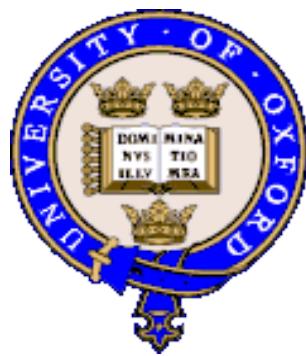
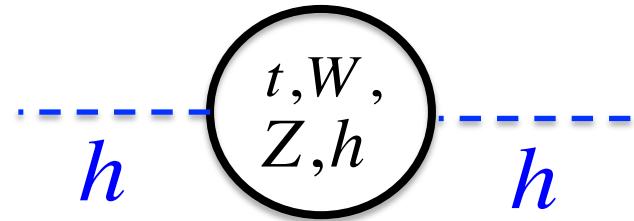


Naturalness: SUSY and other solutions.

G. Ross, PACTS, Tallinn, June 2018



Hierarchy problem?



$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2} \left(4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2 \right) \Lambda^2$$

Solutions (?)

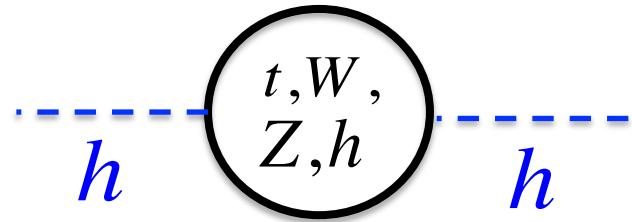
Symmetry:

Scale
SUSY
Nambu-Goldstone

Dynamics:

Scanning - Relaxion

Scale Invariance



$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2} \left(4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2 \right) \Lambda^2$$

Field theory: δm^2 not measurable

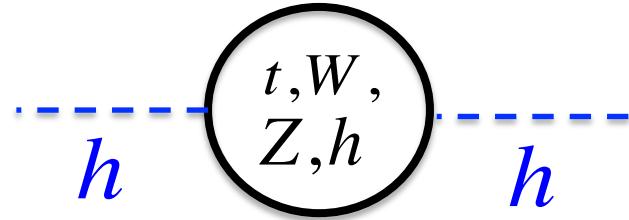
...only $m^2 = m_0^2 + \delta m^2$ "physical"

Only $m^2 = 0$ special

$$\frac{d m_h^2}{d \ln \mu} = \frac{3m_h^2}{8\pi^2} \left(2\lambda + y_t^2 - \frac{3g_2^2}{4} - \frac{3g_1^2}{20} \right)$$

Wheeler
Wetterich
Bardeen

Scale Invariance



$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2} \left(4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2 \right) \Lambda^2$$

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$$\frac{d m_h^2}{d \ln \mu} = \frac{3m_h^2}{8\pi^2} \left(2\lambda + y_t^2 - \frac{3g_2^2}{4} - \frac{3g_1^2}{20} \right)$$

... but

What about Gravity?

How is scale symmetry broken?

Brans Dicke gravity:

$$S = \int \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4} \phi^4 - \frac{1}{12} \alpha \phi^2 R \right)$$

(globally) Weyl (scale) invariant

$$\left\{ \begin{array}{l} \phi(x) \rightarrow e^\varepsilon \phi(x) \\ g_{\mu\nu}(x) \rightarrow e^{-2\varepsilon} g_{\mu\nu}(x), \quad \det(-g(x)) \rightarrow e^{-4\varepsilon} \det(-g(x)) \end{array} \right.$$

Brans Dicke gravity:

$$S = \int \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4} \phi^4 - \frac{1}{12} \alpha \phi^2 R \right)$$

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Noether current

$$K_\mu = \frac{1}{\sqrt{\det(-g)}} \frac{\delta S}{\delta \partial_\mu \phi} = (1-\alpha) \phi \partial_\mu \phi \equiv \partial_\mu K,$$

$$FRW: \quad D^\mu K_\mu = \ddot{K} + 3H\dot{K} = 0$$

$$K(t) = c_1 + c_2 \int_{t_0}^t \left(\frac{dt'}{a(t')^3} \right) \rightarrow \text{constant}$$

Inertial symmetry breaking

$$K = \frac{1}{2} (1-\alpha) \phi^2 \rightarrow \text{constant}$$

$$M_{Planck}^2 = -\frac{1}{6} \alpha \langle \phi \rangle^2$$

Scale breaking order parameter
independent of potential!
- set by initial conditions

Ferreira, Hill, GGR

$$\left(KG: \quad (1-\alpha) \left[\phi D^2 \phi + \partial^\mu \phi \partial_\mu \phi \right] = \phi \frac{\partial V(\phi)}{\partial \phi} - 4V(\phi) = 0 \right)$$

Hierarchy generation:

$$S = \int \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4} \phi^4 - \frac{1}{12} \alpha \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{\xi}{4} \chi^4 + \frac{\delta}{2} \phi^2 \chi^2 - \frac{1}{12} \beta \chi^2 R \right)$$


2nd singlet models Higgs sector

Noether current

$$K_\mu = \frac{1}{\sqrt{\det(-g)}} \frac{\delta S}{\delta \partial_\mu \varepsilon} = (1-\alpha)\phi \partial_\mu \phi + (1-\beta)\chi \partial_\mu \chi \equiv \partial_\mu K,$$

Inertial symmetry breaking

$$K = \frac{1}{2}(1-\alpha)\phi^2 + \frac{1}{2}(1-\alpha)\chi^2 \rightarrow \text{constant}$$

$$M_{Planck}^2 = -\frac{1}{6}\alpha\phi^2 - \frac{1}{6}\beta\chi^2$$

Hierarchy

$$\lambda \ll \delta \ll \xi$$

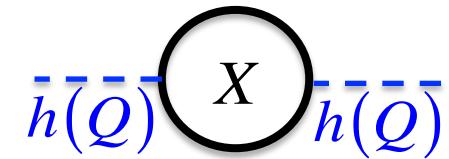
$$\frac{\chi_{IR}^2}{\phi_{IR}^2} = \frac{4\lambda\beta - 2\alpha\delta}{4\alpha\xi - 2\beta\delta}$$

IR fixed point 

(The model can also give realistic inflation)

Shaposhnikov, Zenhausern
Garcia-Bellido, Rubio, Shaposhnikov, Zenhausern

Tests?

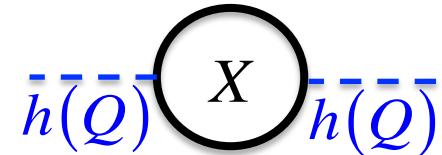


Heavy states, eg GUTs: $\delta m_h^2 \propto M_X^2$

⇒ no significant coupling to heavies allowed

i.e. BSM structure must come from light states

Tests?



Heavy states, eg GUTs: $\delta m_h^2 \propto M_X^2$

⇒ no significant coupling to heavies allowed

i.e. BSM structure must come from light states

Inflation ✓

Dilaton - decouples

Neutrino masses and baryogenesis - e.g. vMSM[†], $m(v_R^i) \sim \text{KeV} - \text{GeV}$

Strong CP problem - Axion

Dark matter - v_R and/or axion

Landau pole? Giudice, Isidori, Salvio, Strumia
Strong gravity?

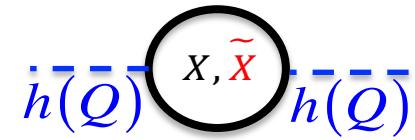
For reviews see:

A.Boyarsky et al 0901.011

M.Shaposhnikov, Subnucl.Ser. 47 (2011) 167-207

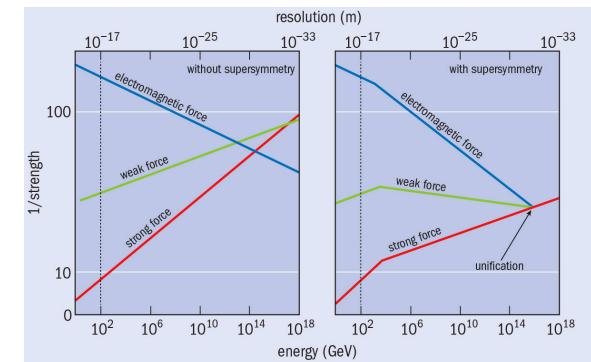
SUSY (and GUTs)

Elementary Higgs ✓



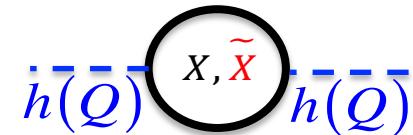
Solves big hierarchy problem $\delta m_h^2 \propto M_{SUSY}^2$ ✓

Gauge coupling unification ✓



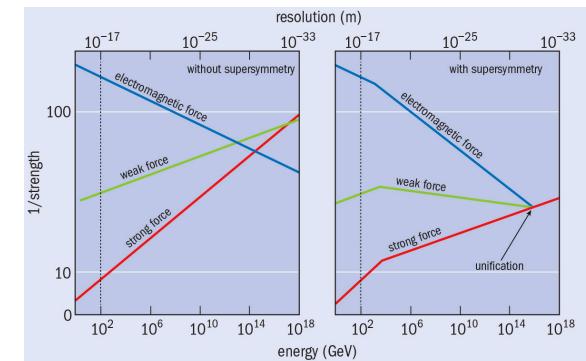
SUSY

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Gauge coupling unification ✓



Low scale SUSY ?

