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Semiclassical Higgsplosion

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IPPP Durham

- VVK & Spannowsky 1704.03447, 1707.01531
- VVK, Reiness, Spannowsky, Waite 1709.08655
- & with Reiness, Scholtz & Spannowsky 1803.05441
- VVK 1806.05648
- VVK 1705.04365

- In this talk: I'll imagine n~150 of Higgs bosons being produced in a final state at n lambda >> 1. Kinematically possible for scattering at E ~100 TeV

- HIGGSPLOSION: n-particle rates computed in a weakly-coupled theory can become unsuppressed above certain critical values of n and E.

- will consider an intrinsically Non-perturbative — semiclassical set-up

- it incorporates correctly the tree-level results and

- the leading-order quantum effects = leading loops

already known

In this talk:

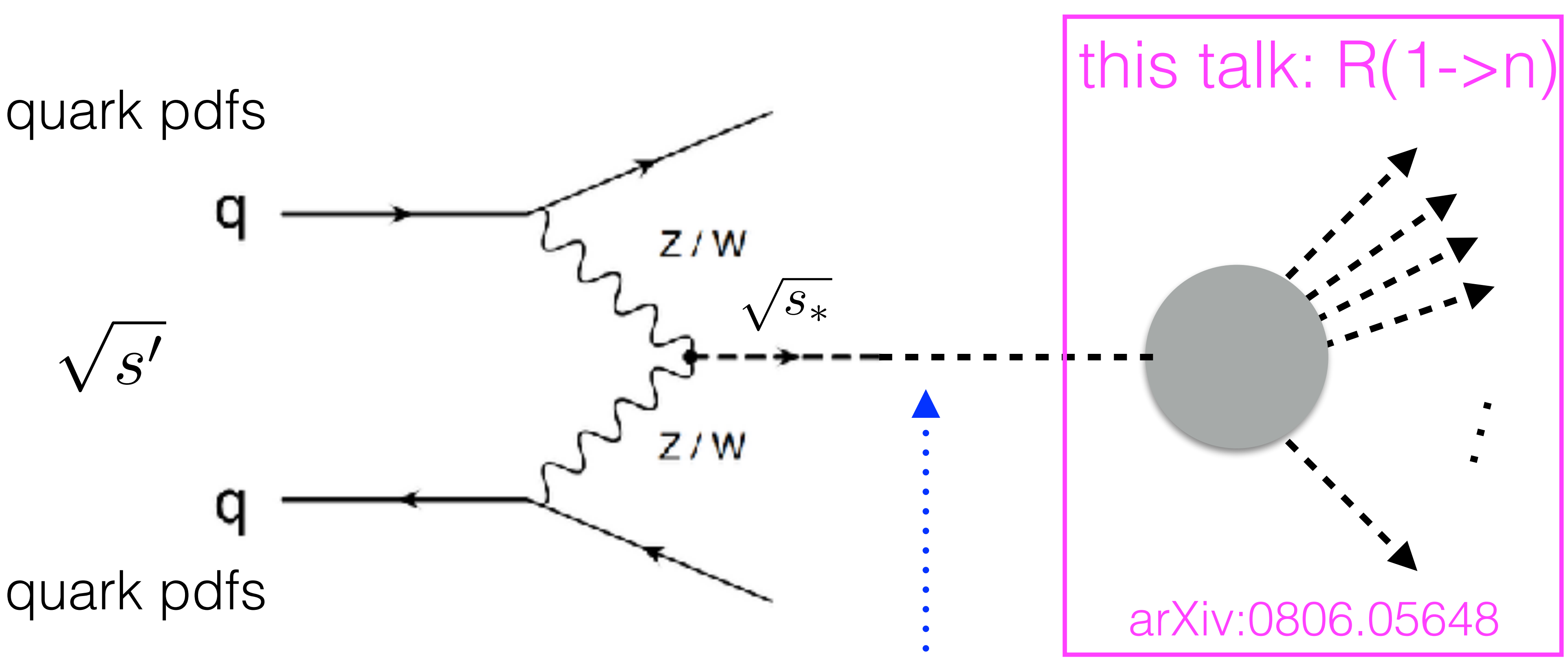
- compute quantum effects in the large lambda n limit

new

1->n processes of interest

for Higgsplosion

e.g.:Vector boson fusion in high-energy pp collisions at ~100 TeV



n non-relativistic Higgses
Higgsplosion at $\sqrt{s_*}$

$$\frac{i}{s_* - m_h^2 - Re\tilde{\Sigma}(s_*) + i m_h \Gamma(s_*)}$$ Propagator with Higgspersion at $\sqrt{s_*}$

- VVK & Spannowsky 1704.03447,1707.01531

Factorial growth of tree-level amplitudes at thresholds:

$$\boxed{\mathcal{L} \,=\, \frac{1}{2}\,\partial^\mu h\,\partial_\mu h \,-\, \frac{\lambda}{4}\,\left(h^2 - v^2\right)^2}$$

prototype of the Higgs
in the unitary gauge

The classical equation for the spatially uniform field $h(t)$,

$$d_t^2 h \,=\, -\lambda\, h^3 + \lambda v^2\, h\,,$$

has a closed-form solution with correct initial conditions $h_{\rm cl} = v + z + \ldots$

$$\boxed{h_0(z_0;t) \,=\, v\,\left(\frac{1 + z_0\, e^{imt}/(2v)}{1 - z_0\, e^{imt}/(2v)}\right)\,, \quad m = \sqrt{2\lambda}v}$$

$$h_0(z) \,=\, v \,+\, 2v\sum_{n=1}^{\infty}\left(\frac{z}{2v}\right)^n\,, \quad z = z(t) = z_0\, e^{imt}$$

