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Spacetime curvature and vacuum stability during and after inflation

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Based on:

- Herranen, Markkanen, Nurmi & AR, PRL113 (2014) 211102
- Herranen, Markkanen, Nurmi & AR, PRL115 (2015) 241301
- AR & Stopyra, PRD95 (2017) 025008
- AR & Stopyra, PRD97 (2018) 025012
- Figueroa, AR & Torrenti, arXiv:1709.00398
- Markkanen, Nurmi, AR & Stopyra, JHEP 1806 (2018) 040

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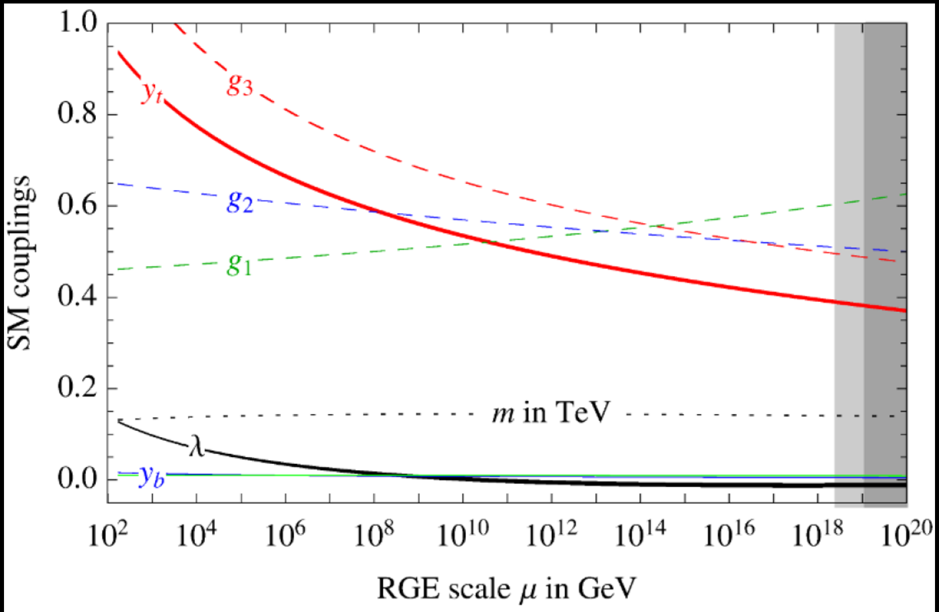
The Standard Model

- Six leptons, six quarks, three gauge fields + Higgs scalar ϕ
- All renormalisable terms allowed by symmetries in Minkowski space
- 19 parameters – all have been measured
- Can be extrapolated all the way to the Planck scale



(Buttazzo et al 2013)

Vacuum Instability

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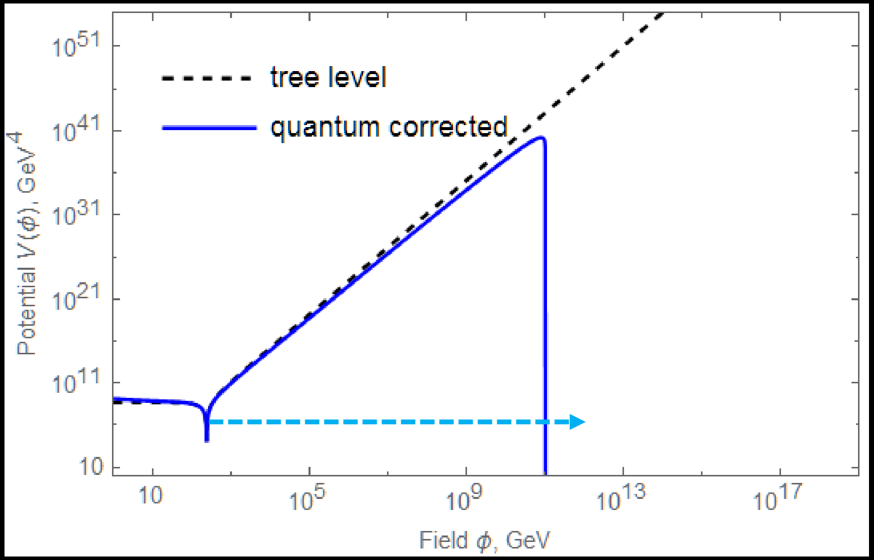
- Higgs effective potential

$$V(\phi) \approx \lambda(\phi)\phi^4$$

- Becomes negative at $\phi > \phi_c \approx 10^{10}\mathrm{GeV}$
- True vacuum at Planck scale?
- Current vacuum metastable against quantum tunneling

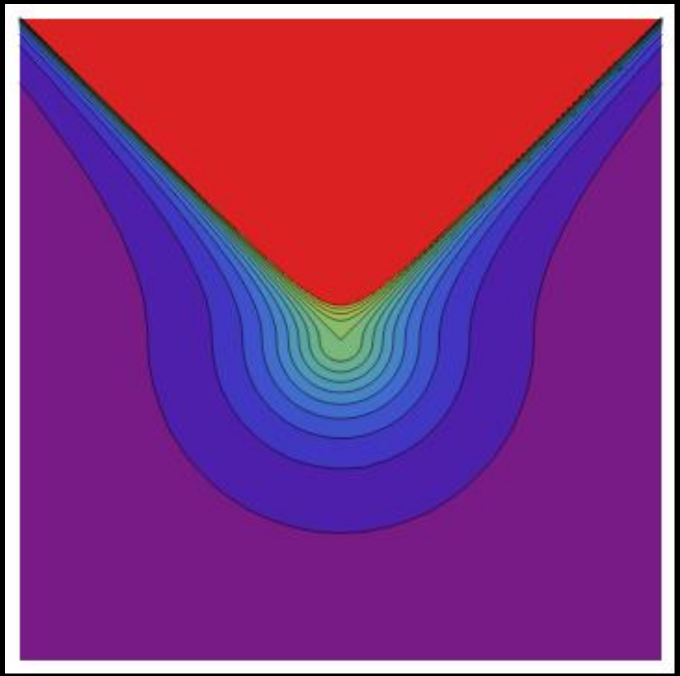


Tunneling Rate

- Bubble nucleation rate: $\Gamma \sim e^{-B}$, where $B =$ action of the "bounce" solution (Coleman 1977)
- Solve Euclidean eqs of motion
- Constant $\lambda < 0$:

