What did HERA teach us about low-x?

A M Cooper-Sarkar

We have the final HERA combined data on inclusive neutral and charge current cross-sections for e^+p and e^-p scattering, for 4 different centre of mass energies.
We also have final data on charm and beauty production.
And there is final inclusive jet, di-jet and tri-jet data from both H1 and ZEUS

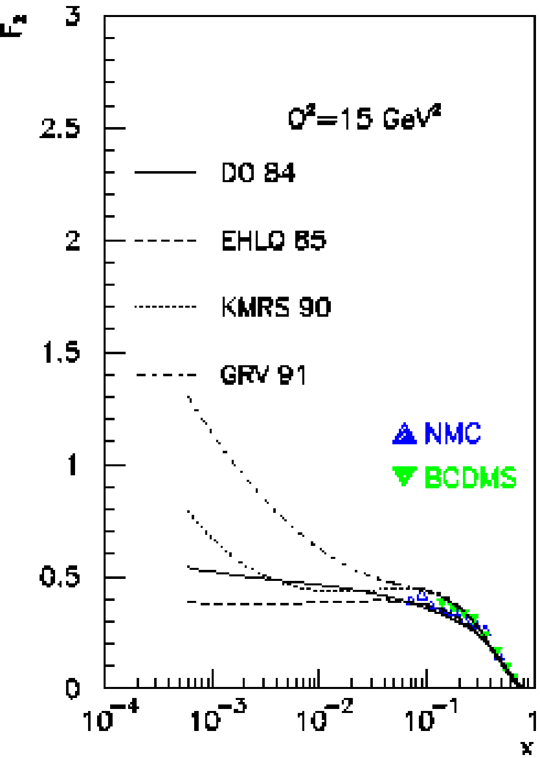
This has been used to extract the Parton Distribution Functions HERAPDF
AND is the back-bone of all the other PDFs –CT14, MMHT14, NNPDF3.0, ABM- most of which do not YET have the final HERA data

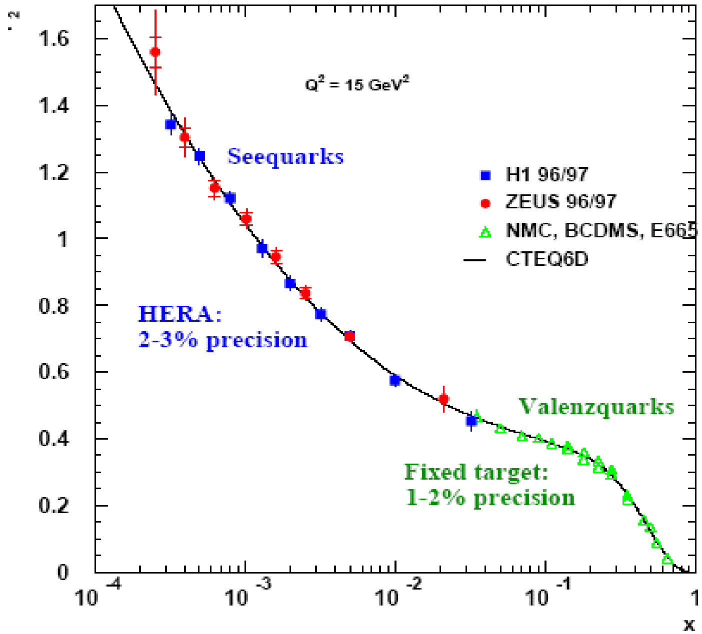
PDF fitting assumes the validity of conventional QCD DGLAP evolution, which sums $\ln(Q^2)$ diagrams
This has served us very well

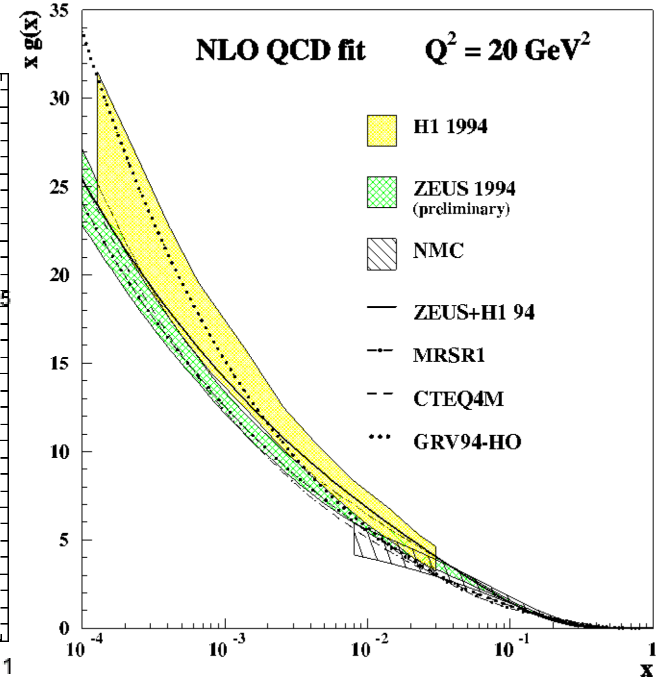
But we have always had suspicions that at low-x we should also be re-summing $\ln(1/x)$ diagrams AND that we could be heading into a new kinematic regime of high-density partons in which we need non-linear evolution, which could lead to saturation.

The FINAL combined HERA data that are used for pQCD fits also extend to very low Q^2
The transition from perturbative to non-perturbative region using these legacy data
repeats old ground, but with some new insights?

