euclid, LSST, SKA *etc*) will hopefully provide sufficiently large datasets to enable definitive tests

NB: In the framework of the ΛCDM model, we are a <1% likely observer