$$\phi(r) = \sqrt{\frac{2}{|\lambda|}}\frac{2R}{r^2+R^2}$$

- Action $B = \frac{8\pi^2}{3|\lambda|}$



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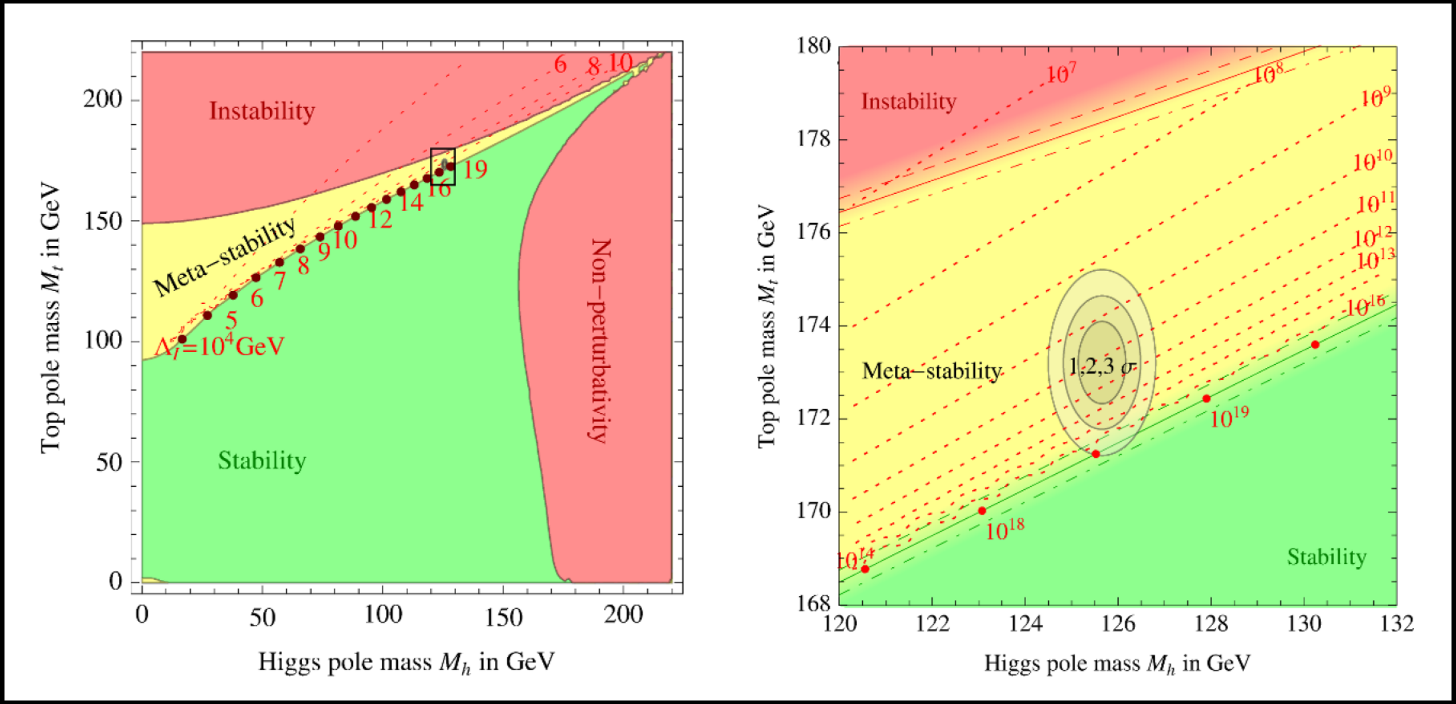
Instability Bounds



(Buttazzo et al. 2013)

- Vacuum lifetime $\sim \exp(B) \sim \exp\left(\frac{8\pi^2}{3|\lambda|}\right)$
- Longer than the age of the Universe

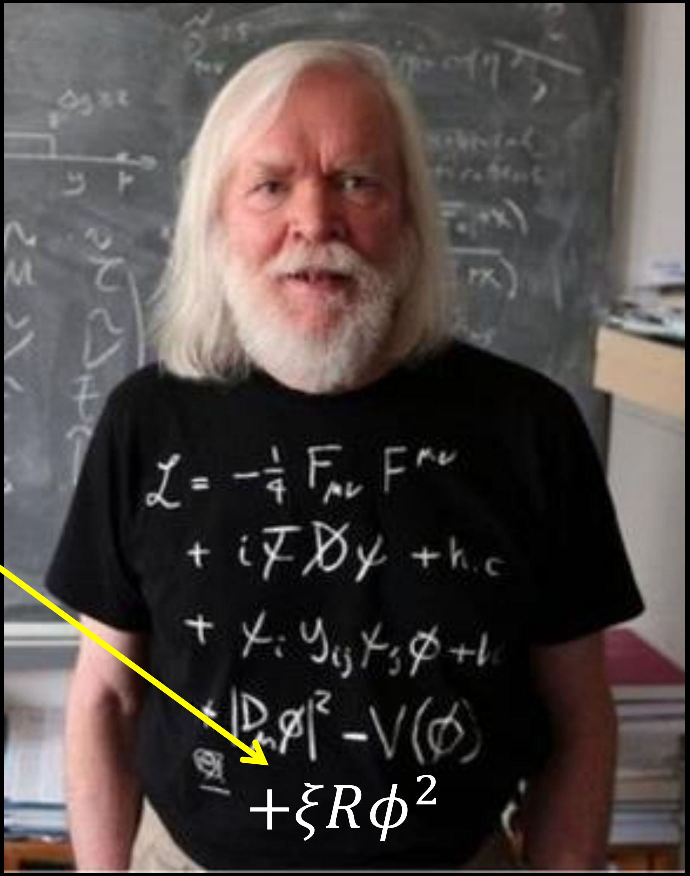
Higgs-Curvature Coupling

- Curved spacetime:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \xi R \phi^\dagger \phi$$

(Chernikov&Tagirov 1968)

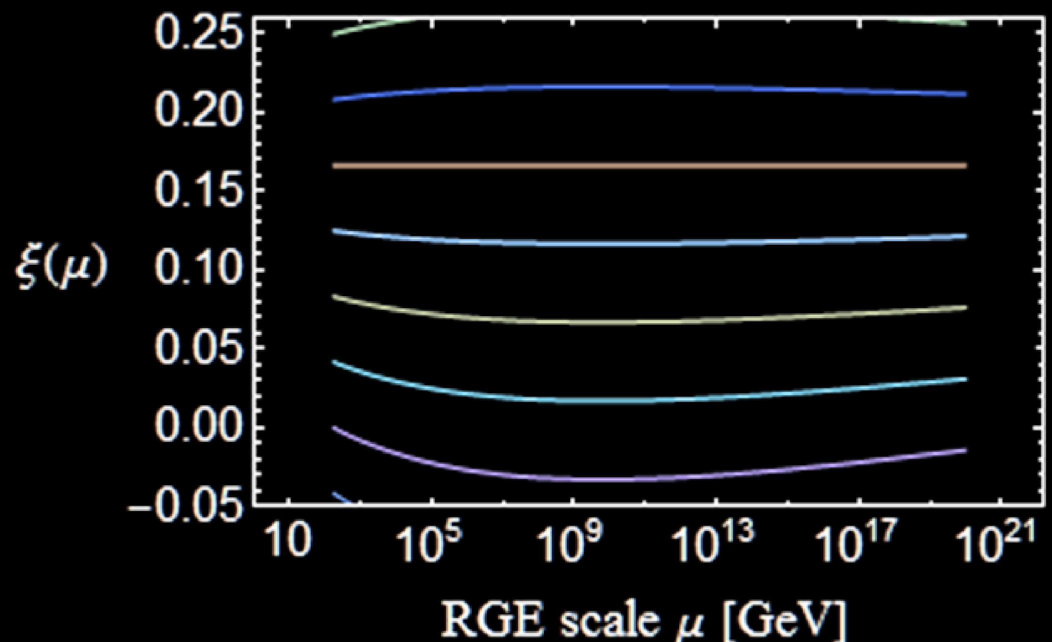
- Symmetries allow one more renormalisable term: Higgs-curvature coupling ξ
- Last unknown parameter in the Standard Model



Running ξ

$$\mu \frac{d\xi}{d\mu} = \left(\xi - \frac{1}{6}\right) \frac{12\lambda + 6y_t^2 - \frac{3}{2}g'^2 - \frac{9}{2}g^2}{16\pi^2}$$

- Cannot be set to zero
- Becomes negative if $\xi_{\mathrm{EW}} = 0$
- Conformal value $\xi = 1/6$ RG invariant at 1 loop but not beyond



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Measuring ξ

- Curved spacetime:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \xi R \phi^\dagger \phi$$

- Ricci scalar R very small today
  $\Rightarrow$ Difficult to measure ξ
- Colliders: Suppresses Higgs couplings (Atkins&Calmet 2012)
  - LHC Bound $|\xi| \lesssim 2.6 \times 10^{15}$
  - Future (?) ILC: $|\xi| \lesssim 4 \times 10^{14}$
- In contrast, R was high in the early Universe