Let us look at low-x physics at HERA– because the connection to EIC is strongest





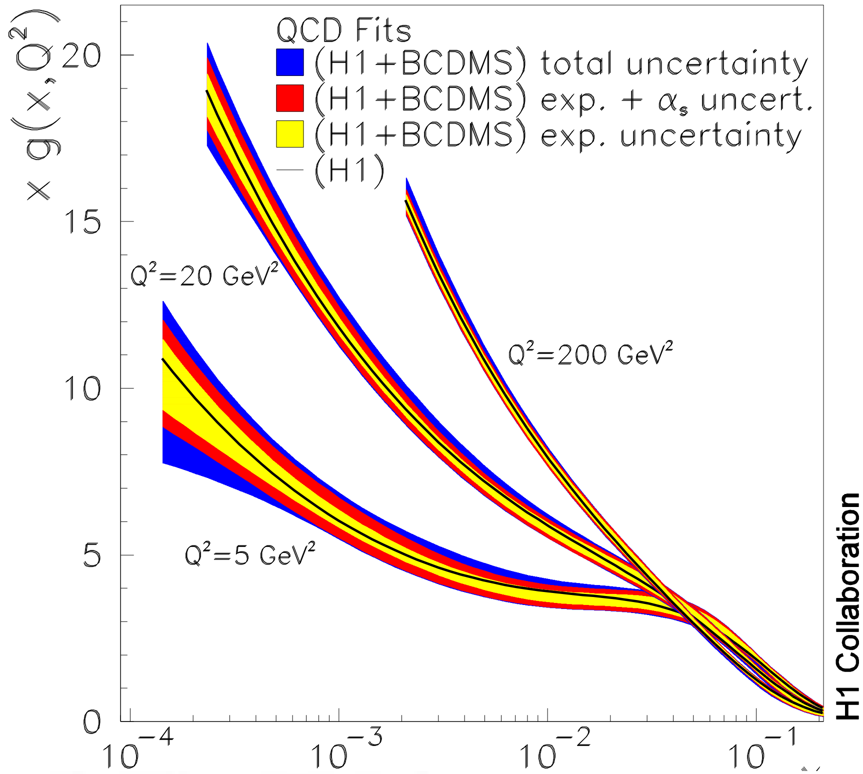


Before the HERA measurements most of the predictions for low-x behaviour of the structure functions and the gluon PDF were wrong – most theoreticians expected it to flatten out. It actually rises steeply

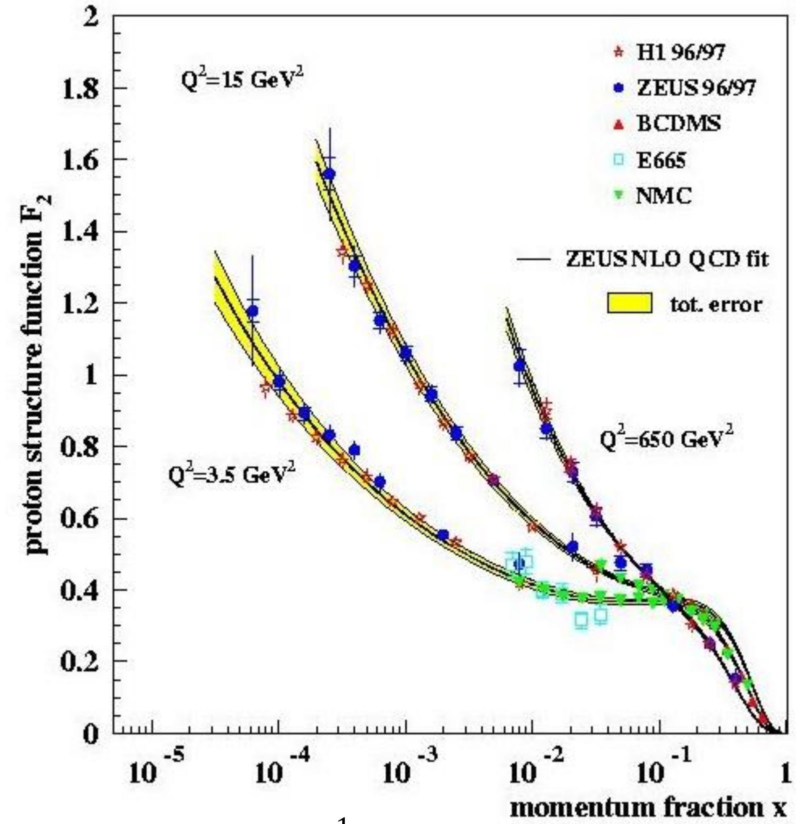
AND YET—DGLAP does predict the rise that we saw!

Now it seems that the conventional DGLAP formalism works TOO WELL !

(we think there **should be** $\ln(^1/_x)$ corrections and/or non-linear high density corrections for $x < 5 \times 10^{-3}$)



Low-x



$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 \frac{dy}{y}\left[\Sigma_q Pgq(z)q(y,Q^2) + Pgg(z)g(y,Q^2)\right]$$

At small x,

small z=x/y

$$Pgq \to \frac{C_F}{z}, \quad Pgg \to \frac{2C_A}{z}$$

Gluon splitting functions become singular

$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 \frac{dy}{y}\frac{6}{z}g(y,Q^2)$$

$xg(x,Q^2) \sim x^{-\lambda g}$

$$\lambda_g = \left(\frac{12\ln(t/t_0)}{\beta_0 \ln(1/x)}\right)^{\frac{1}{2}},$$