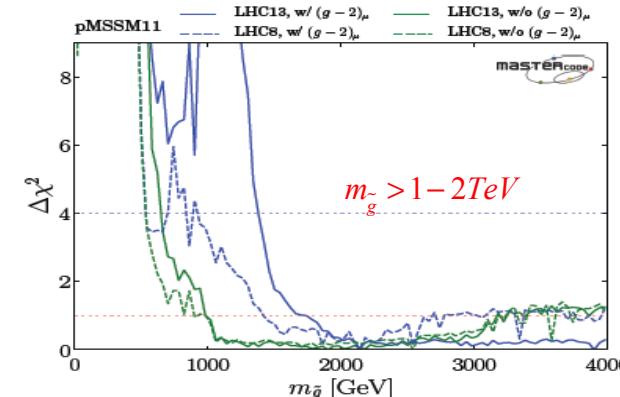
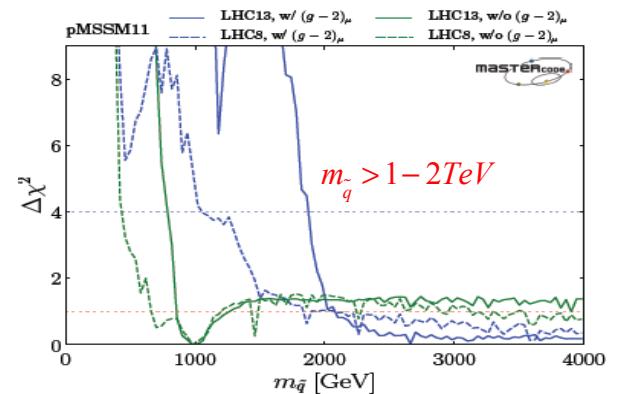
Can low scale SUSY improve on SM? Dark matter, $(g-2)_\mu \dots$?

MSSM: 105 +(19) Parameters! ... need simplification

pMSSM11 fit LHC 13 + DM

Parameter	$g - 2$	$\cancel{g - 2}$
M_1	0.25 TeV	- 1.3 TeV
M_2	0.25 TeV	2.3 TeV
M_3	- 3.86 TeV	1.9 TeV
$m_{\tilde{q}}$	4.0 TeV	0.9 TeV
$m_{\tilde{q}_3}$	1.7 TeV	2.0 TeV
$m_{\tilde{\ell}}$	0.35 TeV	1.9 TeV
$m_{\tilde{\tau}}$	0.46 TeV	1.3 TeV
M_A	4.0 TeV	3.0 TeV
A	2.8 TeV	- 3.4 TeV
μ	1.33 TeV	- 0.95 TeV
$\tan \beta$	36	33
$\chi^2/\text{d.o.f.}$	22.1/20	20.88/19
p-value	0.33	0.34
$\chi^2(HS)$	68.01	68.06

χ^2 Probability 34% 34%



Bagnaschi et al 1710.11091 MasterCode
(see also talk by S.King)

The Little hierarchy problem in SUSY

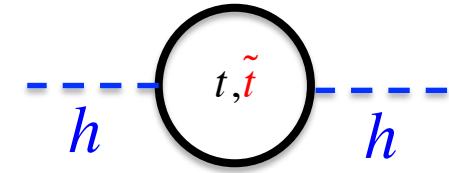
Low scale SUSY ?

Tension -

$$\left\{ \begin{array}{l} m_h^2 = M_Z^2 + \frac{3m_t^2 h_t^2}{4\pi^2} \left(\ln \left(\frac{m_{stop}^2}{m_t^2} \right) + \delta_t \right) + \dots \simeq 126 GeV \\ \delta m_{H_u}^2 = -\frac{3y_t^2}{4\pi^2} \left(m_{stop}^2 + \frac{g_s^2}{3\pi^2} m_{gluino}^2 \log \left(\frac{\Lambda}{m_{gluino}} \right) \right) \log \left(\frac{\Lambda}{m_{stop}} \right) \end{array} \right.$$

$\Lambda \sim M_{GUT}$?

$$m_{\tilde{t}, \tilde{g}} < 1 TeV ??$$



The Little hierarchy problem in SUSY

Low scale SUSY ?

Tension -

$$\left\{ \begin{array}{l} m_h^2 = M_Z^2 + \frac{3m_t^2 h_t^2}{4\pi^2} \left(\ln \left(\frac{m_{stop}^2}{m_t^2} \right) + \delta_t \right) + \dots \simeq 126 GeV \\ \delta m_{H_u}^2 = -\frac{3y_t^2}{4\pi^2} \left(m_{stop}^2 + \frac{g_s^2}{3\pi^2} m_{gluino}^2 \log \left(\frac{\Lambda}{m_{gluino}} \right) \right) \log \left(\frac{\Lambda}{m_{stop}} \right) \\ m_{\tilde{t}, \tilde{g}} < 1 TeV ?? \end{array} \right. \quad \Lambda \sim M_{GUT} ?$$

Fine tuning at the electroweak scale - Δ_{EW}

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \sim -m_{H_u}^2 - \Sigma_u^u - \mu^2$$

$$\Delta_{EW, \mu}^{loop} \equiv \frac{\partial \ln M_Z^2}{\partial \ln \mu^2} \sim \frac{2\mu^2}{2^\dagger M_Z^2}$$

SUSY parameters at EW scale
No GUT logs

Top loop contribution

Baer et al 1207.3343

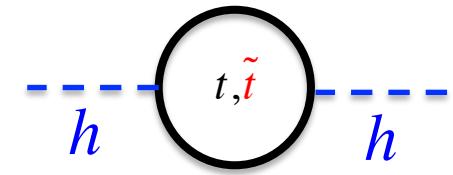
Ellis, Enquist, Nanopoulos, Zwirner
Barbieri, Giudice

$$\Delta_{EW} = 30 \Rightarrow \mu = 500 GeV \sim M_{\tilde{h}}$$

$$\delta \left(\frac{\chi^2}{d.f.} \right) < 0.5$$

Light Higgsino

[†] Staub, Schmidt-Hoberg, GGR



Fine tuning at the GUT scale

$$M_Z^2 = \sum_{\tilde{q}, \tilde{l}} a_i \tilde{m}_i^2 + \sum_{\tilde{g}, \tilde{W}, \tilde{B}} b_i \tilde{M}_i^2 + \dots$$

Large logs $\Rightarrow b = O(1)$

$$M_{\tilde{g}} > 2TeV \Rightarrow \Delta > b \frac{\tilde{M}^2}{M_Z^2} \sim 400$$

\Rightarrow correlations between parameters $b \ll 1$?

Fine tuning at the GUT scale

$$M_Z^2 = \sum_{\tilde{q}, \tilde{l}} a_i \tilde{m}_i^2 + \sum_{\tilde{g}, \tilde{W}, \tilde{B}} b_i \tilde{M}_i^2 + \dots$$

Large logs $\Rightarrow b = O(1)$

$$M_g \sim 2 TeV \Rightarrow \Delta > b \frac{\tilde{M}^2}{M_Z^2} \sim 400$$

\Rightarrow correlations between parameters $b \ll 1$?

Focus points - e.g. gaugino focus point - NUGM 

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3 \left(2 |y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2) + 2 |a_t|^2 \right) - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2$$

Focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \approx |M_3|^2$ at M_{SUSY}

... cancels large log - $b \ll 1$

Horton, GGR

(Also improves precision of gauge coupling unification)

Shifman, Roszkowski
Krippendorf, Nilles, Ratz, Winkler



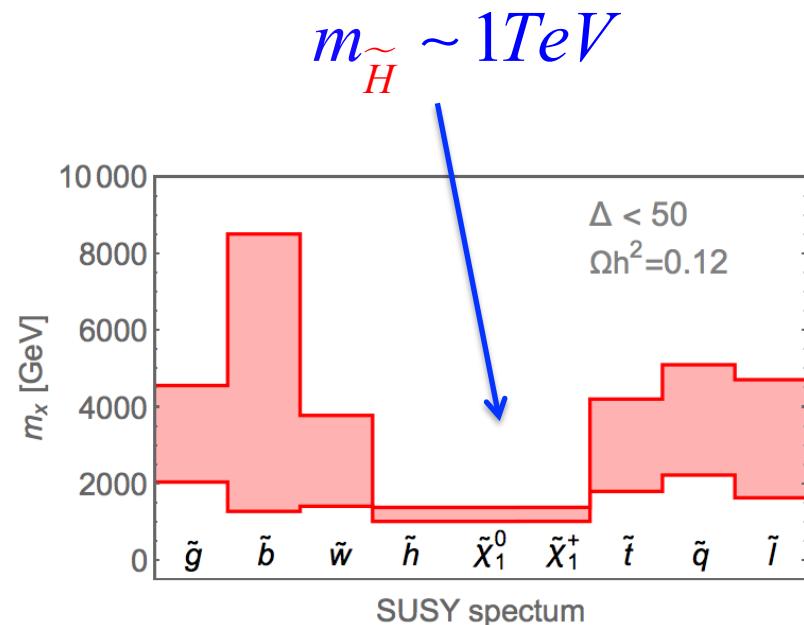
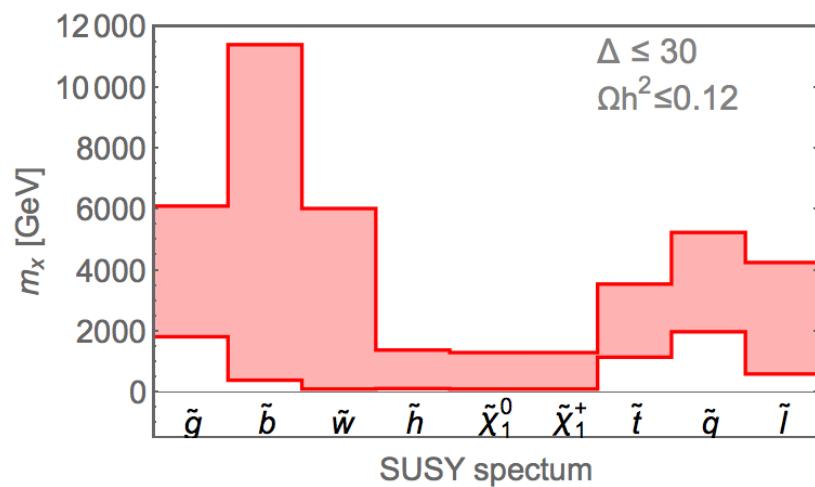
Non-universal gaugino masses at GUT scale (orbifolds, mirage mediation...)

