Recent developments in bimetric & multimetric theories of gravity

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Outline of the talk

Introduction

Motivation & challenges

Bimetric theory: gravity with an extra spin-2 field

Uniqueness and the local structure of spacetime

Ghost-free multi spin-2 theories

Discussion

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Bimetric & multimetric theories:

Gravity ($g_{\mu\nu}$) coupled to extra spin-2 fields ($f_{\mu\nu}, \cdots$).

Spectrum:

A massless spin-2 state + massive spin-2 states

Why interesting? What are they?

(Only formal, classical aspects discussed here)

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Relevance for today's talks: Spin-2 dark matter

Babichev, Marzola, Raidal, Schmidt-May, Urban, Veerme, von Strauss [arXiv:1604.08564, 1607.03497]
Marzola, Raidal, Urban [arXiv:1708.04253]

Aoki, Mukohyama [arXiv:1604.06704]
Aoki, Maeda [arXiv:1707.05003]

Chu, Garcia-Cely [arXiv:1708.06764]

Gonzlez Albornoz, Schmidt-May, von Strauss [arXiv:1709.05128]

Outline of the talk

Motivation:

Recall: Spin (s) $\Rightarrow$ basic structure of field equations

Klein-Gordon (s=0) $\sim$ *Maxwell (s=1)* $\sim$ *Einstein-Hilbert (s=2)*

Standard Model: multiplets of $s = 0, \frac{1}{2}, 1$ fields.

Additional structures and mixings crucial for viability (e.g. QED)

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General relativity: the simplest $s = 2$ theory.

No additional structure!

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No additional structure!

Unexplored theories: gravity with more spin-2 fields

Can such consistent theories exist? Or is GR unique?
Consequences?

Challenge: the ghost problem

Ghost: A field with negative kinetic energy

Example:

$$\mathcal{L} = T - V = (\partial_t \phi)^2 \cdots \qquad (\textit{healthy})$$

But

$$\mathcal{L} = T - V = -(\partial_t \phi)^2 \cdots \qquad (\textit{ghostly})$$

Consequences:

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Consequences:

- Instability: unlimited energy transfer from ghost to other fields
- Negative probabilities, violation of unitarity in quantum theory

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GR with a generic spin-2 field

A dynamical theory for the metric $g_{\mu\nu}$ & spin-2 field $f_{\mu\nu}$

$$\mathcal{L} = m_p^2 \sqrt{|g|} R - m^4 \sqrt{|g|}\, V(g^{-1} f) + \mathcal{L}(f, \nabla f)$$

GR with a generic spin-2 field

A dynamical theory for the metric $g_{\mu\nu}$ & spin-2 field $f_{\mu\nu}$

$$\mathcal{L} = m_p^2 \sqrt{|g|} R - m^4 \sqrt{|g|}\, V(g^{-1} f) + \mathcal{L}(f, \nabla f)$$

- what is $V(g^{-1} f)$?
- what is $\mathcal{L}(f, \nabla f)$?
- proof of absence of the Boulware-Deser ghost

Digression: elementary symmetric polynomials $e_n(S)$

Consider matrix S with eigenvalues $\lambda_1, \cdots, \lambda_4$. Then,

$$
\begin{aligned}
e_0(S) &= 1\,,\\
e_1(S) &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4\,,\\
e_2(S) &= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4,\\
e_3(S) &= \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\,,\\
e_4(S) &= \lambda_1\lambda_2\lambda_3\lambda_4\,.
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$$

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\begin{aligned}
e_0(S) &= 1\,, \\
e_1(S) &= \mathrm{Tr}(S) \equiv [S]\,, \\
e_2(S) &= \tfrac{1}{2}([S]^2 - [S^2]), \\
e_3(S) &= \tfrac{1}{6}([S]^3 - 3[S][S^2] + 2[S^3])\,, \\
e_4(S) &= \det(S)\,, \qquad e_n(S) = 0 \quad (n > 4)\,.
\end{aligned}
$$

$$\det(\mathbb{1} + S) = \sum_{n=0}^{4} e_n(S)$$

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$$V(S) = \sum_{n=0}^{4} \beta_n \, e_n(S)$$

Where:

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$$V(S) = \sum_{n=0}^{4} \beta_n \, e_n(S)$$

Where:

$$S^\mu_{\ \nu} = \left(\sqrt{g^{-1} f}\right)^\mu_{\ \nu}$$

(square root of the matrix $g^{\mu\lambda} f_{\lambda\nu}$)