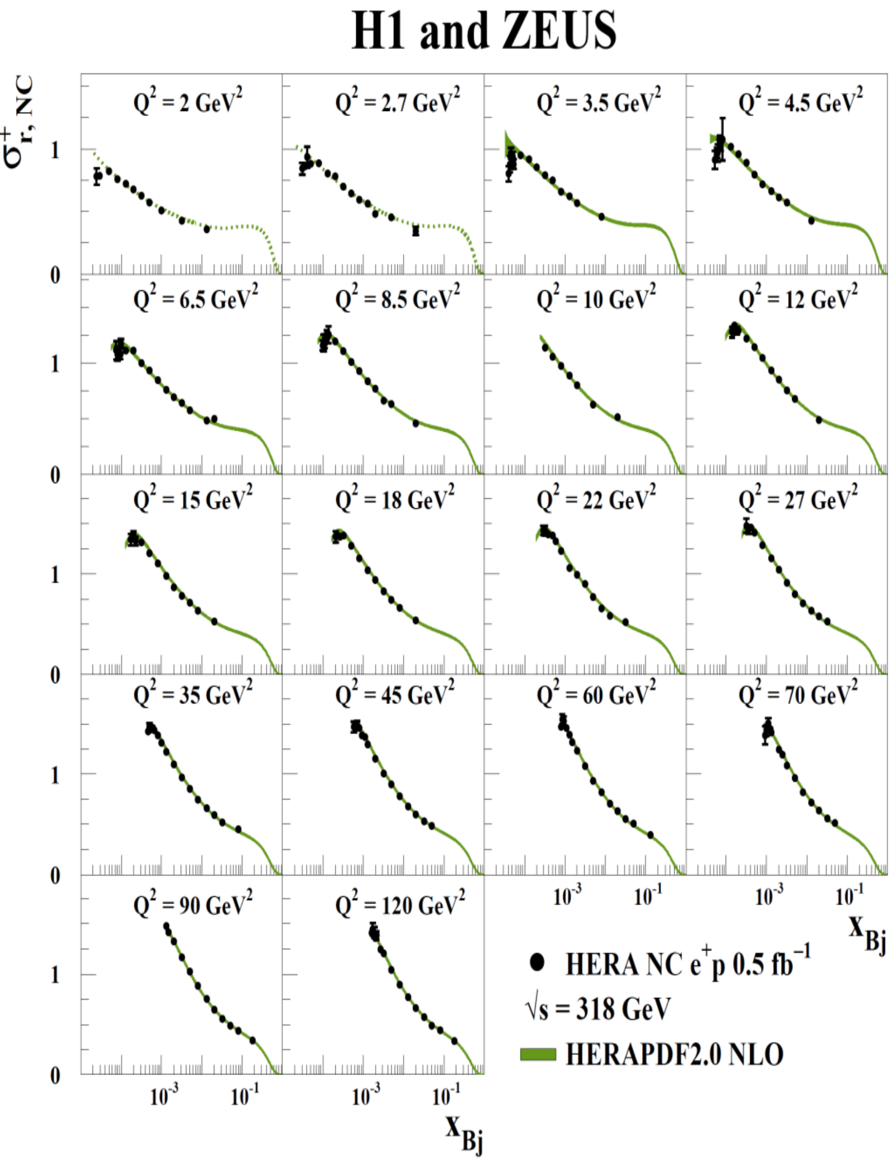
$t = \ln Q^2/\Lambda^2$

$\alpha_s \sim 1/\ln Q^2/\Lambda^2$

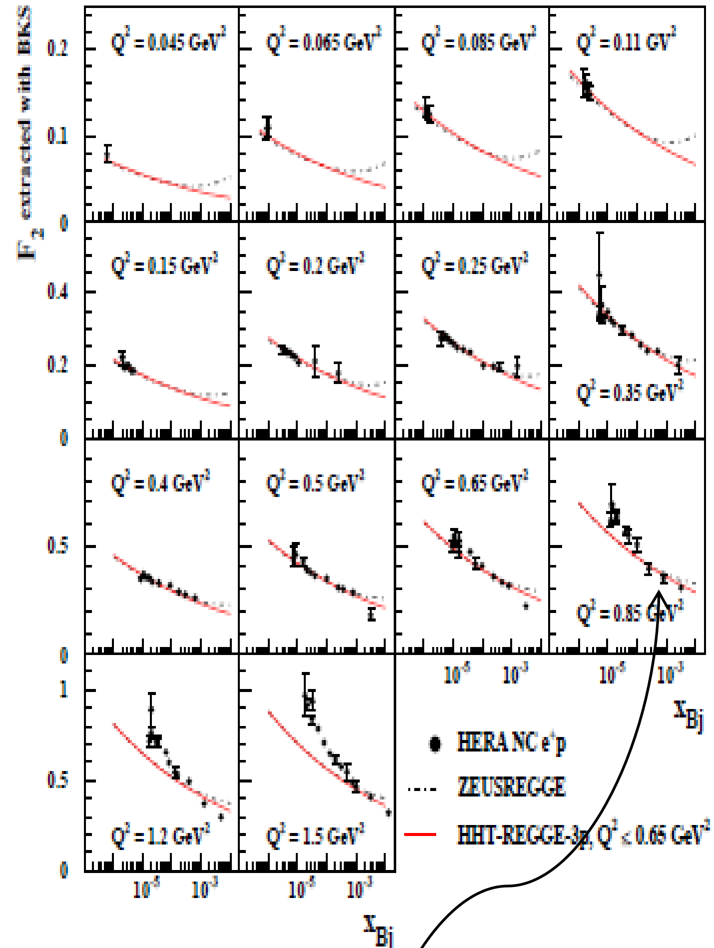
A flat gluon at low Q^2 becomes very steep **AFTER Q^2 evolution** AND F_2 becomes gluon dominated

$F_2(x,Q^2) \sim x^{-\lambda s}$, $\lambda s = \lambda g - \varepsilon$

The point is that steepness should set in AFTER evolution, so at higher Q^2



NEW Low Q^2 plot from 1704.03187



So it was a surprise to see F$_2$ steep at small x even for low Q^2, Q^2 <~5 GeV^2
and even more of a surprise to see it steep down to Q^2 ~ 1 GeV^2

Should perturbative QCD work? α_s is becoming large - α_s at Q^2 ~ 1 GeV^2 is ~ 0.4

There is another reason why the application of conventional DGLAP at low x is questionable:

The splitting functions, $P(x) = P^0(x) + P^1(x)\,\alpha_s(Q^2) + P^2(x)\,\alpha_s^2(Q^2)$

have contributions,

$$P^n(x) = \frac{1}{x}\left[a_n \ln^n\left(\frac{1}{x}\right) + b_n \ln^{n-1}\left(\frac{1}{x}\right)\right]$$

dominant at small x

Their contribution to the PDF comes from,

$$\frac{\partial q(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} P(x)\, q(y, Q^2)$$

→ and thus give rise to contributions to the PDF of the form,

$$\alpha_s^P(Q^2)\,(\ln Q^2)^q\left(\ln\frac{1}{x}\right)^r$$

Leading log(Q^2):

conventionally in LO DGLAP: p = q ≥ r ≥ 0 LL(Q^2)

NLO: p = q +1 ≥ r ≥ 0 NLL(Q^2)

But if $\ln(^1/_x)$ is large, we should also consider,

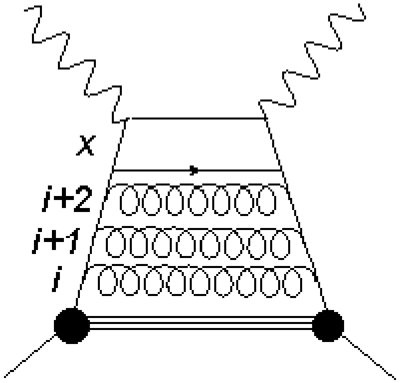
Leading log(1/x):

p = r ≥ q ≥ 1 LL(1/x)

p = r+1 ≥ q ≥ 1 NLL(1/x)

This is what is meant by BFKL summation.

Diagrammatically,



Leading $\log Q^2 \rightarrow$ *strong* p_t ordering

$$Q^2 >> p_{t_i}^2 >> p_{t_{i-1}}^2 \ldots >> p_{t_1}^2$$

and at small *x* we also have strong ordering in *x*

$$x << x_i << x_{i-1} \ldots << x_1$$

$$\Rightarrow \text{leading } \ln(1/x)$$

$\rightarrow$ double leading logs $\alpha_s \ln Q^2 \ln(1/x)$ at small x (double asymptotic scaling)
But why not sum up $\alpha_s \ln(1/x)$ independent of Q^2?

