PACTS 2018: Particle, Astroparticle and Cosmology Tallinn Symposium
21 June 2018

# Classifying single-field inflationary models

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LJ, Kristjan Kannike, Luca Marzola, Antonio Racioppi, Martti Raidal, Mihkel Rünkla, Margus Saal, Hardi Veermäe

Phys. Rev. Lett. 118 (2017) 151302, arXiv:1612.06863

+ review of some follow-up and related work
+ work in progress

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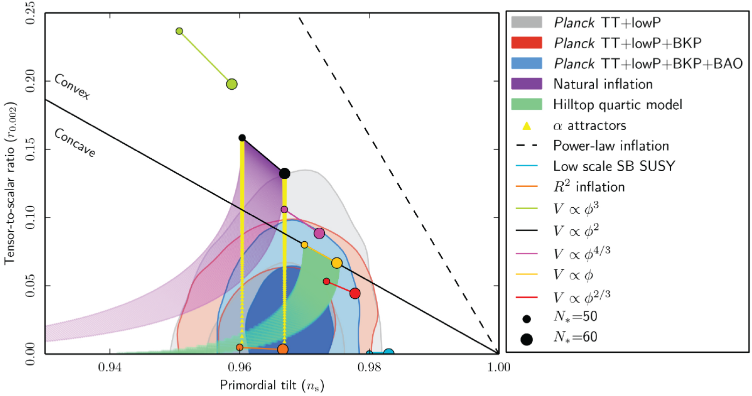
# Outline

1. Classification of inflationary models
   - The formalism of invariants in scalar-tensor gravity
   - Computing the inflationary observables and classifying the models
2. Comment on the issue of frames
3. Anatomy of inflationary attractors

# Introduction




Planck Collaboration 1502.02114

# Motivation

Seemingly very different models, e. g.

- $\alpha-\beta$ model Ferrara, Kallosh, Linde, Porrati 1307.7096
(Starobinsky 1979 $R+R^2$ model has $\alpha=1$)

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[-\frac{M_{\mathrm{Pl}}^2}{2}R+\frac{1}{2}g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi-M^4\left(1-e^{-\sqrt{\frac{2}{3\alpha}}\frac{\Phi}{M_{\mathrm{Pl}}}}\right)^2\right],$$

- E-type $\alpha$-attractors Kallosh, Linde, Roest 1311.0472

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[-\frac{M_{\mathrm{Pl}}^2}{2}R+\frac{3\alpha}{4}\frac{M_{\mathrm{Pl}}^2}{\Phi^2}g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi-M^4\left(1-\frac{\Phi}{M_{\mathrm{Pl}}}\right)^2\right],$$

- Special $\xi$-attractor Galante, Kallosh, Linde, Roest 1412.3797

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[-\frac{M_{\mathrm{Pl}}^2+\xi\Phi^2}{2}R+\frac{1}{2}\frac{\xi\Phi^2}{M_{\mathrm{Pl}}^2+\xi\Phi^2}g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi-\lambda\xi^2\Phi^4\right],$$

give the same inflationary observables to leading order

$$n_s=1-\frac{2}{N}\,,\qquad r=\frac{12\alpha}{N^2}$$

(identifying $\alpha=1+\frac{1}{6\xi}$).

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$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[ -\frac{M_{\mathrm{Pl}}^2}{2} R + \frac{3\alpha}{4} \frac{M_{\mathrm{Pl}}^2}{\Phi^2} g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi - M^4 \left( 1 - \frac{\Phi}{M_{\mathrm{Pl}}} \right)^2 \right],$$

- Special $\xi$-attractor Galante, Kallosh, Linde, Roest 1412.3797

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[ -\frac{M_{\mathrm{Pl}}^2 + \xi \Phi^2}{2} R + \frac{1}{2} \frac{\xi \Phi^2}{M_{\mathrm{Pl}}^2 + \xi \Phi^2} g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi - \lambda \xi^2 \Phi^4 \right],$$

give the same inflationary observables to leading order

$$n_s = 1 - \frac{2}{N}, \qquad r = \frac{12\alpha}{N^2}$$

**WHY?**

(identifying $\alpha = 1 + \frac{1}{6\xi}$.)

