Spacetime and dark matter from spontaneous breaking of Lorentz symmetry

Tom Złośnik[1],* Federico Urban[1],† Luca Marzola[2],‡ and Tomi Koivisto[3§]
[1] *Institute of Physics of the Czech Academy of Sciences, Na Slovance 1999/2, 182 21, Prague*
[2] *KBFI, Akadeemia Tee 23, Tallinn, Estonia*
[3] *NORDITA, Roslagstullsbacken 23, 106 91 Stockholm, Sweden*

Nordita - the Nordic Institute for Theoretical Physics

PACTS 2018:
Particle, Astroparticle and Cosmology Tallinn Symposium

KBFI NICPB

Principles

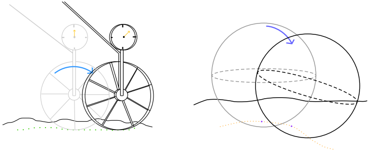
"...the Standard Model has been "ruled in" because it was based on **principles**, in particular the *gauge* principle and *renormalisability*..."

Prof. Hassan, @PACTS 2018, June 18, Tallinn

We'd like to build the theory of gravity on these principles. Thus we'll adopt *Cartan's gauge geometry* and will write down only *polynomial* actions.

Cartan geometry

- The idea: a distance can be measured by monitoring how much a wheel has rotated when rolled without slipping.



- The maths: generalise the tangent space $\mathbb{R}^n$ by G/H
  - Transitive action of the Lie group G
  - Stabiliser $H = \{g \in G | gx = x\}$ given $x \in G/H$
- Example: the $SO(1,4)/SO(1,3)$

Gravity with ~~more or~~ less variables

We'll gauge the $SO(3,1)$ and break it. We'll need

- the spin connection ω^{IJ}
- the symmetry-breaking field τ^I

The standard approach assumes also the vierbein e^I

In e.g. Poincaré gauge theory, MAG, TEGR, etc., the relation of the vierbein and the translational gauge potential θ is

$$\theta^I = e^I - D\tau^I \,.$$

The τ^I is known as e.g. "the generalised Higgs field", the "Poincaré coordinates", the "Cartan's generalised radius vector" etc but is (almost) always hidden in the gauge $D\tau^I = 0$ or $\tau^I = 0$.

The action

$$S_G[\omega,\tau] = \int \mathcal{L}_G = \int P_{IJKL} D\tau^I D\tau^J R^{KL}$$

- Products: with the $\wedge$ e.g. $D\tau^I D\tau^J \equiv D\tau^I \wedge D\tau^J = D_\mu \tau^I D_\nu \tau^J dx^\mu \wedge dx^\nu$
- The projector $P_{IJKL} \equiv \frac{1}{2}\left(\frac{1}{2}\epsilon_{IJKL} + \frac{1}{\gamma}\eta_{IK}\eta_{JL}\right)$
  - ϵ_{IJKL} is the completely antisymmetric $SO(3,1)$ invariant
  - η_{IJ} is the symmetric $SO(3,1)$ invariant
  - γ is the Barbero-Immirzi -parameter
- The curvature $R^{IJ} = \frac{1}{2} R^{IJ}{}_{\mu\nu} dx^\mu \wedge dx^\nu$
  - components: $R^I{}_{J\alpha\beta} \equiv 2\partial_{[\alpha|}\omega^I{}_{J|\beta]} + 2\omega^I{}_{K[\alpha|}\omega^K{}_{J|\beta]}$
- The gradient: $D\tau^I = D_\mu \tau^I dx^\mu$
  - components: $D_\mu \tau^I \equiv \partial_\mu \tau^I + \omega^I{}_{J\mu}\tau^J$

Equations of motion

The action

$$
\begin{aligned}
S_G[\omega,\tau] &= \int \mathcal{L}_G = P_{IJKL}\int D\tau^I D\tau^J R^{KL} \\
&= \frac{1}{2}\int\left(\frac{1}{2}\tau^2\epsilon_{IJKL} - \frac{1}{\gamma}\tau_I\tau_K\eta_{JL}\right)R^{IJ}R^{KL}
\end{aligned}
$$

The connection EoM

$$P_{IJLM}R^I{}_N\tau^N D\tau^J - \tau_{[L}P_{M]IJK}D\tau^I R^{JK} = 0$$

The scalar EoM

$$P_{IJKL}R^J{}_M\tau^M R^{KL} = 0$$

Summary of the theory

Variables: ω^{IJ} and τ^I

Now the "vierbein" is $D\tau^I$, and the "torsion" is $DD\tau^I = R^I{}_J\tau^J$.

