# Gravity with more or less variables

## and a route to unification

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# Outline

## Five formulations of GR: EG

$$S_{\mathrm{EG}}(g) = Z_N \int d^4x \,\sqrt{|g|}\, R \qquad \text{where} \qquad Z_N = \frac{1}{16\pi G}\ .$$

Invariant under *Diff M*

## Five formulations of GR: DG

$S_{\mathrm{DG}}(g, \phi) = S_{\mathrm{EG}}(\bar{g})$ where $\bar{g}_{\mu\nu} = \frac{\alpha}{Z_N}\phi^2 g_{\mu\nu}$

$$S_{\mathrm{DG}}(g, \phi) \;\; = \;\; \alpha \int d^4x \sqrt{|g|} \left[\phi^2 R - 6\phi\nabla^2\phi\right]$$

Invariant under *Diff* $\ltimes$ *Weyl*

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu} \; , \qquad \phi \to \Omega^{-1}\phi \; .$$

## Five formulations of GR: UG

$$\sqrt{|g|} = \omega$$

$$S_{\mathrm{UG}}(g) = Z_N \int d^4x\, \omega\, R\,.$$

Invariant under *SDiff M*

# Five formulations of GR: UD

$$S_{\rm UD}(g,\phi) = \alpha \int d^4x\, \omega\, \left[\phi^2 R - 6\phi\nabla^2\phi\right] .$$

Invariant under $Diff * M$

## Five formulations of GR: *WTDiff*

$$\phi = \phi_0 \left(\frac{|g|}{\omega^2}\right)^{-\frac{1}{8}} .$$

$$S_X(g) = Z_N \int d^4x \, |g|^{\frac{1}{4}} \, \omega^{\frac{1}{2}} \left[ R + \frac{3}{32} \left( |g|^{-1} \nabla |g| - 2\omega^{-1} \nabla \omega \right)^2 \right]$$

Invariant under *SDiff M* $\ltimes$ *Weyl*

## Five formulations of GR: summary

$$
\begin{tikzcd}[column sep=large, row sep=huge]
 & \begin{array}{c}\textbf{DG}\\ (g,\phi);\ \textit{Diff} \ltimes \textit{Weyl}\end{array} \arrow[dl, "\phi=\phi_0"'] \arrow[d, "\phi=\phi_0\left(\frac{|g|}{\omega^2}\right)^{-\frac{1}{8}}"] \arrow[dr, "\sqrt{|g|}=\omega"] & \\
\begin{array}{c}\textbf{EG}\\ (g);\ \textit{Diff}\end{array} \arrow[dr, "\sqrt{|g|}=\omega"] & \begin{array}{c}\textit{WTDiff}\\ (g);\ \textit{SDiff} \ltimes \textit{Weyl}\end{array} \arrow[d, "\sqrt{|g|}=\omega"'] & \begin{array}{c}\textbf{UD}\\ (g_U,\phi);\ \textit{Diff}^*\end{array} \arrow[dl, "\phi=\phi_0"] \\
 & \begin{array}{c}\textbf{UG}\\ (g_U);\ \textit{SDiff}\end{array} &
\end{tikzcd}
$$

## Hamiltonian formulation

| | DG | EG | UG |
|---|---|---|---|
| fields | $q_{ij}$,$N_i$,$N$,$\phi$ | $q_{ij}$,$N_i$,$N$ | $q_{ij}$,$N_i$ |
| momenta | $p^{ij}$,$P^i$,$P$,$\pi$ | $p^{ij}$,$P^i$,$P$ | $p^{ij}$,$P^i$ |
| # can. variables | 22 | 20 | 18 |
| primary constr. | $P^i$, $P$, $C$ | $P^i$, $P$ | $P^i$ |
| secondary constr. | $\mathcal{H}_i$,$\mathcal{H}$ | $\mathcal{H}_i$,$\mathcal{H}$ | $\mathcal{H}_i$,$\mathcal{H}_\Lambda$ |
| # 1st cl. constr. | 9 | 8 | 7 |
| # canonical d.o.f. | 4 | 4 | 4 |

R. De Leon Ardon, S. Gielen, R. P., arXiv:1805.11626 [gr-qc]

## Which is best?

Gauge invariances needed in order to deal with local d.o.f.
The gauge group of all except UG is unnecessarily large.