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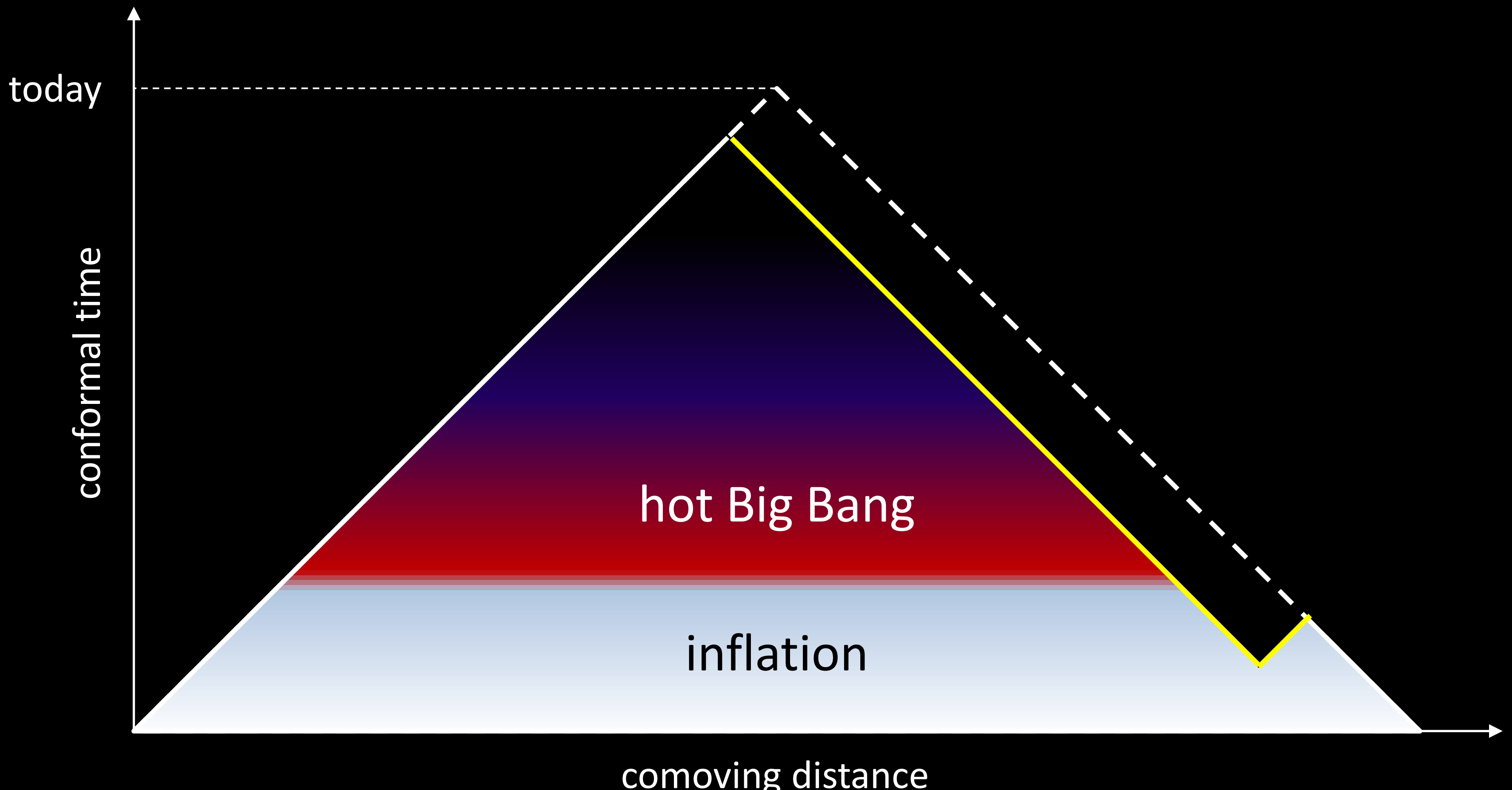
Past Light Cone



Early Universe

- Assume:
  - Light, subdominant Higgs
  - Inflaton decoupled from the Higgs
- Effective Higgs mass term $m_{\rm eff}^2(t) = m_{\rm H}^2 + \xi R(t)$
- Ricci scalar in FRW spacetime:

$$R = 6\left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right) = 3(1-3w)H^2$$

  - Radiation dominated $w = 1/3$ $R = 0$
  - Matter dominated $w = 0$ $R = 3H^2$
  - Inflation / de Sitter $w = -1$ $R = 12H^2$

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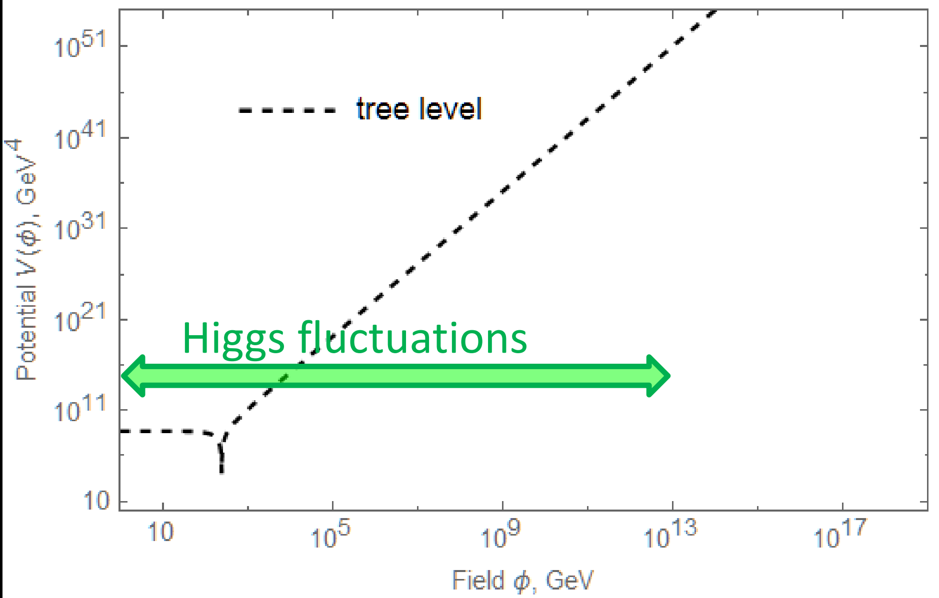
Higgs Fluctuations from Inflation

- Inflation: $H \lesssim 9 \times 10^{13}$ GeV (Planck+BICEP2 2015)
- Equilibrium field distribution (Starobinsky&Yokoyama 1994)

$$P(\phi) \propto \exp\left[-\frac{8\pi^2}{3H^4}V(\phi)\right]$$

- Tree-level potential $V(\phi) = \lambda(\phi^2 - v^2)^2$
- Nearly scale-invariant fluctuations with amplitude $\phi \sim \lambda^{-1/4}H$



