The Case against Ghosts in Fundamental Theory

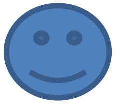
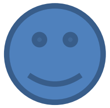
RP Woodard
University of Florida

PACTS: June 22, 2018

Three Quick Questions & Answers
(Witnessing to the heathen)

1. What is a ghost?
   - Particle with negative KE
2. Why should we avoid ghosts?
   - Interacting ghosts blow up the universe!
3. Why do people (here!) nonetheless consider ghosts?
   - They want to quantize gravity
   - Stelle (1977) ➔ $R + R^2 + C^2$ is renormalizable
   - Higher ∂`s in C^2 give ghosts!

How Lower Derivatives Work

- Dynamical variable $q(t)$ & Lagrangian $L(q,\dot{q})$
  - $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}$ initial conditions ➔ 2 canonical variables
- Canonical formulation
  - $Q = q$ & $P = \frac{\partial L}{\partial \dot{q}}$ ➔ $\dot{q} = v(Q,P)$ (nondegeneracy)
  - $H(Q,P) = Pv(Q,P) - L(Q,v(Q,P))$
- Hamilton's equations generate time evolution
  - $\dot{Q} = \frac{\partial H}{\partial P} = v + P\frac{\partial v}{\partial P} - \frac{\partial L}{\partial \dot{q}}\frac{\partial v}{\partial P} = v$ 
  - $\dot{P} = -\frac{\partial H}{\partial Q} = -P\frac{\partial v}{\partial Q} + \frac{\partial L}{\partial q} + \frac{\partial L}{\partial \dot{q}}\frac{\partial v}{\partial Q} = \frac{\partial L}{\partial q}$ 
- $H(Q,P)$ can be bounded below

Higher Derivatives (Ostrogradsky 1850)

- Lagrangian $L(q,\dot{q},\ddot{q})$
  - $\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}} - \frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{q}}\right)\right] = \frac{\partial L}{\partial q}$ 4 initial conditions ➔ 4 canonical coordinates
- Canonical Formulation
  - $Q_1 = q\ ,\ \ P_1 = \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{q}}\right)\ ,\ \ Q_2 = \dot{q}\ ,\ \ P_2 = \frac{\partial L}{\partial \ddot{q}}$ ➔ $\ddot{q} = a(\vec{Q}, P_2)$ (ND)
  - $H(\vec{Q},\vec{P}) = P_1 Q_2 + P_2 a(\vec{Q}, P_2) - L(Q_1, Q_2, a(\vec{Q}, P_2))$
- Hamilton's equations generate time evolution
  - $\dot{Q}_1 = \frac{\partial H}{\partial P_1} = Q_2$, $\dot{Q}_2 = \frac{\partial H}{\partial P_2} = a + P_2 \frac{\partial a}{\partial P_2} - \frac{\partial L}{\partial \ddot{q}}\frac{\partial a}{\partial P_2} = a$
  - $\dot{P}_2 = -\frac{\partial H}{\partial Q_2} = -P_1 - P_2 \frac{\partial a}{\partial Q_2} + \frac{\partial L}{\partial \dot{q}} + \frac{\partial L}{\partial \ddot{q}}\frac{\partial a}{\partial Q_2} = \frac{\partial L}{\partial \dot{q}} - P_1$
  - $\dot{P}_1 = -\frac{\partial H}{\partial Q_1} = -P_2 \frac{\partial a}{\partial Q_1} + \frac{\partial L}{\partial q} + \frac{\partial L}{\partial \ddot{q}}\frac{\partial a}{\partial Q_1} = \frac{\partial L}{\partial q}$
- H is **linear** in P_1 ➔ not bounded below (or above)

Why this is bad

- No guaranteed problem w/o interactions
  - Problem is energy flow from $KE < 0$ to $KE > 0$
- No guaranteed problem w/o continuum DoF's
  - Instability is driven by vast d^3k UV phase space
    - Overwhelms even the weakest nonzero coupling
  - Decay is instantaneous
    - $\tau \neq 0$ results only come from imposing a UV cutoff
- Power & simplicity of the result
  - Requires only non-degenerate HD's
    - Non-perturbative &independent of interactions
  - This is the strongest constraint on Fundamental Theory!
    - "Newton got it right about $F = ma$"

Common Misconceptions

- “No problem at any constant $q(t)$”
  - Problem is pathological **time dependence**
- “Quantization might help”
  - This is a large phase space problem
- “Problem is unitarity, not instability”
  - Regards negative KE $a^{\dagger}(\vec{k})$ as positive KE a $(\vec{k})$
- “High mass ghosts decouple at low energies”
  - They actually couple more strongly!
- “No problem if HD’s confined to interactions”
  - Problem is non-perturbative
- “No problem from entire functions of ∂^2”
  - Only works perturbatively in Euclidean momentum space

Alternate Quantizations Sacrifice Classical Correspondence Limit

- $L = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}m\omega^2 q^2$ ➔ $H = \frac{1}{2m}P^2 + \frac{1}{2}m\omega^2 Q^2$
  - $a = \sqrt{\frac{m\omega}{2\hbar}}\left[Q + \frac{i}{m\omega}P\right] \to \sqrt{\frac{m\omega}{2\hbar}}\left[q + \frac{\hbar}{m\omega}\frac{\partial}{\partial q}\right]$
  - $a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left[Q - \frac{i}{m\omega}P\right] \to \sqrt{\frac{m\omega}{2\hbar}}\left[q - \frac{\hbar}{m\omega}\frac{\partial}{\partial q}\right]$
- Normal Quantization: $\Omega(q) \propto \exp\left[-\frac{m\omega}{2\hbar}q^2\right]$
  - $H \cdot \left(a^\dagger\right)^N \Omega = \left(N + \frac{1}{2}\right)\hbar\omega \times \left(a^\dagger\right)^N \Omega$
- Alternate Quantization: $\overline{\Omega}(q) \propto \exp\left[+\frac{m\omega}{2\hbar}q^2\right]$
  - $H \cdot a^N \overline{\Omega} = -\left(N + \frac{1}{2}\right)\hbar\omega \times a^N \overline{\Omega}$
- ONLY data from low E gravity is classical GR
  - Dangerous to give this up
    - You won't get a local, metric theory ➔ causality? Strong fields? Cosmology?
    - IF everything worked ➔ just START with this and forget about HDG!

Only Hope is Constraints

- Constraints compromise non-degeneracy
- $R \rightarrow f(R)$ gravity ok
  - Ostrogradsky ➔ new DoF of opposite KE
  - But Newtonian potential has negative KE in GR
  - Hence new $f(R)$ DoF has positive KE
    - NB This is <u>not</u> a counter-example to Ostrogradsky!
- But there are only so many gauge symmetries
- Could always try for ad hoc constraints
  - But at odds with interacting QFT
  - Same field carries both $\pm$ DoF's

Lessons from Pop Culture

- "You can't always get what you want"
  - Face it: C^2 just isn't viable as a fundamental theory
- "But if you try, sometimes you just might find, that you get what you need"
  - $C\ln(\Box)C$ occurs in Γ_{1loop}
  - Coefficient finite & fixed
  - Stronger in the IR than C^2

Conclusions

- Ostrogradsky Thm is the strongest constraint on fundamental theory
- Need to distinguish effective field theory from fundamental theory
  - Fundamental ghosts present at *all* scales
  - Nonlocal EFT effects stronger than local
- Alternate quantization schemes discard the Correspondence Principle
  - This is not acceptable for gravity!