

DM or MOND

From Galactic Rotation Curves

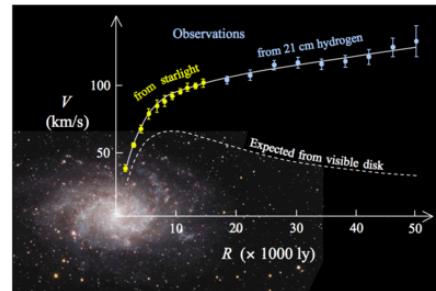
Mads Toudal Frandsen

Arxiv:1805.10706, 1710.03096 with J. Petersen

Missing mass problem

Galactic scales

Rotation curves of stars and gas
(Freeman '70, Bosma '78, Rubin et al '78)



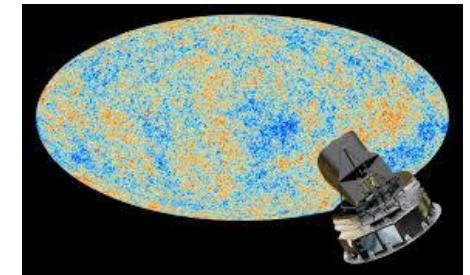
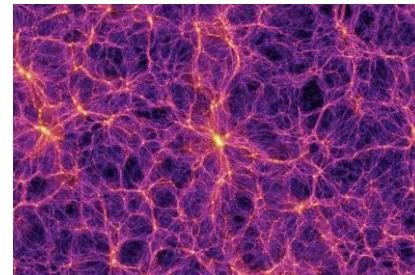
Cluster Scales

Galaxy velocity dispersions,
Cluster mergers
(Zwicky '33, Clowe et al '06)



Cosmological scales

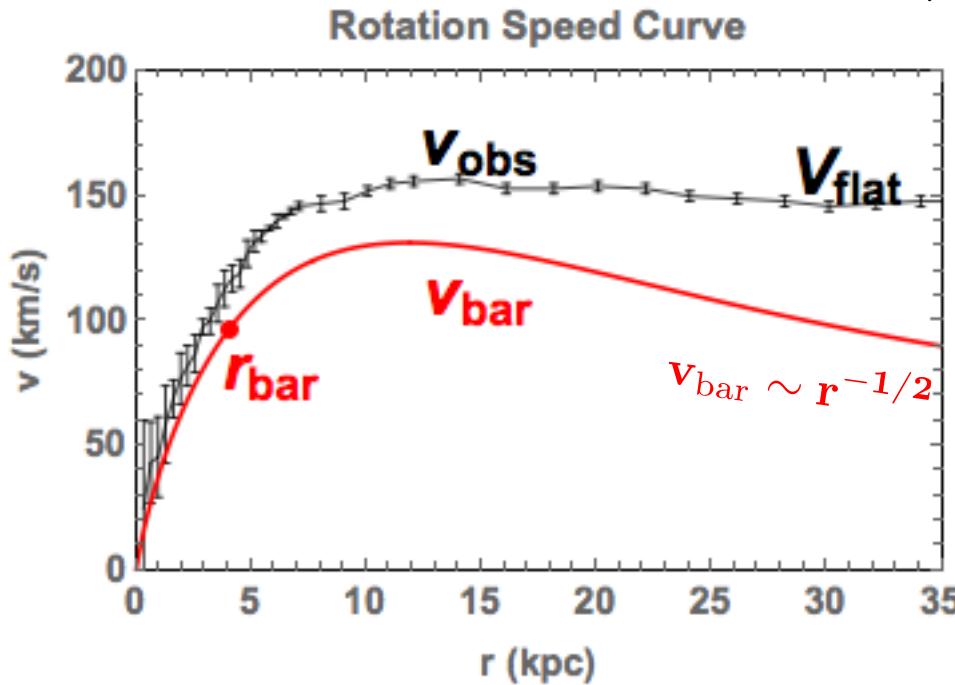
CMB and LSS
(Davis et al '82, Peebles '82)



Rotation speed curves

Baryonic and observed speeds and accelerations

$$g = \frac{v^2}{r}$$



Radii r_{bar} of maximum baryonic acceleration

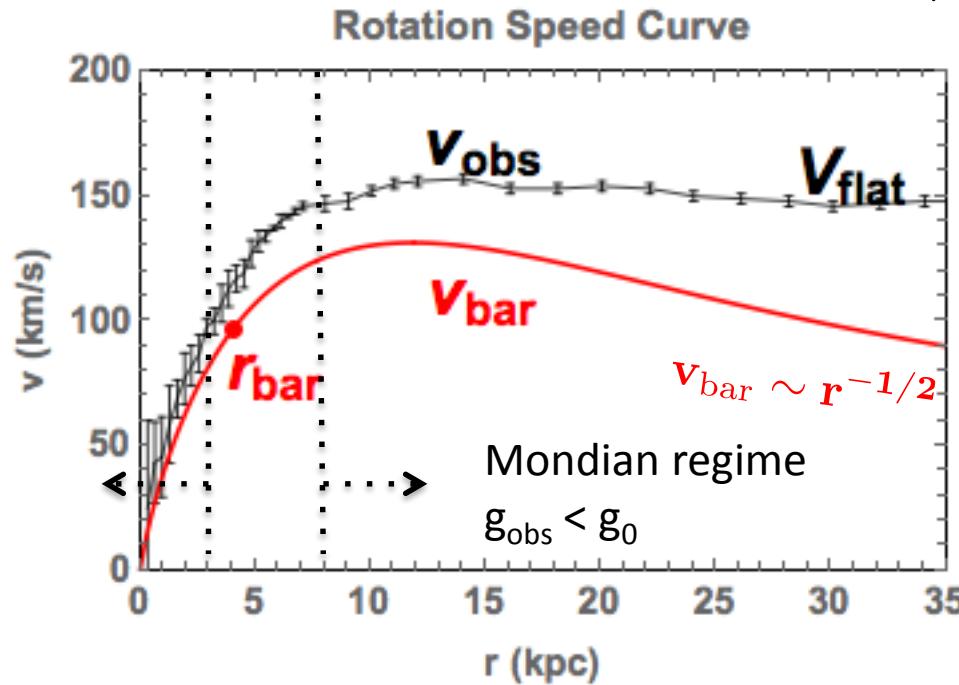
Rotation speed curves

Baryonic and observed speeds and accelerations

$$g = \frac{v^2}{r}$$

Newtonian
regime

$$g_{\text{obs}} \sim g_{\text{bar}}$$



Radii r_{bar} of maximum baryonic acceleration

$g_0 \sim 10^{-10} \text{ m/s}^2$ is characteristic transition regime

Mondian regime

$$g_{\text{obs}}^2 \sim g_{\text{bar}}$$

Baryonic
Tully-Fisher relation

$$V_f^4 \sim M_{\text{bar}}$$

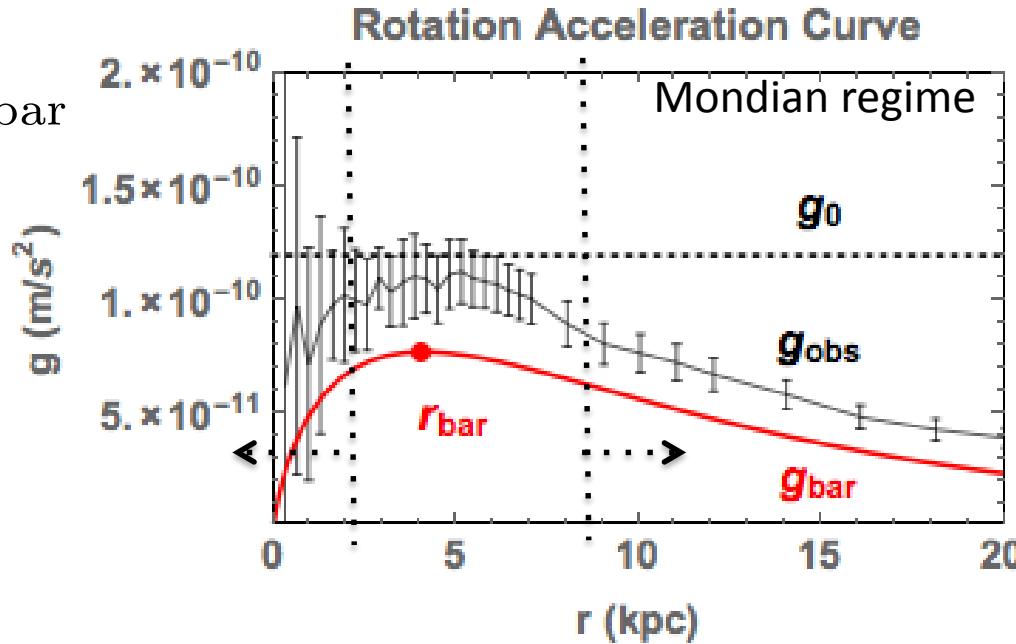
Rotation acceleration curves

Baryonic and observed speeds and accelerations

$$g_{\text{obs}} = \frac{v_{\text{obs}}^2}{r} \quad g_{\text{bar}} = \left| \frac{\partial \Phi_{\text{bar}}}{\partial r} \right|$$