On the other hand:

1. extending the gauge group is useful to recognize equivalences between different formulations
2. larger gauge group means that certain singular configurations could only be gauge artifacts (e.g. big bang)
3. suggest route to unification

# Further extension to $GL(4)$

$GL(4)$-invariant formulation:

$$g_{\mu\nu} = \theta^a{}_\mu \, \theta^b{}_\nu \, \gamma_{ab}$$

$$\theta \mapsto \Lambda^{-1}\theta \; , \qquad \gamma \mapsto \Lambda^T \gamma \Lambda$$

Different gauge fixings of $GL(4)$:

- $\theta^a_\mu = \delta^a_\mu$ metric formulation
- $\gamma_{ab} = \eta_{ab}$ vierbein formulation

## Reformulation of GR

$$S(\theta, \gamma) = S_{EG}(g)$$

This is the linking theory for metric and tetrad gravity.

Shows that $\theta$ and $\gamma$ are Goldstone bosons.

## Higgsless Higgs mechanism

Reconcile mass with gauge invariance
Goldstone bosons $\sigma \in G/H$ coupled to $G$-YM fields $A_\mu$.

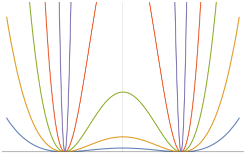
$$-\frac{1}{2}D\sigma^2 \qquad \text{where} \qquad D\sigma = \partial\sigma + A^i K_i(\sigma)$$

$\mathcal{L}(G) = \mathcal{L}(H) \oplus \mathcal{P}$ $A = A|_{\mathcal{L}(H)} + A|_{\mathcal{P}}$
In unitary gauge $\sigma = \sigma_0$
$D\sigma_0 = A^i|_{\mathcal{P}} K_i(\sigma_0)$

T. Appelquist, C.W. Bernard, Phys.Rev.D22:200,1980.
A.C. Longhitano, Phys.Rev.D22:1166,1980.

## Higgsful Higgs mechanism

Higgs field $\phi \in V$, and $\sigma$ parametrize orbit of $G$ in $V$
$W = \frac{\lambda}{4}(\rho^2 - \rho_0^2)^2$, with $\rho = |\phi|$.
$\lim_{\lambda\to\infty} W$ with $\rho_0$=const



- for $p \ll m_\rho$, $\rho = \rho_0$ (Higgsless model as low energy EFT)
- for $p \ll m_A$, $A|_{\mathcal{P}} = 0$ or $D\sigma = 0$

# Independent connection

Allow dynamical connection:
Palatini and Einstein-Cartan are off-shell reformulations of EG:
Connection is dynamically set equal to Levi-Civita.

Consider more general dynamics,
e.g. curvature squared terms.

## $GL(4)$-Gravity with independent connection

Spacetime manifold $M$, dim$M = 4$
$E$ real vectorbundle with fiber dimension 4
local bases $\{\partial_\mu\}$ in $TM$ and $\{e_a\}$ in $E$

- pseudo-fiber metric in $E$, $\gamma_{ab}$ signature $+, +, +, -$
- soldering form $\theta^a{}_\mu$, det$\theta \neq 0$
- linear connection in $E$, $A_\mu{}^a{}_b$

# Induced structures in $TM$

- $g_{\mu\nu} = \theta^a{}_\mu \, \theta^b{}_\nu \, \gamma_{ab}$
- $\Gamma_\lambda{}^\mu{}_\nu = \theta^{-1}{}_a{}^\mu A_\lambda{}^a{}_b \theta^b{}_\nu + \theta_a^{-1}{}^\mu \partial_\lambda \theta^a{}_\nu$