Fine tuning measure, Δ_{GUT}

<i>Model</i>	<i>LHC & Higgs</i>	<i>soft DM</i>	<i>strong DM</i>
<i>CMSSM</i>	204	266	290
<i>MSSM – NUGM</i>	11	11	110
<i>MSSM – NUGM + μ'</i> †	10	11	27
<i>GNMSSM – NUGM</i>	10	10	30

$\Delta_{GUT} \sim \Delta_{EW}$

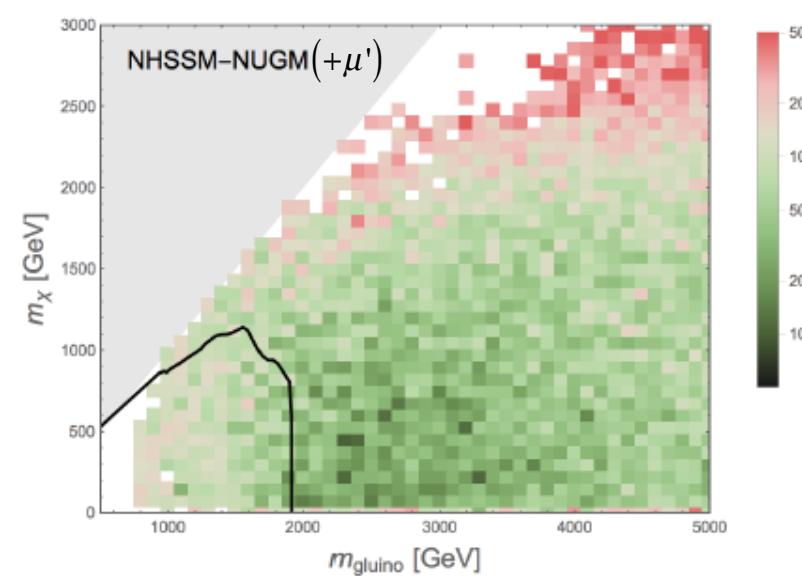
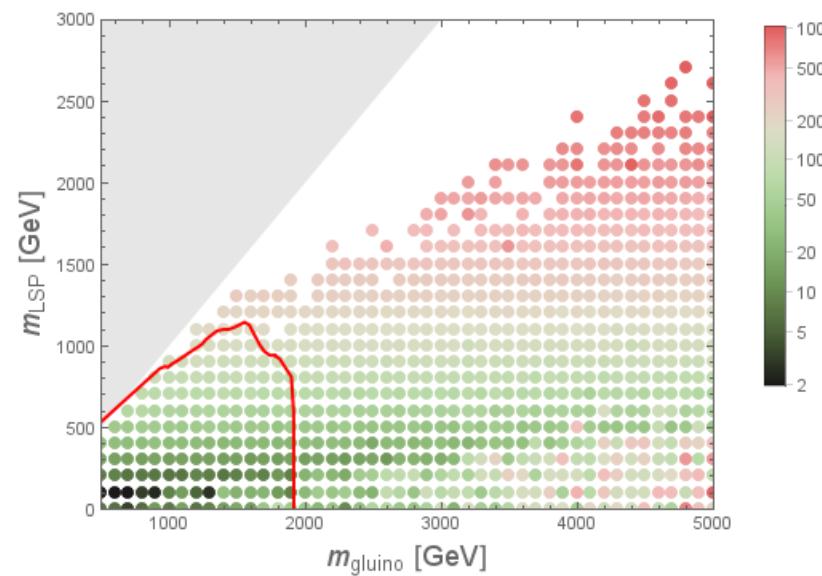
Spectrum



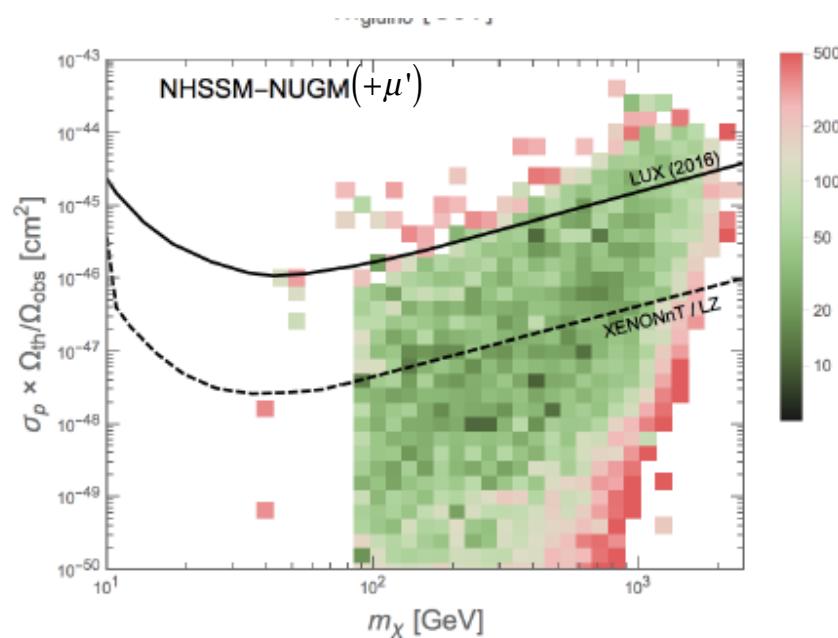
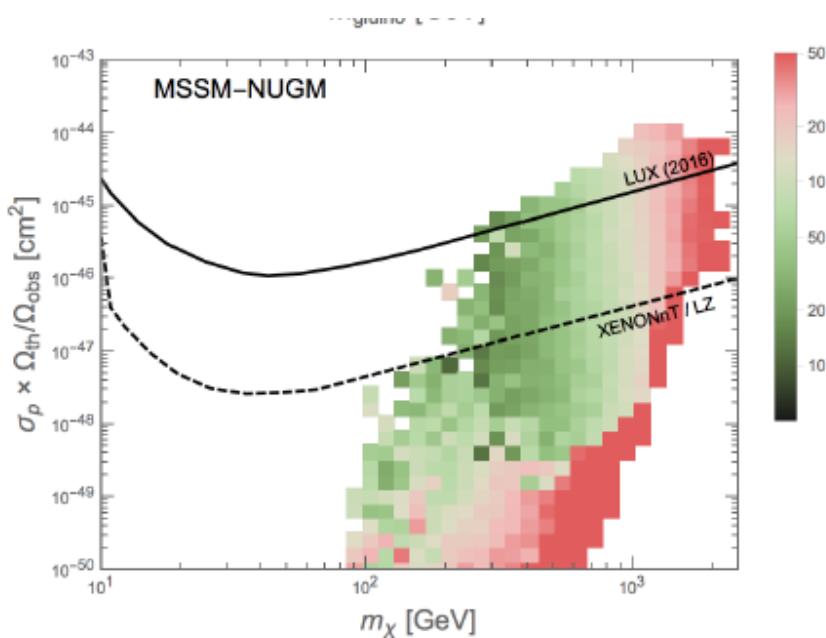
†

Soft Higgsino mass $m_H = 0$, $m_{\tilde{H}} = \mu'$

Staub, Schmidt-Hoberg, GGR



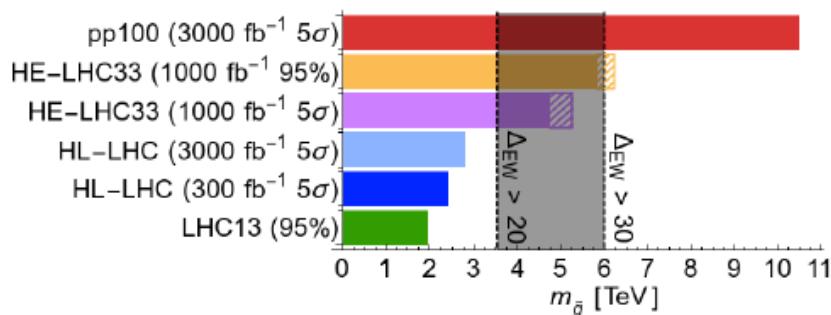
DM searches



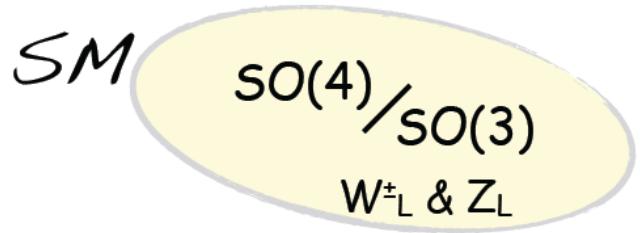
Prospects: Fine tuning reach

CMSSM							
	current			prospects			
Cut	LHC & Higgs	soft DM	strong DM	$m_{\tilde{g}} \geq 3$ TeV	DD	$m_{\tilde{g}} \geq 5$ TeV	DD
Δ_{\min}	134	216	276	231	271	686	-
CNHSSM							
	current			prospects			
Cut	LHC & Higgs	soft DM	strong DM	$m_{\tilde{g}} \geq 3$ TeV	DD	$m_{\tilde{g}} \geq 5$ TeV	DD
Δ_{\min}	114	116	166	227	264	665	677
MSSM-NUGM							
	current			prospects			
Cut	LHC & Higgs	soft DM	strong DM	$m_{\tilde{g}} \geq 3$ TeV	DD	$m_{\tilde{g}} \geq 5$ TeV	DD
Δ_{\min}	11	11	117	17	17	29	29
NHSSM-NUGM (+ μ')							
	current			prospects			
Cut	LHC & Higgs	soft DM	strong DM	$m_{\tilde{g}} \geq 3$ TeV	DD	$m_{\tilde{g}} \geq 5$ TeV	DD
Δ_{\min}	10	10	23	11	11	23	23