$$g_{\text{obs}} \sim g_{\text{bar}}$$

Newtonian
regime



Radii r_{bar} of maximum baryonic acceleration

$g_0 \sim 10^{-10} \text{ m/s}^2$ is characteristic transition regime

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Baryonic
Tully-Fisher relation

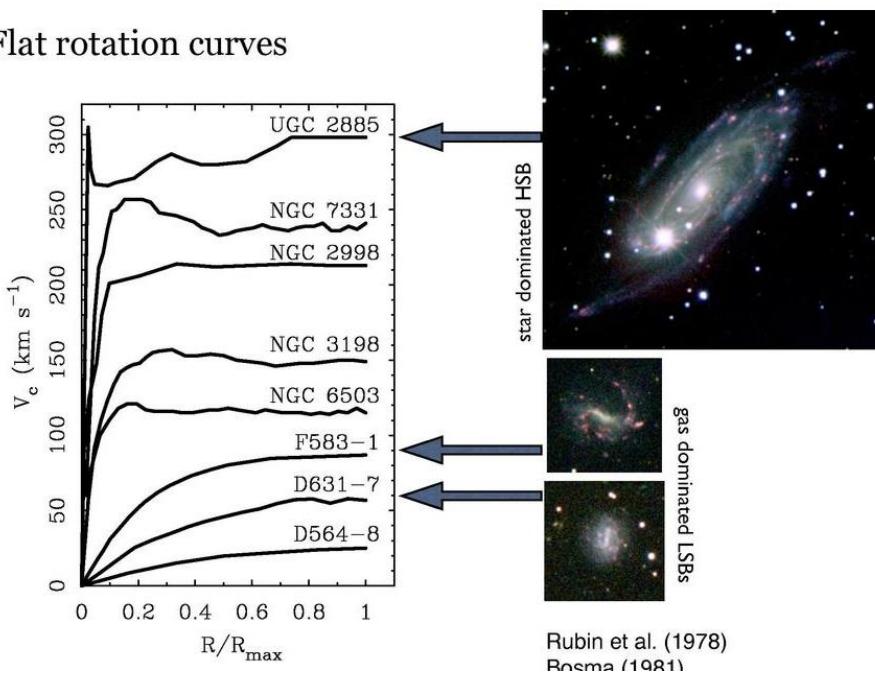
$$V_f^4 \sim M_{\text{bar}}$$



175 late-type spirals and irregulars.

Stellar mass range 5 dex, surface brightnesses 4 dex, range of gas fractions

1. Flat rotation curves



(From McGaugh, KITP DM workshop '18)

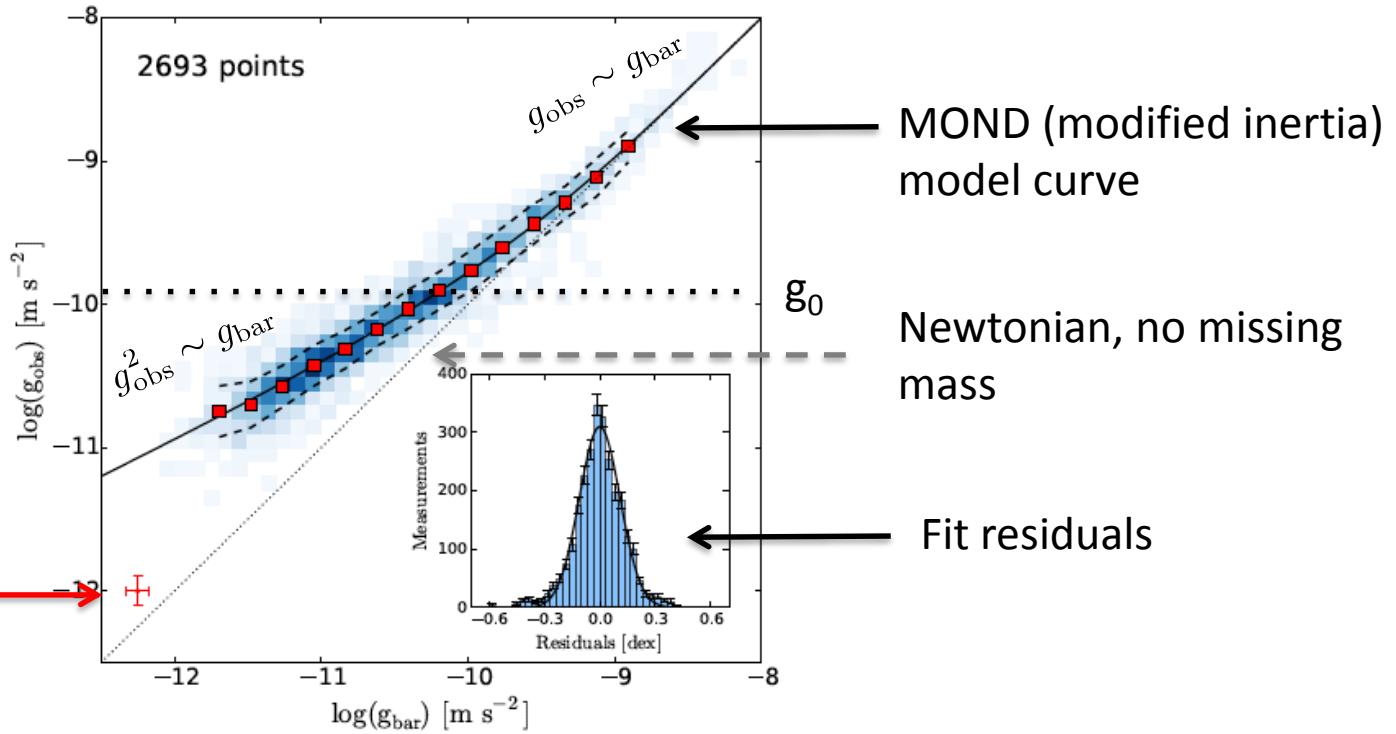
Radial Acceleration Relation

Correlation between baryonic acceleration and total acceleration with 175 galaxies from the SPARC database (McGaugh, Lelli & Schombert '16)

$$g_{\text{obs}} = \frac{v_{\text{obs}}^2}{r}$$

$$g_{\text{bar}} = \left| \frac{\partial \Phi_{\text{bar}}}{\partial r} \right|$$

g2-space curve



Modified Newtonian Dynamics

Newtonian acceleration modified below $g_0 \sim 1.2 \times 10^{-10} \text{ m/s}^2$
to account for flat rotation curves

(Milgrom '83
Bekenstein & Milgrom '85)

Bekenstein-Milgrom MOND

$$\vec{\nabla} \cdot \left(\mu \left(\frac{g_M}{g_0} \right) \vec{g}_M \right) = 4\pi G \rho_{\text{bar}} = \vec{\nabla} \cdot \vec{g}_{\text{bar}}$$

MOND acceleration

Newtonian Poisson Equation

Baryonic matter distribution ρ_{bar}

Newtonian acceleration

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MOND acceleration Baryonic matter distribution ρ_{bar} Newtonian acceleration