Higgs Fluctuations from Inflation

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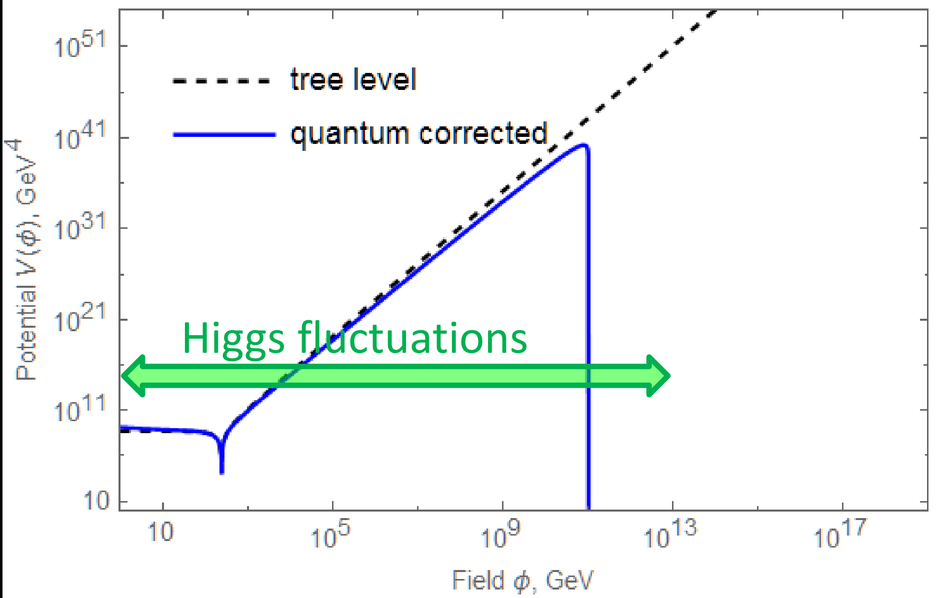
- Equilibrium $P(\phi) \propto \exp\left[-\frac{8\pi^2}{3H^4}V(\phi)\right]$
- Running λ:
  Fluctuations take the Higgs over the barrier if $H \gtrsim \phi_c \approx 10^{10}\mathrm{GeV}$
  (Espinosa et al. 2008;
  Lebedev&Westphal 2013;
  Kobakhidze&Spencer-Smith 2013;
  Fairbairn&Hogan 2014;
  Hook et al. 2014)
- Does this imply $H \lesssim 10^{10}\mathrm{GeV}$?



Higgs During Inflation

- Inflation: Constant $R = 12H^2$
- Effective mass term
$$m_{\mathrm{eff}}^2 = m_{\mathrm{H}}^2 + \xi R = m_{\mathrm{H}}^2 + 12\xi H^2$$
- Tree level: (Espinosa et al 2008)
  - $\xi > 0$: Increases barrier height
    Makes the low-energy vacuum more stable
  - $\xi < 0$: Decreases barrier height
    Makes the low energy vacuum less stable
- H contributes to loop corrections:
  For $H \gg \phi, V(\phi) \approx \lambda(H)\phi^4$ ⇨ No barrier! (HMNR 2014)

Potential in Curved Spacetime

- One-loop computation in de Sitter:

$$V_{\rm SM}^{\rm eff}(\varphi(\mu)) = -\frac{1}{2}m^2(\mu)\varphi^2(\mu) + \frac{\xi(\mu)}{2}R\varphi^2(\mu) + \frac{\lambda(\mu)}{4}\varphi^4(\mu) + V_\Lambda(\mu) - 12\kappa(\mu)H^2 + \alpha(\mu)H^4 + \frac{1}{64\pi^2}\sum_{i=1}^{31}\left\{n_i\mathcal{M}_i^4(\mu)\left[\log\left(\frac{|\mathcal{M}_i^2|}{\mu^2}\right) - d_i\right] + n_i' H^4 \log\left(\frac{|\mathcal{M}_i^2(\mu)|}{\mu^2}\right)\right\}. \quad (5.3)$$

Ψ	i	n_i	d_i	n_i'	$\mathcal{M}_i^2$
$W^\pm$	1	2	3/2	$-34/15$	$m_W^2 + H^2$
	2	6	5/6	$-34/5$	$m_W^2 + H^2$
	3	-2	3/2	4/15	$m_W^2 - 2H^2$
Z^0	4	1	3/2	$-17/15$	$m_Z^2 + H^2$
	5	3	5/6	$-17/5$	$m_Z^2 + H^2$
	6	-1	3/2	2/15	$m_Z^2 - 2H^2$
q	$7-12$	-12	3/2	38/5	$m_q^2 + H^2$
l	$13-15$	-4	3/2	38/15	$m_l^2 + H^2$
h	16	1	3/2	$-2/15$	$m_h^2 + 12(\xi - 1/6)H^2$
χ_W	17	2	3/2	$-4/15$	$m_\chi^2 + \zeta_W m_W^2 + 12(\xi - 1/6)H^2$
χ_Z	18	1	3/2	$-2/15$	$m_\chi^2 + \zeta_Z m_Z^2 + 12(\xi - 1/6)H^2$
c_W	19	-2	3/2	4/15	$\zeta_W m_W^2 - 2H^2$
c_Z	20	-1	3/2	2/15	$\zeta_Z m_Z^2 - 2H^2$

Ψ	i	n_i	d_i	n_i'	$\mathcal{M}_i^2$
γ	21	1	3/2	$-17/15$	H^2
	22	3	5/6	$-17/5$	H^2
	23	-1	3/2	2/15	$-2H^2$
g	24	8	3/2	$-136/15$	H^2
	25	24	5/6	$-136/5$	H^2
	26	-8	3/2	16/15	$-2H^2$
ν	$27-29$	-2	3/2	19/15	H^2
c_γ	30	-1	3/2	2/15	$-2H^2$
c_g	31	-8	3/2	16/15	$-2H^2$

(MNRS 2018)

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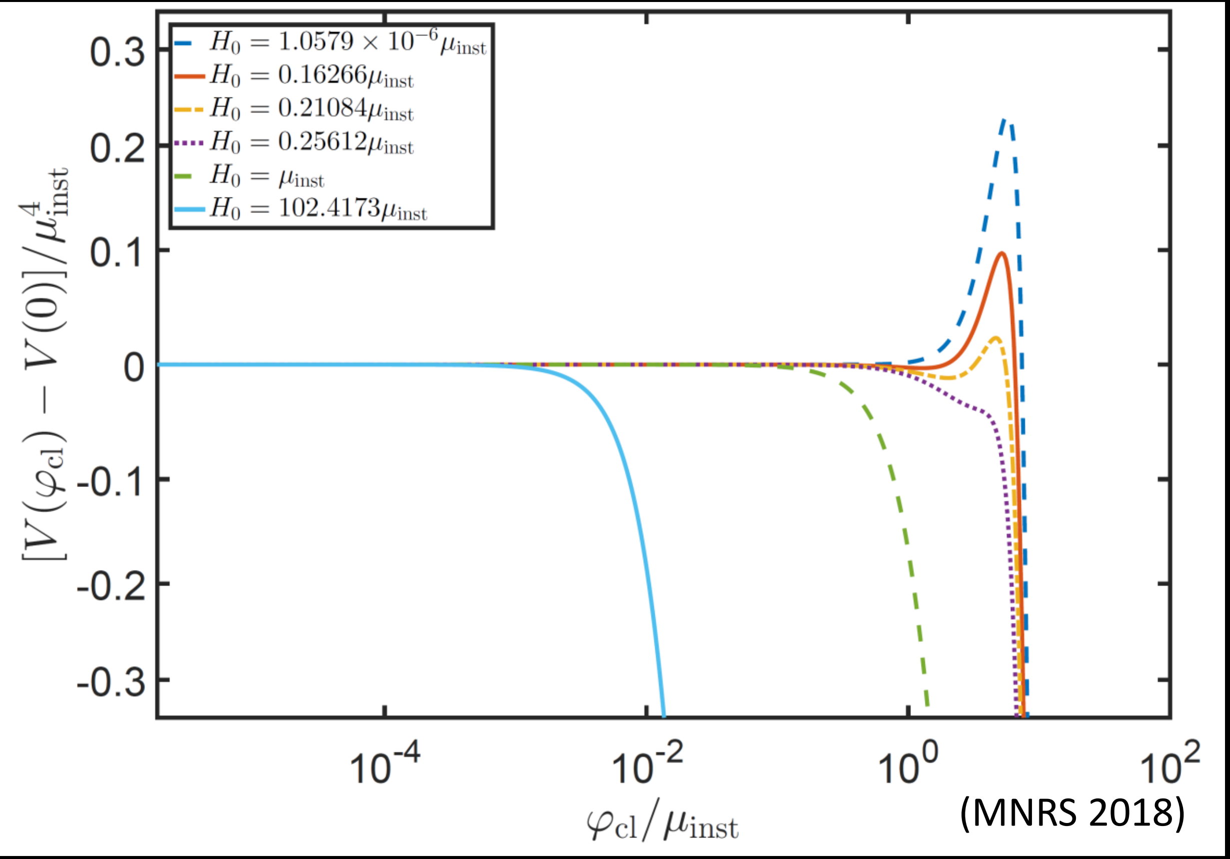
Potential in Curved Spacetime