$\rightarrow$ Diagrams ordered in *x*, but *not* in p_t
BFKL formalism

$\rightarrow$

$$x\,g(x, Q^2) \sim x^{-\lambda}$$

$$\lambda = \frac{\alpha_s}{\pi} C_A \ln 2 \simeq 0.5$$

for $\alpha_s \sim 0.25$ (low Q^2)

$\rightarrow$ A singular gluon behaviour even at low-ish Q^2
$\rightarrow$ Is this the reason for the steep behaviour of F_2 at low-x ?
IS there a "BFKL Pomeron"-

However we all know that this steep behaviour was modified once NLO BFKL calculations were made. It has proved very difficult to get 'smoking gun' evidence for anything beyond DGLAP at HERA

Furthermore if the **gluon density becomes large** there maybe **non-linear effects**

Gluon recombination $g\, g \rightarrow g$

$\sigma \sim \alpha_s^2 \rho^2/Q^2$

may compete with gluon evolution $g \rightarrow g\, g$

$\sigma \sim \alpha_s\, \rho$

where ρ is the gluon density

~

Non-linear evolution equations – GLR

$$\frac{d^2xg(x,Q2)}{d\ln Q2\, d\ln 1/x} = \frac{3\alpha_s}{\pi}\, xg(x,Q2) - \frac{\alpha_s^2\, 81}{16Q^2R^2}\, [xg(x,Q2)]^2$$

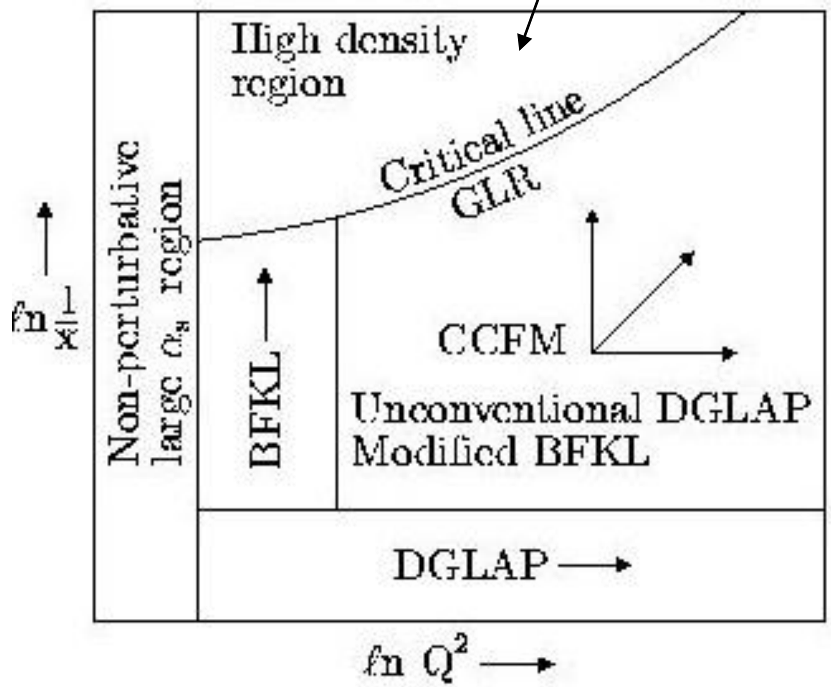
$\alpha_s\, \rho$ $\qquad$ $\alpha_s^2\, \rho^2/Q^2$

The non-linear term slows down the evolution of $xg(x,Q^2)$ and thus tames the rise at small x

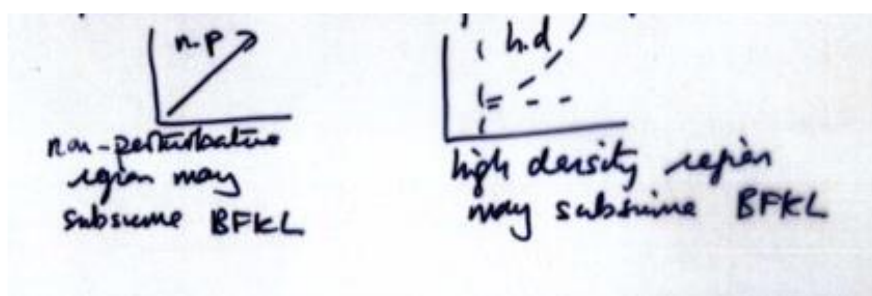
The gluon density may even saturate

(-respecting the Froissart bound)

Colour Glass Condensate, JIMWLK, BK etc. etc. At higher Q^2 the region moves to lower and lower x



Extending the conventional DGLAP equations across the x, Q2 plane.
Plenty of debate about the positions of these lines!



There are various reasons to worry that conventional $\ln(Q^2)$ summations – as embodied in the DGLAP equations may be inadequate

It was a surprise to see F_2 steep at small x - even for very low Q^2, $Q^2 \sim 1$ GeV^2

1. Should perturbative QCD work? α_s is becoming large - α_s at $Q^2 \sim 1$ GeV^2 is ~ 0.4

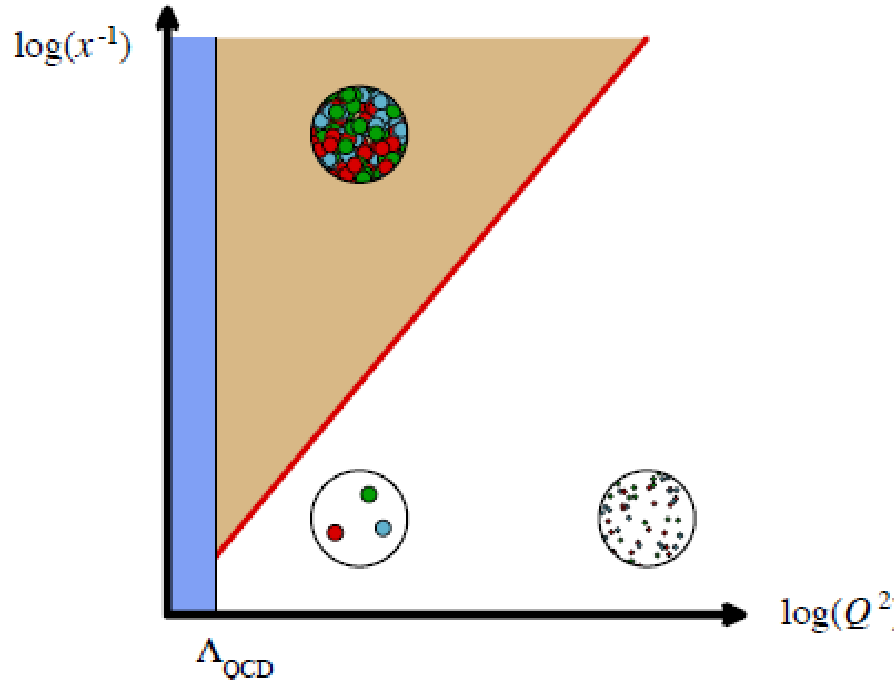
2. There hasn't been enough lever arm in Q^2 for evolution, but even the starting distribution is steep- **the HUGE rise at low-x makes us think**

3. there **should be** $\ln(1/x)$ resummation (BFKL) as well as the traditional $\ln(Q^2)$ DGLAP resummation- BFKL predicted $F_2(x,Q^2) \sim x^{-\lambda s}$, with λ_s=0.5, even at low Q^2