# Torsion and Nonmetricity

- $\Theta_\mu{}^a{}_\nu = \partial_\mu \theta^a{}_\nu - \partial_\nu \theta^a{}_\mu + A_\mu{}^a{}_b\, \theta^b{}_\nu - A_\nu{}^a{}_b\, \theta^b{}_\mu$
- $\Delta_{\lambda ab} = -\partial_\lambda \gamma_{ab} + A_\lambda{}^c{}_a\, \gamma_{cb} + A_\lambda{}^c{}_b\, \gamma_{ac}$

## Gravitational Higgs mechanism v.I

$$
\begin{aligned}
S_m &= \int d^4x \sqrt{|\det g|}\left[A^{\mu}{}_{a}{}^{\nu\rho}{}_{b}{}^{\sigma}\Theta_{\mu}{}^{a}{}_{\nu}\Theta_{\rho}{}^{b}{}_{\sigma}\right. \\
&\quad \left.+B^{\mu ab\nu cd}\Delta_{\mu ab}\Delta_{\nu cd}+C^{\mu}{}_{a}{}^{\nu\rho cd}\Theta_{\mu}{}^{a}{}_{\nu}\Delta_{\rho cd}\right]
\end{aligned}
$$

expanding around flat background: $A = 0$, $\theta = \mathbf{1}$, $\gamma = \eta$

$$
\begin{aligned}
\Theta_{\mu}{}^{a}{}_{\nu} &= A_{\mu}{}^{a}{}_{\nu} - A_{\nu}{}^{a}{}_{\mu} \\
\Delta_{\mu ab} &= A_{\mu ab} + A_{\mu ba}
\end{aligned}
$$

$S$ contains

$$
S_m = \frac{1}{2}\int d^4x\ \sqrt{|\det g|}\ Q^{\mu}{}_{a}{}^{b\nu}{}_{c}{}^{d}\, A_{\mu}{}^{a}{}_{b}\, A_{\nu}{}^{c}{}_{d}
$$

## Levi–Civita Connection

given $\theta$, $\gamma$, there is a unique $\bar{A}$ such that $\bar{\Theta} = 0$, $\bar{\Delta} = 0$

$$\bar{A} = \frac{1}{2}\left(\theta^{-1}{}_c{}^{\lambda}\,\partial_\lambda\kappa_{ab} + \theta^{-1}{}_a{}^{\lambda}\,\partial_\lambda\kappa_{bc} - \theta^{-1}{}_b{}^{\lambda}\,\partial_\lambda\kappa_{ac}\right) + \frac{1}{2}\left(C_{abc} + C_{bac} - C_{cab}\right)$$

where $C_{abc} = \gamma_{ad}\,\theta^d{}_\lambda\left(\theta^{-1}{}_b{}^{\mu}\,\partial_\mu\theta^{-1}{}_c{}^{\lambda} - \theta^{-1}{}_c{}^{\mu}\,\partial_\mu\theta^{-1}{}_b{}^{\lambda}\right)$

## Gravitational Higgs mechanism v.II

Any connection $A$ can be split uniquely in $A = \bar{A} + \Phi$
then $S(A, \gamma, \theta) = S(\bar{A}(\theta, \gamma) + \Phi, \theta, \gamma) = S'(\Phi, \theta, \gamma)$

$$\begin{aligned} \Theta_\mu{}^a{}_\nu &= \Phi_\mu{}^a{}_\nu - \Phi_\nu{}^a{}_\mu \\ \Delta_{\mu ab} &= \Phi_{\mu ab} + \Phi_{\mu ba} \end{aligned}$$

therefore

$$S_m = \frac{1}{2} \int d^4x \, \sqrt{|\det g|} \, Q^\mu{}_a{}^{b\nu}{}_c{}^d \, \Phi_\mu{}^a{}_b \, \Phi_\nu{}^c{}_d$$

For $p \ll m$, $\Theta = \Delta = 0$ and therefore $A = \bar{A}(\theta, \gamma)$

## Lesson and questions

The fact that the connection is a composite of the metric/vierbein is a feature of the low energy EFT.

Main questions for quantum theory of spacetime:

- why is the metric nondegenerate?
- what is the dynamical origin of the Planck scale?