Field equations from the action $S_G[\omega,\tau] = \int P_{IJKL} D\tau^I D\tau^J R^{KL}$

Now the δ_ω gives field equation, and the δ_τ a Bianchi identity.

Symmetric solution

We immediately obtain an exact solution: $\tau^I = 0$, $\Omega^{IJ} =$ anything.

Solutions with spacetime

When $D\tau^I \neq 0$, we'll obtain a nonzero $g_{\mu\nu} = D_\mu\tau^I D_\nu\tau_I$.

Gauge fixing

- **A partial gauge fixing**:

$$\bar{\mathcal{D}}_a\tau^0 = \bar{d}\tau^0 + \bar{\omega}^{0i}\tau_i \overset{*}{=} 0$$

where i, j, k are in $SO(3) \in SO(3,1)$.

- **Assume a time-like norm**:

$$-\tau^2 \equiv \tau^I\tau_I < 0$$

This breaks $SO(3,1)$ down to a residual $SO(3)$ invariance.

The spatial triad appears within the connection

$D\tau^0 = d\tau\,, \quad D\tau^i = \omega^i{}_0\tau^0 \equiv E^i\,.$

Connection

The pullback $\bar{\omega}^{IJ}$ of ω^{IJ} to surface Σ of constant τ:

$$\bar{\omega}^{IJ} \overset{*}{=} \begin{pmatrix} 0 & \frac{1}{\tau}E^i \\ -\frac{1}{\tau}E^i & \epsilon^{ijk}\left(\Gamma_k - \frac{1}{\gamma}\left(K_k - \frac{1}{\tau}E_k\right)\right) \end{pmatrix}$$

- $\epsilon^{ijk}\Gamma_k$ is the torsion-free E^i-compatible $SO(3)$-connection
- K_i is the extrinsic curvature form
- Set spatial coordinates x^a on Σ, define $h_{ab} = \delta_{ij}E^i_a E^j_b$:
  - $K_{ab} \equiv \frac{1}{2}\mathcal{L}_{(n^\mu)}h_{ab}$ where n^μ is the unit normal to Σ
  - Relation to the to the curvature form $K_i = K_{ab}E^b_i dx^a$
- The metric part of $\bar{\omega}^{0i}$ describes distances on Σ
- The metric part of $\bar{\omega}^{ij}$ related to the intrinsic curvature
- The torsionful part of $\bar{\omega}^{ij}$ related to the extrinsic curvature

GR recovered only when $\gamma^2 = -1$

With a general value of γ, the equations of motion for the spatial metric h_{ab} are produced from the following Lagrangian

$$L = \sqrt{h}\left(R^{(3)} + \frac{1}{\gamma^2}\left(K^2 - K^{ab}K_{ab} \right) \right)$$

- This is valid in the frame $t = \tau$
- $R^{(3)}$ is the Ricci scalar corresponding to h_{ab}
- K_{ab} is the extrinsic curvature

Iff $\gamma^2 = -1$ we recover the ADM Lagrangian with unit lapse.

Different from LQG

where the Barbero-Immirzi parameter is not fixed.

The self-dual curvature

- Any SO(3,1)-form can be decomposed as $f^{IJ} = f^{+IJ} + f^{-IJ}$
  - $f^{+IJ} = \frac{1}{2}(f^{IJ} - \frac{i}{2}\epsilon^{IJ}{}_{KL} f^{KL})$
  - $f^{-IJ} = \frac{1}{2}(f^{IJ} + \frac{i}{2}\epsilon^{IJ}{}_{KL} f^{KL})$
- These are the *self-dual* and *anti-self-dual* pieces
  - $\frac{1}{2}\epsilon^{IJ}{}_{KL} f^{+KL} = i f^{+IJ}$
  - $\frac{1}{2}\epsilon^{IJ}{}_{KL} f^{-KL} = -i f^{-IJ}$
- When $\gamma = \pm i$, the P_{IJKL} is the projector
  - $\gamma = i$: $P^{IJ}{}_{KL} f^{KL} = \frac{1}{2}\epsilon^{IJ}{}_{KL} f^{+KL}$
  - $\gamma = -i$: $P^{IJ}{}_{KL} f^{KL} = \frac{1}{2}\epsilon^{IJ}{}_{KL} f^{-KL}$