4. and/or there should be non-linear high density corrections for x < 5 10^{-3}

5. In nuclei these could be enhanced by $A^{1/3}$

Colour Glass Condensate, JIMWLK, BK etc. At higher Q^2 the region moves to lower and lower x



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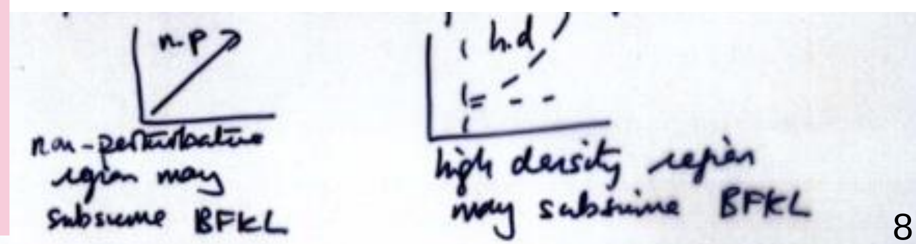


Does the data *need* unconventional explanations?

- $\ln(^1/_x)$ terms in the splitting factors
- CCFM
- modified BFKL

Afficionados claim χ^2 improvements over conventional NLLO DGLAP..

But, one seems to be able to use DGLAP by absorbing unconventional behaviour in the boundary conditions i.e. the unknown shapes of the non-perturbative parton distributions at Q_0^2

We measure, $F_2 \sim x\,q$

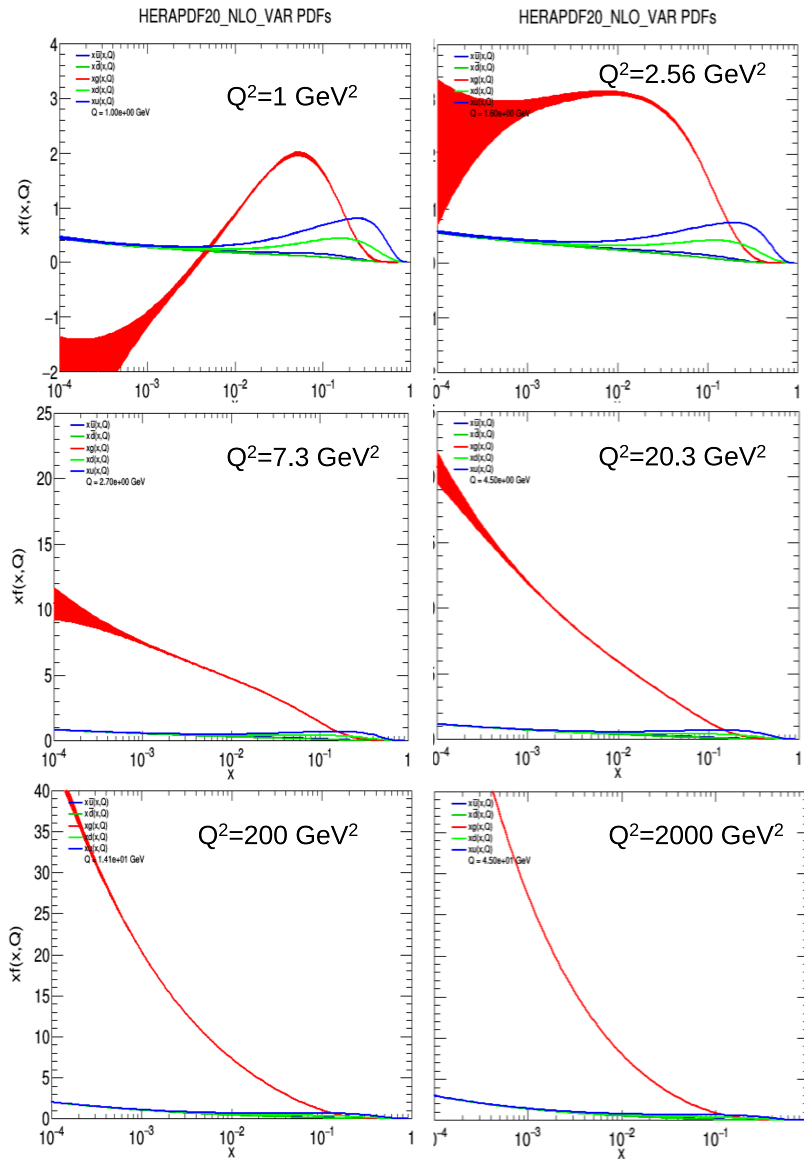
$$\frac{dF_2}{d\ln Q^2} \sim P_{qg} \cdot x\,g$$

we can explain unusually steep $\frac{dF_2}{d\ln Q^2}$ by:

unusual $P_{qg} \rightarrow$ eg $\ln(^1/_x)$, BFKL

OR unusual $x\,g(x,Q_0^2) \rightarrow$ “valence-like” gluon etc.

$\rightarrow$ need to measure other gluon sensitive quantities at low x: F_L



Conventional NLO-DGLAP needs a valence-like gluon but a singular sea at lowQ^2
This does not get better at NNLO

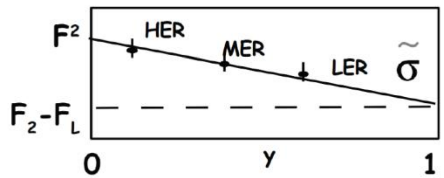
Longitudinal Structure Function

Longitudinal structure function FL is a pure QCD effect:

—> an independent way to probe sensitivity to gluon

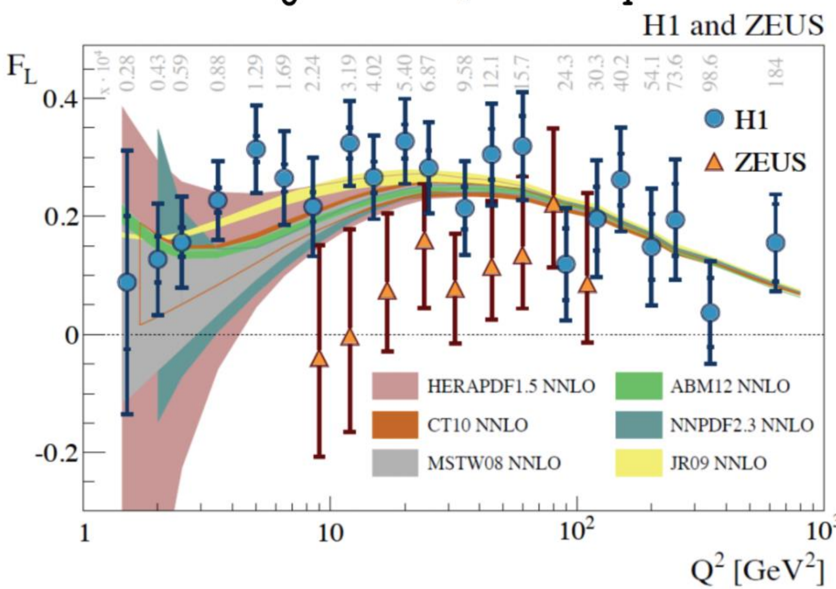
$$F_L = \frac{\alpha_s}{4\pi} x^2 \int_x^1 \frac{dz}{z^3}\left[\underbrace{\frac{16}{3}F_2}_{\text{quarks radiating a gluon}} + 8\sum_q \underbrace{e_q^2\left(1-\frac{x}{z}\right)zg(z)}_{\text{gluons splitting into quarks}}\right]$$