# Non/minimally coupled scalar field

Scalar-tensor gravity (STG) action in the most general form for one scalar field $\Phi$ and no derivative couplings

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\mathcal{A}(\Phi)M_{\mathrm{Pl}}^2 R + \frac{1}{2}\mathcal{B}(\Phi)g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi - \mathcal{V}(\Phi)\right] + S_{\mathrm{m}}\left(\mathrm{e}^{2\sigma(\Phi)}g_{\mu\nu}, \chi\right), \tag{1}$$

- Four arbitrary functions, allowing
  - $\mathcal{A}(\Phi)$ nonminimal coupling to gravity
  - $\mathcal{B}(\Phi)$ noncanonical kinetic term,
  - $\mathcal{V}(\Phi)$ potential
  - $e^{2\alpha(\Phi)}$ nonminimal coupling to matter fields $\chi$,

  $M_{\mathrm{Pl}}$ reduced Planck mass

# Frames and parametrizations

The action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\mathcal{A}(\Phi)M_{\mathrm{Pl}}^2 R + \frac{1}{2}\mathcal{B}(\Phi)g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi - \mathcal{V}(\Phi)\right] + S_{\mathrm{m}}\left(\mathrm{e}^{2\sigma(\Phi)}g_{\mu\nu},\chi\right) \tag{1}$$

- is invariant under
  - conformal rescaling of the metric (changes the frame)

$$g_{\mu\nu} = e^{2\bar{\gamma}(\bar{\Phi})}\bar{g}_{\mu\nu} \tag{2}$$

  - reparametrization of the scalar field (changes the parametrization)

$$\Phi = \bar{f}(\bar{\Phi})\,. \tag{3}$$

- Thus can define frames and parametrizations, e.g.
  - Einstein frame $\mathcal{A}(\Phi) = 1$, canonical parametrization $\mathcal{B}(\Phi) = 1$,
  - Jordan frame $\sigma(\Phi) = 1$, Brans-Dicke like parametrization $\mathcal{A}(\Phi) = \Phi$, $\mathcal{B}(\Phi) = \frac{\omega(\Phi)}{\Phi}$.

# Transformation rules

The action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[ -\frac{1}{2}\mathcal{A}(\Phi)M_{\mathrm{Pl}}^2 R + \frac{1}{2}\mathcal{B}(\Phi)g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi - \mathcal{V}(\Phi)\right] + S_{\mathrm{m}}\left(\mathrm{e}^{2\sigma(\Phi)}g_{\mu\nu},\chi\right)\,, \tag{1}$$

preserves its structure, up to a boundary term, under the composition of

$$g_{\mu\nu} = e^{2\bar{\gamma}(\bar{\Phi})}\bar{g}_{\mu\nu}\,, \qquad \Phi = \bar{f}(\bar{\Phi})\,, \tag{2, 3}$$

provided that the functions of the scalar field transform according to Flanagan gr-qc/0403063 :

$$\begin{aligned}
\bar{\mathcal{A}}(\bar{\Phi}) &= \mathrm{e}^{2\bar{\gamma}(\bar{\Phi})}\mathcal{A}\left(\bar{f}(\bar{\Phi})\right)\,, \\
\bar{\mathcal{V}}(\bar{\Phi}) &= \mathrm{e}^{4\bar{\gamma}(\bar{\Phi})}\,\mathcal{V}\left(\bar{f}(\bar{\Phi})\right)\,, \\
\bar{\sigma}(\bar{\Phi}) &= \sigma\left(\bar{f}(\bar{\Phi})\right) + \bar{\gamma}(\bar{\Phi})\,, \\
\bar{\mathcal{B}}(\bar{\Phi}) &= \mathrm{e}^{2\bar{\gamma}(\bar{\Phi})}\left[\left(\bar{f}'\right)^2\mathcal{B}\left(\bar{f}(\bar{\Phi})\right) - 6M_{\mathrm{Pl}}^2\left(\bar{\gamma}'\right)^2\mathcal{A}\left(\bar{f}(\bar{\Phi})\right) - 6M_{\mathrm{Pl}}^2\bar{\gamma}'\bar{f}'\mathcal{A}'\right]\,,
\end{aligned} \tag{4}$$