If $\gamma = i$, the action involves the *self-dual curvature*

$$S_G[\omega, \tau] = P_{IJKL} \int D\tau^I D\tau^J R^{KL} = \frac{1}{2}\epsilon_{IJKL} \int D\tau^I D\tau^J R^{+KL}$$

Self-dual connection

The pullback $\bar{\omega}^{IJ}$ of ω^{IJ} to surface Σ of constant τ:

$$\bar{\omega}^{IJ} \overset{*}{=} \begin{pmatrix} 0 & \frac{1}{\tau} E^i \\ -\frac{1}{\tau} E^i & \epsilon^{ijk}\left(\Gamma_k - i\left(K_k - \frac{1}{\tau} E_k\right)\right) \end{pmatrix}$$

$$\omega^{\pm IJ} = \frac{1}{2} \begin{pmatrix} 0 & \pm i\left(\Gamma^i - iK^i\right) + (1 \mp 1)\frac{E^i}{\tau} \\ \mp i\left(\Gamma^i - iK^i\right) - (1 \mp 1)\frac{E^i}{\tau} & \epsilon^{ijk}\left(\Gamma_k - iK_k\right) + i(1 \mp 1)\epsilon^{ijk}\frac{E_k}{\tau} \end{pmatrix}$$

Compare with the ECSK theory:

$$\bar{\omega}^{IJ}_{(ECSK)} \overset{*}{=} \begin{pmatrix} 0 & K^i \\ -K^i & \epsilon^{ijk}\Gamma_k \end{pmatrix}, \quad \bar{e}^I_{(ECSK)} \overset{*}{=} \begin{pmatrix} 0 \\ E^i \end{pmatrix} \tag{1}$$

$$\omega^{+IJ} = \frac{1}{2} \begin{pmatrix} 0 & i\left(\Gamma^i - iK^i\right) \\ -i\left(\Gamma^i - iK^i\right) & \epsilon^{ijk}\left(\Gamma_k - iK_k\right) \end{pmatrix} = \omega^{+IJ}_{(ECSK)} \tag{2}$$

1+3

- Assume topology $R \times \Sigma$ for some submanifold Σ
- Σ corresponds $t = \mathrm{cst.}$ where $t(x^\mu)$ is a global time function
  - Introduce spatial coordinates x^a covering Σ
  - Introduce $\bar{d}$ the exterior derivative according to the x^a
- Decompose the fields and their exterior derivatives as:

$$\begin{aligned}
\omega^{IJ} &= \omega^{IJ} dt + \bar{\omega}^{IJ} \\
d\tau^I &= \partial_t \tau^I dt + \bar{d}\tau^I \\
d\omega^{IJ} &= \partial_t \bar{\omega}^{IJ}{}_a dt dx^a + \bar{d}\omega^{IJ} dt + \bar{d}\bar{\omega}^{IJ}
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_G \stackrel{b}{=} & P_{IJKL} dt \left(2\bar{\mathcal{D}}\tau^J \bar{R}^{KL} \partial_t \tau^I + \bar{\mathcal{D}}\tau^I \bar{\mathcal{D}}\tau^J \partial_t \bar{\omega}^{KL}{}_a dx^a \right) \\
& + P_{IJKL} dt \left(2\Omega^{IM} \tau_M \bar{\mathcal{D}}\tau^J \bar{R}^{KL} + \Omega^{KL} \bar{\mathcal{D}} \left(\bar{\mathcal{D}}\tau^I \bar{\mathcal{D}}\tau^J \right) \right)
\end{aligned}$$