Direct measurement of F_L at HERA required differential cross sections at same x and Q^2 but different y —> different beam energies: Ep= 460, 575, 920 GeV

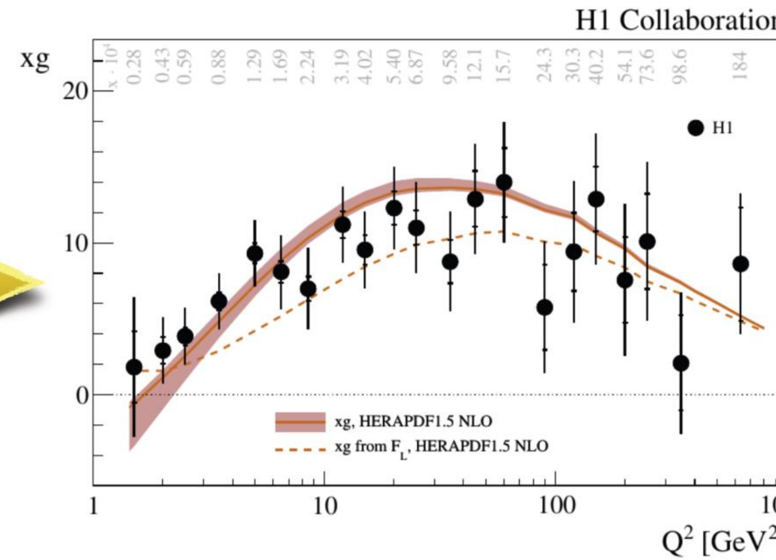


$$\sigma_{NC}(x,Q^2,y) \propto F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2}F_L(x,Q^2)$$

$$xg(x,Q^2) \approx 1.77\frac{3\pi}{2\alpha_S(Q^2)}F_L(ax,Q^2)$$

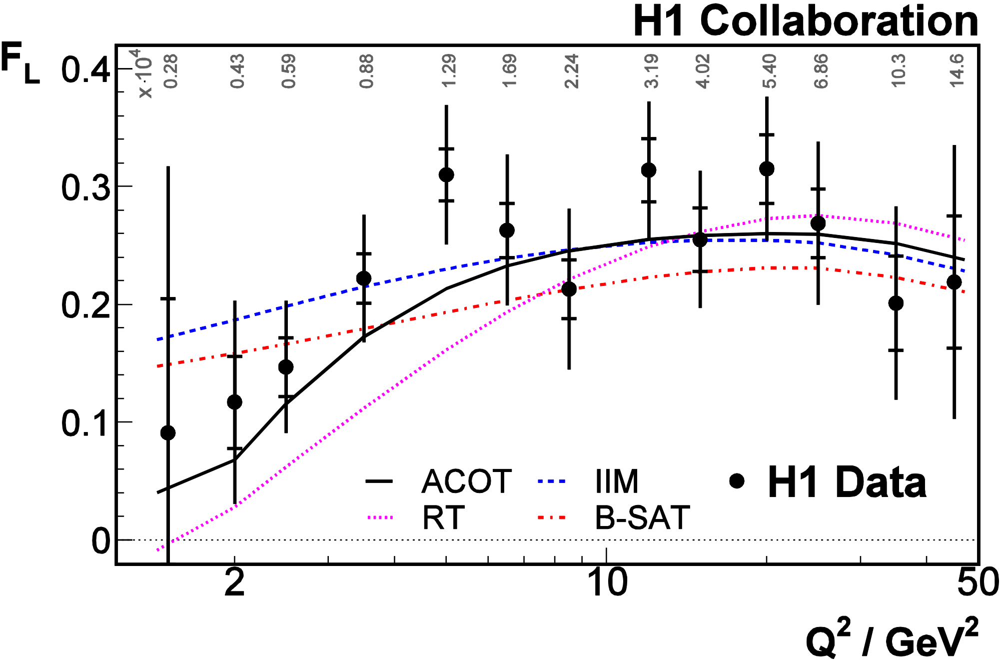


Consistency of H1 and ZEUS FL was checked accounting for corr. unc: $\chi 2/\mathrm{ndf} = 11/8$ (p-value = 20%)



Eur. Phys. J. C 74 (2014) 2814 [arXiv:1312.4821]

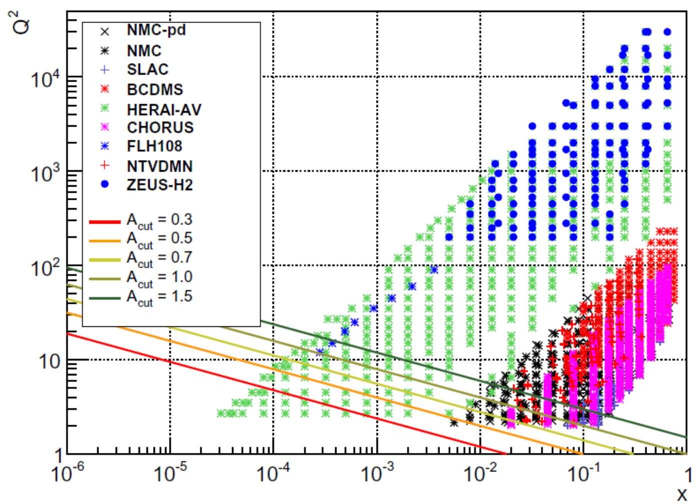
These are the final data on FL from ZEUS and H1