where prime denotes the differentiation of the corresponding quantity with respect to its argument, for instance $\bar{f}' \equiv \mathrm{d}\bar{f}(\bar{\Phi})/\mathrm{d}\bar{\Phi}$ and $\mathcal{A}' \equiv \mathrm{d}\mathcal{A}(\Phi)/\mathrm{d}\Phi$.

# Invariant quantities

Can construct quantitites, invariant under conformal rescaling and transform as scalar functions under reparametrization
LJ, Kuusk, Saal, Vilson 1411.1947 :

$$\mathcal{I}_{\mathrm{m}}(\Phi) \equiv \frac{\mathrm{e}^{2\sigma(\Phi)}}{\mathcal{A}(\Phi)} \tag{5}$$

$$\mathcal{I}_{\mathcal{V}}(\Phi) \equiv \frac{\mathcal{V}(\Phi)}{(\mathcal{A}(\Phi))^2} \tag{6}$$

$$\mathcal{I}_{\phi}(\Phi) \equiv \frac{1}{\sqrt{2}} \int \left( \frac{2\mathcal{A}\mathcal{B} + 3\left(\mathcal{A}'\right)^2 M_{\mathrm{Pl}}^2}{\mathcal{A}^2} \right)^{\frac{1}{2}} \mathrm{d}\Phi \tag{7}$$

- Can define infinitely many more invariants using

$$\mathcal{I}_i \equiv \mathfrak{f}(\mathcal{I}_j)\,, \qquad \mathcal{I}_m \equiv \frac{\mathcal{I}_k'}{\mathcal{I}_l'}\,, \qquad \mathcal{I}_r \equiv \int \mathcal{I}_n \mathcal{I}_p' d\Phi\,.$$

- Expect the physical observables be given in terms of these invariants.

# Meaning of the invariants

$$S = \int \mathrm{d}^4x \sqrt{-g}\left[-\frac{1}{2}\mathcal{A}(\Phi)M_{\mathrm{Pl}}^2 R + \frac{1}{2}\mathcal{B}(\Phi)g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi - \mathcal{V}(\Phi)\right] + S_{\mathrm{m}}\left(\mathrm{e}^{2\sigma(\Phi)}g_{\mu\nu}, \chi\right) \tag{1}$$

- Matter coupling invriant

$$\mathcal{I}_{\mathrm{m}}(\Phi) \equiv \frac{\mathrm{e}^{2\sigma(\Phi)}}{\mathcal{A}(\Phi)} \tag{5}$$

$\frac{d\mathcal{I}_{\mathrm{m}}}{d\Phi} \not\equiv 0$ means nonminimal coupling (in "veiled" general relativity $\frac{d\mathcal{I}_1}{d\Phi} \equiv 0$)

- Potential invariant

$$\mathcal{I}_{\mathcal{V}}(\Phi) \equiv \frac{\mathcal{V}(\Phi)}{(\mathcal{A}(\Phi))^2} \tag{6}$$

$\frac{d\mathcal{I}_{\mathcal{V}}}{d\Phi} \not\equiv 0$ means nontrivial mass or self-interactions

- Field invariant

$$\mathcal{I}_\phi(\Phi) \equiv \frac{1}{\sqrt{2}} \int \left(\frac{2\mathcal{A}\mathcal{B} + 3\left(\mathcal{A}'\right)^2 M_{\mathrm{Pl}}^2}{\mathcal{A}^2}\right)^{\frac{1}{2}} \mathrm{d}\Phi \tag{7}$$