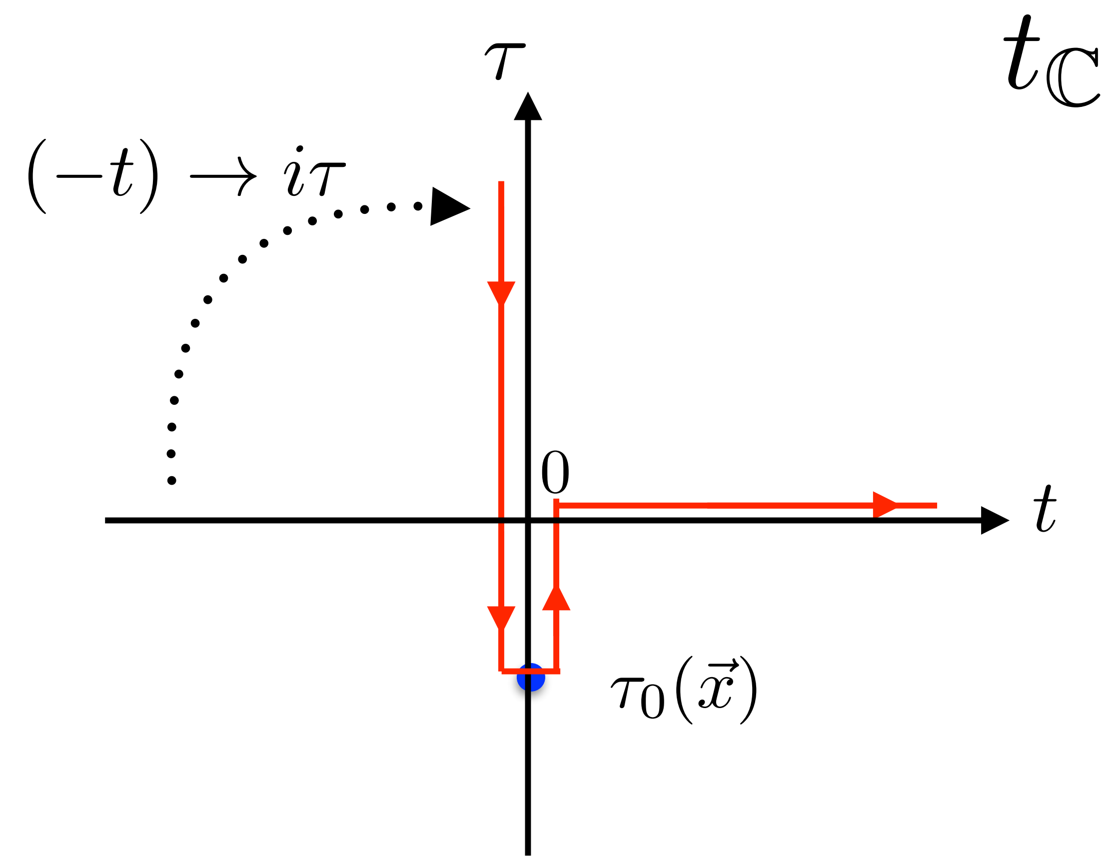
$$\mathcal{A}_{1\to n} \,=\, \left(\frac{\partial}{\partial z}\right)^n h_{\rm cl}\Big|_{z=0} = \boxed{n!\,(2v)^{1-n}}$$

Factorial growth

L. Brown 9209203

Analytic continuation & singularities in complex time:

$$t \longrightarrow t_{\mathbb{C}} = t + i\tau$$

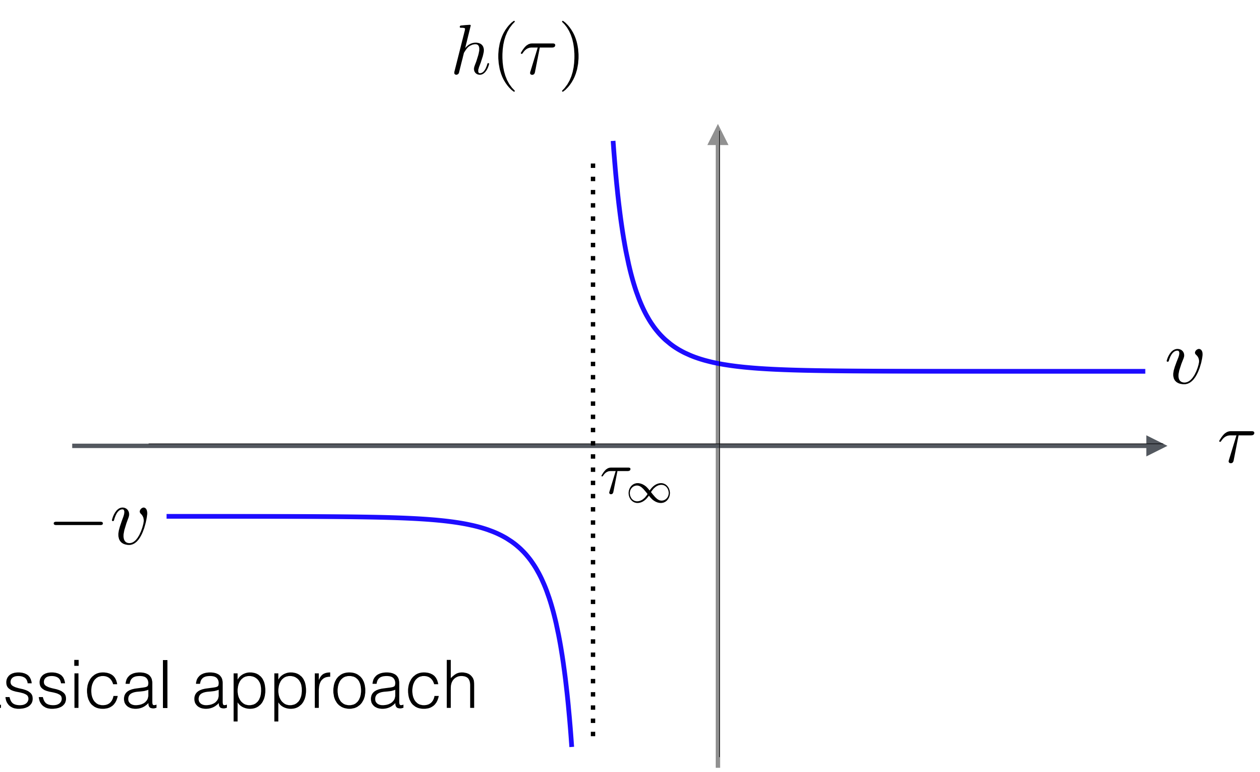


$$h_0(t_{\mathbb{C}}) \;=\; v\left(\frac{1 \;+\; e^{im(t_{\mathbb{C}}-i\tau_\infty)}}{1 \;-\; e^{im(t_{\mathbb{C}}-i\tau_\infty)}}\right),$$

$$\tau_\infty := \frac{1}{m}\log\left(\frac{z_0}{2v}\right)$$

Our simple example of a classical solution

$$h_0(\tau) \;=\; v\left(\frac{1 \;+\; e^{-m(\tau-\tau_\infty)}}{1 \;-\; e^{-m(\tau-\tau_\infty)}}\right)$$