## Grand Unification to do list

i. identify GUT group $G$
ii. fit particles in irreps of $G$
iii. write $\mathcal{G}$-invariant action
iv. explain symmetry breaking (select order parameter, orbit, potential)
v. check that new particles not seen at low energy have high mass

# GraviGUT II

$G_1 = SO(1,3)$, $G_2 = SO(10)$, $G = SO(1,13)$ or $G = SO(3,11)$

keep dim$M$=4, enlarge fibers of $E$ to have dimension $N > 4$
order parameter is soldering form

$$\gamma = \begin{bmatrix} \eta & 0 \\ 0 & \mathbf{1}_{N-4} \end{bmatrix} \qquad , \qquad \theta \text{ is } 4 \times N \text{ matrix, e.g. } \langle\theta\rangle = \begin{bmatrix} \mathbf{1}_4 \\ 0 \end{bmatrix}$$

## GraviGUT III

Gravitational Higgs phenomenon:

$$A = \begin{bmatrix} A^{(4)} & H \\ H^T & A^{(10)} \end{bmatrix}$$

kinetic term of $\theta$ gives mass to $A^{(4)}$, $H$, $SO(10)$ remains unbroken

R.P. Phys. Lett. B 144, 37 (1984), Nucl. Phys. B 353, 271, (1991).

## Fermions I

$SO(3,11)$ has Majorana-Weyl representation $\mathbf{64_R}$ that decomposes under $SO(3,1)\times SO(10)$ as

$$\mathbf{64_R} = \mathbf{2_C} \times \mathbf{16_C}$$

Remark: $SO(1,13)$ has Weyl $\mathbf{64_C}$
decomposing as $\mathbf{64_C} = \mathbf{2_C} \times \mathbf{16_C} + \overline{\mathbf{2}}_\mathbf{C} \times \overline{\mathbf{16}}_\mathbf{C}$

F. Nesti, R.P., Phys. Rev. D 81, 025010 (2010) arXiv:0909.4537 [hep-th]

# Fermions II

$$D_\mu \psi_{L+} = \left( \partial_\mu + \frac{1}{2} A^{ij}_\mu \Sigma^{(3,11)}_{L\, ij} \right) \psi_{L+}$$

let $\Sigma^\dagger_{ij} A = -A\, \Sigma_{ij}$
then $\psi^\dagger_{L+} (A\gamma^i)_L D\psi_{L+}$ is one-form in **14** of $SO(3,11)$

$$\mathcal{S} = \int \psi^\dagger_{L+} (A\gamma^i)_L D\psi_{L+} \wedge \theta^j \wedge \theta^k \wedge \theta^\ell \, \phi_{ijk\ell} \, .$$

## Fermions III

Assuming the following VEVs:

$$\begin{cases} \phi_{mnrs} = \epsilon_{mnrs} \\ \phi_{ijk\ell} = 0 \quad \text{otherwise} \end{cases} \qquad \begin{cases} \theta^m_\mu = M e^m{}_\mu \\ \theta^a_\mu = 0 \quad \text{otherwise} \end{cases}$$

we find after some work

$$S = \int d^4x \, \eta^\dagger \sigma^\mu \nabla_\mu \eta \, ,$$

where now $\nabla_\mu = D^{(10)}_\mu = \partial_\mu + \frac{1}{2} A^{ab}_{\mu\,(10)} \Sigma^{(10)}_{ab}$

## Status of GraviGUT

- kinematics well understood
- bosonic action for broken phase can be written
- fermionic content and dynamics ok
- hard to write action that works in both phases

K. Krasnov, R.P., CQG (to appear) arXiv:1712.03061 [hep-th]

## Conclusions

- Recent work seek dynamical origin of Planck scale in a scale-invariant theory
- View $m_P^{-2}$ as the VEV of the metric, which is clearly an order parameter distinguishing phases
- The low energy EFT presents a “Higgsless Higgs phenomenon” giving mass to the connection
- This suggests a unification of the SM and gravitational interactions