(DD: Direct detection DM)

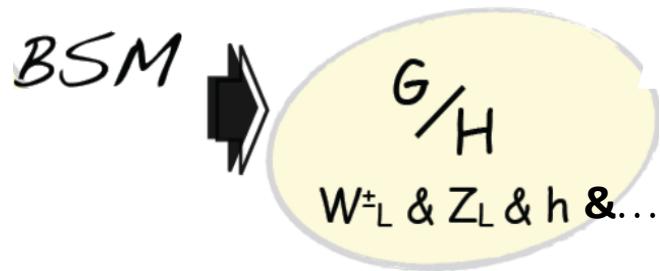


(Pseudo) Nambu Goldstone



$$H^\dagger H = \sum_1 h_i^2$$

$$SO(4) \sim SU(2)_L \times SU(2)_R \xrightarrow{v} SO(3) \sim SU(2)_C$$



e.g.

$$SO(5)/SO(3): W, Z, h$$

$$SO(6)/SO(3): W, Z, h, a$$

...

$$SO(5)/SO(3): W, Z, h$$

$$SO(5)/SO(4): h^{a=1..4} \quad \left(SO(5) \xrightarrow{f} SO(4) \right)$$

$$SO(4)/SO(3): W_L^\pm, Z_L$$

$$\Sigma(x) = \Sigma_0 e^{\Pi(x)/f} \qquad \begin{aligned} \Sigma_0 &= (0, 0, 0, 0, 1) \\ \Pi(x) &= -iT^{\hat{a}}h^{\hat{a}}(x)\sqrt{2} \end{aligned}$$

$$\Sigma = \frac{\sin(h/f)}{h} \left(h^1, h^2, h^3, h^4, h \cot(h/f) \right), \quad h \equiv \sqrt{(h^{\hat{a}})^2}$$

$$\mathcal{L}_H = \frac{1}{2} (\partial_\mu h)^2 + V(h) + \frac{v^2}{4} \text{Tr} \left[(D_\mu \Sigma)^\dagger (D_\mu \Sigma) \right] \left(1 + 2a \frac{h}{v} + b \frac{h^2}{v^2} + \dots \right)$$

$$a = \sqrt{1 - \xi}, \quad \quad b = 1 - 2\xi \quad \quad \quad \xi \equiv \frac{v^2}{f^2}$$

Precision tests $T \left(\equiv \Delta \rho \right) \checkmark \text{(Custodial symmetry)}, \quad S \Rightarrow \left(\frac{v}{f} \right)^2 \equiv \xi \leq 0.1$

Fine tuning

$$V(h) = -\gamma \sin^2(h/f) + \beta \sin^4(h/f),$$

Fermion resonance contribution

$$\gamma_f = \frac{N_{cv} M_f^4}{16\pi^2 g_f^2} \left[c_2 \varepsilon^2 \frac{\Lambda^2}{M_f^2} + c_0 \varepsilon^4 \log \frac{\Lambda^2}{M_f^2} + \text{finite} \right]$$

$$\beta_f = \frac{N_{cv} M_f^4}{16\pi^2 g_f^2} \left[c'_0 \varepsilon^4 \log \frac{\Lambda^2}{M_f^2} + \text{finite} \right]$$

$$M_f = g_f f$$

$$\varepsilon = \varepsilon_{qfH} \text{ Yukawa}$$

Fine tuning

$$V(h) = -\gamma \sin^2(h/f) + \beta \sin^4(h/f),$$

Fermion resonance contribution

$$\left. \begin{aligned} \gamma_f &= \frac{N_{cv} M_f^4}{16\pi^2 g_f^2} \left[c_2 \varepsilon^2 \frac{\Lambda^2}{M_f^2} + c_0 \varepsilon^4 \log \frac{\Lambda^2}{M_f^2} + \text{finite} \right] \\ \beta_f &= \frac{N_{cv} M_f^4}{16\pi^2 g_f^2} \left[c'_0 \varepsilon^4 \log \frac{\Lambda^2}{M_f^2} + \text{finite} \right] \end{aligned} \right\} \quad \xi \equiv \sin^2(\langle h \rangle / f) = \frac{\gamma}{2\beta}$$

$\frac{\Lambda^2}{M_f^2 \log \frac{\Lambda^2}{M_f^2}}$

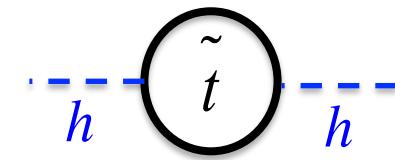
Tuning

$$\Delta \simeq \frac{1}{\xi} \cdot \frac{\Lambda^2}{M_f^2 \log \frac{\Lambda^2}{M_f^2}} \rightarrow \Delta_{\min} = \frac{1}{\xi} \geq 10$$

e.g. G/H symmetric space
 $\Sigma \rightarrow g \Sigma g^\dagger$

Csaki, Ma, Shu

Top quark partners

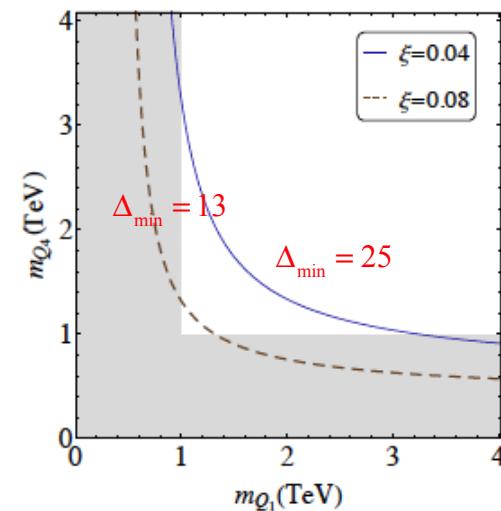


$$\delta m_h^2 = \frac{3}{\sqrt{2}\pi^2} G_F m_t^2 \Lambda^2 \Rightarrow \Delta \geq \frac{\delta m_h^2}{m_h^2} = \left(\frac{\Lambda}{400 \text{ GeV}} \right)^2 \left(\frac{125 \text{ GeV}}{m_h} \right)^2$$

$m_{\tilde{t}} < 1 \text{ TeV?}$

$SO(5)/SO(3):W,Z,h$

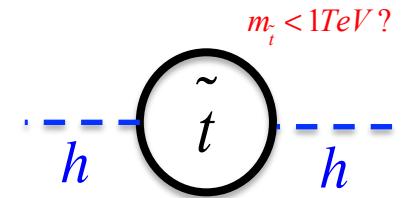
$SO(4)|_{\tilde{t}} : 4+1$



Banerjee, Bhattacharyya, Ray

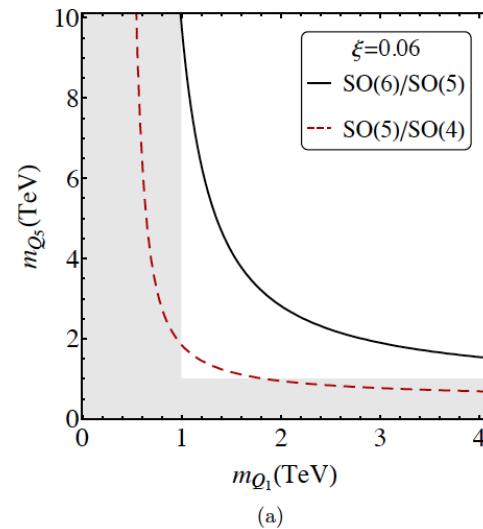
Top quark partners

$$\delta m_h^2 = \frac{3}{\sqrt{2}\pi^2} G_F m_t^2 \Lambda^2 \Rightarrow \Delta \geq \frac{\delta m_h^2}{m_h^2} = \left(\frac{\Lambda}{400 \text{ GeV}} \right)^2 \left(\frac{125 \text{ GeV}}{m_h} \right)^2$$