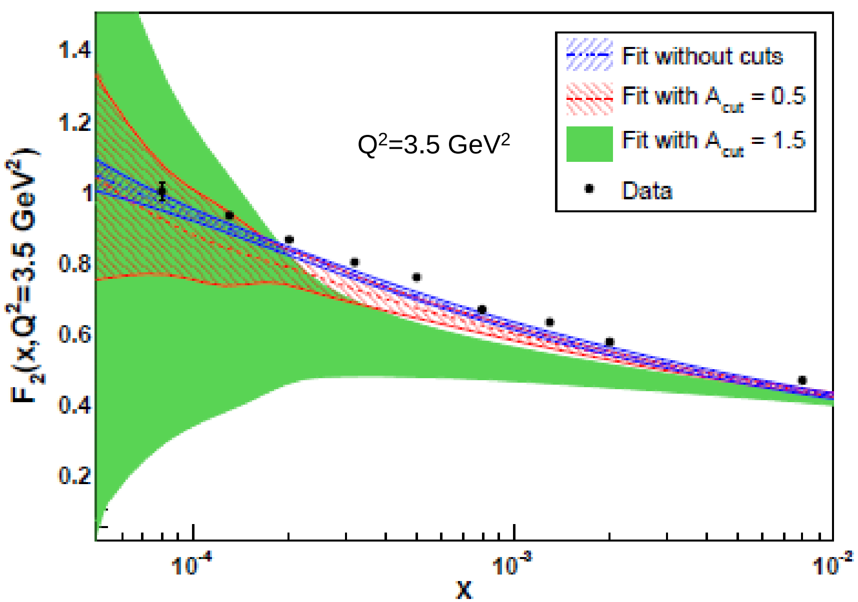


A slightly earlier version illustrating some saturation models and
different DGLAP predictions –
(ACOT and RT differ mostly in their treatment of FL to O(α_s) and O(α_s^2) respectively.)

It is not possible to tell if models beyond DGLAP such as saturation are needed.

Are there clever ways of looking at the inclusive data to uncover hints of something beyond DGLAP?





Caola *et. Al.*, arXiv:1007.5405 observe that the combined HERA-I data shows tension as cuts are made to cut out low-x,Q^2 data.
Cut $Q^2 > A_{cut}\ x^{-0.3}$
Such a cut is 'saturation inspired' At higher Q^2 the region moves to lower and lower x

If all is well then a fit done with harder cuts should be compatible with fits done without cuts (though obviously the uncertainties grow larger) when evolved backwards

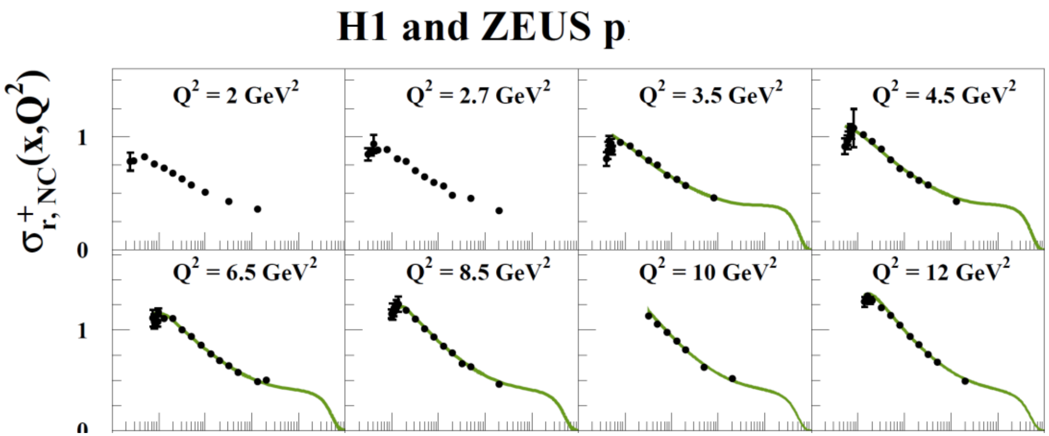
The fit with harder cuts undershoots the data, thus the this fit wanted more evolution of F_2 between Q^2=3.5 GeV^2 and Q^2=10 GeV^2 than is seen in the data. The fit was DGLAP at NLO.

NNLO gives even more evolution, which is not what is needed

Ln(1/x) resummation gives less evolution, this could help

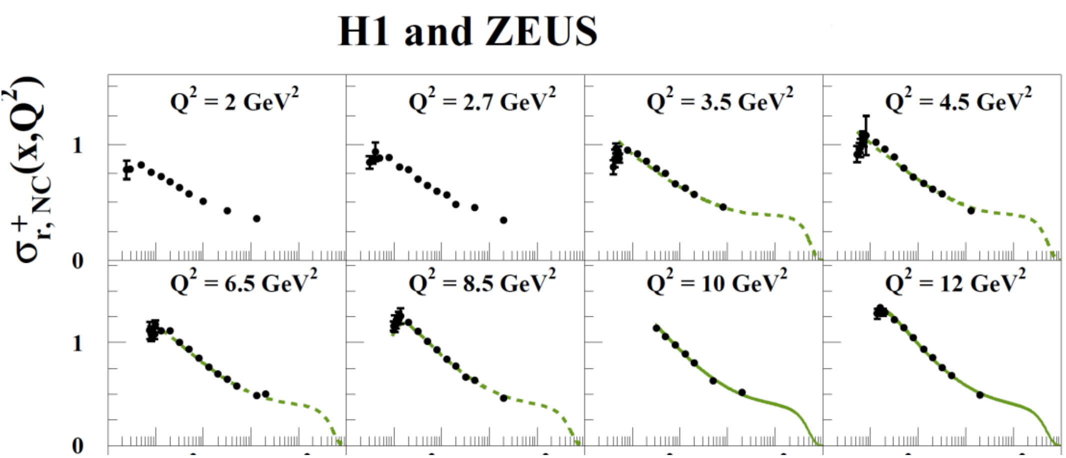
Saturation can also lead to less evolution

Look at the final HERA combined data arXiv:1506.06042



Let's take a look at the lowest Q^2 bins for a fit which includes data down to Q^2= 3.5 GeV^2

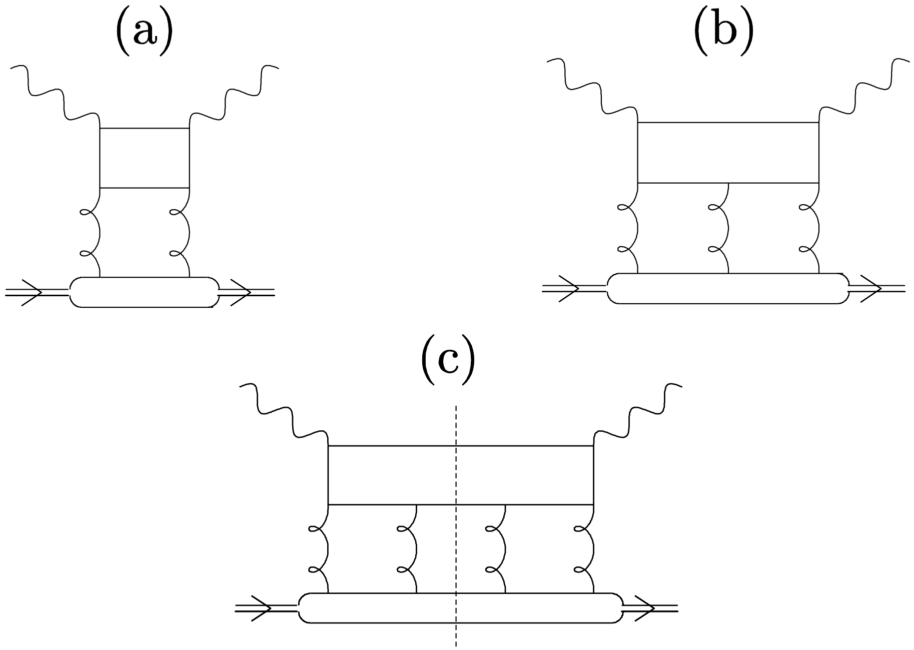
The NLO fit compromises between fitting the high-y turnover and fitting the data at slightly higher x 0.0001< x <0.001