The 1+3 Lagrangian

- Introduce the momenta for τ^0 and τ^i:

$$\pi_0 \equiv 2P_{0jKL}E^j\bar{R}^{KL}$$
$$\pi_i \equiv 2P_{ijKL}E^j\bar{R}^{KL}$$

- Force them with multipliers, vary wrt them and obtain:

$$\pi_0\tau_i - \pi_i\tau_0 = 0$$
$$\pi_{[i}\tau_{j]} = 0$$

- Use these and obtain:

$$\begin{aligned}\mathcal{L}_G \overset{b}{=} & \frac{1}{2}dt\epsilon_{ijKL}E^iE^j\partial_t\bar{\omega}^{+KL}_{\ a}dx^a + \pi_0\frac{\tau}{\tau_0}d\tau \\ & + dt\Omega^{0i+}\epsilon_{0ijk}\bar{\mathcal{D}}^+\left(E^jE^k\right) + P_i\left(E^i - \bar{\mathcal{D}}\tau^i\right) \\ & + \lambda(\pi_0 - 2P_{0jKL}E^j\bar{R}^{KL}) + \lambda^i(\pi_i - 2P_{ijKL}E^j\bar{R}^{KL})\end{aligned}$$

The Hamiltonian

Renaming things:

- Introduce $\mathcal{P}$ via $\pi_0 = (\tau_0/\tau)\mathcal{P}$
- Introduce N via $\lambda = Ndt$
- Introduce N^i via $\lambda^i = N^i dt$
- Use τ instead of $\tau^0 = \sqrt{\tau^2 + \tau^i \tau_i}$

The result becomes:

$$S_G \stackrel{b}{=} \int dt \left(\epsilon_{ijk} E^i E^j \partial_t \bar{\omega}^{+0k}_{\ a} dx^a + \mathcal{P} \partial_t \tau - \mathcal{H} \right)$$

where the Hamiltonian three-form $\mathcal{H}$ is:

$$\begin{aligned} \mathcal{H} = {} & \Omega^{i0+} \epsilon_{ijk} \bar{\mathcal{D}}^+ \left(E^j E^k \right) \\ & + N(\mathcal{P}\sqrt{1 + \partial^a \tau \partial_a \tau} + \epsilon_{ijk} E^i \bar{R}^{+jk}) \\ & + N^i (\mathcal{P} \partial_i \tau + 2\epsilon_{ijk} E^j \bar{R}^{+0k}) \, . \end{aligned}$$

Covariant Ashtekar's self-dual gravity coupled to rotationless dust.

Summary of the analysis

The symmetry-broken phase

The spatial triad appears in the connection: $D\tau^0 = d\tau$, $D\tau^i = E^i$.

The action is completely fixed: $S_G[\omega, \tau] = i \int \epsilon_{IJKL} D\tau^I D\tau^J R^{+KL}$

We need to consider a complex Lorentz group $SO(3, 1, \mathbb{C})$.

The result:

Covariant Ashtekar's self-dual gravity coupled to rotationless dust.

The matter coupling:

Chiral $S_S[\omega, \tau] = i \int \epsilon_{IJKL} D\tau^I D\tau^J D\tau^K \Psi^*_{A'} \sigma^{LA'A} D^+ \Psi_A$ is consistent.

Dark matter

The solutions to the EoMs from $S_G[\omega, \tau]$ include those from

$$S'_G[g,\tau,\rho] = \frac{1}{2}\int \sqrt{-g}d^4x\Big(R - \rho(\partial^\mu\tau\partial_\mu\tau + 1)\Big)$$

So does

- "Mimetic dark matter" (the minimal version)
- "Projectable Horava-Lifshitz gravity" (without a kinetic term)

Caustics?

There is no necessity that τ forms a global time coordinate. It is sufficient that the theory leads to field equations that can be evolved in physical situations of interest. The ability of τ to "tilt" from timelike to null values may be of importance in terms of its ability to evolve past "caustic" situations.

Singularities

A problem with mimetics:

With the shift symmetry $\tau \to \tau + \mathrm{cst.}$, inflation dilutes the ρ_{DM}.