MOND interpolation function

$$\mu(x) = \begin{cases} \mu(x) \simeq 1 & x \gg 1 \text{ Newtonian regime} \\ \mu(x) \simeq x & x \ll 1 \text{ Mondian regime} \end{cases}$$

Modified Gravity vs Inertia

$$\mu\left(\frac{g_M}{g_0}\right)\vec{g}_M + \vec{\nabla} \times \vec{h} = \vec{g}_{\text{bar}}$$

$$\mu\left(\frac{g_M}{g_0}\right)g_M = g_{\text{bar}}$$

Bekenstein-Milgrom MOND
(modified gravity) solution
(Bekenstein & Milgrom '85)
MOND modified inertia
Applied to rotation curves

(Milgrom '83, '93)

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Bekenstein-Milgrom MOND
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Inverse interpolation function ν gives *exact* MOND modified inertia acceleration

$$g_M = \nu\left(\frac{g_{\text{bar}}}{g_0}\right)g_{\text{bar}}$$

Or *simplest* approximation for MOND modified gravity

Example inverse interpolation function: (Famaey and McGaugh '11)

$$g_M = \frac{g_{\text{bar}}}{1 - e^{-\sqrt{\frac{g_{\text{bar}}}{g_0}}}}$$

$$\nu\left(\frac{g_{\text{bar}}}{g_0}\right) = \frac{1}{1 - e^{-\sqrt{\frac{g_{\text{bar}}}{g_0}}}}$$

Radial Acceleration Relation

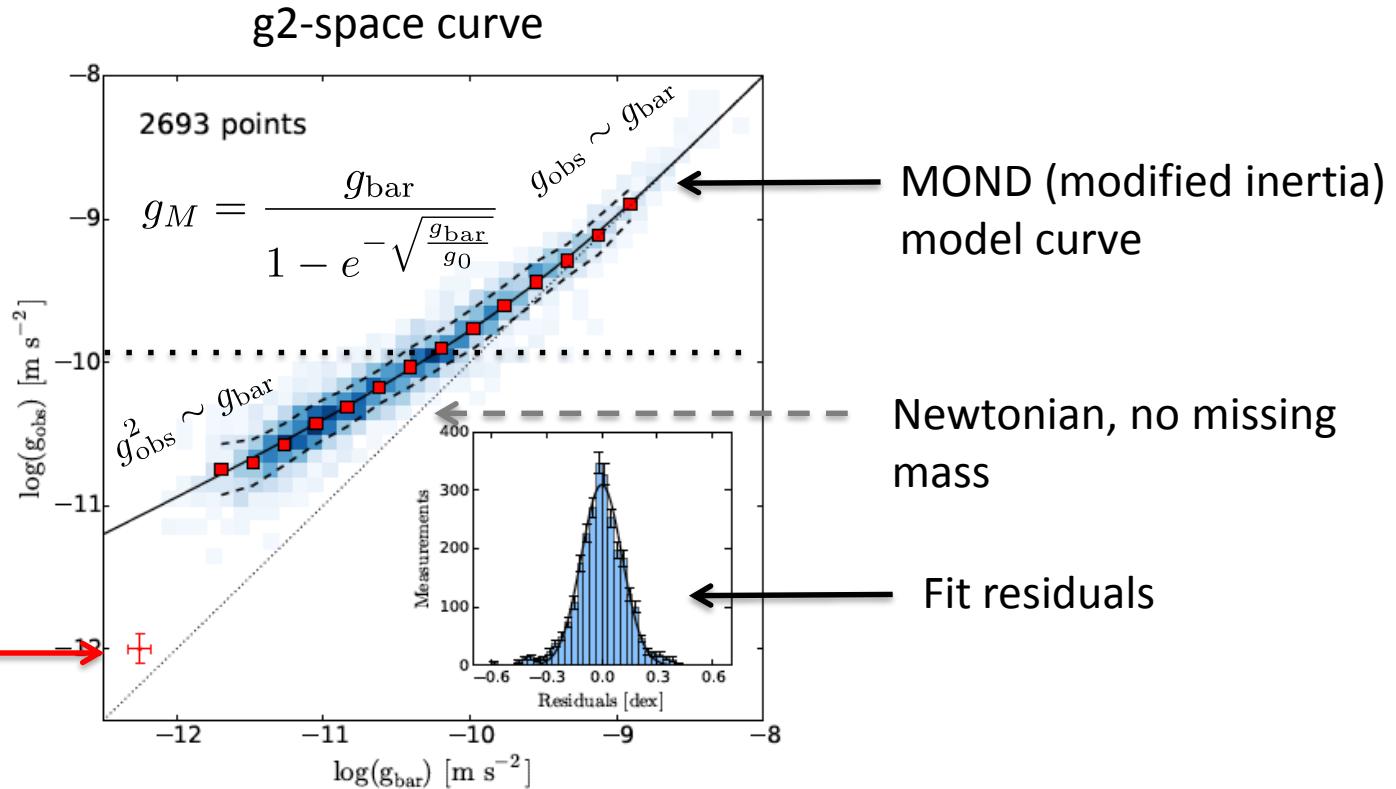
g2-space curve
shows

- 1) Correlation between baryonic acceleration and total acceleration
- 2) MOND modified inertia fit.
- 3) (Simplest) approximation to MOND modified gravity fit.

$$g_{\text{obs}} = \frac{v_{\text{obs}}^2}{r}$$

$$g_{\text{bar}} = \left| \frac{\partial \Phi_{\text{bar}}}{\partial r} \right|$$

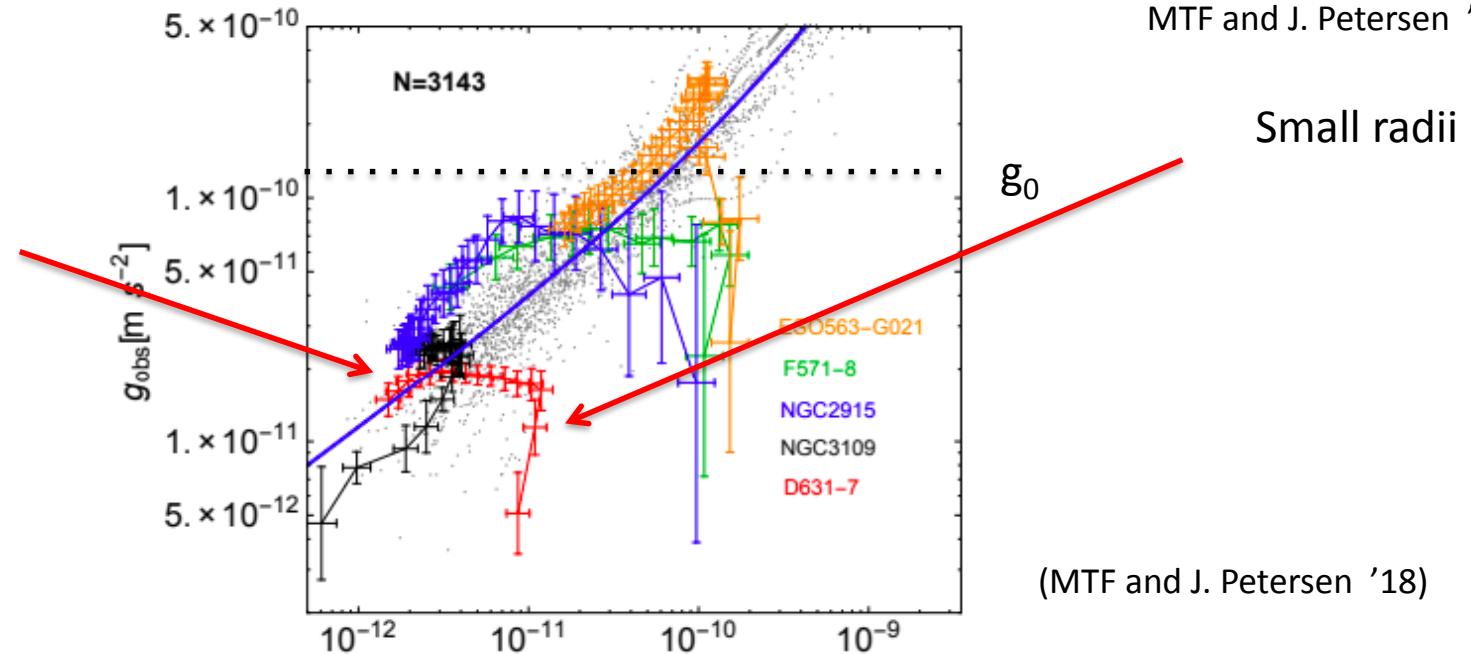
Typical error



SPARC Individual galaxies

(J. Petersen and MTF '17
MTF and J. Petersen '18)

Large radii



Highlighted galaxies don't follow Radial Acceleration Relation at smallest radii

Result of baryonic complexities at small radii?