$\frac{d\mathcal{I}_\phi}{d\Phi} \not\equiv 0$ means $\Phi$ is dynamical ($\omega_{BD} = -\frac{3}{2}$ gives $\frac{d\mathcal{I}_3}{d\Phi} = 0$)
Invariant measure – the integrand in can be interpreted as the volume form of the 1-dimensional space of the scalar field. Kuusk, LJ, Vilson 1509.02903

# Action in terms of the invariants. Classification

- The quantity $\mathcal{I}_\phi(\Phi)$ provides an invariant description of the scalar degree of freedom and has the corresponding dimension.
- Can regard $\mathcal{I}_\phi$ as a new independent degree of freedom in place of $\Phi$, write the action (1) in an invariant fashion
  LJ, Kuusk, Saal, Vilson 1411.1947 :

$$S = \int \mathrm{d}^4x \sqrt{-\hat{g}} \left[ -\frac{M_{\mathrm{Pl}}^2}{2} \hat{R} + \frac{1}{2} \hat{g}^{\mu\nu} \hat{\nabla}_\mu \mathcal{I}_\phi \hat{\nabla}_\nu \mathcal{I}_\phi - \mathcal{I}_\mathcal{V} \right] + S_\mathrm{m} \left( \mathcal{I}_\mathrm{m} \hat{g}_{\mu\nu}, \chi \right) , \tag{8}$$

Here the hatted quantities are functions of the invariant metric

$$\hat{g}_{\mu\nu} \equiv \mathcal{A} g_{\mu\nu} . \tag{9}$$

- Gravitational theories are classified by $\mathcal{I}_\mathcal{V}(\mathcal{I}_\phi)$, $\mathcal{I}_\mathrm{m}(\mathcal{I}_\phi)$.
- Inflationary models are classified by $\mathcal{I}_\mathcal{V}(\mathcal{I}_\phi)$. LJ *et al* 1612.06863

# Slow roll parameters

- Can rephrase the usual expressions for the slow-roll parameters in terms of the invariants
  Kuusk, Rünkla, Saal, 1605.07033

$$\epsilon = \frac{M_{\mathrm{Pl}}^2}{2} \left( \frac{\mathrm{d} \ln \mathcal{I}_{\mathcal{V}}}{\mathrm{d}\mathcal{I}_{\phi}} \right)^2 , \tag{10}$$

$$\eta = \frac{M_{\mathrm{Pl}}^2}{\mathcal{I}_{\mathcal{V}}} \frac{\mathrm{d}^2 \mathcal{I}_{\mathcal{V}}}{\mathrm{d}\mathcal{I}_{\phi}^2} , \tag{11}$$

$$\xi^2 = \frac{M_{\mathrm{Pl}}^4}{\mathcal{I}_{\mathcal{V}}^2} \frac{\mathrm{d}\mathcal{I}_{\mathcal{V}}}{\mathrm{d}\mathcal{I}_{\phi}} \frac{\mathrm{d}^3 \mathcal{I}_{\mathcal{V}}}{\mathrm{d}\mathcal{I}_{\phi}^3} . \tag{12}$$

# Inflationary observables

Inflationary observables in the slow-roll approximation LJ *et al* 1612.06863

- the scalar spectral index

$$n_s = 1 - 3M_{\rm Pl}^2 \left(\frac{\mathrm{d}\ln\mathcal{I}_\mathcal{V}}{\mathrm{d}\mathcal{I}_\phi}\right)^2 + 2\frac{M_{\rm Pl}^2}{\mathcal{I}_\mathcal{V}}\frac{\mathrm{d}^2\mathcal{I}_\mathcal{V}}{\mathrm{d}\mathcal{I}_\phi^2}\,,$$

- the tensor-to-scalar ratio $r$,

$$r = 8M_{\rm Pl}^2 \left(\frac{\mathrm{d}\ln\mathcal{I}_\mathcal{V}}{\mathrm{d}\mathcal{I}_\phi}\right)^2\,,$$

