

# **From fakeons and Lee-Wick models to quantum gravity**

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I formulate a consistent theory of quantum gravity,  
which has very good chances to be the right one

Action:

$$S_{\text{HD}} = -\frac{\mu^{-\varepsilon}}{2\kappa^2} \int \sqrt{-g} \left[ 2\Lambda_C + \zeta R + \alpha \left( R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2 \right) - \frac{\xi}{6}R^2 \right]$$

$\zeta > 0, \alpha > 0, \xi > 0$

New quantization:

the would-be ghosts are turned into  
fake particles ("fakeons")

A consistent projection allows us to define  
the space of physical states

D. A., On the quantum field theory of the gravitational interactions,

J. High Energy Phys. 06 (2017) 086 and arXiv:1704.07728 [hep-th]

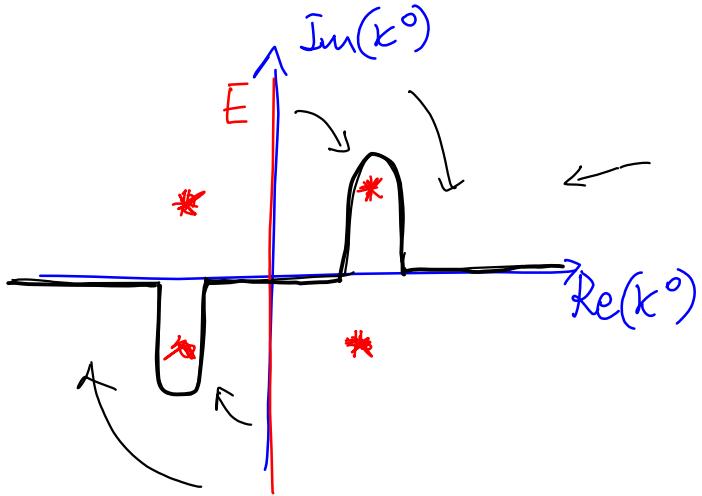
The idea is an extension of a notion that emerged from the reformulation of the Lee-Wick models

T.D. Lee and G.C. Wick, Negative metric and the unitarity of the S-matrix, Nucl. Phys. B 9 (1969) 209;  
T.D. Lee and G.C. Wick, Finite theory of quantum electrodynamics, Phys. Rev. D 2 (1970) 1033

as nonanalytically Wick rotated Euclidean theories

D. A. and M. Piva, A new formulation of Lee-Wick quantum field theory,  
J. High Energy Phys. 06 (2017) 066 and arXiv:1703.04584 [hep-th]

D. A. and M. Piva, Perturbative unitarity of Lee-Wick quantum field theory,  
Phys. Rev. D 96 (2017) 045009 and arXiv:1703.05563 [hep-th]



Lee-Wick contour  
 prescription for  
 the integral  $\int \frac{dk^0}{2\pi}$   
 on the loop energy

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The Lee-Wick models have been surrounded by skepticism (Lorentz invariance & analyticity are in jeopardy)

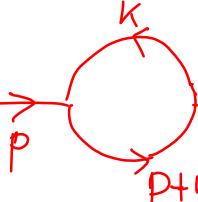
The LW prescription is ambiguous.

Additional prescriptions have been proposed,

e.g. R.E. Cutkosky, P.V Landshoff, D.I. Olive, J.C. Polkinghorne, A non-analytic S matrix,  
Nucl. Phys. B12 (1969) 281

but did not remove the ambiguities completely and did not uncover what is going on

The Wick rotation itself is highly nontrivial (and not analytic), but ultimately helps understanding how to correctly formulate the models

