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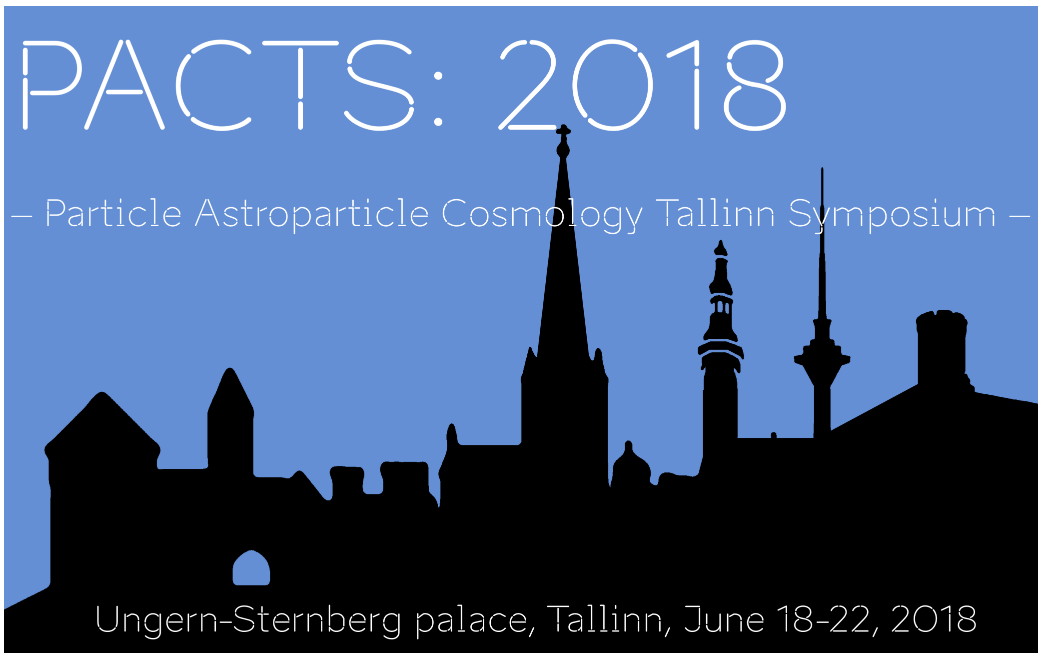
Conformal symmetry: towards the link between the Fermi and the Planck scales

Mikhail Shaposhnikov

based of works with Andrey Shkerin



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- No significant deviations from the SM have been observed
- With experimental values of the masses of the top quark and of the Higgs boson the SM is a self-consistent effective field theory all the way up to the quantum gravity Planck scale M_P.

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Spell Syllables

Synonyms Examples Word Origin

See more synonyms on Thesaurus.com

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This is unfair to “unnatural” SM as it describes the Nature better than “natural” theories...

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right above EW scale

The original source of the naturalness requirement: hierarchy problem in Grand Unified theories

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Gauge-symmetry hierarchies*

Eldad Gildener
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 15 June 1976)

It is shown that one cannot artifically establish a gauge hierarchy of any desired magnitude by arbitrarily adjusting the scalar-field parameters in the Lagrangian and using the tree approximation to the potential; radiative corrections will set an upper bound on such a hierarchy. If the gauge coupling constant is approximately equal to the electromagnetic coupling constant, the upper bound on the ratio of vector-meson masses is of the order of $\alpha^{-1/2}$, independent of the sclar-field masses and their self-couplings. In particular, the usual assumption that large scalar-field mass ratios in the Lagrangian can induce large vector-meson mass ratios is false. A thus far unsuccessful search for natural gauge hierarchies is briefly discussed. It is shown that if such a hierarchy occurred, it would have an upper bound of the order of $\alpha^{-1/2}$.