[de Rham, Gabadadze, Tolley (2010)]

[SFH, Rosen (2011); SFH, Rosen, Schmidt-May (2011)]

Potential issues:

Real? Unique? Covariant (rank-2 tensor)? more later $\cdots$

Ghost-free "bi-metric" theory

[SFH, Rosen (1109.3515,1111.2070)]

Ghost-free combination of *kinetic* and *potential* terms:

$$\mathcal{L} = m_g^2 \sqrt{|g|} R_g - m^4 \sqrt{|g|} \sum_{n=0}^{4} \beta_n \, e_n(\sqrt{g^{-1} f}) + m_f^2 \sqrt{|f|} \, R_f$$

"bimetric" nature + *minimal coupling of $g_{\mu\nu}$ to matter* forced by the absence of ghost

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Hamiltonian analysis:

$7 = 2 + 5$ nonlinear propagating modes, no BD ghost!

- $C_1 = 0, \qquad C_2 = \frac{d}{dt} C_1 = \{H, C\} = 0$

Detailed analysis in [SFH, A. Lundkvist (arXiv:1802.07267)]

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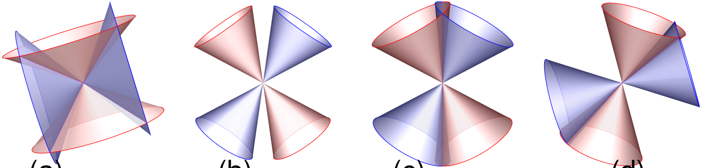
Potential consistency problems

Problem 1: Incompatible spacetimes

$g_{\mu\nu}$ & $f_{\mu\nu}$: Lorentzian metrics, may not admit compatible notions of *space* and *time* (3+1 splits)



(a) (b) (c) (d)

Implications: no consistent time evolution equations,
no Hamiltonian formulation.

Potential consistency problems

Problem 2: Uniqueness, Reality, Covariance

Recall the matrix square root $S^{\mu}_{\nu} = \left(\sqrt{g^{-1}f}\right)^{\mu}_{\ \nu}$:

- Multiple roots (non-unique)
- Possibly non-real
- May not transforms as a $(1,1)$ tensor: non-covariant

Can a **unique, real, covariant** *S be spacified meaingfully?*

Consistent with dynamics?

Answer: Yes. Also solves problem 1

Uniqueness and the local structure of spacetime

[SFH, M. Kocic (arXiv:1706.07806)]

Theorem: Reality + General Covariance $\Rightarrow$

Uniqueness and the local structure of spacetime

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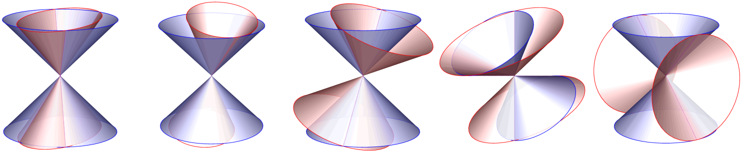
Theorem: Reality + General Covariance $\Rightarrow$

* $S =$ unique real principal square root

* The null cones of $g_{\mu\nu}$ and $f_{\mu\nu}$ intersect in open sets



Type I   Type IIa   Type IIb   Type III   Type IV

Types I-III: proper 3+1 decompositions, allowed.

Type IV: excluded by general covariance

(Implication for accausality arguments in the literature)

Outline of the talk

Ghost-free multi spin-2 theories

Consider N spin-2 fields $g^I_{\mu\nu} = (e_I)^A{}_\mu (e_I)^B{}_\nu \eta_{AB}$, with $I = 1, \cdots, N$

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1) Trivial case: Pairwise bimetric interactions :

$$V(g^1, g^2) + V(g^2, g^3) + \cdots$$

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2) Important step beyond pairwise: multivielbein interactions

[K. Hinterbichler, R. A. Rosen (arXiv:1203.5783)]

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3) Generally, not ghost free except the pairwise bimetric cases

[C. de Rham, A. J. Tolley (arXiv:1505.01450)]

Do genuine ghost-free multi spin-2 interaction exist?