Bubble : 

$$= \int \frac{dk^0}{2\pi} \int_{R^3} \frac{d^3 k}{(2\pi)^3} S(k) S(p+k) =$$

$$= \int_{R^3} \frac{d^3 k}{(2\pi)^3} f(\vec{k}, p)$$

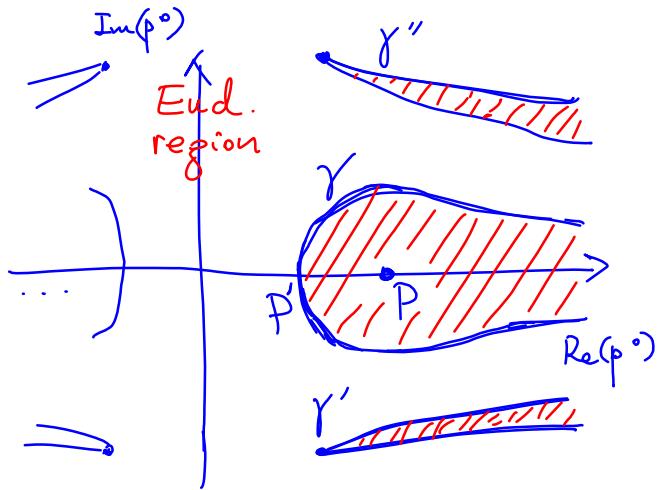
Problems where  
 $f(\vec{k}, p)$  is singular

$$p^0 = \sqrt{\vec{k}^2 \pm iM^2} + \sqrt{(\vec{p}-\vec{k})^2 \pm iM^2}$$

Singularities of  $f(\vec{k}, p)$ :

$$p^0 \in \mathbb{C} \quad \vec{p} \in \mathbb{R}^3$$

$$p^0 = \sqrt{\vec{k}^2 \pm iM^2} + \sqrt{(\vec{p}-\vec{k})^2 \pm iM^2}$$



Inside  $\gamma$  the result is NOT analytic & NOT Lorentz invariant

Beyond Lee-Wick :

$$\int_{\mathbb{R}^3} \frac{d^3 k}{(2\pi)^3} f(\vec{k}, p)$$

The problem is here!

$$= \int \frac{dk^0}{2\pi} \int \frac{d^3 k}{(2\pi)^3} S(k) S(p+k)$$

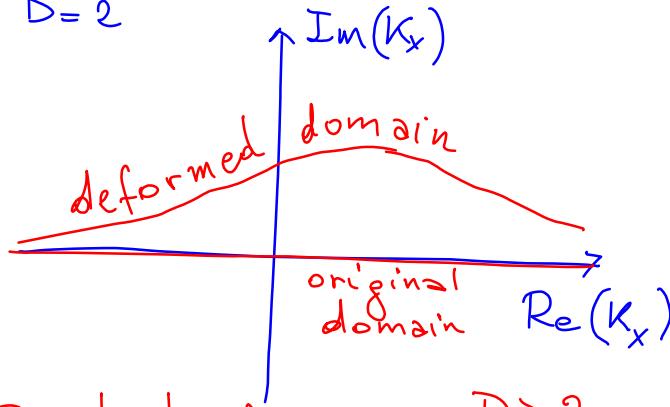
LW  $D_3$  → deformed domain on the loop  
(Complexified) space momenta

D. A., Fakeons and Lee-Wick models,

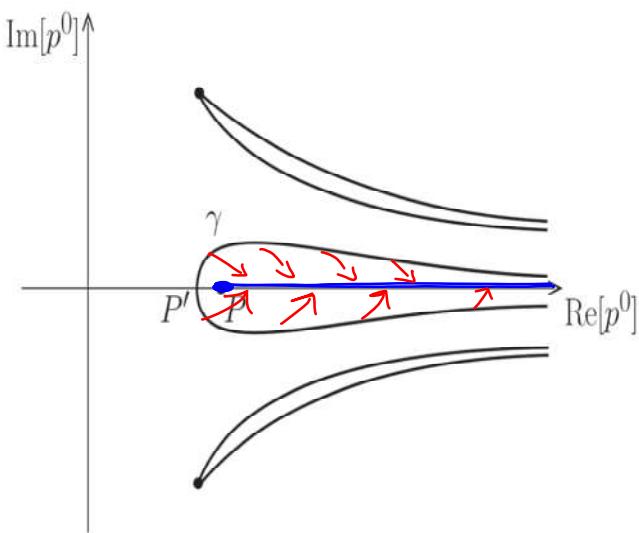
J. High Energy Phys. 02 (2018) 141 and arXiv:1801.00915 [hep-th]