- the running of the index

$$\frac{\mathrm{d}n_s}{\mathrm{d}\ln k} = 2M_{\rm Pl}^4 \frac{1}{\mathcal{I}_\mathcal{V}}\frac{\mathrm{d}\ln\mathcal{I}_\mathcal{V}}{\mathrm{d}\mathcal{I}_\phi}\left[4\frac{\mathrm{d}\ln\mathcal{I}_\mathcal{V}}{\mathrm{d}\mathcal{I}_\phi}\frac{\mathrm{d}^2\mathcal{I}_\mathcal{V}}{\mathrm{d}\mathcal{I}_\phi^2} - 3\mathcal{I}_\mathcal{V}\left(\frac{\mathrm{d}\ln\mathcal{I}_\mathcal{V}}{\mathrm{d}\mathcal{I}_\phi}\right)^3 - \frac{\mathrm{d}^3\mathcal{I}_\mathcal{V}}{\mathrm{d}\,\mathcal{I}_\phi^3}\right],$$

- the number of e-folds (till $\epsilon(\mathcal{I}_\phi^{\rm end}) = 1$ )

$$N(\mathcal{I}_\phi^N) = \frac{1}{M_{\rm Pl}^2}\int_{\mathcal{I}_\phi^{\rm end}}^{\mathcal{I}_\phi^N} \mathcal{I}_\mathcal{V}(\mathcal{I}_\phi)\left(\frac{\mathrm{d}\mathcal{I}_\mathcal{V}(\mathcal{I}_\phi)}{\mathrm{d}\mathcal{I}_\phi}\right)^{-1}\mathrm{d}\mathcal{I}_\phi\,,$$

# Classification of inflationary models

- the amplitude of the scalar power spectra

$$A_s = \frac{\mathcal{I}_{\mathcal{V}}}{12\pi^2 M_{\rm Pl}^6} \left( \frac{\mathrm{d} \ln \mathcal{I}_{\mathcal{V}}}{\mathrm{d}\mathcal{I}_\phi} \right)^{-2} .$$

Main points:

- The observables are clearly invariant quantities.
- The observables depend on the invariant potential $\mathcal{I}_{\mathcal{V}}(\mathcal{I}_\phi)$ only.
- Any class of theories with the same functional form of $\mathcal{I}_{\mathcal{V}}(\mathcal{I}_\phi)$ delivers the same phenomenology.
- Algorithm to compute the observables for any model.

LJ *et al* 1612.06863

Invariant matter coupling $\mathcal{I}_{\rm m}(\mathcal{I}_\phi)$ may still play a role in further distinguishing between the inflation models through observables which depend on the couplings of the inflaton to matter (the reheating temperature of the Universe, the baryon asymmetry generated, the thermal production of Dark Matter, ...).

# Algorithm to analyze any model

Take for example Higgs inflation Bezrukov, Shaposhnikov 0710.3755 [hep-th]

1. Identify the functions of the model

$$\mathcal{A}(\Phi) = \frac{M^2 + \xi\Phi^2}{M_{\rm Pl}^2}\,, \quad \mathcal{B}(\Phi) = 1\,, \quad \mathcal{V}(\Phi) = \frac{\lambda}{4}(\Phi^2 - v^2)^2\,.$$

2. Compute (7)

$$\mathcal{I}_\phi(\Phi) = \sqrt{6}M_{\rm Pl}\ln\left(\frac{\sqrt{\xi}\Phi}{M_{\rm Pl}}\right)\,,$$

where set $\mathcal{I}_\phi(M_{\rm Pl}/\sqrt{\xi}) = 0$. Invert to $\Phi(\mathcal{I}_\phi)$.