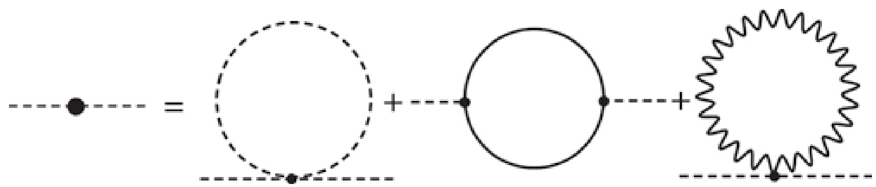
Extra GUT particles beyond the SM – leptoquarks (vector and scalar) must be very heavy, $M_X > 10^{15}$ GeV

- this is required by the gauge coupling unification
- this is needed for stability of matter, proton lifetime $\tau_p > 10^{34}$ years

Hierarchy: $(\frac{M_X}{M_W})^2 \simeq 10^{28}$

Two faces of hierarchy

- Ad hoc tuning between the parameters (masses and couplings of different multiplets) at the tree level with an accuracy of 26 orders of magnitude
- Stability of the Higgs mass against radiative corrections Gildener, '76



$$\delta m_H^2 \simeq \alpha_{GUT}^n M_X^2$$

Tuning is needed up to 14th order of perturbation theory!

Proposed solutions

Stability of EW scale – requirement of “naturalness”: absence of quadratic divergencies in the Higgs mass

- Low energy SUSY: compensation of bosonic loops by fermionic loops
- Composite Higgs boson - new strong interactions
- Large extra dimensions

All require new physics right above the Fermi scale, which was expected to show up at the LHC

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An attempt towards this direction: M.S., Shkerin, arXiv:1803.08907 + arXiv:1804.06376

Higgs-Planck hierarchy: the ratio of the two scales is exponentially small,

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Requirements to the theory

- Perturbatively $m_H = 0$, symmetry protection (?)
- Existence of (semi) classical configurations leading to $\langle H \rangle \neq 0, \ \langle H \rangle \ll M_P$

Conformal SM

If the mass of the Higgs boson is put to zero in the SM, the classical Lagrangian has a wider symmetry: it is scale and conformally invariant:

Dilatations - global scale transformations ($\sigma = const$)

$$\Psi(x) \;\to\; \sigma^n \Psi(\sigma x) \; ,$$

$n = 1$ for scalars and vectors and $n = 3/2$ for fermions.

Conformal anomaly

Lagrangian is invariant at the classical level, and scale symmetry is broken by quantum corrections (conformal anomaly) a'la Coleman-Weinberg: Linde '76; Weinberg '76; Buchmuller, Dragon '88; Hempfling '96; Meissner, Nicolai '06; Foot et al '07, '11; Iso, et al '09; Boyle et al '11; Wetterich '11, Salvio, Strumia '14; Lindner et al, '14, '15, '17

Does not work for the SM:

- If the top quark mass $m_t \lesssim 172$ GeV, then the minimum of the effective potential is generated at $\langle H \rangle \simeq 100$ MeV due to chiral symmetry breaking in QCD
- If the top quark mass $m_t \gtrsim 172$ GeV, then an extra minimum of the effective potential is generated at $\langle H \rangle \gtrsim M_P$ due to top quark loops

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Gravity + conformally invariant SM is an ideal playground for looking at non-perturbative generation of the weak scale

The action

To search for non-perturbative effects, consider only the scalar sector of the SM (unitary gauge, one scalar degree of freedom, $|H| = h$), other SM fields are "spectators":

$$\frac{\mathcal{L}_{h,g}}{\sqrt{-g}} = G_4(h)R + G_2(h, \partial h^2) \ ,$$

where the functions G_4, G_2 are chosen so that to reproduce the SM Higgs kinetic term, the Higgs field potential with $m_H = 0$, and GR in the low energy limit. Part of Horndeski action, leading to second order equations of motion $\Longrightarrow$ no new degrees of freedom are introduced, we have just massless Higgs and graviton. We will also require asymptotic scale invariance at large h.

What to compute

The Higgs vev:

$$\langle h \rangle \sim \int \mathcal{D}\mathcal{A}\mathcal{D}h\mathcal{D}g_{\mu\nu} h e^{-S_E} \ .$$

$\mathcal{A}$ – collection of fields of the model under consideration, other than h and $g_{\mu\nu}$, and S_E is the euclidean action of the model.

Remarks:

- Euclidean path integral for gravity may not be well defined due to the problem with the conformal factor of the metric
- We will ignore this problem and follow the crowd: Hawking; Coleman, de Luccia; Veneziano; ..., Isidori, Rychkov, Strumia, Tetradis; ... Branchina, Messina, Sher;...