Example :

$$D = 2$$

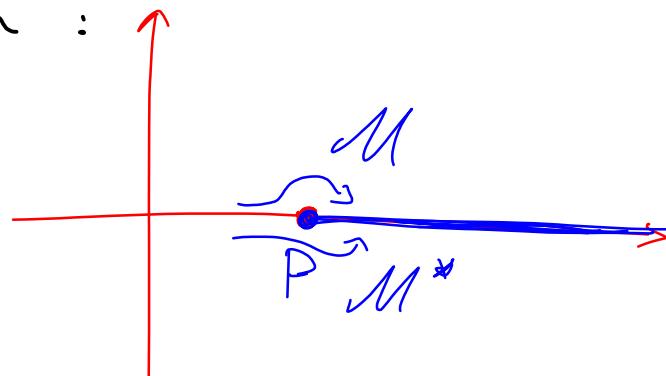


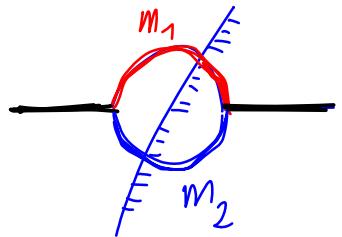
Similarly in every  $D \geq 2$



After the deformation :

$$\int_{D_3} \frac{\partial^3 K}{(2\pi)^3} f(\vec{k}, p)$$





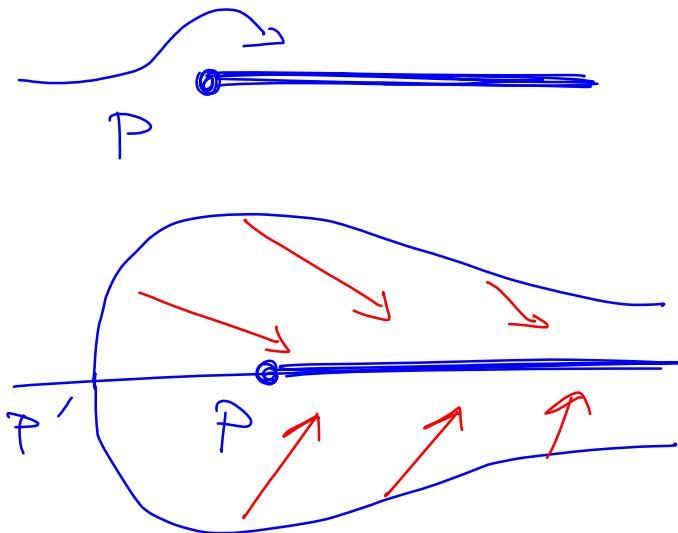
$$P: p^2 = (m_1 + m_2)^2$$

Optical  
theorem (?)

$$2 \operatorname{Im} M = \int d\tau \begin{cases} \text{phase} \\ \text{Space} \end{cases} |m_2|^2$$

$M$  violates unitarity:  $P$  is the threshold  
of an unphysical process  $m_1^2 = \pm i M^2$   $m_2^2 = \pm i M^2$

New possibility : instead of doing

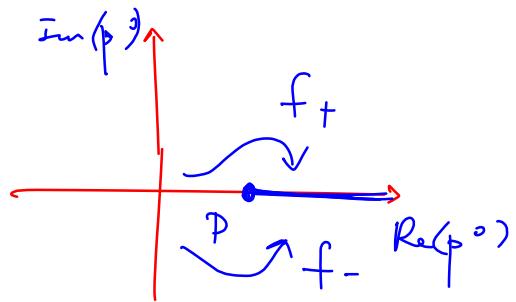


do as follows :

calculate inside,  
then deform the  
domain to  $D_3$

Analyticity and Lorentz invariance are recovered at the end of the deformation,  
but NO imaginary part, no process ("fakreon")