MOND models in g2-space

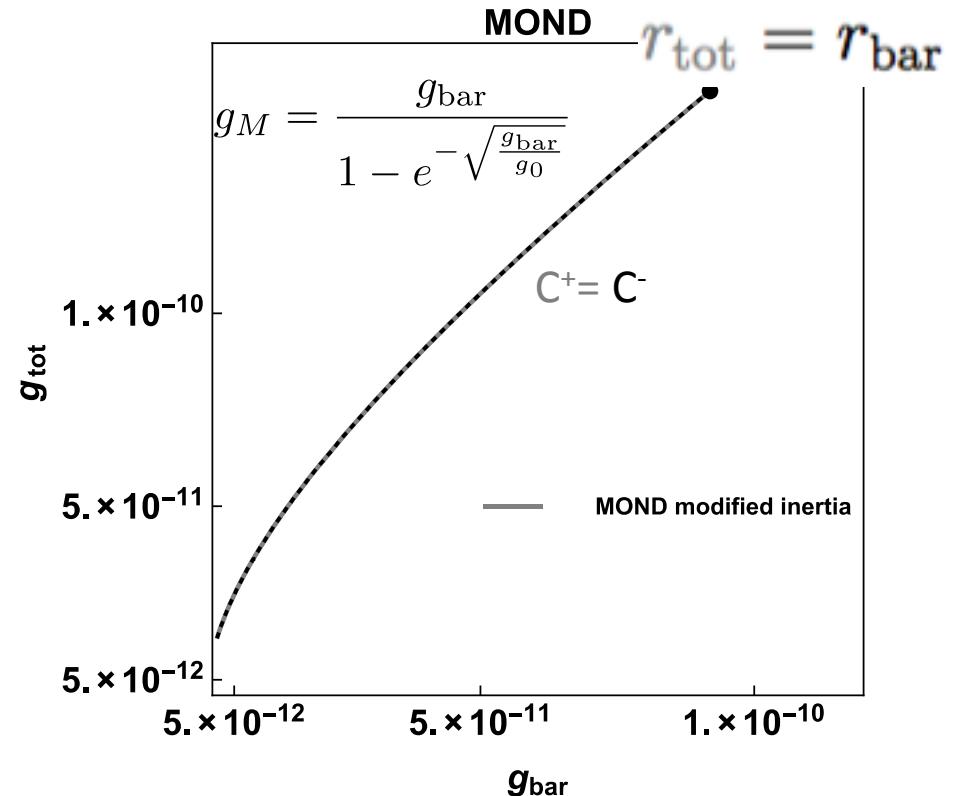
(MTF and J. Petersen '18)

Radii $r_{\text{bar,tot}}$ of maximum baryonic and total accelerations

Curve segment C^+ at radii $r > r_{\text{bar}}$
curve segment C^- at radii $r < r_{\text{bar}}$

MOND modified Inertia predictions:

$r_{\text{tot}} = r_{\text{bar}}$ and $C^+ = C^-$



MOND models in g2-space

(MTF and J. Petersen '18)

Radii $r_{\text{bar,tot}}$ of maximum baryonic and total accelerations

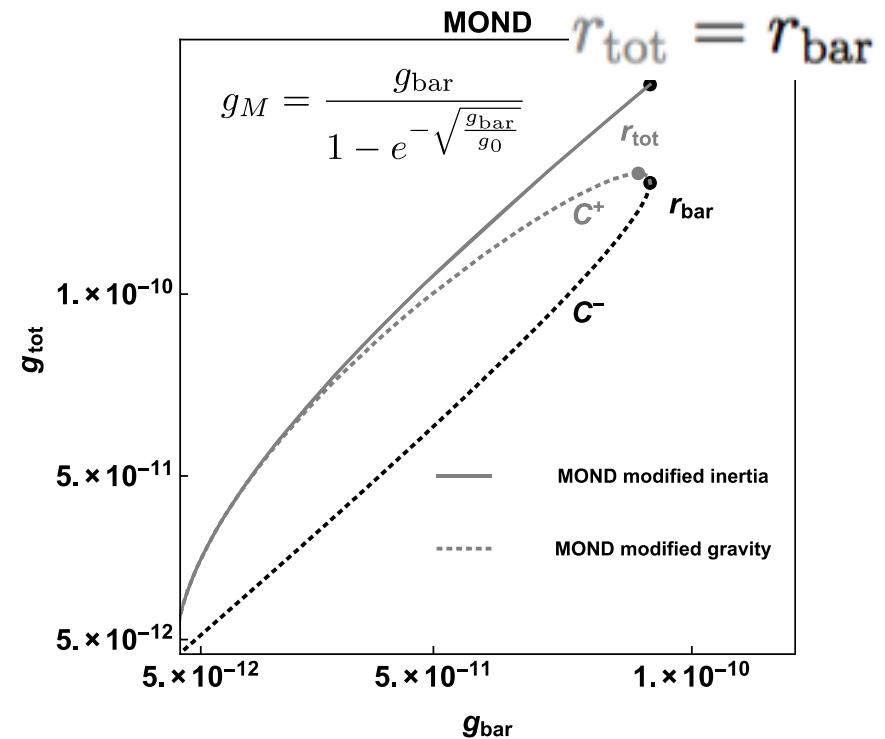
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MOND modified Inertia predictions:

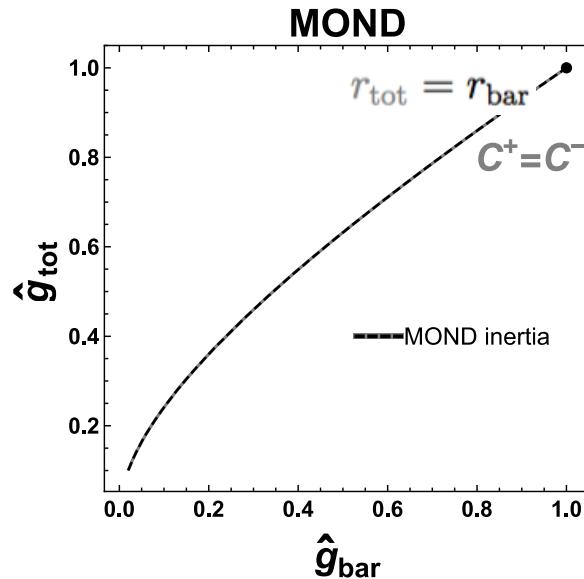
$r_{\text{tot}} = r_{\text{bar}}$ and $C^+ = C^-$

MOND modified gravity predictions (approximation Brada & Milgrom '95)