- One-loop computation for $\xi = 0$ (in units of $\mu_{\text{inst}} \approx 6.6 \times 10^9$ GeV)



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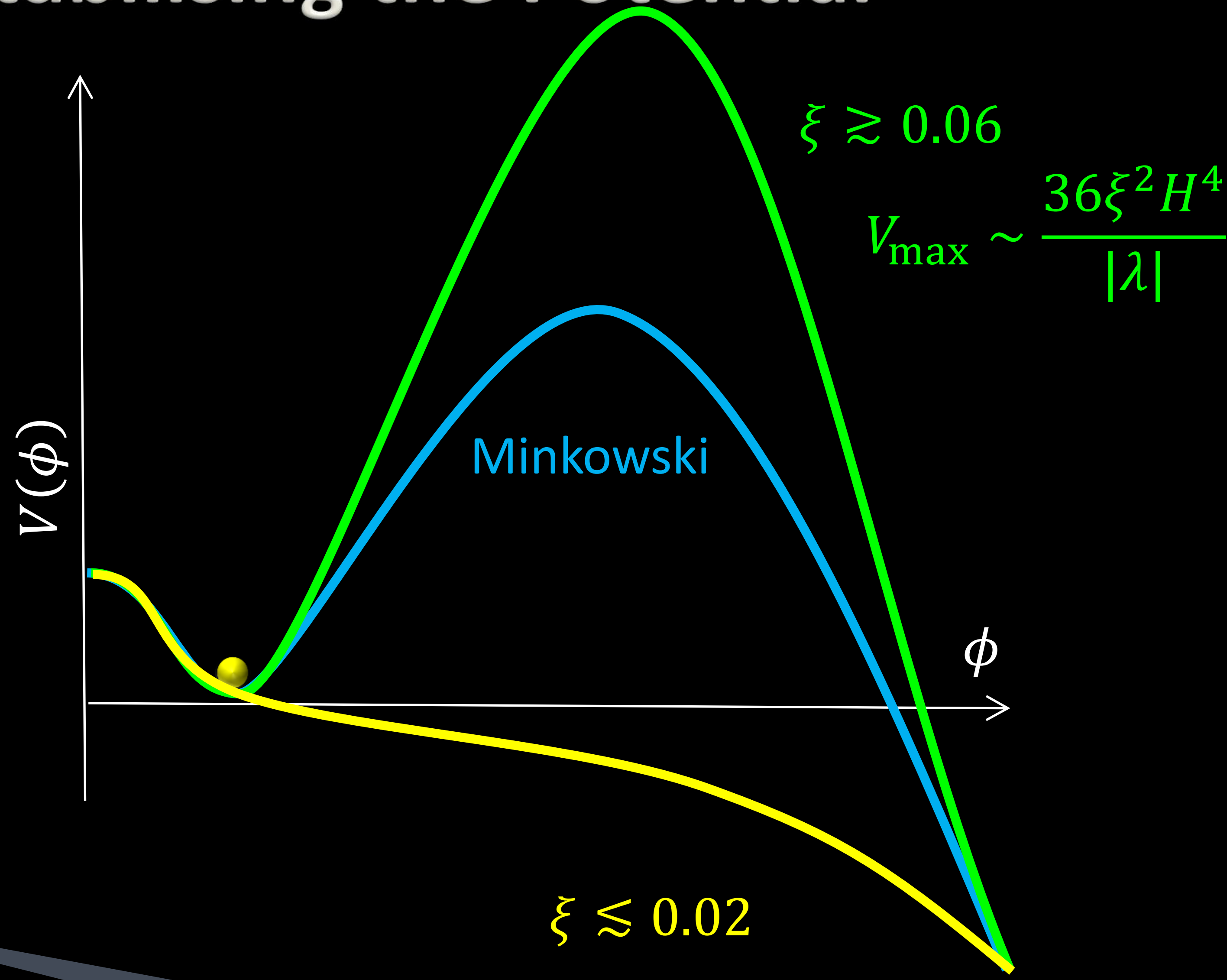
(De-)Stabilising the Potential



(De)Stabilising the Potential

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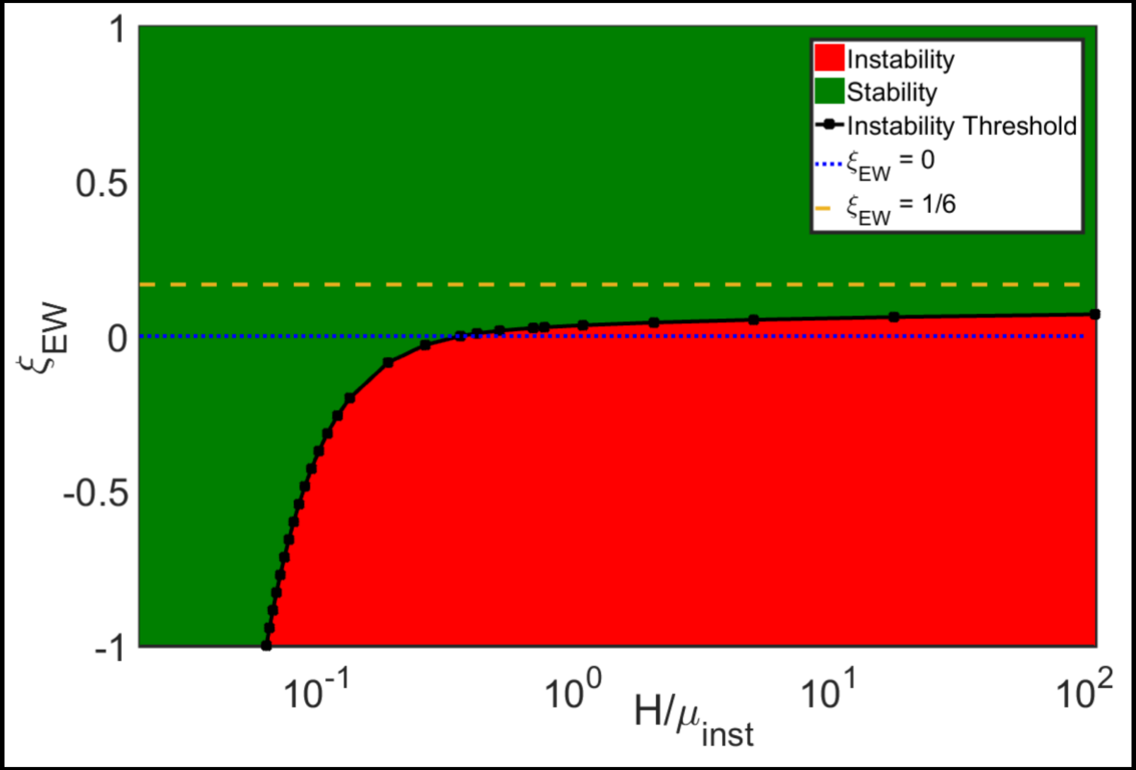