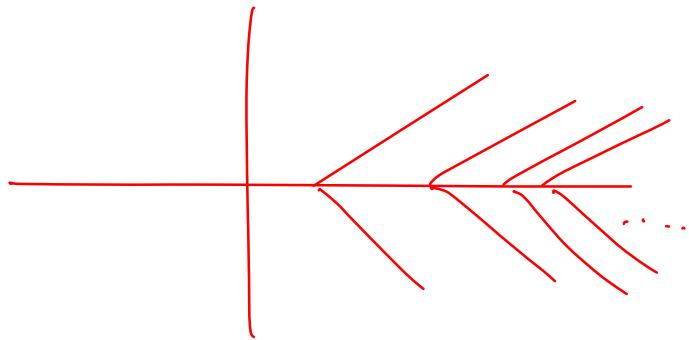
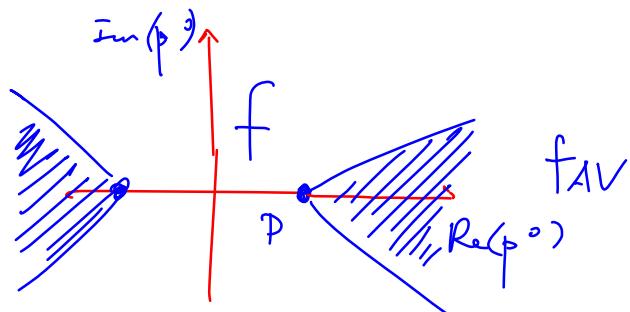
The result is the arithmetic average of the two analytic continuations



$$\text{Result: } f_{AV} = \frac{f_+ + f_-}{2}$$

Analyticity & Lorentz invariance are recovered.

The complex  $\rho^o$  plane is divided into disjoint regions of analyticity



$f_{AV}$  has no absorptive part, so the optical theorem holds :

$$2 \operatorname{Im} M = \int_{\text{phase space}} d\bar{N} \left| \begin{array}{c} f \\ \hline f' \end{array} \right|^2 = 0$$

$\operatorname{Im} M = 0 \Rightarrow$  we can consistently

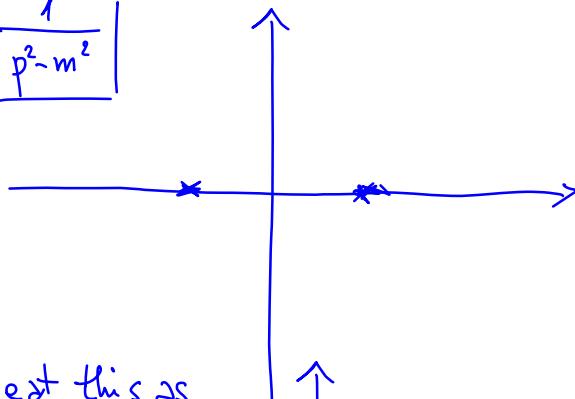
project away the final states  $f, f'$

of squared masses  $\pm iM^2$  from the physical space

The ideas can be generalized to non  
Lee-Wick models, by defining a new (real!!)  
quantization prescription

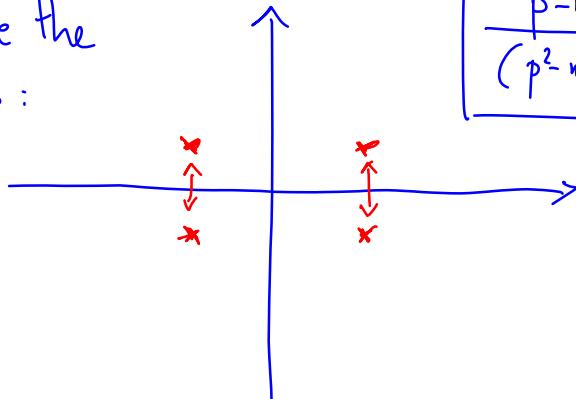
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$$\frac{1}{p^2 - m^2}$$