Known classical solutions

- Bounce: an indication of the vacuum instability Coleman, de Luccia
- Hawking-Moss instanton: dominates transitions between vacua at high temperature
- Gravitational instantons (Taub-NUT, Eguchi-Hanson, Gibbons-Hawking): pure gravity, no relation to scalar field
- Giddings-Strominger instanton: gravity + axion field: wormholes
- Hawking-Turok instanton: creation of Universe from nothing ?

None works for our purpose!

New solutions

Choice of G_4: Higgs field in general must have non-minimal coupling to gravity:

$$S_G = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R - \frac{\xi h^2}{2}R\right\}$$

Conjecture: contribution of large Higgs fields $h > M_P/\sqrt{\xi}$ to path integral is better to be found in the Einstein frame. Conformal transformation:

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}\ , \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

Redefinition of the Higgs field to make canonical kinetic term

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi_h^2 h^2/M_P^2}{\Omega^4}} \Longrightarrow \begin{cases} h \simeq \chi & \text{for } h < M_P/\xi \\ h \simeq \frac{M_P}{\sqrt{\xi}} \exp\left(\frac{\chi}{\sqrt{6}M_P}\right) & \text{for } h > M_P/\sqrt{\xi} \end{cases}$$

Resulting kinetic part of the action

$$S_{kin} = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} \right\}$$

Most important:

$$\langle h(x) \rangle \sim \int \mathcal{D}\mathcal{A}\mathcal{D}h(x)\mathcal{D}g_{\mu\nu} h(x) e^{-S_E} \Longrightarrow \int \mathcal{D}\mathcal{A}\mathcal{D}\chi\mathcal{D}\hat{g}_{\mu\nu} e^{\frac{\chi(x)}{\sqrt{6}M_P} - S_E}$$

Modification of the action and equations of motion!

Equations of motion for χ contain a source term $\delta(x)$ $\Longrightarrow$ new classical solutions. Similar to computation of $\langle \exp(\int A_\mu dv^\mu)\rangle$ in Polyakov '76. Schematically, modification of the right-hand side for scalar field equation: $\Box\chi + ... = \delta(x)/\sqrt{6}M_P$

Path integral:

$$\int_{h \gtrsim M_P/\sqrt{\xi}} \mathcal{D}h h e^{-S_E} \to M_P \int_{\chi \gtrsim M_P \log(1/\sqrt{\xi})} \mathcal{D}\chi J e^{-W} \,,$$

where $W = \chi(0)/\sqrt{6}M_P + S_E$ and J is the corresponding Jacobian. Conjecture: Higgs vev

$$\langle h\rangle \approx M_P e^{-\bar{W}} \,,$$

is much smaller than M_P because the action $\bar{W}$ on the saddle point

$$\bar{W} \gg 1 \,.$$

Example of computation

Field theory in the Jordan frame:

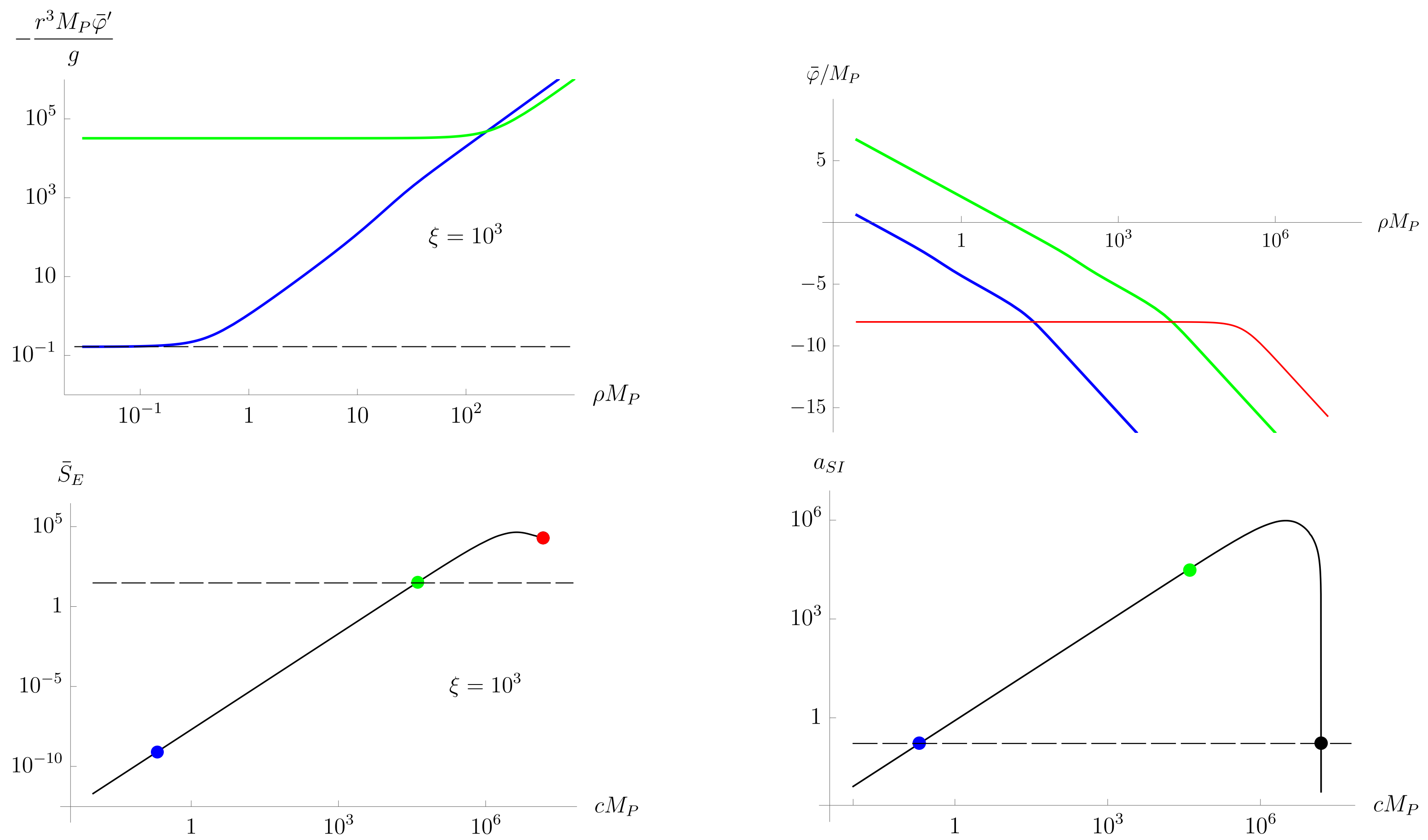
$$\frac{\mathcal{L}_{h,g}}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \xi h^2)R + \frac{1}{2}(\partial h)^2 + V(h)$$

Field equations in the Einstein frame for maximally $O(4)$ symmetric metric $d\tilde{s}^2 = g^2(\rho)d\rho^2 + \rho^2 d\Omega_3^2$ in the large χ regime:

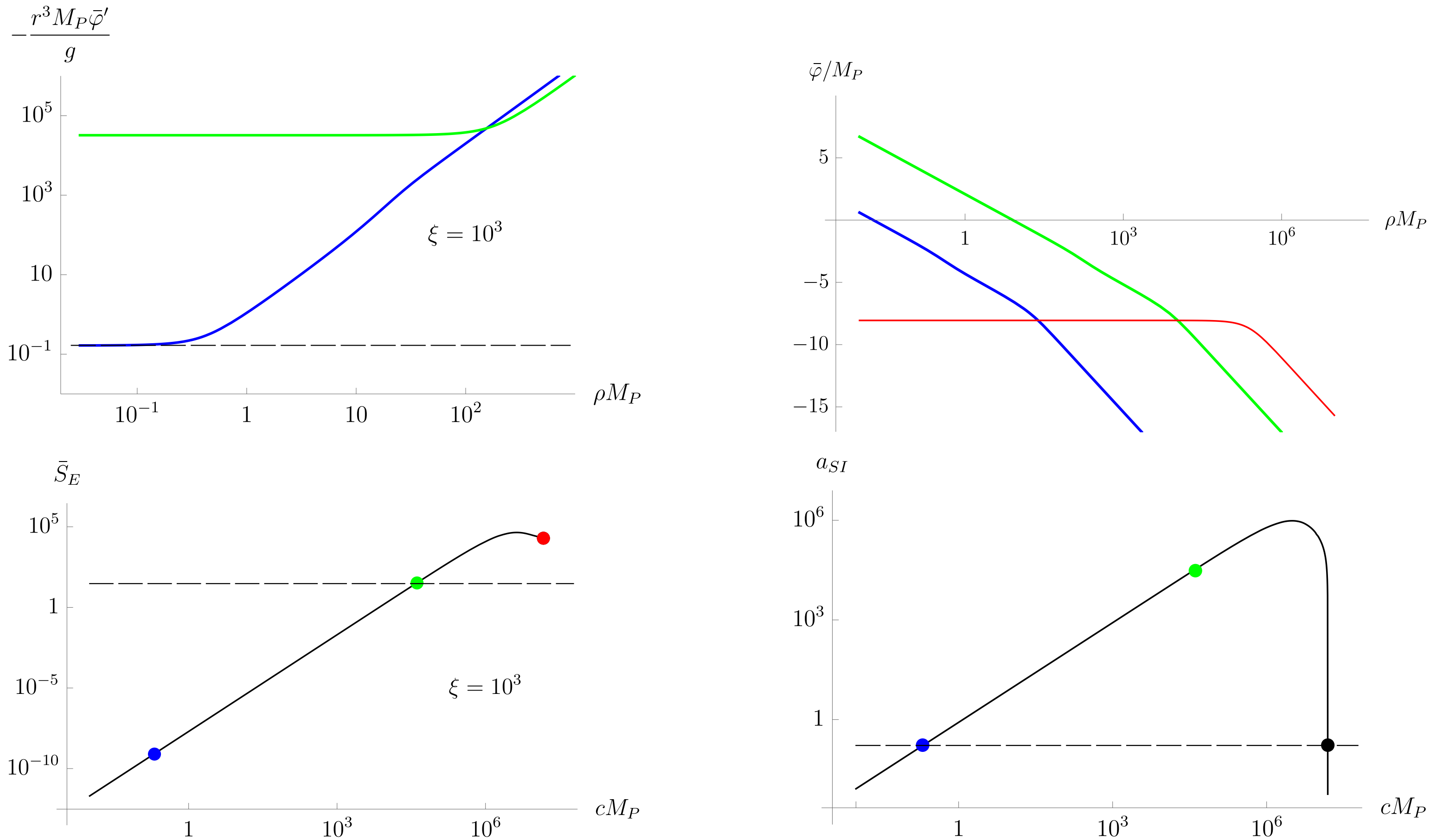
$$\partial_\rho \left(\frac{\rho^3 \bar{\chi}'}{g a_{SI}}\right) = -\frac{1}{M_P}\delta(\rho)\ , \quad g^2 = 1 - \frac{\rho^2 \bar{\chi}'^2}{6 a_{SI} M_P^2}\ , \quad a_{SI} = \frac{1}{1/\xi + 6}$$

Vacuum boundary conditions at infinity $\rho \to \infty$:

$g^2(\rho) \to 1\ ,\quad h(\rho) \to 0\ .$ The asymptotic behaviour of the scalar field χ at $\rho \to 0$: $\chi = -M_P\sqrt{6a_{SI}}\log \rho M_P + c$ with c a constant used to match with the asymptotics at large ρ.



blue: configuration obeying the boundary condition imposed by the source. green is the one with the large euclidean action $\bar{W} = 40$. red: the bounce. c: fall-off at infinity, $\chi = c\rho^{-2}$, $\rho \to \infty$.



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The action is too small! Semiclassical approximation does not work, and $\langle H \rangle \sim M_P$ is expected.

The appearance of a scale small compared with M_P is not generic and requires a specific theory in UV, well along with our conjecture.

Modifications of the theory in the UV:

- Include higher dimensional operators, for example

$$\mathcal{O}_4 = \sqrt{g}\,\delta\frac{(\partial h)^4}{(M_P\Omega)^4}, \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

  Ω^2 is needed to keep the scale symmetry at large h. Leads to finite values of the Higgs field in the centre of the instanton!