(MNRS 2018)

- If $H \gtrsim \mu_{\mathrm{inst}} = 6.6 \times 10^9\mathrm{GeV}$ and there is no new physics, vacuum stability during inflation requires $\xi \gtrsim 0.1$

Quantum Tunneling

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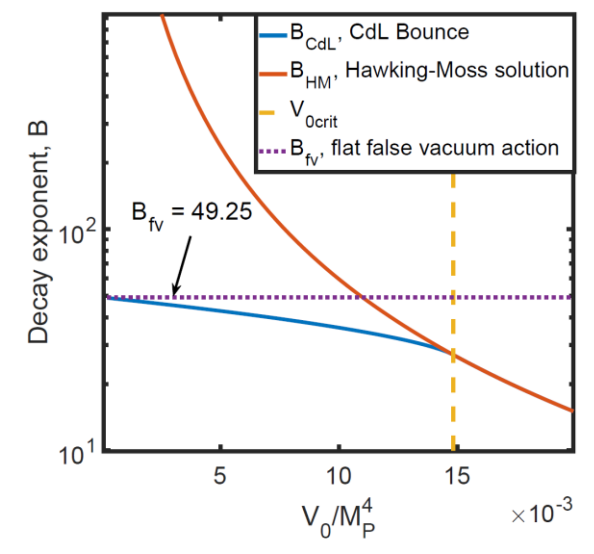
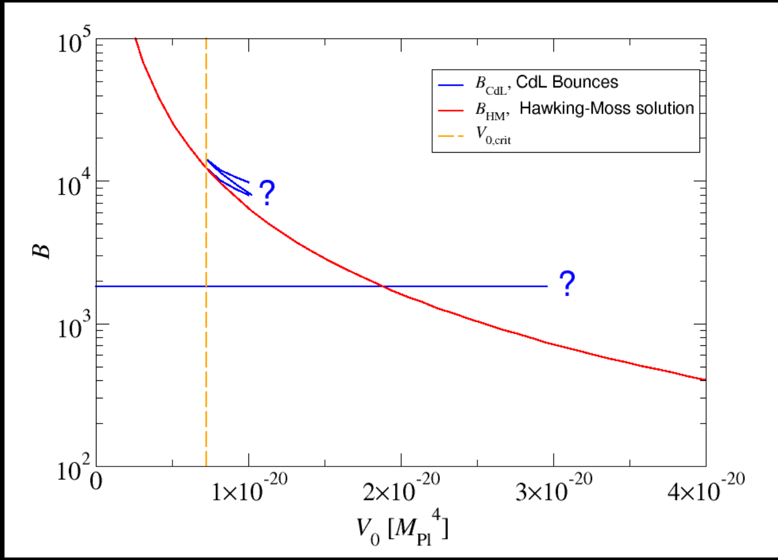
Toy model potential



Standard Model potential



- Multiple coexisting solutions (AR&Stopyra, PRD 2018)
- Tunnelling rate $\Gamma \sim e^{-B}$ nearly constant until Hawking-Moss starts to dominate

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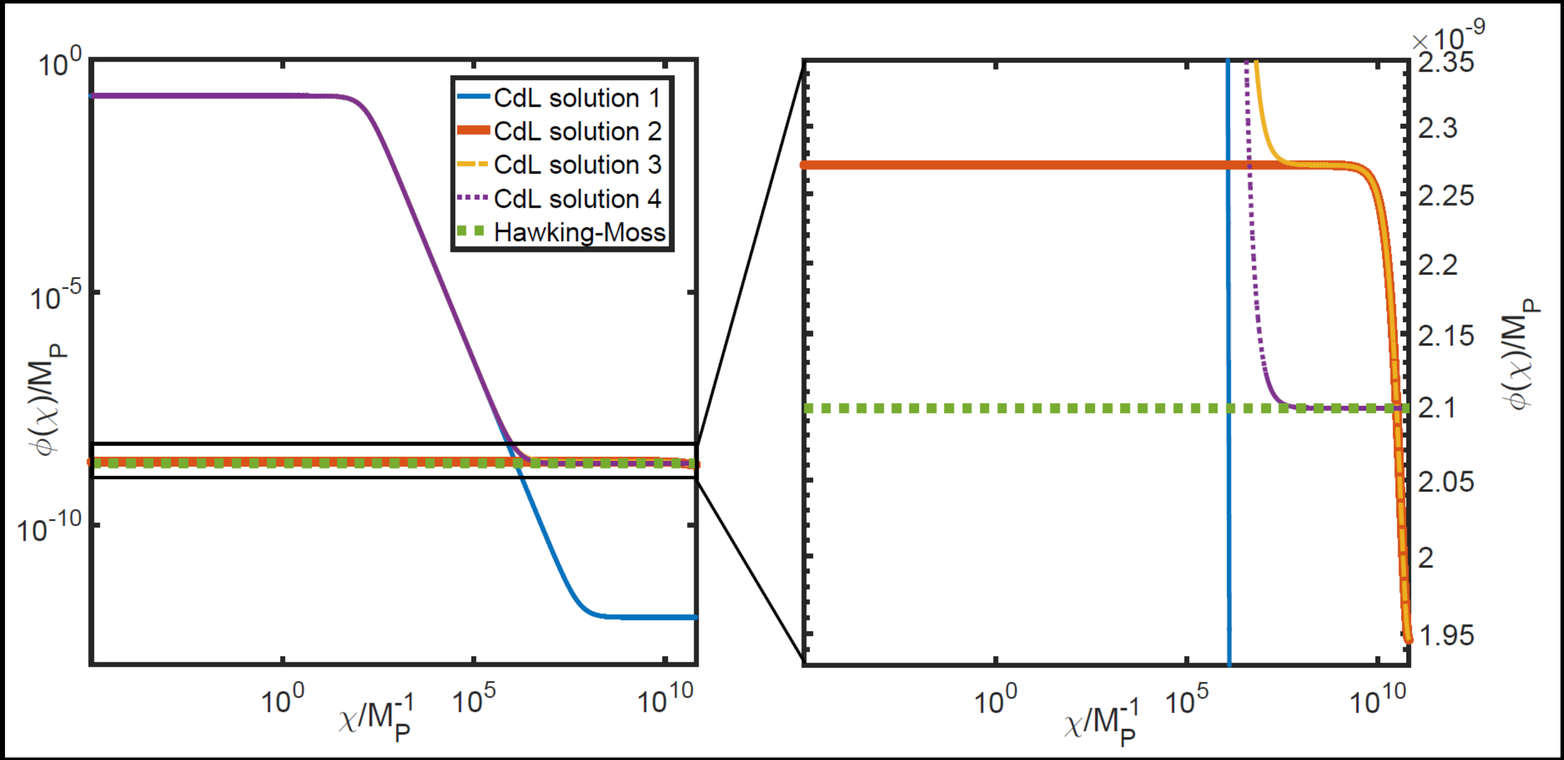
Multiple Solutions



(AR&Stopyra, PRD 2018)

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End of Inflation

- Reheating: Inflation ($R = 12H^2$) → radiation ($R = 0$)