3. Calculate the invariant potential $\mathcal{I}_\mathcal{V}(\Phi(\mathcal{I}_\phi))$ from (6), i.e.

$$\mathcal{I}_\mathcal{V}(\mathcal{I}_\phi) \simeq \frac{\lambda}{4}\frac{M_{\rm Pl}^4}{\xi^2}\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\mathcal{I}_\phi}{M_{\rm Pl}}\right)\right]^2\,.$$

# Algorithm to analyze any model

4. Compute the slow roll parameters (10), (11)

$$\epsilon = \frac{4}{3} \exp\left(-2\sqrt{\frac{2}{3}}\frac{\mathcal{I}_\phi}{M_{\rm Pl}}\right)\left[1-\exp\left(-\sqrt{\frac{2}{3}}\frac{\mathcal{I}_\phi}{M_{\rm Pl}}\right)\right]^{-2},$$
$$\eta = \left[2-\exp\left(\sqrt{\frac{2}{3}}\frac{\mathcal{I}_\phi}{M_{\rm Pl}}\right)\right]\epsilon\,.$$

Solving now for $\epsilon(\mathcal{I}_\phi^{\rm end}) = 1$ yields $\mathcal{I}_\phi^{\rm end} \simeq 0.94 M_{\rm Pl}$. The number of $e$-folds is then given by

$$N \simeq \frac{3}{4}\left[\exp\left(\sqrt{\frac{2}{3}}\frac{\mathcal{I}_\phi}{M_{\rm Pl}}\right) - \sqrt{\frac{2}{3}}\frac{\mathcal{I}_\phi}{M_{\rm Pl}}\right] - 1.$$

Invert $\mathcal{I}_\phi(N)$.

5. Finally get

$$n_s \simeq 1 - \frac{2}{N+1} - \frac{9-3\ln\left[\frac{4}{3}(N+1)\right]}{2(N+1)^2}\,, \quad r \simeq \frac{12}{(N+1)^2}\left\{1+\frac{3-3\ln\left[\frac{4}{3}(N+1)\right]}{2(N+1)}\right\}.$$

To leading order matches the Einstein and Jordan frame results Bezrukov, Shaposhnikov 0710.3755 [hep-th]; van de Bruck, Longden 1512.04768 .

# Can also find new models

1. For example take

$$\mathcal{A} = 1\,, \quad \mathcal{B} = e^{-\frac{\Phi^2}{M_{\rm Pl}^2}}\,, \quad \mathcal{V} = M^4 e^{-\frac{b\Phi}{M_{\rm Pl}}}$$

2. resulting in

$$\mathcal{I}_\phi = \sqrt{\frac{\pi}{2}} M_{\rm Pl}\, \mathrm{Erf}\left(\frac{1}{\sqrt{2}} \frac{\Phi}{M_{\rm Pl}}\right)\,,$$

where $\mathrm{Erf}$ is the "error function" usually appearing in statistics. As its inverse function, $\mathrm{InvErf}$, is also known,

3. we obtain

$$\mathcal{I}_{\mathcal{V}}(\mathcal{I}_\phi) = M^4 \exp\left(-\sqrt{2} b\, \mathrm{InvErf} \sqrt{\frac{2}{\pi}} \frac{\mathcal{I}_\phi}{M_{\rm Pl}}\right)\,. \qquad (2)$$

4.-6. can go on computing the slow roll parameters and inflationary observables, the model is marginally consistent with Planck data.

LJ *et al* 1612.06863

# Recap

The formalism of invariants in scalar-tensor gravity helps to

- find the equivalent inflationary models and thus classify them,
- quickly compute the inflationary observables for any scalar-tensor model.

LJ *et al* 1612.06863

# Some further developments

Einstein and Jordan frame spectral indices are equivalent up to

- second order Kuusk, Rünkla, Saal, Vilson 1605.07033
- third order Karam, Pappas, Tamvakis 1707.00984

in the slow roll expansion.

However, the number of e-folds in these frames is related by Karam, Pappas, Tamvakis 1707.00984

$$dN_J = dN_E + \frac{1}{2} d \ln \mathcal{I}_{\mathrm{m}}$$

The difference gives slightly different predictions for the observables, subleading effect (comparable to the difference between first, second or third order approximation).