Ghost-free multi spin-2 theories

[SFH, Angnis Schmidt-May (arXiv:1804.09723)]

Starting with simple pairwise interactions, genuine multi spin-2 interactions for $(e_I)^A_{\ \mu}$ can be obtained:

$$\sum_{I=1}^{N} m_I^2 \sqrt{|g^I|} R(g^I) - M^4 \det\Big(\sum_{I=1}^{N} \beta^I e_I\Big)$$

Has the correct number of constraints to eliminate the ghosts.

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Relation to the multivielbein actions of Hinterbichler and Rosen:

$$V = M^4 \sum_{I,J,K,L=1}^{\mathcal{N}} \beta^{IJKL} \, \epsilon_{ABCD} \, e_I^A \wedge e_J^B \wedge e_K^C \wedge e_L^D \,,$$

For no ghosts: $\beta^{IJKL} = \beta^I \beta^J \beta^K \beta^L$ (Generalizations possible).
Systematics of multispin-2 interactions?

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Discussion

The beginning of understanding theories of spin-2 fields beyond General Relativity.

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The beginning of understanding theories of spin-2 fields beyond General Relativity.

- Causal structure (some progress)
- Superluminality? (yes, not necessarily harmful, *replacement for inflation?*)
- Forced mixings
- Systematics of multispin-2 interactions
- A more fundamental formulation (*a la* Higgs)
- Application to cosmology, blackholes, GW, etc.
- Extra symmetries $\Rightarrow$ Modified kinetic terms? much less understood

Thank you!

EXTRA MATERIAL

Mass spectrum & Limits

$$\bar{f} = c^2 \bar{g}\,, \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}\,, \quad f_{\mu\nu} = \bar{f}_{\mu\nu} + \delta f_{\mu\nu}$$

Linear modes:

Massless spin-2: $$\delta G_{\mu\nu} = \left(\delta g_{\mu\nu} + \frac{m_f^2}{m_g^2}\,\delta f_{\mu\nu}\right) \qquad (2)$$

Massive spin-2 : $$\delta M_{\mu\nu} = \left(\delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}\right) \qquad (5)$$

$g_{\mu\nu}$, $f_{\mu\nu}$ are mixtures of *massless* and *massive* modes

The General Relativity limit: $m_g = M_P\,, \quad m_f/m_g \to 0$
(more later)

Massive gravity limit: $m_g = M_P\,, \quad m_f/m_g \to \infty$

Can Bimetric be a fundamental theory?

- Similar to Proca theory in curved background,

$$\sqrt{|\det g|}(F_{\mu\nu}F^{\mu\nu} - m^2\, g^{\mu\nu}A_\mu A_\nu + R_g)$$

- May need the equivalent of Higgs mechanism with the extra fields for better quantum or even classical behaviour

GR limit

The General Relativity limit:

$$m_g = M_P\,, \quad \alpha = m_f/m_g \to 0$$

Cosmological solutions in the GR limi:

$$3H^2 = \frac{\rho}{M_{Pl}^2} - \frac{2}{3}\frac{\beta_1^2}{\beta_2}m^2 - \alpha^2\frac{\beta_1^2}{3\beta_2^2}H^2 + \mathcal{O}(\alpha^4)$$

The GR approximation breaks down at sufficiently strong fields

The Hojman-Kuchar-Teitelboim Metric

General Relativity in 3+1 decomposition ($g_{\mu\nu} : \gamma_{ij}, N, N_i$):

$$\sqrt{g}R \sim \pi^{ij}\partial_t\gamma_{ij} - NR^0 - N_iR^i$$

Constraints: $R^0 = 0\,, R^i = 0$.
Algebra of General Coordinate Transformations (GCT):

$$\{R^0(x), R^0(y)\} = -\left[R^i(x)\frac{\partial}{\partial x^i}\delta^3(x-y) - R^i(y)\frac{\partial}{\partial y^i}\delta^3(x-y)\right]$$

$$\{R^0(x), R_i(y)\} = -R^0(y)\frac{\partial}{\partial x^i}\delta^3(x-y)$$

$$\{R_i(x), R_j(y)\} = -\left[R_j(x)\frac{\partial}{\partial x^i}\delta^3(x-y) - R_i(y)\frac{\partial}{\partial y^j}\delta^3(x-y)\right]$$

$R_i = \gamma_{ij}R^j$, γ_{ij}: metric of spatial 3-surfaces.

- Any generally covariant theory contains such an algebra.
- HKT: The tensor that lowers the index on R^i is the physical metric of 3-surfaces.