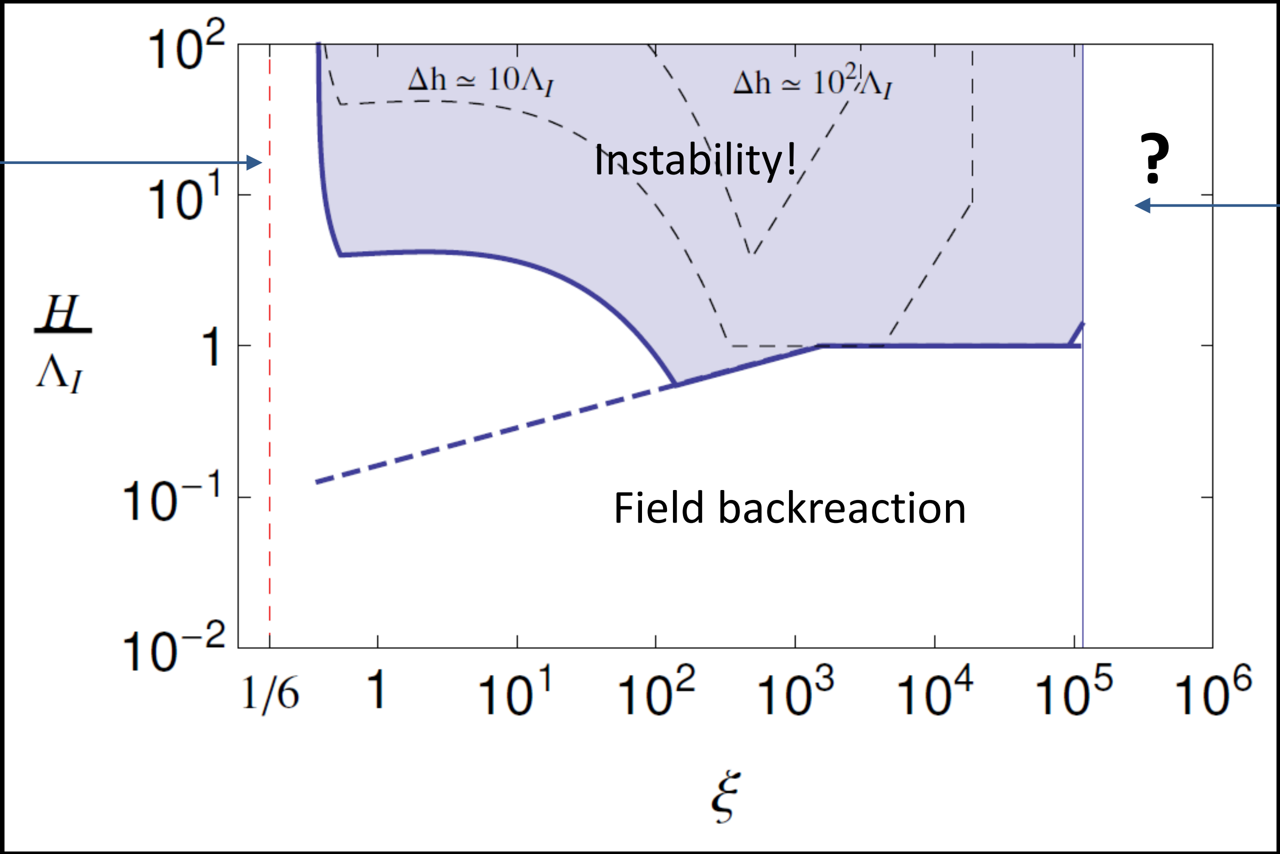
$$R(t) = \frac{2m_\phi^2\phi^2 - \dot{\phi}^2}{M_{\rm Pl}^2}$$

- Effective Higgs mass $m_{\rm eff}^2 = m_{\rm H}^2 + \xi R$ oscillates:
  - "Geometric preheating" (Bassett&Liberati 1998, Tsujikawa et al. 1999)
- R goes negative when $\phi \sim 0$
  - If $\xi > 0$, Higgs becomes tachyonic (HMNR 2015)
  - Exponential amplification

$$\langle\phi^2\rangle_H \sim \frac{2}{3\sqrt{3}\xi}\left(\frac{H}{2\pi}\right)^2 e^{\frac{2\sqrt{\xi}\chi_{\rm ini}}{M_{\rm Pl}}} \sim \frac{H^2}{\xi} e^{2\sqrt{\xi}}$$