Such solutions will emerge in the semiclassical approach

Main idea of the semiclassical approach

$\mathcal{R}_n(E)$ is the probability rate for a local operator $\mathcal{O}(0)$ to create n particles of total energy E from the vacuum,

$$\mathcal{R}_n(E) \;=\; \int \frac{1}{n!} d\Phi_n \, \langle 0|\, \mathcal{O}^\dagger \, S^\dagger \, P_E |n\rangle\langle n|\, P_E \, S \, \mathcal{O}\, |0\rangle$$

P_E is the projection operator on states with fixed energy E.

$$\mathcal{O} \;=\; e^{j\, h(0)}\,,$$

and the limit $j \to 0$ is taken in the computation of the probability rates,

$$\mathcal{R}_n(E) \;=\; \lim_{j\to 0} \int \frac{1}{n!} d\Phi_n \, \langle 0|\, e^{j\, h(0)}{}^\dagger \, S^\dagger \, P_E |n\rangle\langle n|\, P_E \, S \, e^{j\, h(0)}\, |0\rangle\,.$$

Note: non-dynamical (non-propagating) initial state $\mathcal{O}|0\rangle$.
The semi-classical (steepest descent) limit:

$$\varepsilon = \frac{E-nm}{nm}$$

$$\boxed{\lambda \to 0\,, \quad n \to \infty\,, \quad \text{with} \quad \lambda n = \text{fixed}\,, \quad \varepsilon = \text{fixed}\,.}$$

Evaluate the path integral in this double-scaling limit.
n enters via the coherent state formalism.

Rubakov & Tinyakov; Son '95 …

The Semiclassical formalism of Son: results in four steps

1. Solve the classical equation without the source-term:

$$\frac{\delta S}{\delta h(x)} = 0$$

   a complex-valued solution $h(x)$ with a point-like singularity at $x^\mu = 0$. The singularity is due to $\mathcal{O}(x=0)$.

2. Impose the initial and final-time boundary conditions:

$$\lim_{t\to-\infty} h(x) \;=\; v + \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}}\, a^\dagger_{\mathbf{k}}\, e^{ik_\mu x^\mu}$$

$$\lim_{t\to+\infty} h(x) \;=\; v + \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \left(b_{\mathbf{k}}\, e^{\omega_{\mathbf{k}}T-\theta}\, e^{-ik_\mu x^\mu} + b^\dagger_{\mathbf{k}}\, e^{ik_\mu x^\mu} \right)$$

- Son hep-ph/055338

The Semiclassical formalism of Son: results in four steps

3. Compute E and n of the final state using the $t \to +\infty$ asymptotics

$$E = \int d^3k \; \omega_{\mathbf{k}} \, b^\dagger_{\mathbf{k}} \, b_{\mathbf{k}} \, e^{\omega_{\mathbf{k}} T - \theta} \,, \qquad n = \int d^3k \; b^\dagger_{\mathbf{k}} \, b_{\mathbf{k}} \, e^{\omega_{\mathbf{k}} T - \theta}$$

At $t \to -\infty$ the energy and the particle number are vanishing.
The energy changes discontinuously from 0 to E at the singularity at $t = 0$.

4. Eliminate the T and θ parameters in favour of E and n.
Finally, compute the function $W(E,n)$

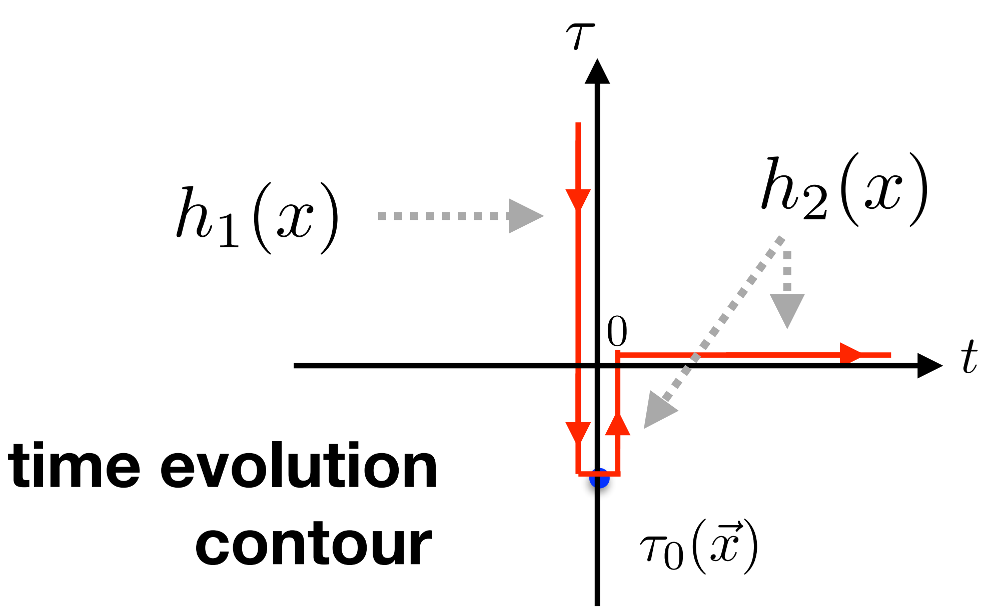
$$W(E,n) = ET - n\theta - 2\mathrm{Im}S[h]$$