$r_{\text{tot}} > r_{\text{bar}}$ and $C^+ \neq C^-$



MOND geometry in g2-space

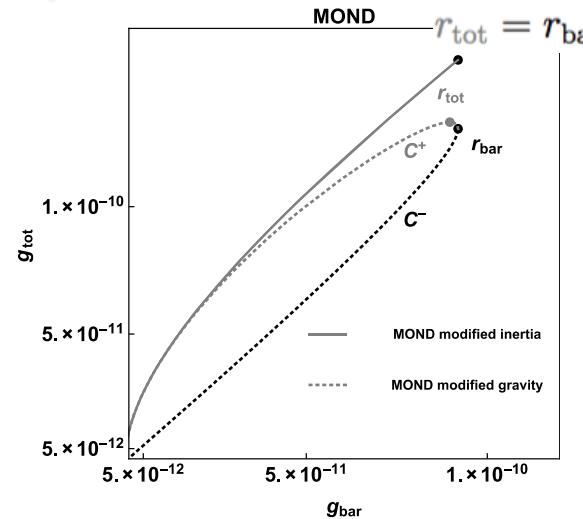


Geometric Classification

MOND Modified Inertia

Models	Reference radii	Curve segments	Curve Area
MOND-MI	$r_{\text{tot}} = r_{\text{bar}}$	$\mathcal{C}^+ = \mathcal{C}^-$	$\mathcal{A}(\mathcal{C}) = 0$

MOND geometry in g2-space



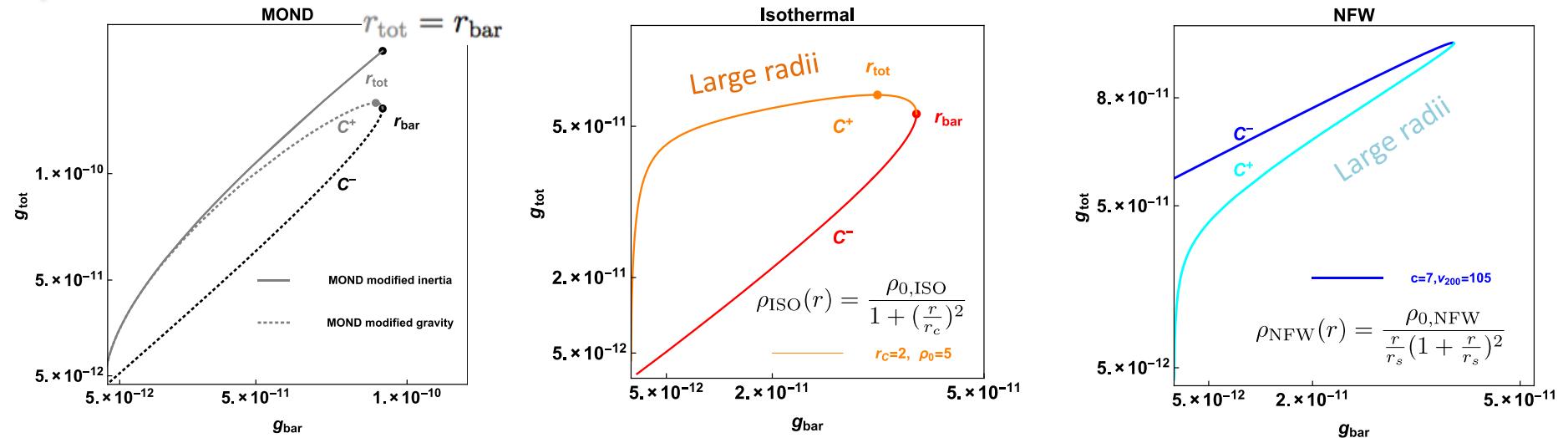
Geometric Classification

Mond Modified Inertia

MOND Modified Gravity

Models	Reference radii	Curve segments	Curve Area
MOND-MI	$r_{\text{tot}} = r_{\text{bar}}$	$\mathcal{C}^+ = \mathcal{C}^-$	$\mathcal{A}(\mathcal{C}) = 0$
MOND-MG	$r_{\text{tot}} > r_{\text{bar}}$	$\mathcal{C}^+ > \mathcal{C}^-$	$\mathcal{A}(\mathcal{C}) > 0$

MOND and DM geometry in g2-space



Geometric Classification

Mond Modified Inertia

MOND Modified Gravity

DM Pseudo-Isothermal

DM Navarro-Frenk-White

Models	Reference radii	Curve segments	Curve Area ^a
MOND-MI	$r_{\text{tot}} = r_{\bar{r}}$	$C^+ = C^-$	$\mathcal{A}(C) = 0$
MOND-MG	$r_{\text{tot}} > r_{\bar{r}}$	$C^+ > C^-$	$\mathcal{A}(C) > 0$
DM-ISO	$r_{\text{tot}} > r_{\bar{r}}$	$C^+ > C^-$	$\mathcal{A}(C) > 0$
DM-NFW	$r_{\text{tot}} < r_{\bar{r}}$	$C^+ < C^-$	Curves open

Comparison to SPARC data

(MTF and J. Petersen '18)

(NB: For data $g_{\text{tot}} \rightarrow g_{\text{obs}}$)

$g_{\text{obs,bar}}$ in terms of measured quantities:

$$g_{\text{obs}}(r_j) = \frac{v_{\text{obs}}^2(r_j)}{r_j}, \quad g_{\text{bar}}(r_j) = \frac{(v_{\text{gas}}^2(r_j) + \Upsilon_{\text{disk}} v_{\text{disk}}^2(r_j) + \Upsilon_{\text{bul}} v_{\text{bul}}^2(r_j))}{r_j}$$

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\mathbf{g}_{obs} uncertainties: random $\delta \mathbf{v}_{\text{obs}}$, Inclination angle δi , Galaxy distance δD

$$\delta g_{\text{obs}}(r_j) = g_{\text{obs}}(r_j) \sqrt{\left[\frac{2\delta v_{\text{obs}}(r_j)}{v_{\text{obs}}(r_j)} \right]^2 + \left[\frac{2\delta i}{\tan(i)} \right]^2 + \left[\frac{\delta D}{D} \right]^2}$$

\mathbf{g}_{bar} uncertainties: $\delta \mathbf{v}_{\text{gas}}$, disk and bulge mass to light ratios $\delta \Upsilon_{\text{disk,bulge}}$

$$\delta g_{\text{bar}}(r_j) = \frac{\sqrt{(2v_{\text{gas}}(r_j))^2 \delta v_{\text{gas}}^2 + v_{\text{disk}}^4(r_j) \delta \Upsilon_{\text{disk}}^2 + v_{\text{bulge}}^4(r_j) \delta \Upsilon_{\text{bulge}}^2}}{r_j}$$

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g_{bar} uncertainties: δv_{gas} , disk and bulge mass to light ratios $\delta \Upsilon_{\text{disk,bulge}}$

$$\delta g_{\text{bar}}(r_j) = \frac{\sqrt{(2v_{\text{gas}}(r_j))^2 \delta v_{\text{gas}}^2 + v_{\text{disk}}^4(r_j) \delta \Upsilon_{\text{disk}}^2 + v_{\text{bulge}}^4(r_j) \delta \Upsilon_{\text{bulge}}^2}}{r_j}$$

Galaxy specific uncertainties δi , δD , $\delta \Upsilon_{\text{disk,bulge}}$ dominate the RAR scatter

(Li, Lelli, McGaugh & Schombert '18)

Also these will depend on radius

Acceleration Ratios in \hat{g} -space

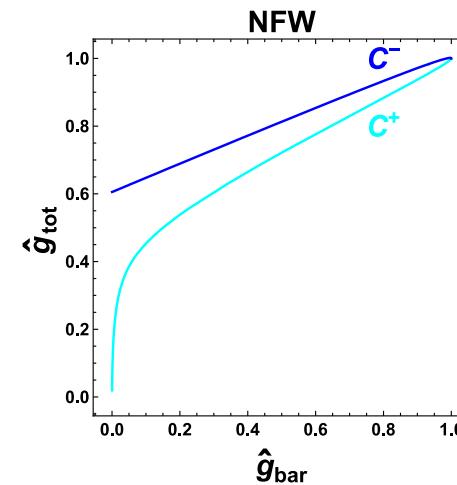
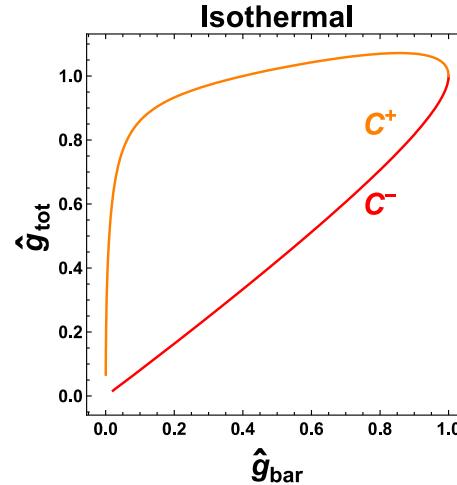
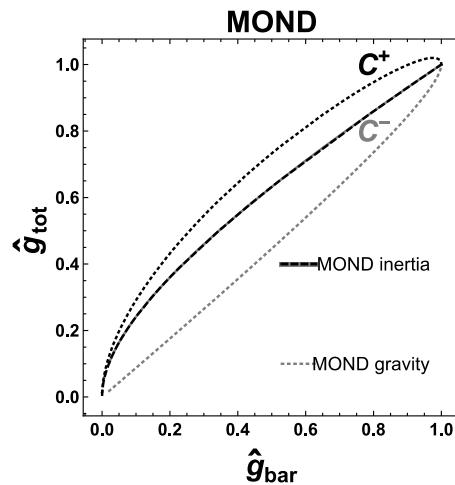
(MTF and J. Petersen '18)