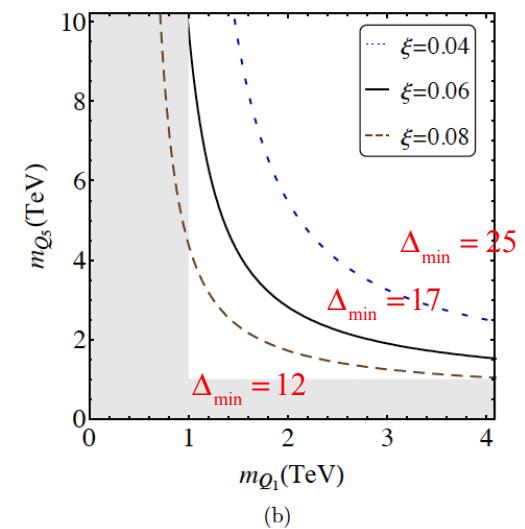


$SO(6)/SO(3):W,Z,h,a$

$SO(6)|_{\tilde{t}} : 6$



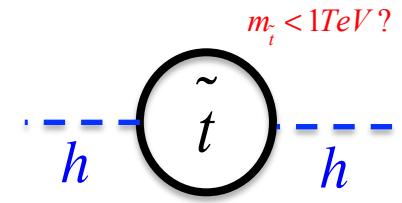
(a)



(b)

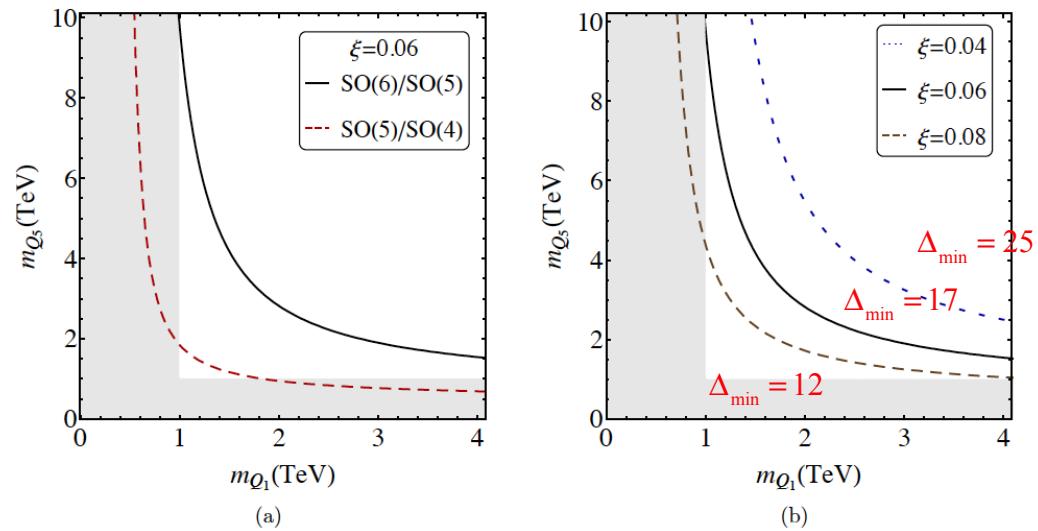
Top quark partners

$$\delta m_h^2 = \frac{3}{\sqrt{2}\pi^2} G_F m_t^2 \Lambda^2 \Rightarrow \Delta \geq \frac{\delta m_h^2}{m_h^2} = \left(\frac{\Lambda}{400 \text{ GeV}} \right)^2 \left(\frac{125 \text{ GeV}}{m_h} \right)^2$$



$SO(6)/SO(3):W,Z,h,a$

$SO(6)|_{\tilde{t}} : 6$



...but in twin Higgs models, $SM \times SM'$, \tilde{t} not coloured!

Chacko, Goh, Harnik

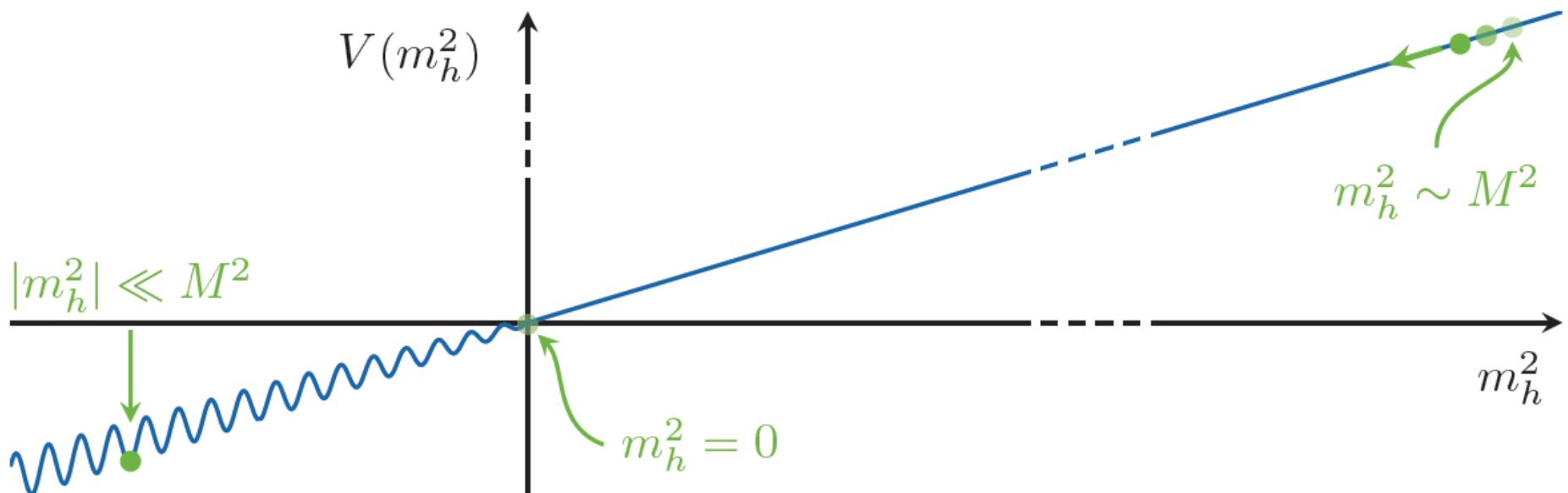
$$V = -\mu^2 (|H_1|^2 + |H_2|^2 + |\textcolor{blue}{H}_3|^2 + |\textcolor{blue}{H}_4|^2) + \lambda (|H_1|^2 + |H_2|^2 + |\textcolor{blue}{H}_3|^2 + |\textcolor{blue}{H}_4|^2)^2$$

Scanning - Relaxion I

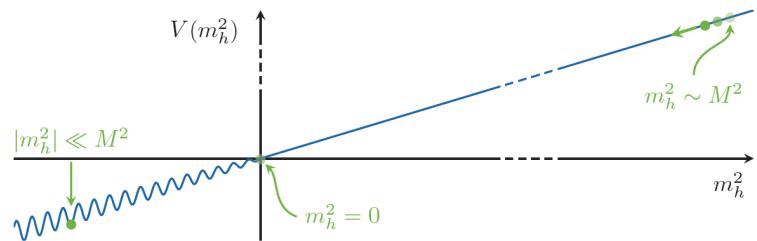
$m_h^2 = 0$ special - boundary between broken and unbroken phase

Scanning : $m_h^2 = m_h^2(\phi)$ "Relaxion"

Higgs mass minimised close to $m_h^2 = 0$ when minimising scalar potential



Scanning - Relaxion I



Slow roll - inflation needed

Graham, Kaplan, Rajendran

$$m_h^2(\phi)|_{t=0} > 0$$

$\underbrace{}$

$$\left(-M^2 + g\phi \right) |h|^2 + (gM^2\phi + g^2\phi^2 + \dots) + \Lambda^4 \cos(\phi/f)$$

hierarchy problem term

$$\Lambda^4 \sim \Lambda^3 h \quad c.f. f_\pi^2 m_\pi^2 \propto m_q \propto h$$

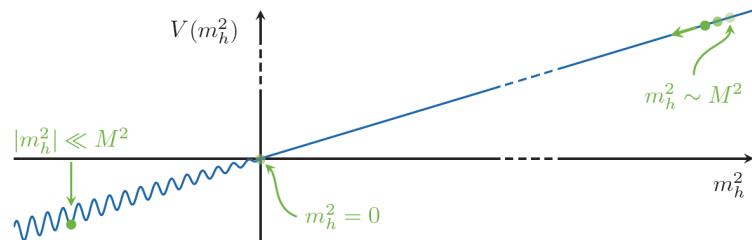
$$\Lambda^4 \cos(\phi/f)$$

QCD (rel)axion

$$\frac{1}{32\pi^2} \frac{\phi}{f} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

g small controlled by shift symmetry: $\phi \rightarrow \phi + c$

Scanning - Relaxion I



Slow roll - inflation needed

Graham, Kaplan, Rajendran

$$\Lambda_{QCD} > H_I > \frac{M^2}{M_P}$$

$$(-M^2 + g\phi) |h|^2 + (gM^2\phi + g^2\phi^2 + \dots) + \Lambda^3 h \cos(\phi/f)$$

$M < 10^7 \text{ GeV}$

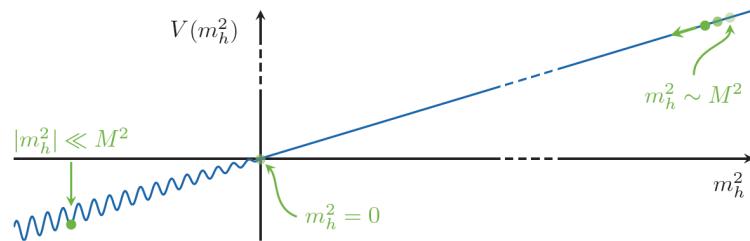
UV completion: SUSY, composite..