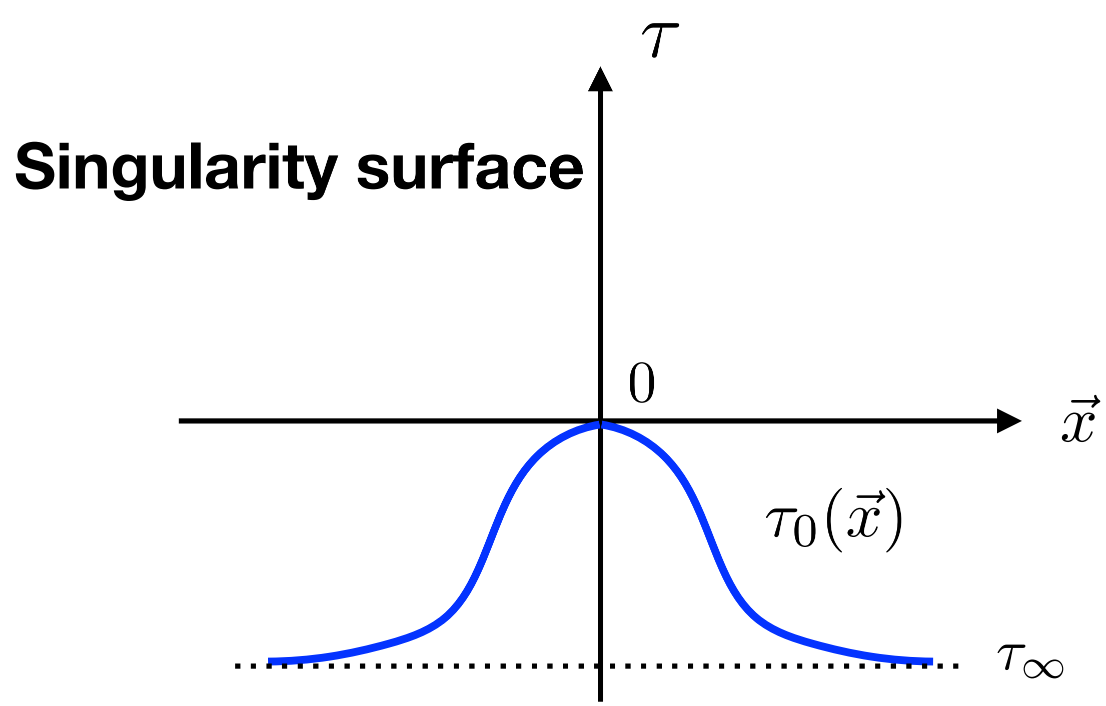
on the set $\{h(x), T, \theta\}$ and fine the semiclassical rate $\mathcal{R}_n(E) = \exp\left[W(E,n)\right]$

- Son hep-ph/055338

Refining the method in complex time

- In the Euclidean space-time, $(\tau, \vec{x})$ the solution will be singular a 3-dimensional hypersurface $\tau = \tau_0(\vec{x})$ located at $t = 0$.





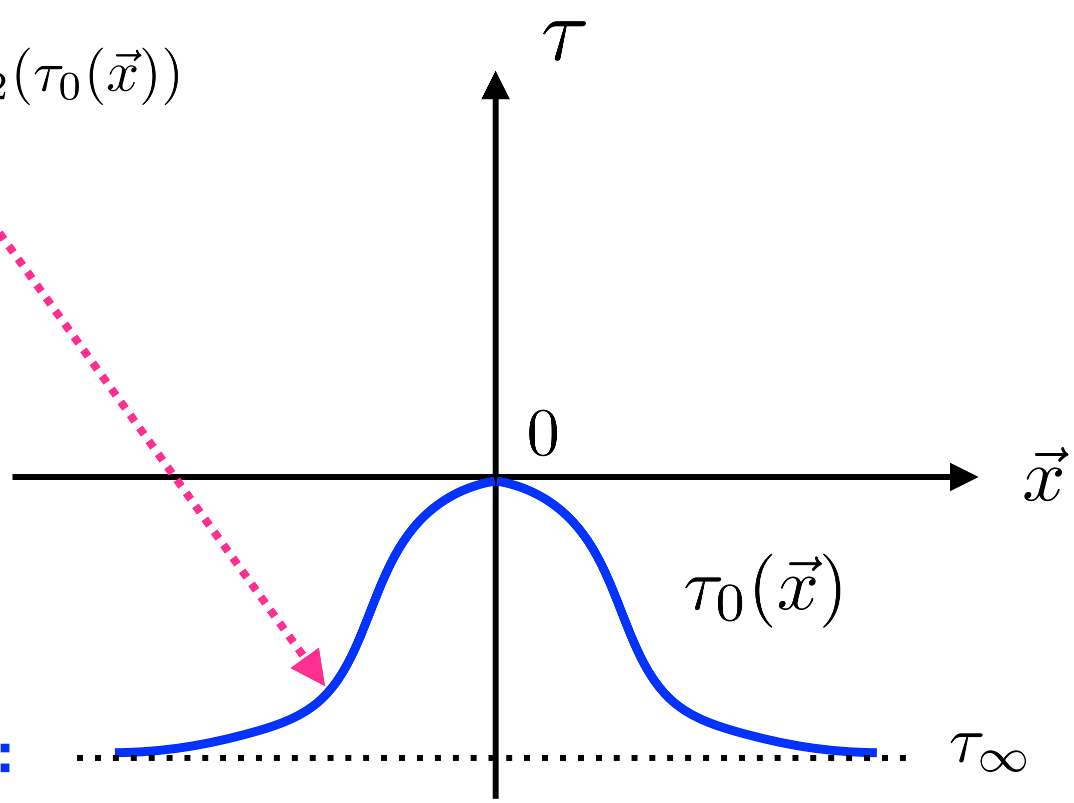
- Find a classical trajectory $h_1(\tau, \vec{x})$ on the first segment $+\infty > \tau > \tau_0(\vec{x})$
- Find another classical solution $h_2(\tau, \vec{x})$ on the remaining part of the contour that at $\tau \to \tau_0(\vec{x})$ is singular and $h_2(\tau_0, \vec{x}) = h_1(\tau_0, \vec{x})$.

- For the combined configuration $h(x)$ to solve classical equations everywhere, including the τ_0 surface:

need to extremize the action integral over all singularity surfaces $\tau = \tau_0(\vec{x})$ containing the point $t = 0 = \vec{x}$.

$$iS[h] = \int d^3x \left(\left| \int_{+\infty}^{\tau_0(\vec{x})} d\tau \, \mathcal{L}_{\text{Eucl}}(h_1) \right| - \left| \int_{\tau_0(\vec{x})}^{0} d\tau \, \mathcal{L}_{\text{Eucl}}(h_2) \right| + i \int_0^{\infty} dt \, \mathcal{L}(h_2) \right)$$