- Action of the instanton depends on a_{SI}, $S \propto \sqrt{a_{SI}}$. The parameter a_{SI} is a combination of a non-minimal coupling to gravity and scalar kinetic term. They can be changed for large h to make the action large, without affecting low energy physics

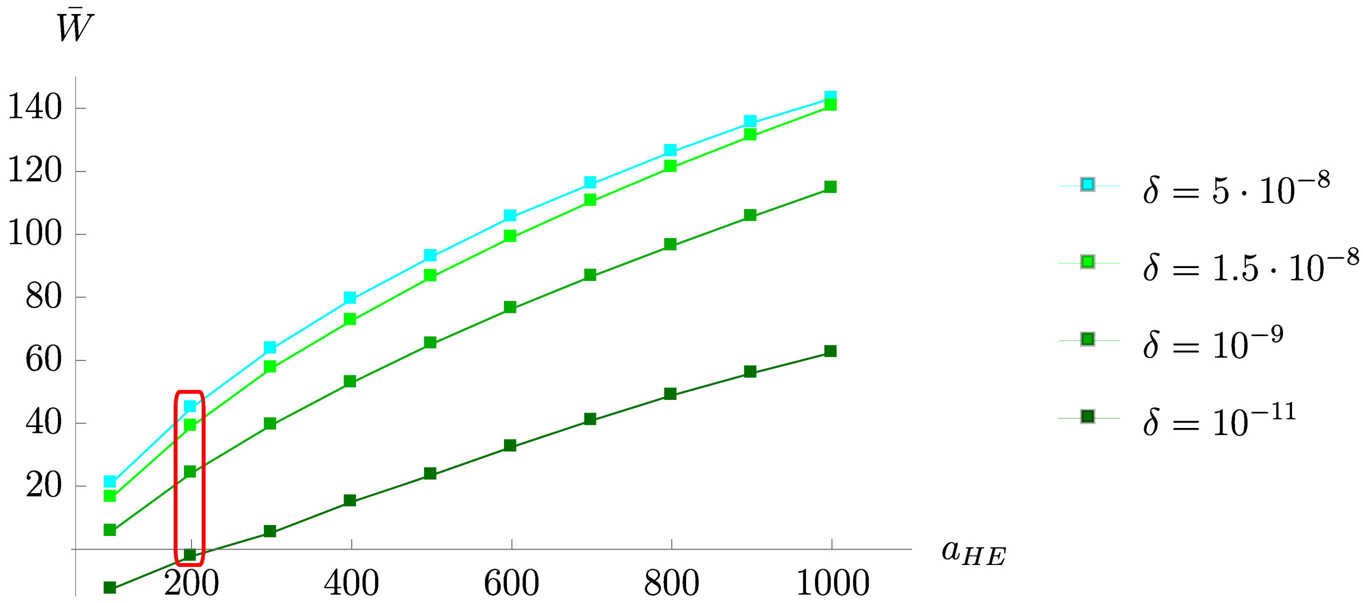
A theory that works

$$\frac{\mathcal{L}_{h,g}}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \xi h^2)R + \frac{1}{2}G(h/M_P)(\partial h)^2 + \delta\xi^2 \frac{(\partial h)^4}{(M_P \Omega)^4} + \frac{\lambda}{4}h^4 \ ,$$

where $G(0) = 1, \ \ G(\infty) = \kappa = const$. Then the asymptotic value of a_{SI} in the large-χ regime modifies to

$$a_{SI} \to a_{HE} = \frac{1}{\kappa/\xi + 6} \ , \quad \rho \to 0 \ , \quad h \gtrsim M_P \ ,$$

Must have $\kappa > -\frac{6}{\xi}$ (absence of ghosts). Taking $\kappa = -\frac{6}{\xi} + \epsilon, \ \ \epsilon > 0, \ \ \epsilon \ll 1$ can make the action large.



The instanton value of the functional W plotted against the coefficient a_{HE} and with the different choices of the parameter δ.

Intriguing fact: large a_{HE} and small δ correspond to an approximate Weyl symmetry.

Conclusions

- Very small m_H/M_P ratio is (perhaps) telling us that
  - There are no new particles with masses between the Fermi and Planck scale
  - The smallness of the Fermi scale is a semiclassical non-perturbative UV effect associated with gravity and new type of instantons
  - The asymptotic theory of the SM at large scalar fields is nearly Weyl invariant
- Open problems
  - Unfortunately, we can make no prediction of the ratio m_H/M_P, as this depends on details of UV theory.
  - We cannot estimate the contribution of the effects other than perturbative and semiclassical.

Remark

Everything can be generalised to a completely scale-invariant theory with spontaneous breaking of scale invariance.

Gravity part:

$$\mathcal{L}_G = -\left(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger \varphi\right) \frac{R}{2} \,,$$

The vev of extra field – dilaton – gives rise to the Planck scale.

For analysis and results see M.S., Shkerin, arXiv:1804.06376.