# Aside: the frame issue

Jordan and Einstein frame are

- as classical theories equivalent, provided one transforms the units
  R. H. Dicke, Mach's principle and invariance under transformation of units , Phys. Rev. 125 (1962) .
- as quantum theories equivalent tree-level and 1-loop on-shell, but not off-shell Kamenshchik, Steinwachs, 1408.5769
  (or are they?) Karamitsos, Pilaftsis 1706.07011, 1801.07151

# Related results

Other observables/features computed in terms of invariants

- Parametrized post-Newtonian parameters for point mass LJ, Kuusk, Saal, Vilson 1411.1947
- Parametrized post-Newtonian parameters for homogeneous sphere Hohmann, Schärer 1708.07851
- Cosmological de Sitter attractor solutions LJ, Kuusk, Saal, Vilson 1411.1947
- Correspondence of frames even at the $\omega_{BD} \to \infty$ limit LJ, Kuusk, Saal, Vilson 1504.02686

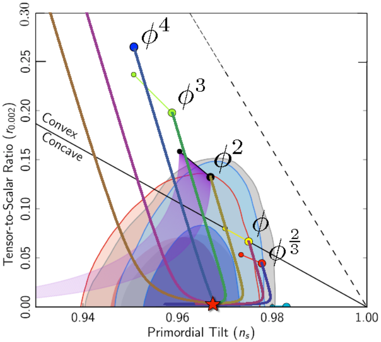
The formalism of invariants has been generalized to

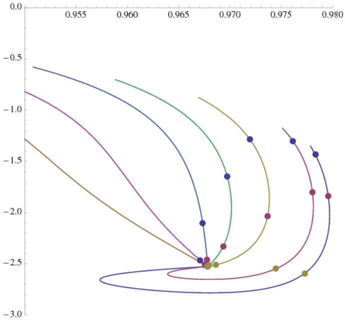
- Multiscalar-tensor gravity fields Kuusk, LJ, Vilson 1509.02903
- Scalar-tensor gravity in the Palatini approach Kozak 1710.09446
- Scalar-torsion gravity Hohmann 1801.06531
- Higher-dimensional scalar-tensor gravity Karam, Lykkas, Tamvakis 1803.04960

A related approach is

- related work Karamitsos, Pilaftsis 1706.07011

# Inflationary attractors




Kallosf, Linde, Roest 1310.3950

With nonminimal coupling on noncanonical kinetic term different potentials lead to the same inflationary predictions.

# Anatomy of inflationary attractors

- Inflationary observables are determined by the invariant potential $\mathcal{I}_{\mathcal{V}}(\mathcal{I}_\phi)$, which depends not only $\mathcal{V}(\Phi)$, but also $\mathcal{A}(\Phi)$ and $\mathcal{B}(\Phi)$.
- The field invariant $\mathcal{I}_\phi(\Phi)$ can very much "stretched out" compared to $\Phi$, and thys "smooth out" $\mathcal{I}_{\mathcal{V}}(\mathcal{I}_\phi)$ compared to $\mathcal{V}(\Phi)$. Hence the attractors.
- From $\mathcal{I}_{\mathcal{V}}(\mathcal{I}_\phi)$ can work "backwards" and construct many equivalent models with $\mathcal{A}(\Phi)$, $\mathcal{B}(\Phi)$, $\mathcal{V}(\Phi)$, that exhibit an attractor behavior.
- The limitation is that not all $\mathcal{I}_{\mathcal{V}}(\mathcal{I}_\phi)$ are in agreement with Planck, and
- not all $\mathcal{A}(\Phi)$, $\mathcal{B}(\Phi)$, $\mathcal{V}(\Phi)$ are appreciated by the particle physics point of view.

LJ, Rünkla, Saal, ... *in progress*

# Summary

The formalism of invariants in scalar-tensor gravity

- helps to find equivalent inflationary models and thus classify them based on invariant potential
- allows toquickly compute the inflationary observables for any model
  LJ, Kristjan Kannike, Luca Marzola, Antonio Racioppi, Martti Raidal, Mihkel Rünkla, Margus Saal, Hardi Veermäe 1612.06863
- helps to understand better and construct new inflationary attractors
  LJ, Rünkla, Saal, ... *in progress*
- can be extended to Palatini theories
- perhaps is useful to study the quantum corrections