QCD (rel)axion

$$\frac{1}{32\pi^2} \frac{\phi}{f} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

g small controlled by shift symmetry: $\phi \rightarrow \phi + c$

Scanning - Relaxion I



Slow roll - inflation needed

Graham, Kaplan, Rajendran

$$m_h^2(\phi)|_{t=0} > 0$$

$$(-M^2 + g\phi)|h|^2 + (gM^2\phi + g^2\phi^2 + \dots) + \Lambda^3 h \cos(\phi/f)$$

$$\text{Inflation ends} \quad gM^2 \sim \frac{\Lambda^3 h}{f}$$

QCD (rel)axion

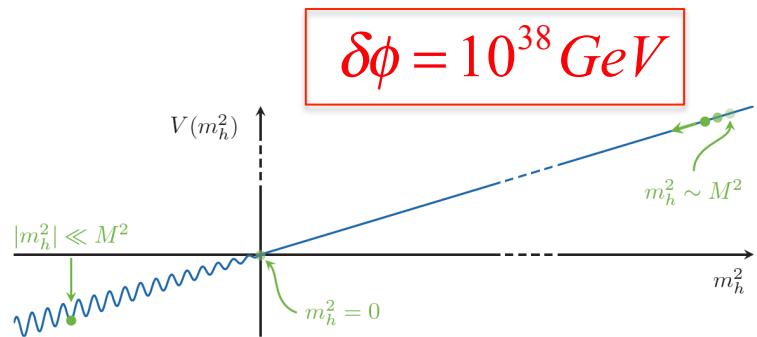
$$\frac{1}{32\pi^2} \frac{\phi}{f} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

g small controlled by shift symmetry: $\phi \rightarrow \phi + c$

$$g < 10^{-25} \text{ GeV}$$

Natural

Scanning - Relaxion I



String monodromy?

Clockwork/multiple axions ✓

c.f. F.Kamenik's talk

Slow roll - inflation needed

Graham, Kaplan, Rajendran

$$m_h^2(\phi)|_{t=0} > 0$$

↙

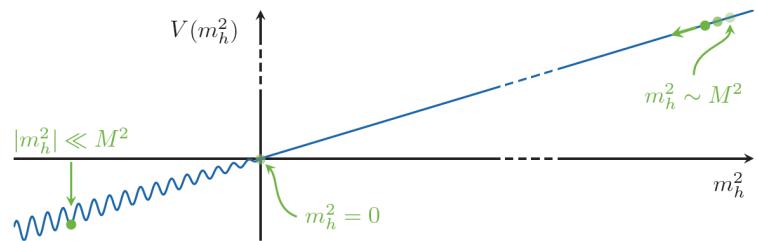
$$(-M^2 + g\phi)|h|^2 + (gM^2\phi + g^2\phi^2 + \dots) + \Lambda^3 h \cos(\phi/f)$$

QCD (rel)axion

$$\frac{1}{32\pi^2} \frac{\phi}{f} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

g small controlled by shift symmetry: $\phi \rightarrow \phi + c$

Scanning - Relaxion I



$$N > 10^{47}$$

$$H_I < \Lambda_{QCD}$$

Inflation potential
fine tuned

Di-Chiara et al 1511.2858

Slow roll - inflation needed

Graham, Kaplan, Rajendran

$$m_h^2(\phi)|_{t=0} > 0$$

$$(-M^2 + g\phi)|h|^2 + (gM^2\phi + g^2\phi^2 + \dots) + \Lambda^3 h \cos(\phi/f)$$

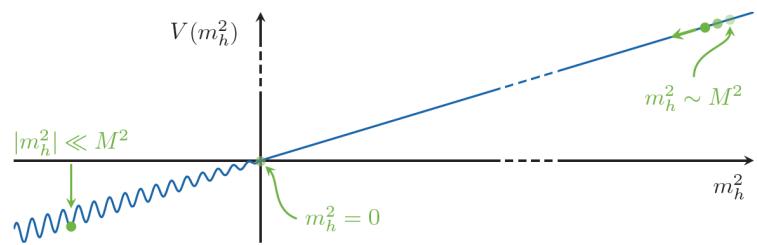
QCD (rel)axion

$$\frac{1}{32\pi^2} \frac{\phi}{f} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

g small controlled by shift symmetry: $\phi \rightarrow \phi + c$

$$\theta_{QCD} = O(1) \times$$

Scanning - Relaxion I b



$$m_h^2(\phi)|_{t=0} > 0$$

$$(-M^2 + g\phi)|h|^2 + (gM^2\phi + g^2\phi^2 + \dots) + \Lambda^3 h \cos(\phi/f)$$

$$N > 10^{47}$$

$$3 \text{GeV} < H_I < 100 \text{GeV}$$

Measure
Problem -
Relaxion
landscape

Slow roll - inflation needed
including thermal effects

Nelson, Prescod-Weinstein

$$\Omega_a \sim \left(\frac{f_a}{10^{11-12} \text{GeV}} \right)$$

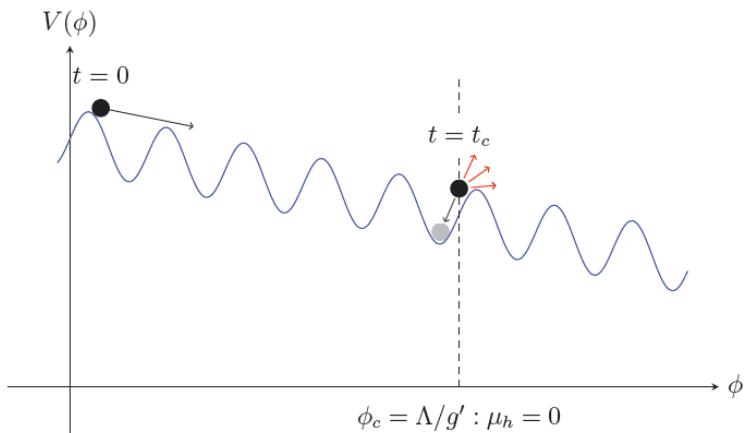
QCD (rel)axion

$$\frac{1}{32\pi^2} \frac{\phi}{f} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

g small controlled by shift symmetry: $\phi \rightarrow \phi + c$

$$\theta_{QCD} = O(10^{-10})$$

Scanning - Relaxion II



Rapid roll - particle production needed

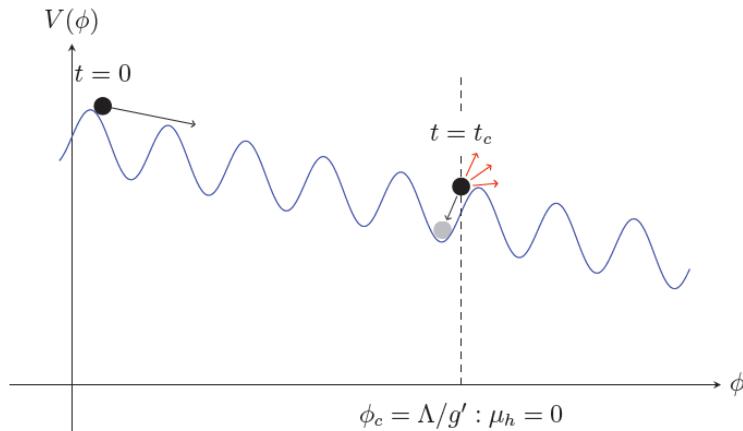
Hook, Marques-Tavares
Fonseca, Morgante,Servant

$$V(\phi, h) = \Lambda^4 - g\Lambda^3\phi + \frac{1}{2}(-\Lambda^2 + g'\Lambda\phi)h^2 + \frac{\lambda}{4}h^4 + \Lambda_b^4 \cos\left(\frac{\phi}{f'}\right)$$

$m_h^2(\phi)|_{t=0} < 0$
}

small controlled by shift symmetry: $\phi \rightarrow \phi + c$

Scanning - Relaxion II



$-\frac{\phi}{4f} \left(g_2^2 W \tilde{W} - g_1^2 B \tilde{B} \right)$: no coupling to photon
(e.g. $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$,
 $SO(6)/SO(5)$)

Rapid roll - particle production needed

$m_h^2(\phi)|_{t=0} < 0$

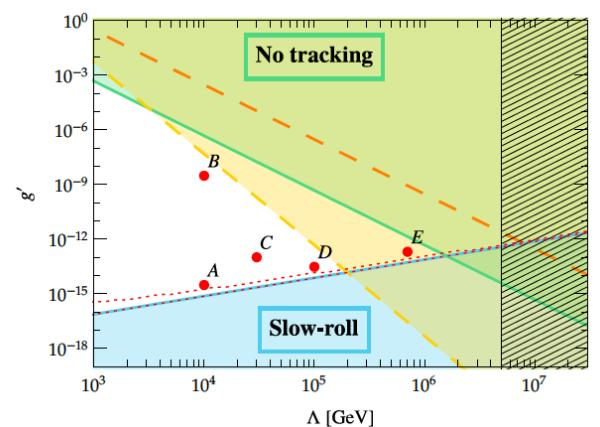
Hook, Marques-Tavares
Fonseca, Morgante, Servant

$$V(\phi, h) = \Lambda^4 - g\Lambda^3\phi + \frac{1}{2}(-\Lambda^2 + g'\Lambda\phi)h^2 + \frac{\lambda}{4}h^4 + \Lambda_b^4 \cos\left(\frac{\phi}{f}\right)$$

$$\ddot{\phi} - g\Lambda^3 + g'\Lambda h^2 + \frac{\Lambda_b^4}{f'} \sin\frac{\phi}{f} + \frac{1}{4f} \langle F \tilde{F} \rangle = 0$$

$$\ddot{V}_\pm + \left(k^2 + m_V^2 \mp k \frac{\dot{\phi}}{f} \right) V_\pm = 0$$