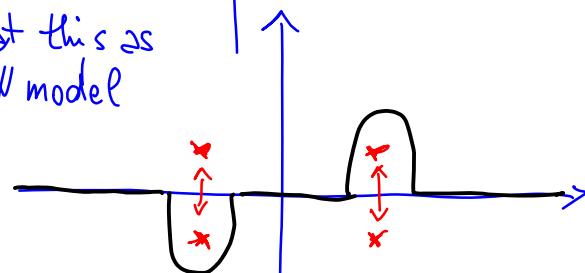


double the  
poles:

$$\frac{p^2 - m^2}{(p^2 - m^2)^2 + \epsilon^4}$$



treat this as  
a LW model

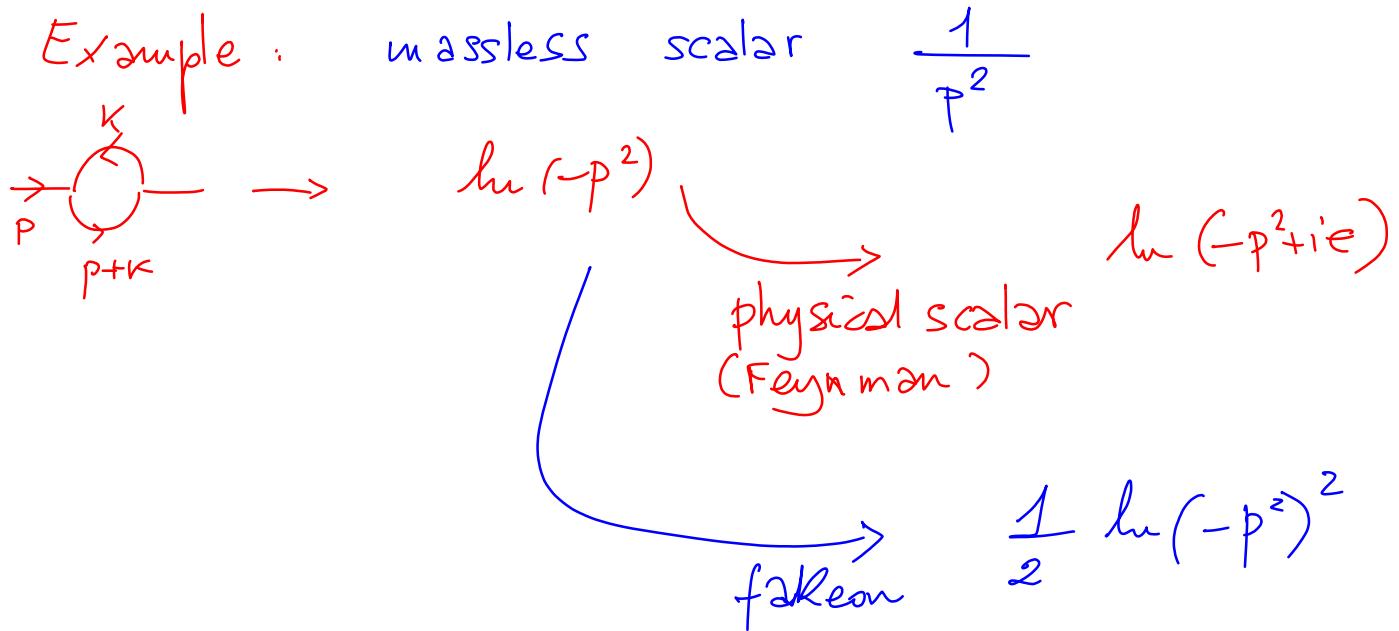


+ deformation  $\mathbb{R}^3 \rightarrow D_3$   
or average continuation

Since the prescription is  
real, there is no absorptive  $\dashrightarrow$   
part, i.e. no particle

[Not the principal  
value!]