$h_1(\tau_0(\vec{x})) = h_2(\tau_0(\vec{x}))$



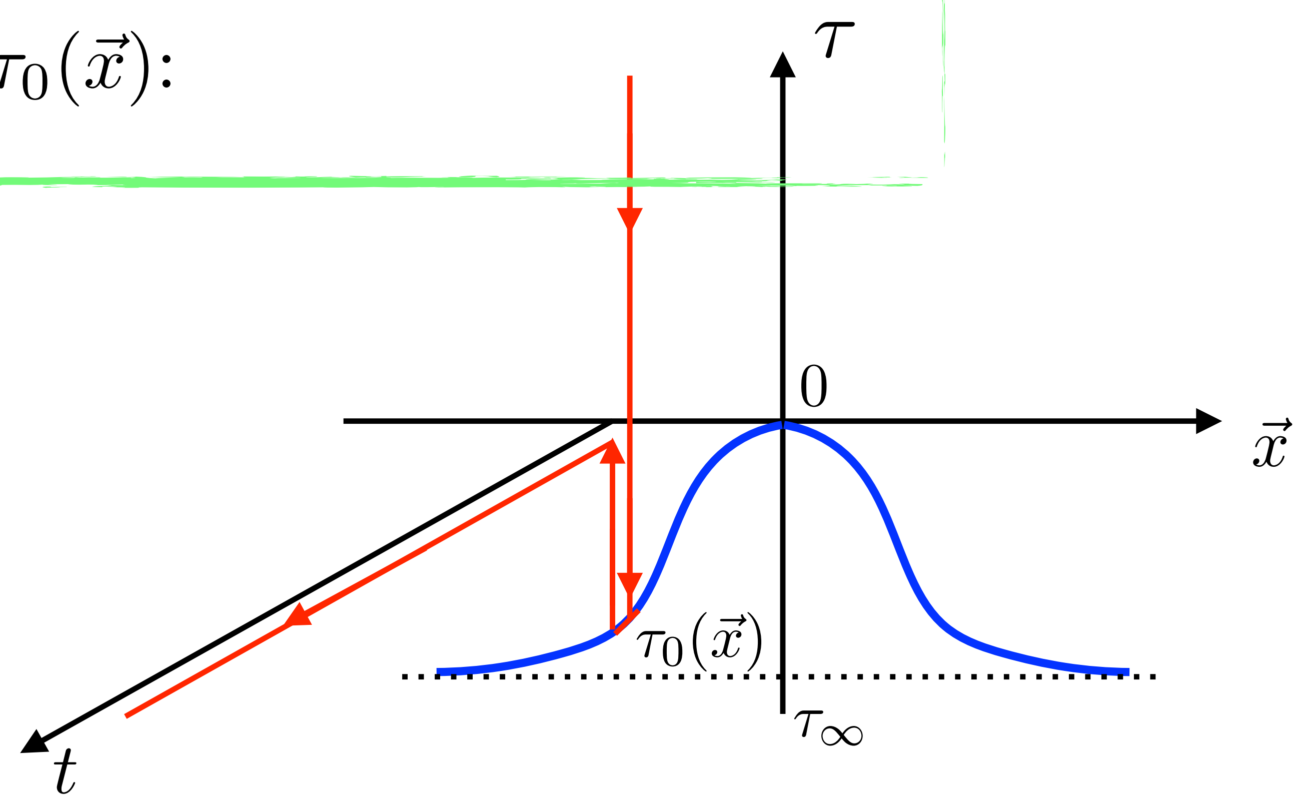
Find <u>Extremum</u> of iS over all such singularity surfaces:

and finally…

- Find the semiclassical rate $\mathcal{R}_n(E) = e^{W(E,n)}$ by evaluating

$$W(E,n) \;=\; ET \;-\; n\theta \;-\; 2\mathrm{Im}S[h]$$

on the *extremal* singular surface $\tau_0(\vec{x})$:



recall that:

Classical solultions h_1 and h_2:

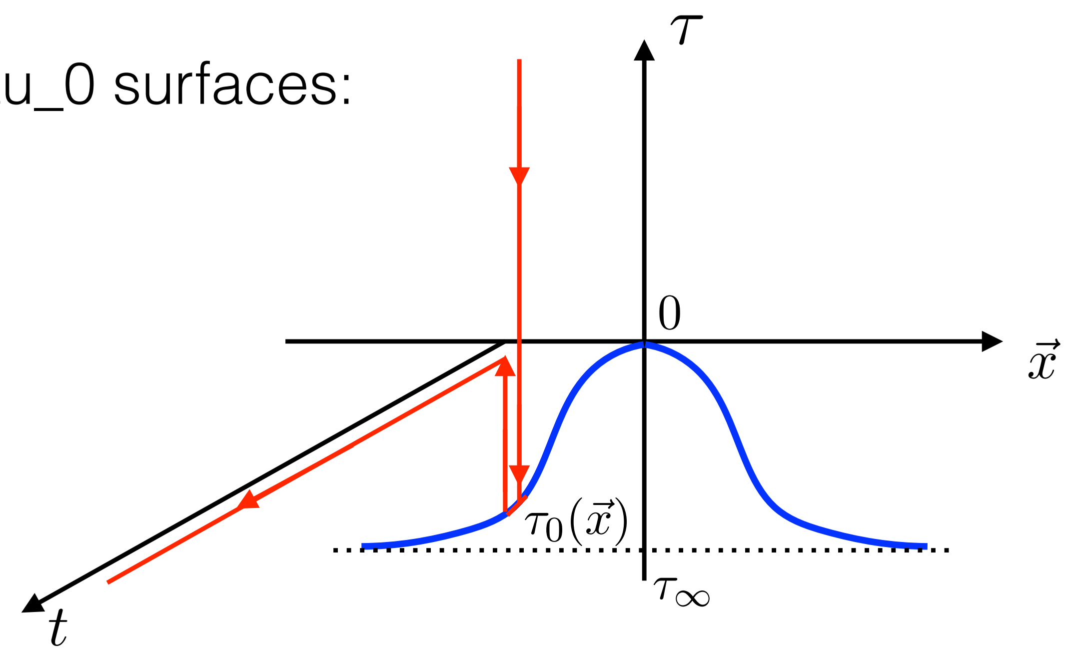
$\lim_{\tau\to+\infty} h_1(\tau,\vec{x}) \,-\, v \;\to\; 0$

$h_2(\tau_0,\vec{x}) \;=\; h_1(\tau_0,\vec{x}) \;=\; \Phi_0(\vec{x}) \;\to\; \infty$

$$\lim_{t\to+\infty} h_2(t,\vec{x}) \,-\, v \;=\; \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \left(b_{\mathbf{k}}\, e^{\omega_{\mathbf{k}} T-\theta}\, e^{-ik_\mu x^\mu} \;+\; b^\dagger_{\mathbf{k}}\, e^{ik_\mu x^\mu} \right)$$