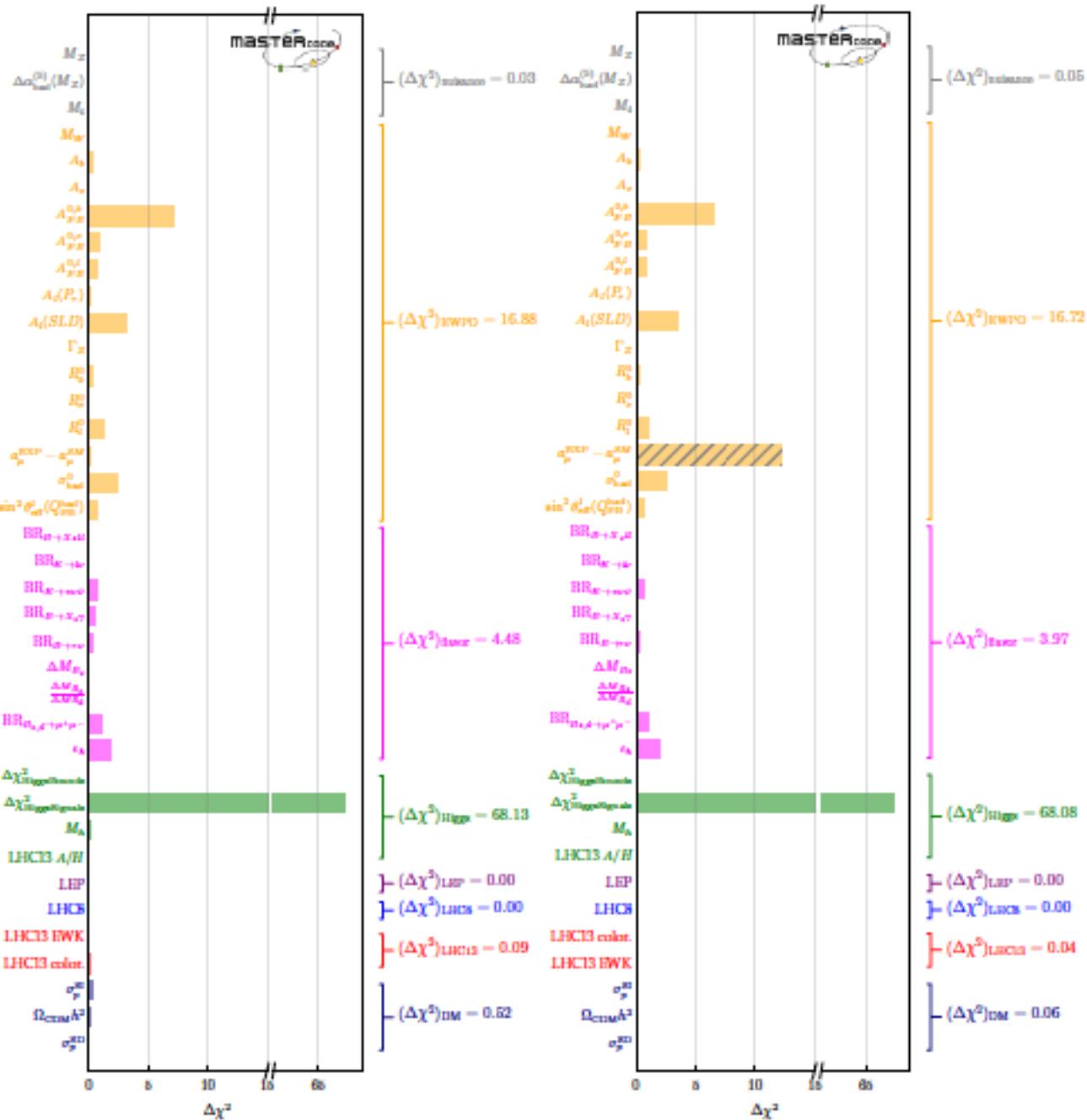
resonant production when $\dot{\phi} \sim \Lambda^2 = 2m_Z f$



$$m_\phi = 0.1 \text{ GeV} - \Lambda \quad \text{Higgs mixing - FCCpp, } \Lambda < 5 \text{ TeV, SHIP, } \Lambda < 10 \text{ TeV}$$

Summary

- Spontaneously broken Scale Invariance ✓
 - Breaking and inflation natural
 - BSM at EW scale
 - Ultraweak couplings, GUT , Landau pole?
- SUSY $\Delta \geq 30$
 - Elementary Higgs
 - UV complete, SUSYGUTS, gauge coupling unification, neutrino masses
 - Not guaranteed to find SUSY at LHC or DM searches
- Pseudo Nambu Goldstone $\Delta \geq 10$
 - Needs UV completion at low scale
 - Top quark partners within LHC range - but might not be coloured
 - Partial compositeness promising for flavour structure
- Relaxion ✓
 - Needs UV completion at intermediate scale
 - $\langle \phi \rangle \gg M_P, N \sim 10^{47}$, ultraweak coupling...technically natural but original version looks contrived
 - Explosive particle production model looks more promising



Observable	Source Th./Ex.	Constraint
$\rightarrow m_t$ [GeV]	[39]	173.34 ± 0.76
$\Delta\alpha_{\text{had}}^{(b)}(M_Z)$	[40]	0.02771 ± 0.00011
M_Z [GeV]	[41, 42]	91.1875 ± 0.0021
Γ_Z [GeV]	[43] / [41, 42]	$2.4952 \pm 0.0023 \pm 0.001_{\text{SUSY}}$
σ_{had}^0 [nb]	[43] / [41, 42]	41.540 ± 0.037
R_l	[43] / [41, 42]	20.767 ± 0.025
$A_{\text{FB}}(\ell)$	[43] / [41, 42]	0.01714 ± 0.00095
$A_\ell(P_\tau)$	[43] / [41, 42]	0.1465 ± 0.0032
R_b	[43] / [41, 42]	0.21629 ± 0.00066
R_c	[43] / [41, 42]	0.1721 ± 0.0030
$A_{\text{FB}}(b)$	[43] / [41, 42]	0.0992 ± 0.0016
$A_{\text{FB}}(c)$	[43] / [41, 42]	0.0707 ± 0.0035
A_b	[43] / [41, 42]	0.923 ± 0.020
A_c	[43] / [41, 42]	0.670 ± 0.027
A_{LR}^e	[43] / [41, 42]	0.1513 ± 0.0021
$\sin^2 \theta_w^\ell(Q_{\text{fb}})$	[43] / [41, 42]	0.2324 ± 0.0012
M_W [GeV]	[43] / [41, 42]	$80.385 \pm 0.015 \pm 0.010_{\text{SUSY}}$
$a_\mu^{\text{EXP}} - a_\mu^{\text{SM}}$	[44] / [45]	$(30.2 \pm 8.8 \pm 2.0_{\text{SUSY}}) \times 10^{-10}$
$\rightarrow M_h$ [GeV]	[46, 47] / [48]	$125.09 \pm 0.24 \pm 1.5_{\text{SUSY}}$
$\rightarrow \text{BR}_{b \rightarrow s\gamma}^{\text{EXP/SM}}$	[49] / [50]	$1.021 \pm 0.066_{\text{EXP}}$ $\pm 0.070_{\text{TH,SM}} \pm 0.050_{\text{TH,SUSY}}$
$\rightarrow R_{\mu\mu}$	[51] / [37, 38]	2D likelihood, MFV
$\rightarrow \text{BR}_{B \rightarrow \tau\nu}^{\text{EXP/SM}}$	[50, 52]	$1.02 \pm 0.19_{\text{EXP}} \pm 0.13_{\text{SM}}$
$\rightarrow \text{BR}_{B \rightarrow X_s \ell\ell}^{\text{EXP/SM}}$	[53] / [50]	$0.99 \pm 0.29_{\text{EXP}} \pm 0.06_{\text{SM}}$
$\rightarrow \text{BR}_{K \rightarrow \mu\nu}^{\text{EXP/SM}}$	[54, 55] / [40]	$0.9998 \pm 0.0017_{\text{EXP}} \pm 0.0090_{\text{TH}}$
$\rightarrow \text{BR}_{K \rightarrow \pi\nu\nu}^{\text{EXP/SM}}$	[56] / [57]	$2.2 \pm 1.39_{\text{EXP}} \pm 0.20_{\text{TH}}$
$\rightarrow \Delta M_{B_s}^{\text{EXP/SM}}$	[54, 58] / [50]	$1.016 \pm 0.074_{\text{SM}}$
$\rightarrow \frac{\Delta M_{B_s}^{\text{EXP/SM}}}{\Delta M_{B_d}^{\text{EXP/SM}}}$	[54, 58] / [50]	$0.84 \pm 0.12_{\text{SM}}$
$\rightarrow \Delta \epsilon_K^{\text{EXP/SM}}$	[54, 58] / [40]	$1.14 \pm 0.10_{\text{EXP+TH}}$
$\rightarrow \Omega_{\text{CDM}} h^2$	[59, 60] / [28]	$0.1186 \pm 0.0020_{\text{EXP}} \pm 0.0024_{\text{TH}}$
$\rightarrow \sigma_p^{\text{SI}}$	[31, 32]	$(m_{\tilde{\chi}_1^0}, \sigma_p^{\text{SI}})$ plane
\rightarrow Heavy stable charged particles	[61]	Fast simulation based on [61, 62]
$\rightarrow \tilde{q} \rightarrow q \tilde{\chi}_1^0, \tilde{g} \rightarrow f \bar{f} \tilde{\chi}_1^0$	[5]	$\sigma \cdot \text{BR}$ limits in the $(m_{\tilde{q}}, m_{\tilde{\chi}_1^0}), (m_{\tilde{g}}, m_{\tilde{\chi}_1^0})$ planes
$\rightarrow H/A \rightarrow \tau^+ \tau^-$	[63–65]	2D likelihood, $\sigma \cdot \text{BR}$ limit