Define ratios of accelerations wrt r_{bar}

$$\hat{g}(r)_{\text{bar,tot}} = \frac{g(r)_{\text{bar,tot}}}{g(r_{\text{bar}})_{\text{bar,tot}}}$$

- Systematic uncertainties on inclination angle δi and galaxy distance δD eliminated
- Systematic uncertainties on mass to light ratios $\delta Y_{\text{disk,bulge}}$ reduced

Caveat: Significant radial dependence of systematic uncertainties



MOND models in \hat{g} -space

(MTF and J. Petersen '18)

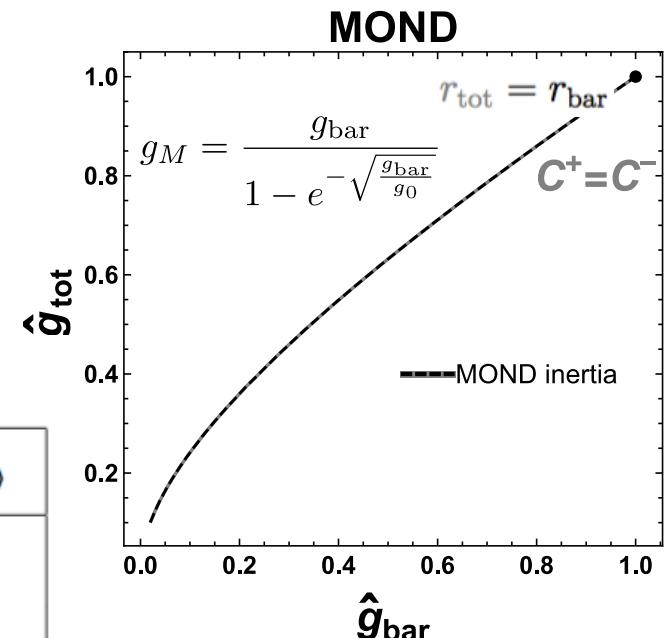
MOND modified Inertia consequences:

$$r_{\text{tot}} = r_{\text{bar}} \text{ and } C^+ = C^- \text{ so } \hat{g}(r_{\text{tot}})_{\text{bar,tot}} = 1$$

Ruled out independent of model $> 5\sigma$

SPARC data analysis for $\hat{g}(r_{\text{tot}})_{\text{bar,tot}}$

Data selection	points	$\langle \hat{g}_{\text{obs}} \pm \delta \hat{g}_{\text{obs}} \rangle$	$\langle \hat{g}_{\text{bar}} \pm \delta \hat{g}_{\text{bar}} \rangle$
$r_j = r_{\text{obs}}$	152	1.39 ± 0.12	0.83 ± 0.01
$r_j = r_{\text{obs}}, \frac{\delta v_{\text{obs}}}{v_{\text{obs}}} < 0.1$	146	1.12 ± 0.02	0.91 ± 0.01



(NB: For data points $g_{\text{tot}} \rightarrow g_{\text{obs}}$, $r_{\text{tot}} \rightarrow r_{\text{obs}}$)

MOND models in \hat{g} -space

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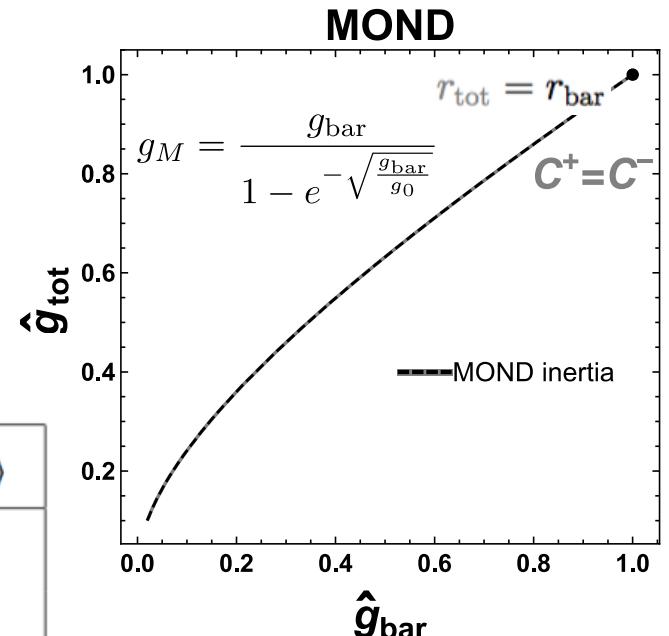
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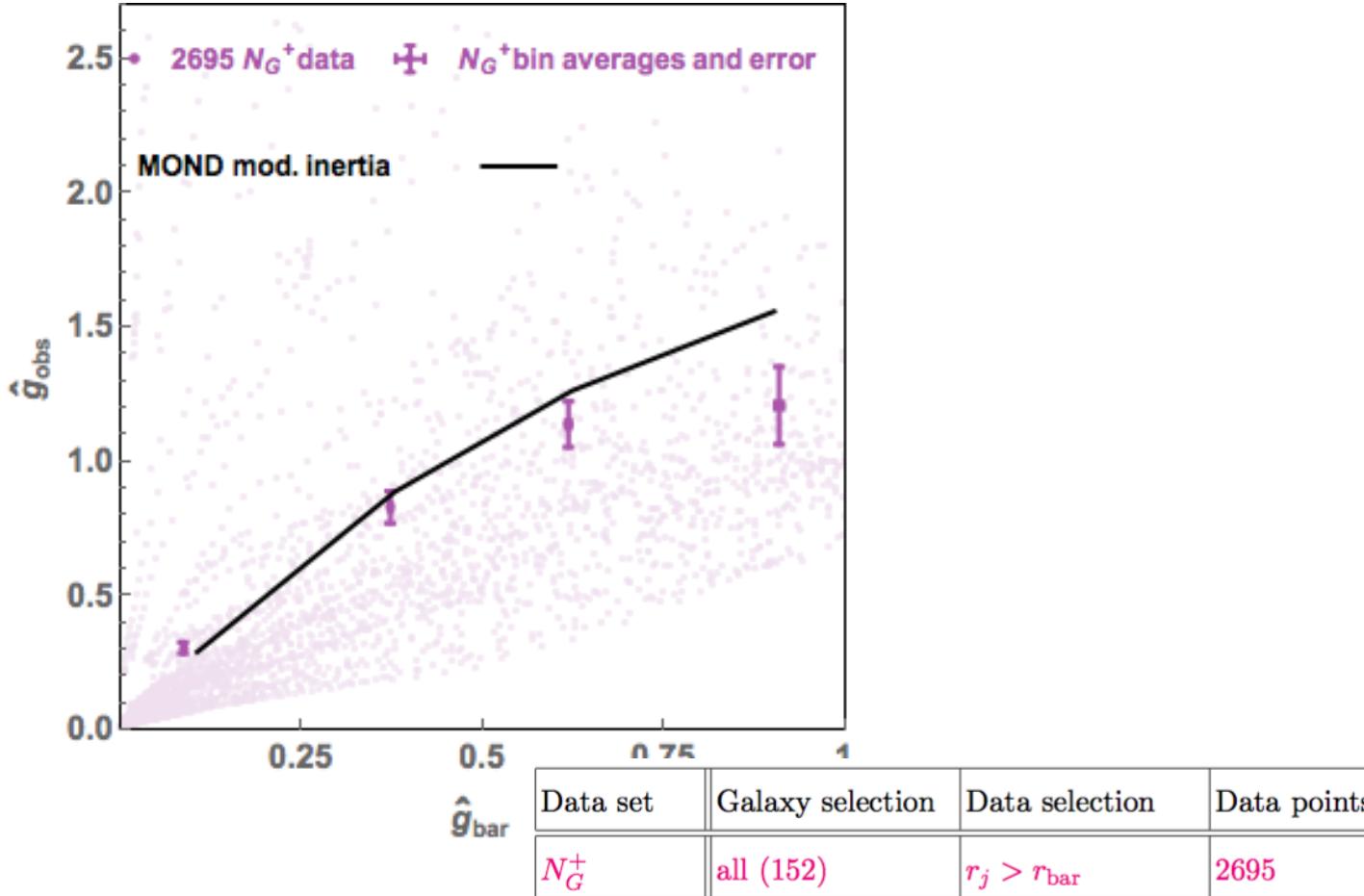
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$r_j = r_{\text{obs}}$	152	1.39 ± 0.12	0.83 ± 0.01
$r_j = r_{\text{obs}}, \frac{\delta v_{\text{obs}}}{v_{\text{obs}}} < 0.1$	146	1.12 ± 0.02	0.91 ± 0.01
$r_{j,\text{bar}} \in \Delta r_{\text{obs,bar}}$	~ 400	1.23 ± 0.04	0.89 ± 0.01