Computing the semiclassical rate

Classical solution singular on a generic tau_0 surfaces:



$$h(t_{\mathbb{C}}, \vec{x}) \;=\; v\left(\frac{1 \,+\, e^{im(t_{\mathbb{C}} - i\tau_\infty)}}{1 \,-\, e^{im(t_{\mathbb{C}} - i\tau_\infty)}}\right) \,+\, \tilde{\phi}(t_{\mathbb{C}}, \vec{x})$$

Find that:

$$W(E,n) \;=\; ET \,-\, n\theta \,-\, 2\mathrm{Re}S_{\mathrm{Eucl}}[h]$$

$$= \; n\,\log\frac{\lambda n}{4} \,+\, \frac{3n}{2}\left(\log\frac{3\pi}{\varepsilon} + 1\right) \,-\, 2nm\,\tau_\infty \,-\, 2\mathrm{Re}S_{\mathrm{Eucl}}[h]$$

$W(E,n)^{\mathrm{tree}}$

agrees with the known result of tree-level contributions

$\Delta W^{\mathrm{quant}}$

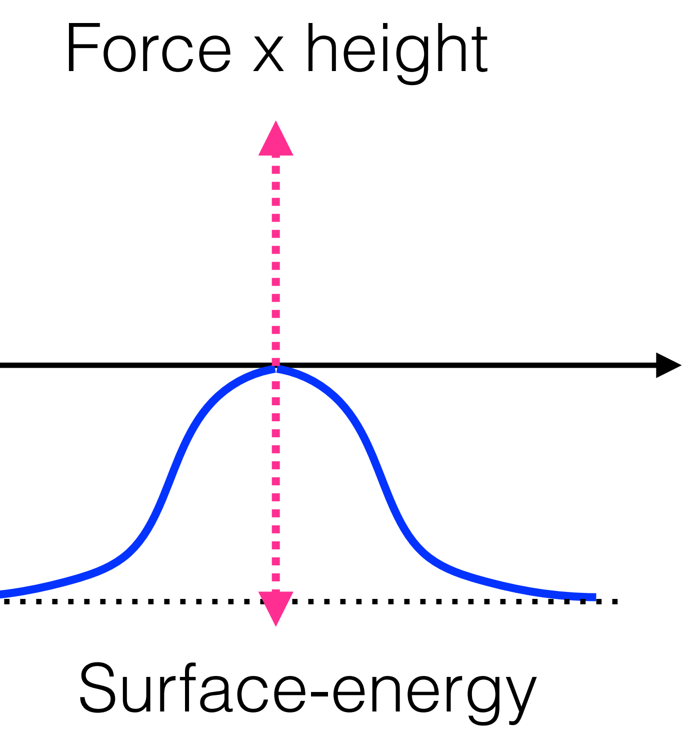
need to compute by extremizing w.r.t tau_0

Computing the semiclassical rate

$$\begin{aligned}\Delta W^{\text{quant}} &= -2nm\,\tau_\infty - 2\text{Re}\,S^{(1,2)}_{\text{Eucl}}\\ &= 2nm\,|\tau_\infty| + 2\int d^3x\left[\int_{\tau_0(\vec{x})}^{+\infty} d\tau\,\mathcal{L}_{\text{Eucl}}(h_1) - \int_{\tau_0(\vec{x})}^{0} d\tau\,\mathcal{L}_{\text{Eucl}}(h_2)\right]\end{aligned}$$

E=0 configuration

E=mn configuration



$$\boxed{\frac{1}{2}\Delta W^{\text{quant}} = nm\,|\tau_\infty| - \underbrace{\int_{A+i\epsilon}^{0+i\epsilon} d\tau\, L_{\text{Eucl}}(h_2;\tau_0(\vec{x}))}_{\equiv S_{\text{Eucl}}[\tau_0(\vec{x})]} + \frac{4\pi}{3}\,\mu R^3}$$

Force x height

Surface-energy

Mechanical analogy: surface at equilibrium/balance of forces

Computing the semiclassical rate

Use *thin wall* approximation:

$$S_{\rm Eucl}[\tau_0(r)] \;=\; \int_{\tau_\infty}^{0} d\tau\, 4\pi\mu\, r^2\sqrt{1+\dot r^2} \;\equiv\; \int_{\tau_\infty}^{0} d\tau\, L(r,\dot r)$$

Surface tension

$$\mu \;=\; \int_{-\infty-i\epsilon}^{+\infty-i\epsilon} d\tau \left(\frac{1}{2}\left(\frac{dh}{d\tau}\right)^2 + \frac{\lambda}{4}\left(h^2-v^2\right)^2\right) \;=\; \frac{m^3}{3\lambda}$$

Conjugate momentum

$$p \;=\; \frac{\partial L(r,\dot r)}{\partial \dot r} \;=\; 4\pi\,\mu\frac{r^2\dot r}{\sqrt{1+\dot r^2}}$$

Hamiltonian => Energy

$$H(p,r) \;=\; L(r,\dot r) \;-\; p\,\dot r$$

$$\frac{1}{2}\Delta W^{\rm quant} \;=\; (E-nm)\tau_\infty \;-\; \int_R^0 p(E)\,dr \;+\; \frac{4\pi}{3}\,\mu R^3$$

Quantum rate on the stationary trajectory:

$$\frac{1}{2}\Delta W^{\rm quant}_{\rm stationary} \;=\; -\int_R^0 p(E)\,dr \;+\; \frac{4\pi}{3}\,\mu R^3\,, \qquad E = nm$$