The HKT metric in bimetric theory

Consider $g_{\mu\nu} = (\gamma_{ij}, N, N_i)$ and $f_{\mu\nu} = (\phi_{ij}, L, L_i)$,

$$\mathcal{L}_{g,f} \sim \pi^{ij}\gamma_{ij} + p^{ij}\phi_{ij} - M\tilde{R}^0 - M_i\tilde{R}^i$$

On the surface of second class Constraints.
GCT Algebra:

$$\left\{\tilde{R}^0(x), \tilde{R}^0(y)\right\} = -\left[\tilde{R}^i(x)\tfrac{\partial}{\partial x^i}\delta^3(x-y) - \tilde{R}^i(y)\tfrac{\partial}{\partial y^i}\delta^3(x-y)\right]$$

$$\left\{\tilde{R}^0(x), \tilde{R}_i(y)\right\} = -\tilde{R}^0(y)\tfrac{\partial}{\partial x^i}\delta^3(x-y)$$

$\tilde{R}_i = \phi_{ij}\tilde{R}^j$, ϕ_{ij}: the 3-metric of $f_{\mu\nu}$, or
$\tilde{R}_i = \gamma_{ij}\tilde{R}^j$, γ_{ij}: the 3-metric of $g_{\mu\nu}$.
The HKT metric of bimetric theory is $g_{\mu\nu}$ or $f_{\mu\nu}$, consistent with ghost-free matter couplings

[SFH, A. Lundkvist [arXiv:1802.07267]]

Solution to the uniqueness problem of $V(S)$

Matrix square roots:

- Primary roots: Max 16 distinct roots, generic
- Nonprimary roots: Infinitely many, non-generic
  (when eigenvalues in different Jordan blocks coincide)

General Covariance: $A^\mu_{\ \nu} = g^{\mu\rho} f_{\rho\nu}$ is a (1,1) tensor,

$$x^\mu \to \tilde{x}^\mu \Rightarrow A \to Q^{-1} A Q \,, \quad \text{for} \quad Q^\mu_{\ \nu} = \frac{\partial x^\mu}{\partial \tilde{x}^\nu}$$

Uniqueness of S

$S^{\mu}_{\nu} = (\sqrt{A})^{\mu}_{\nu}$:

- Primary roots: $\sqrt{A} \to \sqrt{Q^{-1}AQ} = Q^{-1}\sqrt{A}\,Q$
- Nonprimary roots: $\sqrt{Q^{-1}AQ} \neq Q^{-1}\sqrt{A}\,Q$

Step 1:
General covariance $\Rightarrow$ only primary roots are allowed.

A Consequence: Examples of backgrounds with local CTC's correspond to nonprimary roots and are excluded

Step 2:
Only the *principal root* is always primary. Hence, S must be a principle root.

(Nonprinciple roots degenerate to nonprimary roots when some eigenvalues coincide).

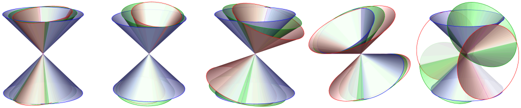
Choice of the square root

Reality + **General Covariance** $\Rightarrow$

* Real principal square root (unique),
* Compatible 3+1 decomposition

$$h_{\mu\nu} = g_{\mu\rho}(\sqrt{g^{-1}f}\,)^{\rho}{}_{\nu}$$

h null-cones for the principal root (except for the last one)



Useful for choosing good coordinate systems,
The specific, existing local CTC's in massive gravity rulled out.

Ghost in Massive Gravity

$$g_{\mu\nu}: \quad N, N_i(4), \gamma_{ij} = g_{ij}(6)$$

General Relativity:

$$\mathcal{L}_{gr} = \sqrt{g}R = \pi^{ij}\dot{\gamma}_{ij} - NR_0 - N^i R_i$$

$R_0 = R_i = 0$, GCT $\Rightarrow$ 2 polarizations

Massive Gravity:

$$\mathcal{L}_{mgr} = \sqrt{g}\left(R - V(g^{-1}f)\right) = \pi^{ij}\dot{\gamma}_{ij} - NR_0 - N^i R_i - \tilde{V}(N, N_i, \gamma, f)$$

No constraints, no GCT $\Rightarrow$ 5 polarizations + 1(BD ghost)

[Boulware, Deser (1972)]

Ghost free massive gravity: with a constraint.

[de Rham, Gabadadze, Tolley (2010)]

[SFH, Rosen (2011); SFH, Rosen, Schmidt-May (2011)]