Vacuum Decay at the End of Inflation

Not enough growth

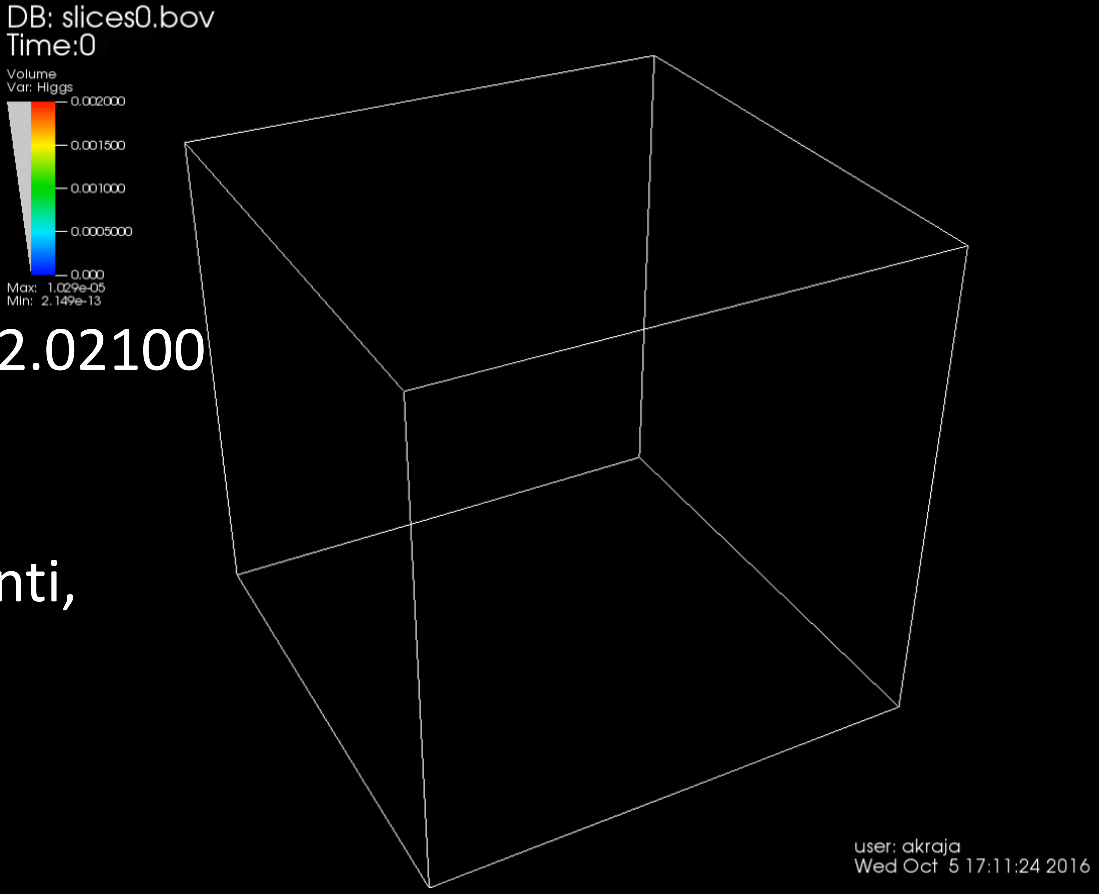


Becomes nonlinear

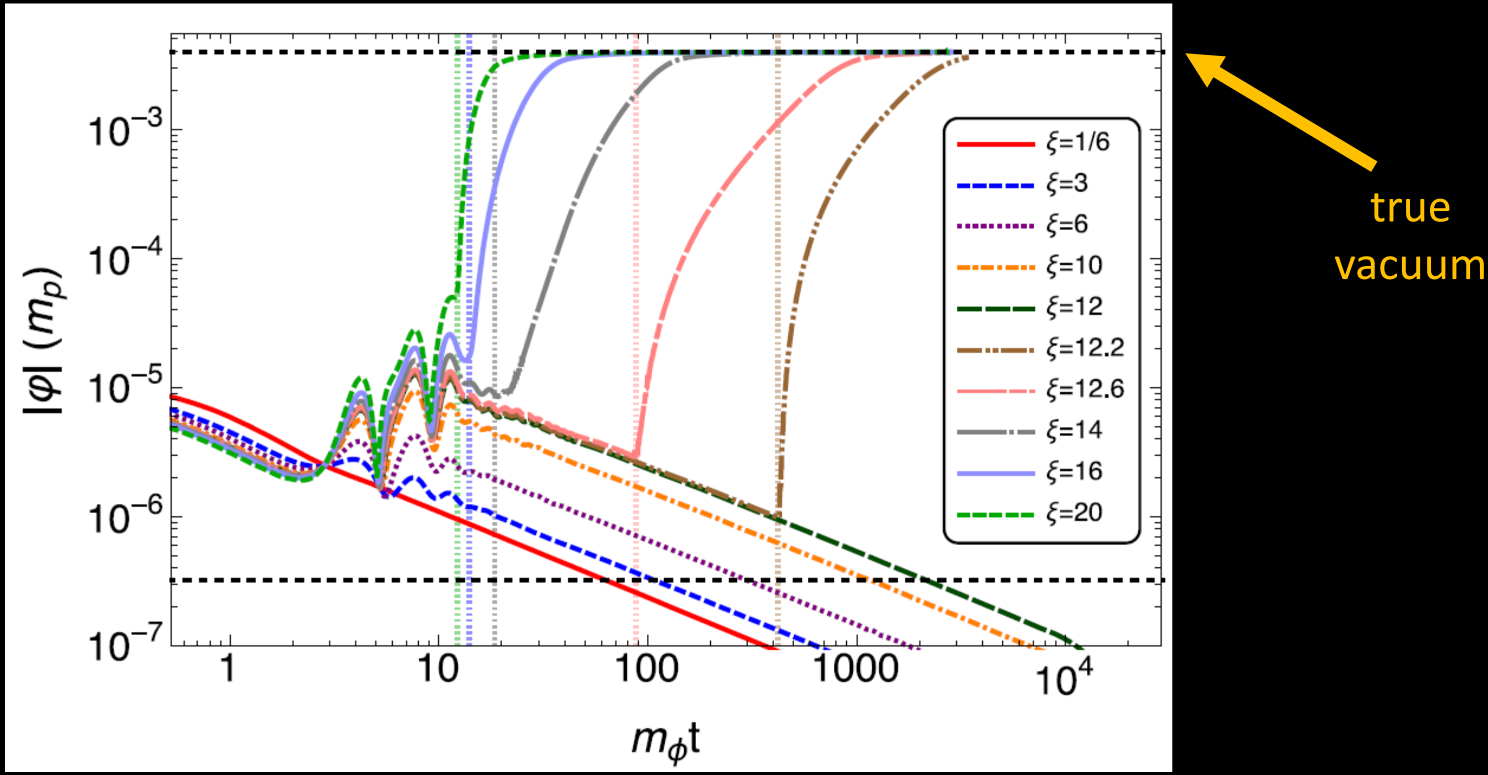
(HMNR 2015)

Detailed Calculations

- Ema et al, arXiv:1602:00483
  - Lattice simulation
  - $\xi \lesssim 10$
- Kohri et al, arXiv:1602.02100
  - Linearised calculation
  - $\xi \lesssim 40$
- Figueroa, AR & Torrenti, arXiv:1709.00398
  - Lattice simulations



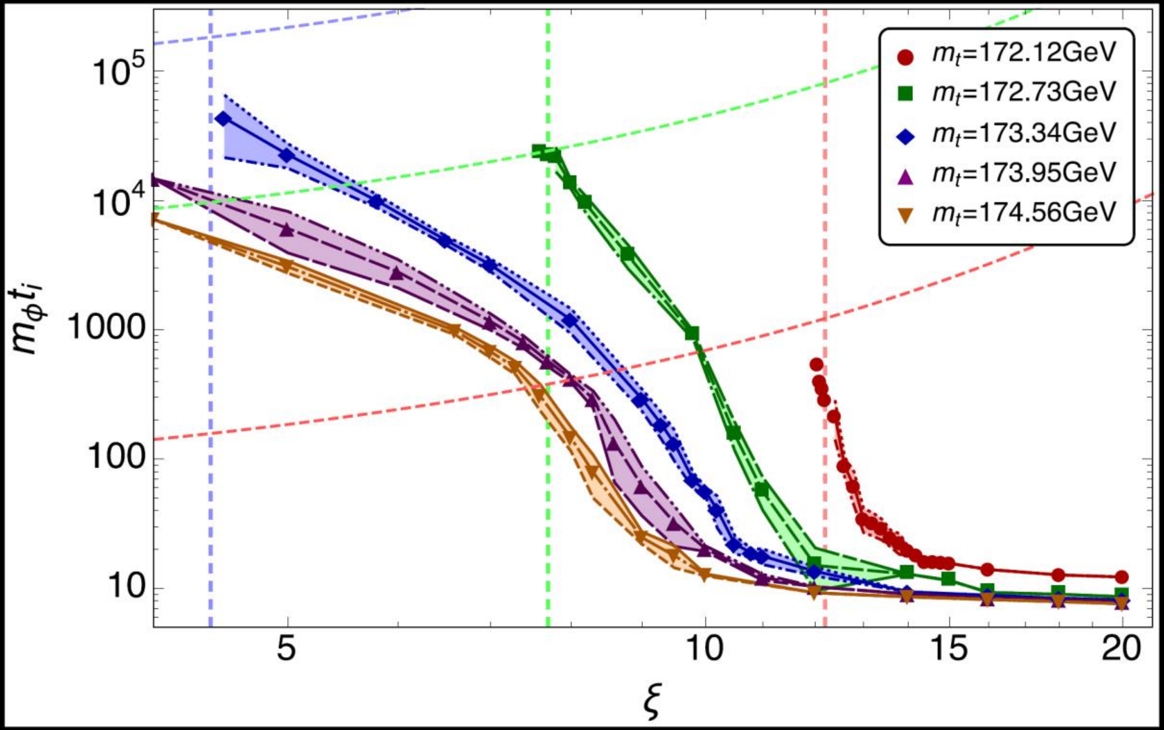
Lattice Simulations



Figueroa, AR & Torrenti, arXiv:1709.00398

- $V(\chi) = \frac{1}{2}m^2\chi^2, M_{\text{top}} = 172.12\ \text{GeV}$

Instability Time



Figueroa, AR & Torrenti, arXiv:1709.00398

- Stability depends on top mass and speed of reheating
- $M_{\text{top}} = 173.34\ \text{GeV}$: vacuum decay before $mt = 100$ if $\xi \gtrsim 9$

Constraints on ξ

- Minimal scenario:
  Standard Model + $m^2\chi^2$ chaotic inflation,
  no direct coupling to inflaton

$$0.06 \lesssim \xi \lesssim 9$$

- 15 orders of magnitude stronger than the LHC bound

$$|\xi| \lesssim 2.6 \times 10^{15}$$

- Caveats:
  - Assumes no direct coupling to inflaton (see Lebedev's talk)
    – Would still need $|\xi| \lesssim O(1)$
  - Assumes no new physics
    – Could stabilise potential altogether
  - Assumes high scale inflation
    – Need $H \gtrsim 10^9$ GeV