Computing the semiclassical rate

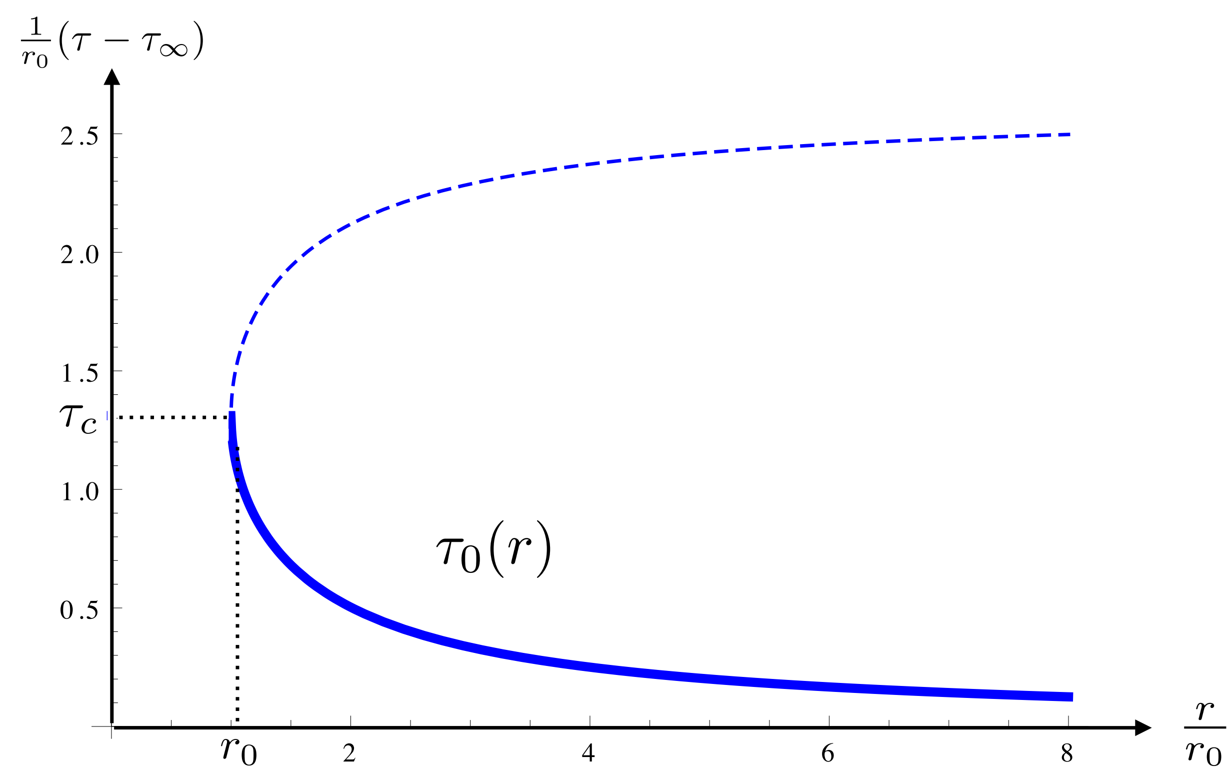
Use *thin wall* approximation:

$$\frac{1}{2}\Delta W^{\text{quant}}_{\text{stationary}} = -\int_R^0 p(E)\,dr + \frac{4\pi}{3}\,\mu R^3\,, \qquad E = nm$$

final result

$$\boxed{\Delta W^{\text{quant}} = \frac{E^{3/2}}{\sqrt{\mu}}\,\frac{2}{3}\,\frac{\Gamma(5/4)}{\Gamma(3/4)} = \frac{1}{\lambda}\,(\lambda n)^{3/2}\,\frac{2}{\sqrt{3}}\,\frac{\Gamma(5/4)}{\Gamma(3/4)} \simeq 0.854\,n\sqrt{\lambda n}}$$

Classical trajectory tau(r):



Justifies the thin wall approximation:

$$rm \geq r_0 m = m\left(\frac{E}{4\pi\mu}\right)^{1/2} \propto \left(\frac{\lambda E}{m}\right)^{1/2} = \sqrt{\lambda n} \gg 1\,, \qquad r_0^2 = \frac{E}{4\pi\mu}$$

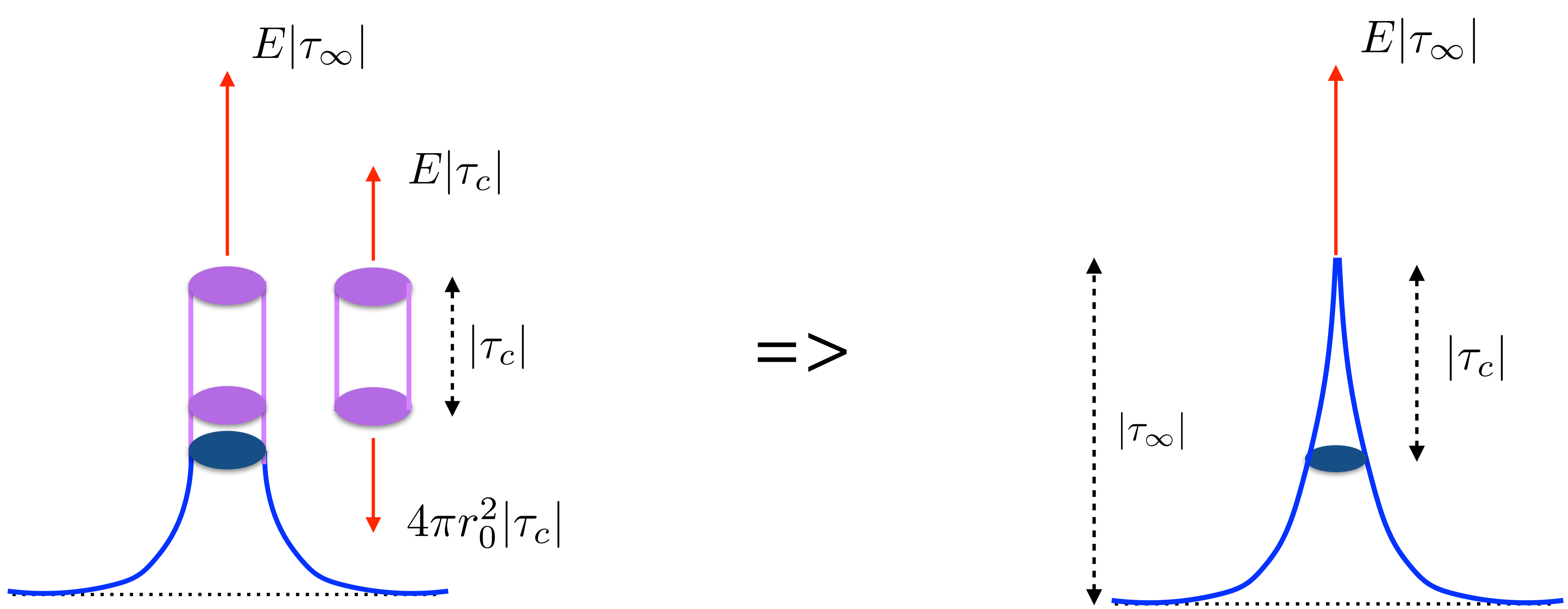
Computing the semiclassical rate

Use *thin wall* approximation:

$$\frac{1}{2}\Delta W^{\rm quant}_{\rm stationary} = -\int_R^0 p(E)\,dr + \frac{4\pi}{3}\,\mu R^3\,, \qquad E = nm$$

final result

$$\boxed{\Delta W^{\rm quant} = \frac{E^{3/2}}{\sqrt{\mu}}\,\frac{2}{3}\,\frac{\Gamma(5/4)}{\Gamma(3/4)} = \frac{1}{\lambda}\,(\lambda n)^{3/2}\,\frac{2}{\sqrt{3}}\,\frac{\Gamma(5/4)}{\Gamma(3/4)} \simeq 0.854\,n\sqrt{\lambda n}}$$