CONSISTENT  
PROJECTION



We can cure the ghosts of

$$S_{\text{HD}} = -\frac{\mu^{-\varepsilon}}{2\kappa^2} \int \sqrt{-g} \left[ 2\Lambda_C + \zeta R + \alpha \left( R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2 \right) - \frac{\xi}{6}R^2 \right]$$

by turning them into fakesons

Graviton multiplet :  $\{h_{\mu\nu}, \phi, X_{\mu\nu}\}$

$$g_{\mu\nu} = \eta_{\mu\nu} + 2k h_{\mu\nu}$$

fluctuation of the metric

massive scalar

spin-2  
fakeon of mass  $m_X$

Fakeon width :

$$\Gamma_X = -\alpha_X C m_X$$

$$C = \frac{N_S + 6N_f + 12N_V}{120}$$

$\Gamma_X < 0$  : causality is violated by  $X_{\mu\nu}$

$$\alpha_X = \left(\frac{m_X}{M_{Pl}}\right)^2$$

D. A. and M. Piva, The ultraviolet behavior of quantum gravity, J. High Energy Phys. 05 (2018) 027 and arxiv:1803.0777 [hep-th]

D. A. and M. Piva, Quantum gravity, fakeons and microcausality, arxiv:1806.03605 [hep-th]

## Physical predictions

- There exists at least one fakeon in nature
- It has spin 2, mass  $m_x \sim 10^{11} \text{ GeV (?)}$  and negative width  $\Gamma_x \sim -1/10^{-20} \text{ sec}$
- Past, present & future, as well as cause & effect lose meaning at energies  $E \gtrsim m_x$  for an amount of time equal to  $1/|\Gamma_x|$  (in the rest frame of the fakeon)

## Summarizing

- Fock space OK
- Hermitian Hamiltonian bounded from below  
(after the projection)
- Perturbative unitarity (optical theorem)  
(at  $\Lambda_c = 0$ )
- Renormalizability, locality, uniqueness
- Lorentz invariance, general covariance
- Causality, but NOT microcausality

- Analyticity in the Euclidean region + Average Continuation  
 $\longrightarrow$  regionwise analyticity
- The low-energy limit is OK
- The high-energy behavior might contain surprises
- No asymptotic freedom
- No positive definiteness in Euclidean space
- The classical limit carries remnants of the forces ( $F = ma \rightarrow \langle F \rangle = ma$ ,

$$\langle F \rangle_t = \int_t^\infty dt' e^{-(t-t')/\tau} F(t') \rightarrow G_{\mu\nu} = \langle \tilde{T}_{\mu\nu} \rangle$$

## PAPERS

### \*\*\* Inconsistency of Minkoswky HD theories \*\*\*

.G. Aglietti and D. A., **Inconsistency of Minkowski higher-derivative theories**,  
Eur. Phys. J. C 77 (2017) 84 and arXiv:1612.06510 [hep-th]

### \*\*\* Reformulation of the Lee-Wick models as nonanalytically Wick rotated Euclidean theories \*\*\*

D. A. and M. Piva, **A new formulation of Lee-Wick quantum field theory**,  
J. High Energy Phys. 06 (2017) 066 and arXiv:1703.04584 [hep-th]

D. A. and M. Piva, **Perturbative unitarity of Lee-Wick quantum field theory**,  
Phys. Rev. D 96 (2017) 045009 and arXiv:1703.05563 [hep-th]

### \*\*\* Fakeons \*\*\*

D. A., **Fakeons and Lee-Wick models**, J. High Energy Phys. 02 (2018) 141 and arXiv:1801.00915 [hep-th]

### \*\*\* Fakeons and quantum gravity \*\*\*

D. A., **On the quantum field theory of the gravitational interactions**,  
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D. A. and M. Piva, **The ultraviolet behavior of quantum gravity**,  
J. High Energy Phys. 05 (2018) 027 and arxiv:1803.0777 [hep-th]

D. A. and M. Piva, **Quantum gravity, fakeons and microcausality**, arxiv:1806.03605 [hep-th]