A possible solution: break the symmetry

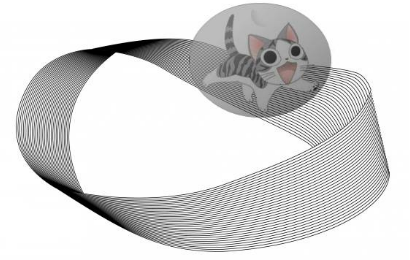
We could add kinetic terms to τ, or non-minimal matter couplings.

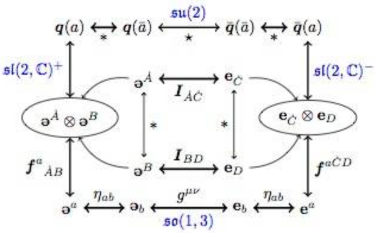
A more interesting possibility: reconsider the Big Bang & inflation

Recall the trivial solution $\tau^I = 0$, ω^{IJ} =anything. Can we (more/less smoothly) join this with the expanding dust solution?

Extensions

- Enlarging the gauge group
  - Double relativism $SO(3,1) \to SO(4,1)$: invariance of both c and M_{Planck}
  - Conformalism $SO(4,1) \to SO(4,2)$: rid of absolute scales
  - et cetera $SO(4,2) \to SO(N, 14-N)$: towards GraviGUTs ála Percacci
- Changing the gauge group
  - Unitary geometry: $SO(4) \simeq SU(2) \otimes SU(2)$
  - Hermitian geometry: $SO(3,1) \simeq SL(2,\mathbb{C})$
  - Spinorial Khronon: $\tau^I = \bar{\Psi}\gamma^I\Psi$





Modifying gravity: a universal recipe

- An example: vector

- Take e^a and a vector V^a
- Write down the polynomials (many are redundant)
- Obtain: the Horndeski when $V_\mu = \partial_\mu \phi$
- If $e^a = D\tau^a$ we'll get "mimetic Horndeski"?

$$
\begin{aligned}
L = {} & a_{abcd}R^{ab}R^{cd} + b_{abc}R^{ab}T^c + c_{ab}T^aT^b \\
& + d_{abcd}R^{ab}e^ce^d + d^v_{abcd}R^{ab}DV^cDV^d \\
& + e_{abc}T^ae^be^c + e^v_{abc}T^aDV^bDV^c \\
& + d'_{abcd}R^{ab}e^cDV^d + e'_{abc}T^ae^bDV^c \\
& + f_{abcd}e^ae^be^ce^d + f'_{abcd}e^ae^be^cDV^d \\
& + f''_{abcd}e^ae^bDV^cDV^d + f'''_{abcd}e^aDV^bDV^cDV^d \\
& + f^v_{abcd}DV^aDV^bDV^cDV^d \,,
\end{aligned} \tag{31}
$$

- Example with 2-form $\Rightarrow$ the "generalised Proca" terms
- Example with two tetrads: bigravity (next talk)
  - The potential interactions: $\epsilon_{abcd}e^ae^be^cf^d$, $\epsilon_{abcd}e^ae^bf^cf^d$, $\epsilon_{abcd}e^af^bf^cf^d$
  - New kinetic interactions? Mimetic bigravity? Chiral bigravity?

The universal recipe

to obtain *all* the consistent actions with *any* field content: covariant, generalised (with torsion) and *complete*. [ArXiv:1807.xxxx].

Summary: "the Cartan Khronon"

- Principle: a 1) minimal 2) polynomial 3) gauge theory

The theory $S_G[\omega,\tau] = i\int \epsilon_{IJKL} D\tau^I D\tau^J R^{+KL}$ is chiral GR + dust

Spacetime and dark matter from spontaneous breaking of Lorentz symmetry

Tom Złośnik[1,*] Federico Urban[1,†] Luca Marzola[2,‡] and Tomi Koivisto[3§]
[1] *Institute of Physics of the Czech Academy of Sciences, Na Slovance 1999/2, 182 21, Prague*
[2] *KBFI, Akadeemia Tee 23, Tallinn, Estonia*
[3] *NORDITA, Roslagstullsbacken 23, 106 91 Stockholm, Sweden*

- Next
  - Resolution of singularities?
  - Complete model of the universe?
- Then
  - Unification?
  - Quantisation?