Fine tuning from a likelihood fit:

SUSY parameters If v included as a “Nuisance” variable

$$L(\text{data} \mid \gamma_i) \propto \int dv \delta(m_z - m_z^0) \delta\left(v - \left(-\frac{m^2}{\lambda}\right)^{1/2}\right) L(\text{data} \mid \gamma_i; v)$$
$$= \frac{1}{\Delta_q} \delta\left(n_q (\ln \gamma_i - \ln \gamma_i^S)\right) L(\text{data} \mid \gamma_i; v_0)$$

Fine tuning measure $\Delta_q = \left(\sum \Delta_{\gamma_i}^2\right)^{1/2}$

Ghilencea, GGR

Probabilistic interpretation:

$$\chi^2_{new} = \chi^2_{old} + 2 \ln \Delta_q$$

$$\Delta_q < 100, \quad \delta\left(\frac{\chi^2}{d.f.}\right) < 1$$

(should be averaged over #df)

Higgsino mass origin

- Sequestering

In strongly coupled near conformal sector hidden sector running can drive Higgs mass to zero leaving Higgsino mass unchanged

Luty Sundum

Dine et al

Murayama, Nomura, Poland

Perez, Roy, Schmaltz

$$\Rightarrow \mu' H_u H_d |_{\theta\theta} - \mu'^2 \left(|H_u|^2 + |H_d|^2 \right) \quad \text{“Soft” even with singlets}$$


SUSY SU(5) fit LHC 13 + DM

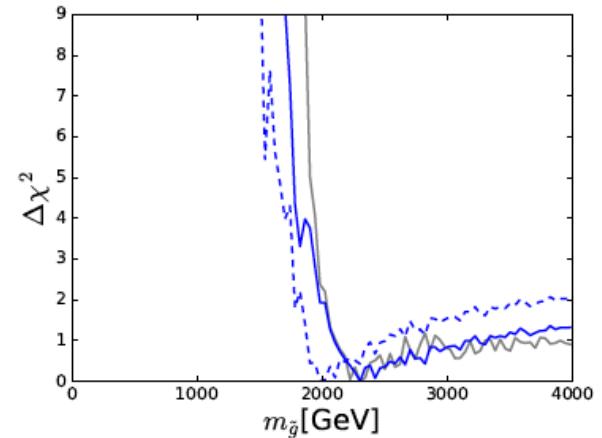
Parameters

$m_{1/2}$	m_5	m_{10}	m_{H_u}	m_{H_d}	A_0	$\tan \beta$
1050 (890)	-220 (-80)	380 (310)	-5210 (-4080)	-4870 (-4420)	-5680 (5020)	12 (11)

+LHC13

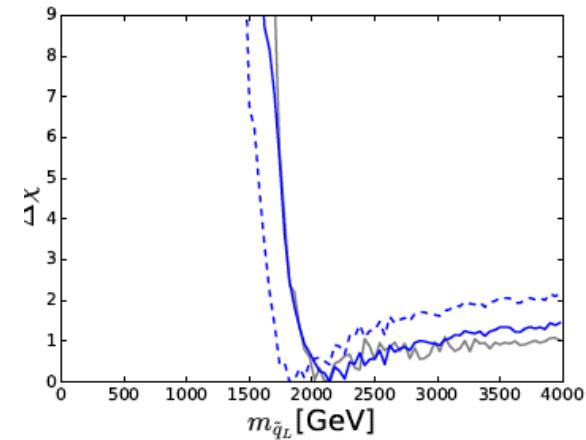
Best fit masses

$\tilde{\tau}_1$	$\tilde{\tau}_2$	\tilde{e}_L	\tilde{e}_R	$\tilde{\nu}_\tau$	\tilde{q}_L	\tilde{t}_1	\tilde{t}_2
470	660	630	678	570	2130	1840	2180
\tilde{b}_1	\tilde{b}_2	\tilde{u}_R	\tilde{d}_R	\tilde{g}	$M_{H,A}$	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0, \tilde{\chi}_1^\pm}$
1940	2090	2000	1980	2310	1620	460	860



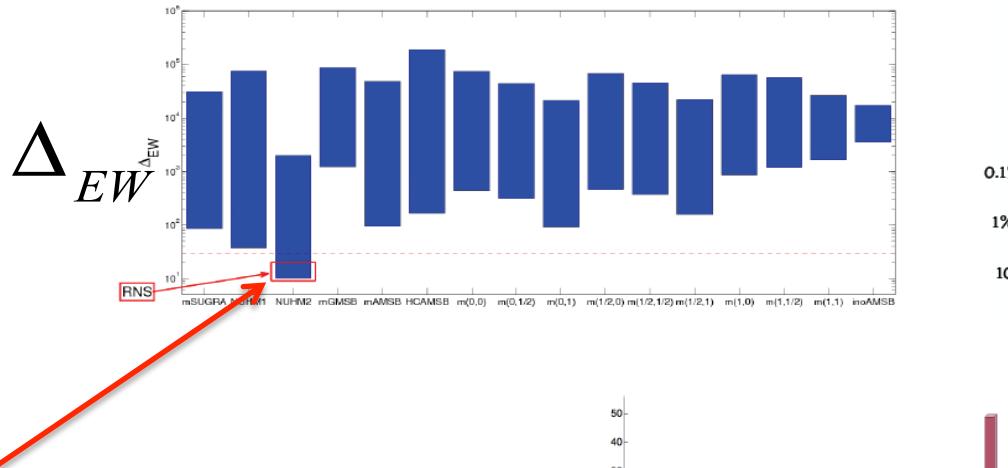
Contributions to χ^2

A_{LR}^e	A_b	$A_{FB}(\ell)$	$A_{FB}(b)$	$A_{FB}(c)$	$A_l(P_\tau)$
3.40	0.35	0.78	6.79	0.82	0.08
R_b	$\text{BR}(b \rightarrow s\gamma)$	$\text{BR}(B_u \rightarrow \tau\nu_\tau)$	$\Omega_{\tilde{\chi}_1^0} h^2$	σ_p^{SI}	$\text{BR}(B_{s,d} \rightarrow \mu^+\mu^-)$
0.26	0.00	0.18	0.00	0.00	2.09
$\sin^2 \theta_{\text{eff}}$	M_W	R_l	$R(K \rightarrow l\nu)$	$(g-2)_\mu$	M_h
0.60	0.07	1.04	0.0	8.28	0.01



$$\left. \frac{\chi^2}{d.f.} \right|_{\mathcal{H}} = \frac{32.4}{23}, \quad \chi^2 \text{ Probability } 9\% \text{ (37%)}$$

Model scan:



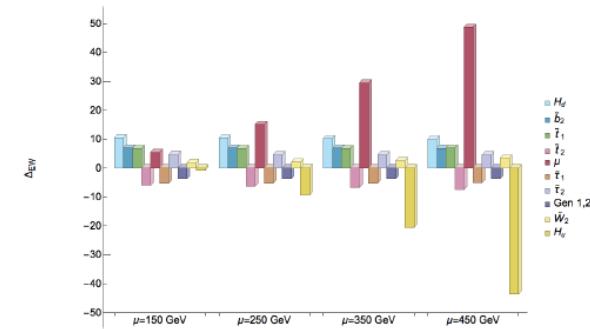
Radiative breaking natural SUSY (RNS)

$NUHM2 : m_0(1,2), m_0(3), m_{1/2}, A_0, \tan\beta, \mu, m_A$

$$m_{\tilde{g}} \leq 5 TeV$$

$$m_{\tilde{t}_1} \leq 3 TeV$$

$$m_{\tilde{W}_1, \tilde{Z}_{1,2}} \leq 300 GeV (500 GeV?) \Rightarrow \cancel{\text{SUSY DM}}$$



Baer et al 1702.06588

Tests

$HL-LHC 3000 fb^{-1} : m_{1/2} \sim 1.2 TeV \equiv \Delta_{EW} < 30$

$$\mu \sim 250 GeV$$

$e^+ e^- \sqrt{s} = 0.5 - 0.7 TeV :$

$pp \rightarrow \tilde{W}_2^\pm \tilde{Z}_4 \rightarrow \text{same sign dibosons}$

$pp \rightarrow \tilde{Z}_1 \tilde{Z}_2 + \text{jet} \rightarrow \tilde{Z}_1 \tilde{Z}_1 l^+ l^- + \text{jet}$

Higgsino pair production