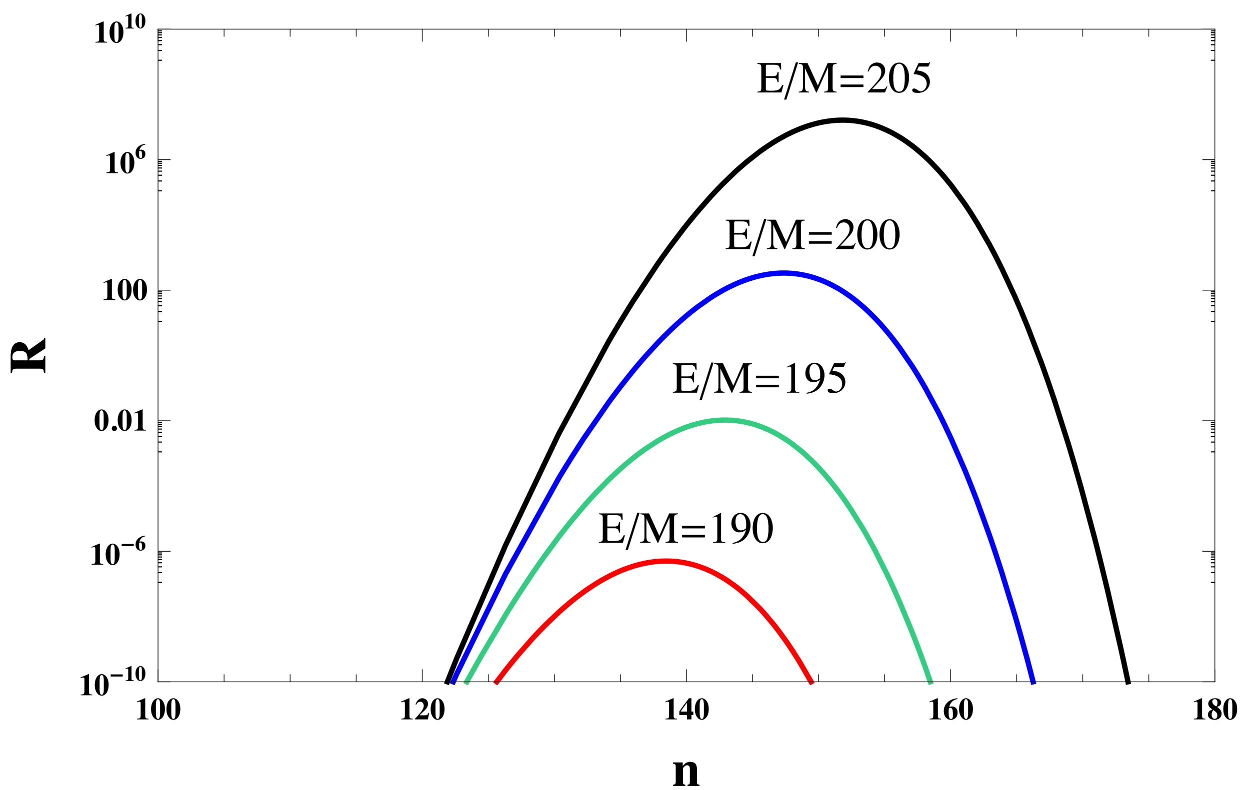
Summary of the main result

- VVK 1806.05648

$$\lambda \to 0\,, \quad n \to \infty\,, \quad \text{with} \quad \lambda n = \text{fixed} \gg 1\,, \quad \varepsilon = \text{fixed} \ll 1$$

$$\mathcal{R}_n(E) \,=\, e^{W(E,n)} \,=\, \exp\left[\frac{\lambda n}{\lambda}\left(\log\frac{\lambda n}{4} + 0.85\sqrt{\lambda n} - 1 + \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right) - \frac{25}{12}\,\varepsilon\right)\right]$$

$$E/m \,=\, (1+\varepsilon)\,n$$



positive (quantum effects)

negative (phase space)

Can always make this term win =>
unsuppressed R at high Energies

Higher order corrections are suppressed by extra powers of $\lambda \to 0$ and $1/n \to 0$ and by $\mathcal{O}(1/\sqrt{\lambda n})$ as well as by $\mathcal{O}(\varepsilon)$.

(Extra slides) looking ahead

- The semiclassical calculation reviewed in the talk was aimed towards developing a theoretical foundation for the mechanism of Higgsplosion

- $$\Delta_R(p) \,=\, \frac{i}{p^2 - m^2 \,-\, \mathrm{Re}\,\Sigma_R(p^2) \,+\, im\Gamma(p^2) \,+ i\epsilon}$$

R $\longleftrightarrow$ Higgsplosion

Loop integrals are effectively cut off at E_* by the exploding width $\Gamma(p^2)$ of the propagating state into the high-multiplicity final states.

The incoming highly energetic state decays rapidly into the multi-particle state made out of soft quanta with momenta $k_i^2 \sim m^2 \lll E_*^2$.

The width of the propagating degree of freedom becomes much greater than its mass: it is no longer a simple particle state.

In this sense, it has become a composite state made out of the n soft particle quanta of the same field ϕ.

- VVK & Spannowsky 1704.03447, 1707.01531

- The Higgsplosion / Higgspersion mechanism makes theory UV finite (all loop momentum integrals are dynamically cut-off at scales above the Higgsplosion energy).

- UV-finiteness => all coupling constants slopes become flat above the Higgsplosion scale => automatic asymptotic safety

- [Below the Higgsplosion scale there is the usual logarithmic running]

- 1. Asymptotic Safety

- 2. No Landau poles for the U(1) and the Yukawa couplings

- 3. The Higgs self-coupling does not turn negative => stable EW vacuum

- No new physics degrees of freedom required — very minimal solution