(NB: For data points $g_{\text{tot}} \rightarrow g_{\text{obs}}$, $r_{\text{tot}} \rightarrow r_{\text{obs}}$)

SPARC data in \hat{g}_2 -space

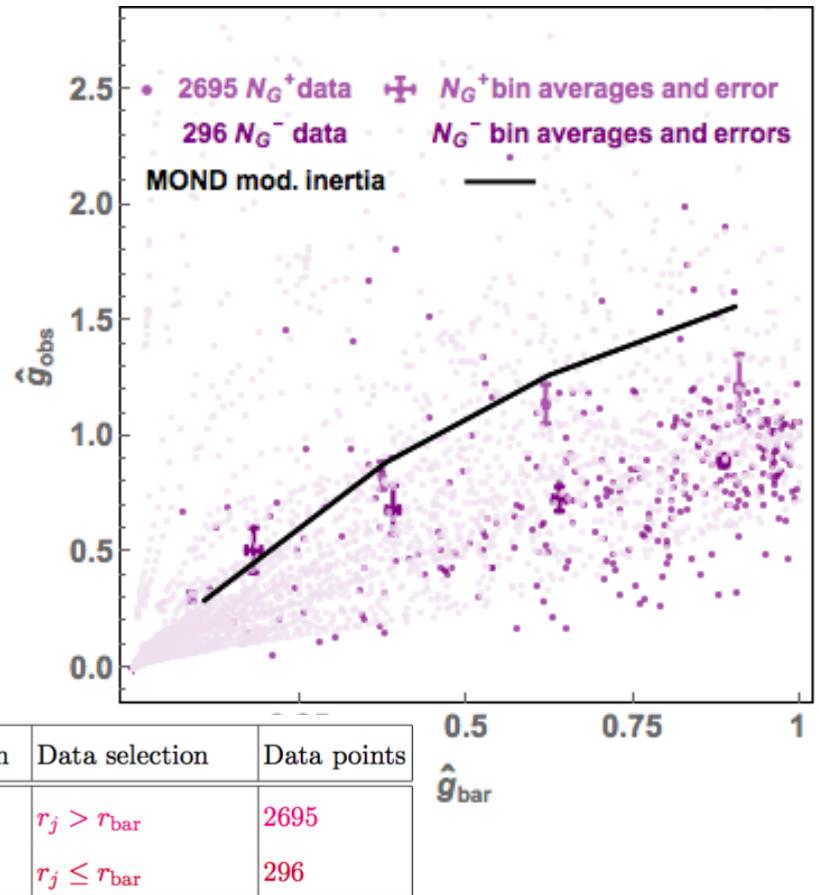
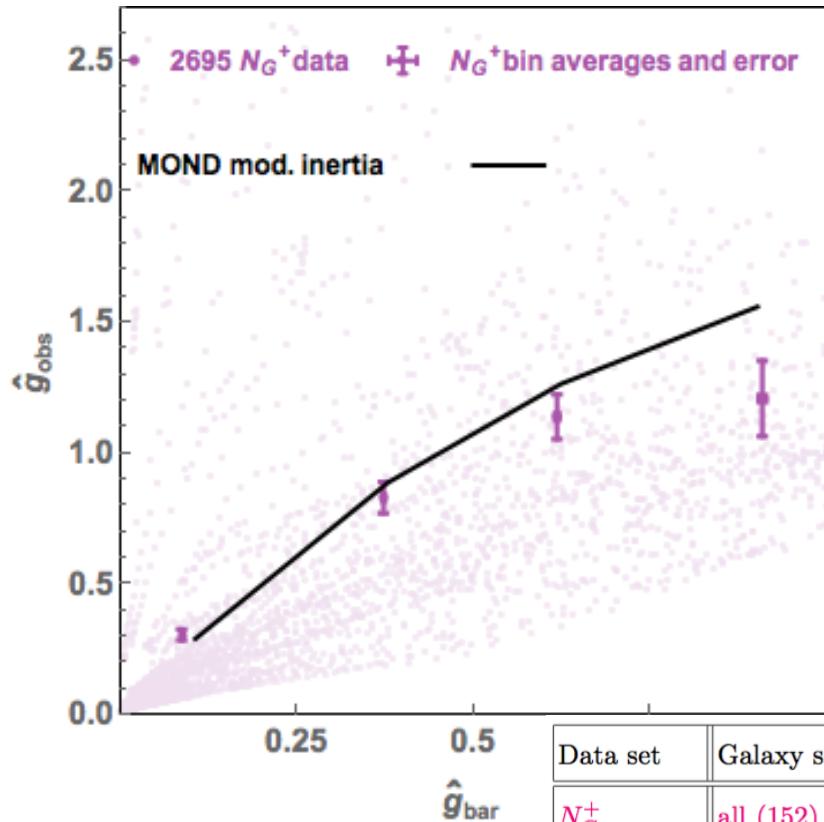
(MTF and J. Petersen '18)



SPARC data in \hat{g}_2 -space

(MTF and J. Petersen '18)

All SPARC data after quality criteria organized wrt r_{bar}

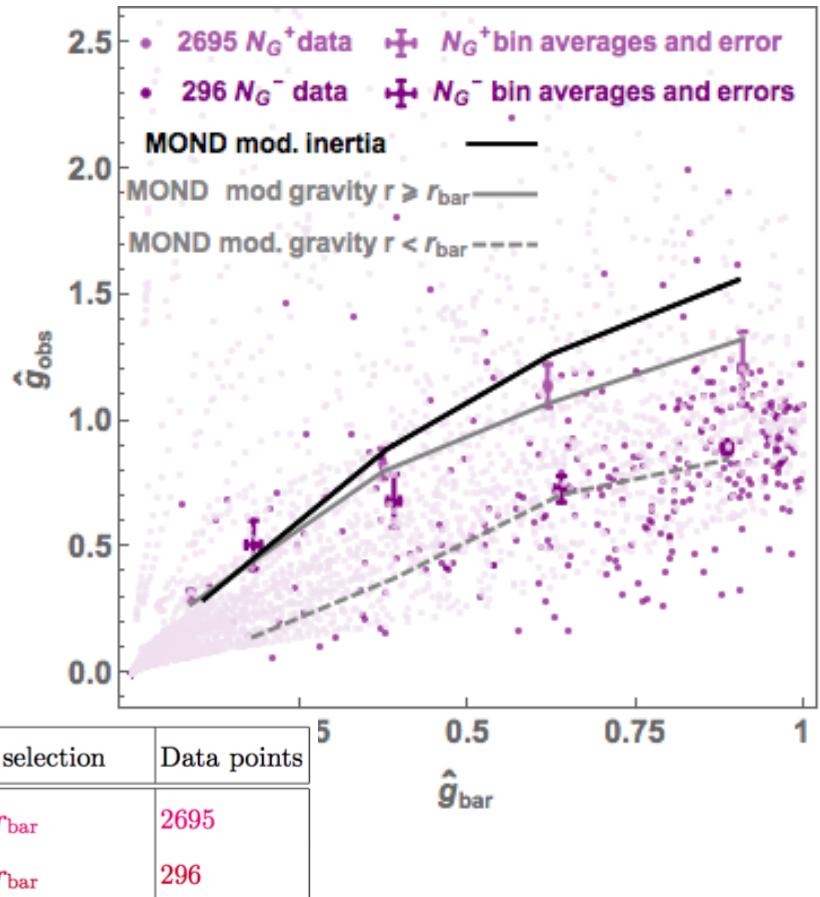
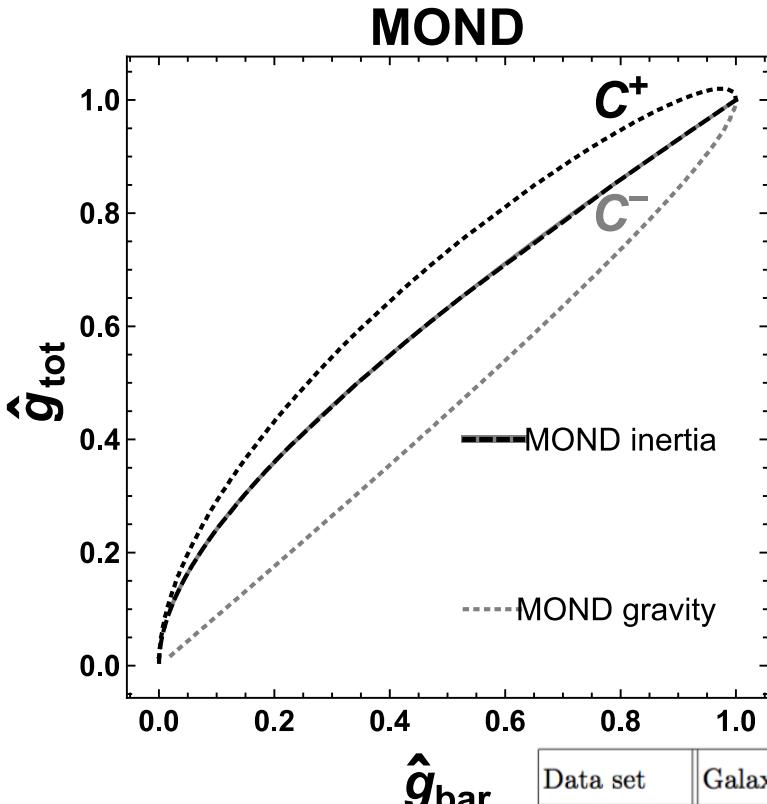


Data set	Galaxy selection	Data selection	Data points
N_G^+	all (152)	$r_j > r_{\text{bar}}$	2695
N_G^-	all (152)	$r_j \leq r_{\text{bar}}$	296

SPARC data in \hat{g}_2 -space

(MTF and J. Petersen '18)

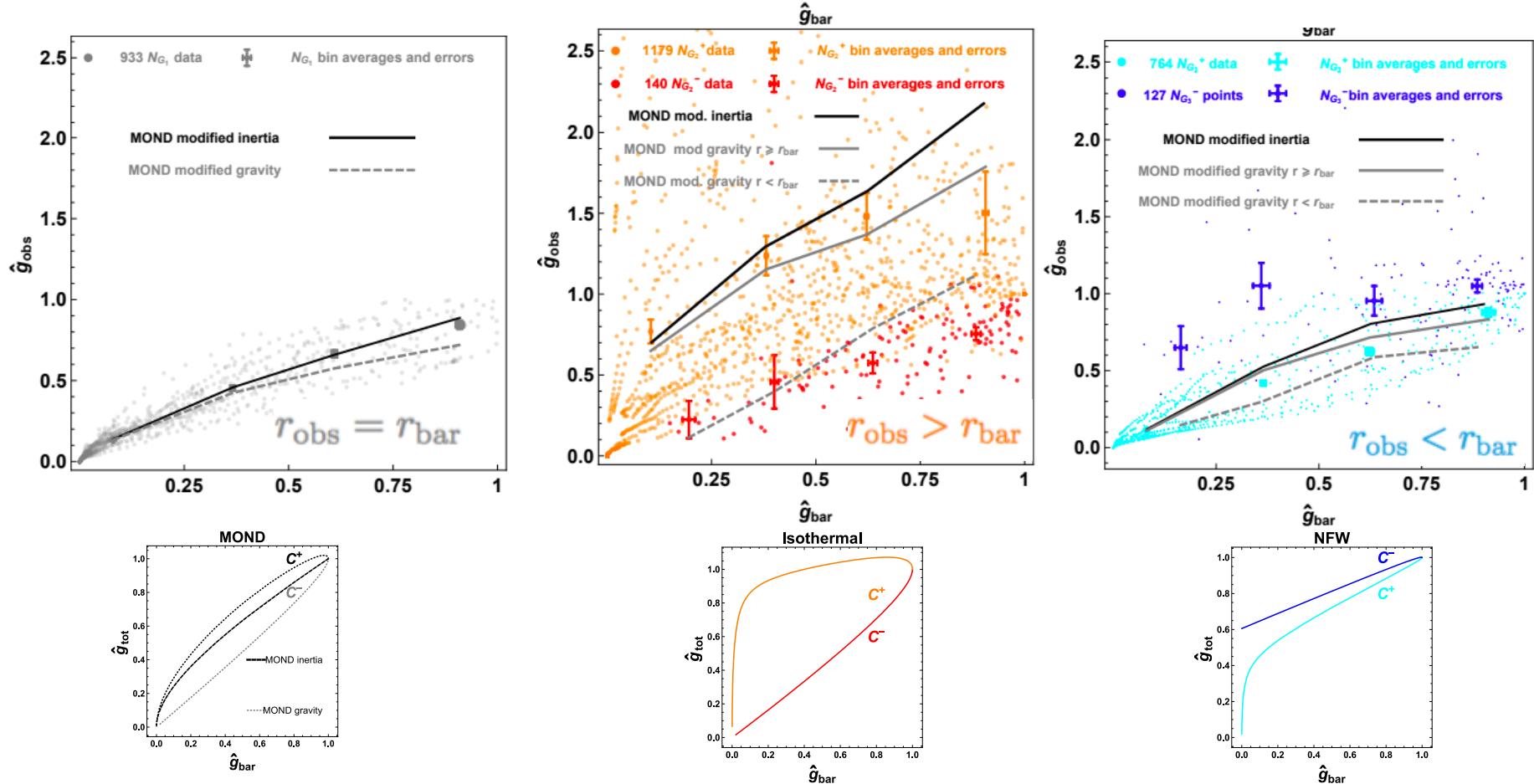
All SPARC data after quality criteria organized wrt r_{bar}



SPARC data in \hat{g}_2 -space

(MTF and J. Petersen '18)

Subdividing galaxies according to overall geometry: r_{obs} vs r_{bar}



Summary

- Geometries of DM and MOND models classified in g2-space
- Rotation curves rule out MOND (modified inertia) $>5\sigma$
Caveat: Significant residual radial dependence of systematic effects
- MOND modified gravity remains to be fully tested
beyond Brada-Milgrom approximation
- Most galaxies display cored geometry
pseudo-isothermal/(approximate) MOND modified gravity
- Diversity from baryonic effects and/or DM self-interactions?