Let's take a look at the lowest Q^2 bins for a fit which includes data down to Q^2= 10 GeV^2 (this cuts out most of the same events as a cut Q^2> 1.0 x $^{-0.3}$) Freed from having to describe these bins the fit undershoots the region 0.0001 to 0.001 more severely . The discrepancies with the fit are systematically worse at lower-x and lower Q^2.
This is similar to the observations of Caola et al on the HERA-I data

These are the NLO fits but NNLO is not better

One approach: (arXiv:1604.02299) consider adding higher twist terms at low-x



Their origin COULD be connected with recombination of gluon ladders- a non-linear evolution effect.

Bartels, Golec-Biernat, Kowalski suggest that such higher twist terms would cancel between σ_L and σ_T in F_2, but remain strong in F_L

Try the simplest of possible modification to the structure functions F_2 and F_L as calculated from HERAPDF2.0 formalism

$$F_{2,L} = F_{2,L}\,(1 + A_{2,L}^{HT}/Q^2)$$

Such a modification of F_L is favoured, whereas for F_2 it is not.

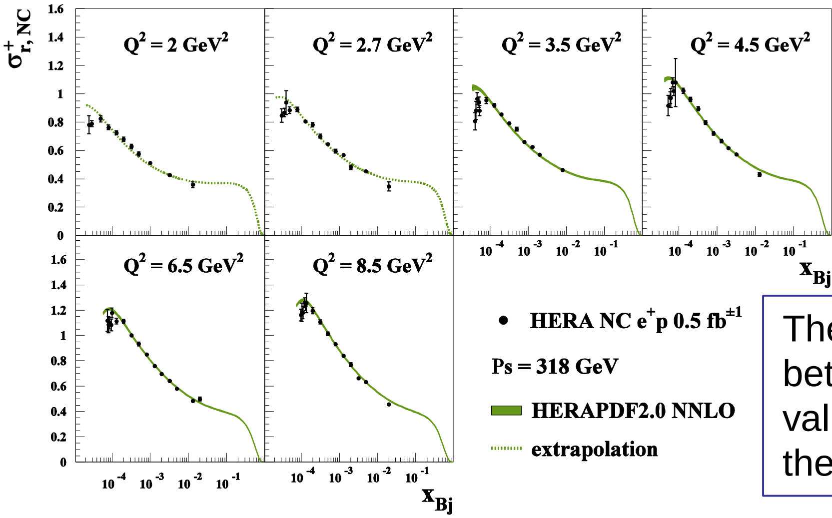
If A_L^{HT} is added $A_L^{HT} = 5.5 \pm 0.6\ \mathrm{GeV}^2$ and $\Delta\chi 2$ =-47

So now let's look at why the Higher Twist fits do so well

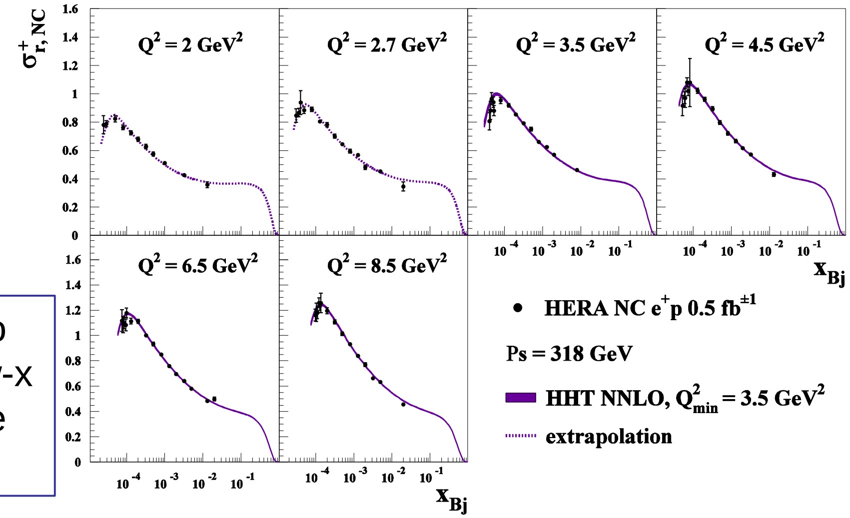
It is because they describe the turn over of the cross section at low x, Q2 much better

$$\sigma_{red} = F_2 - y^2/Y_+ F_L$$

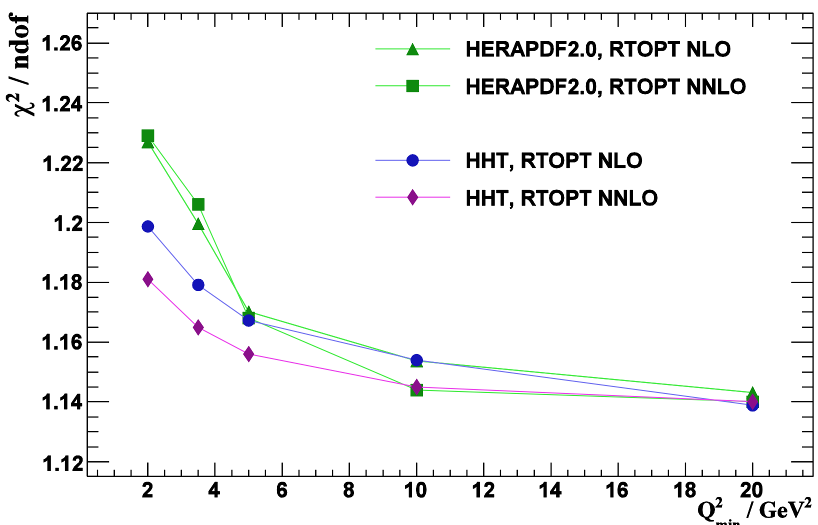
The data clearly wants a larger F_L and this is what the higher twist term provides



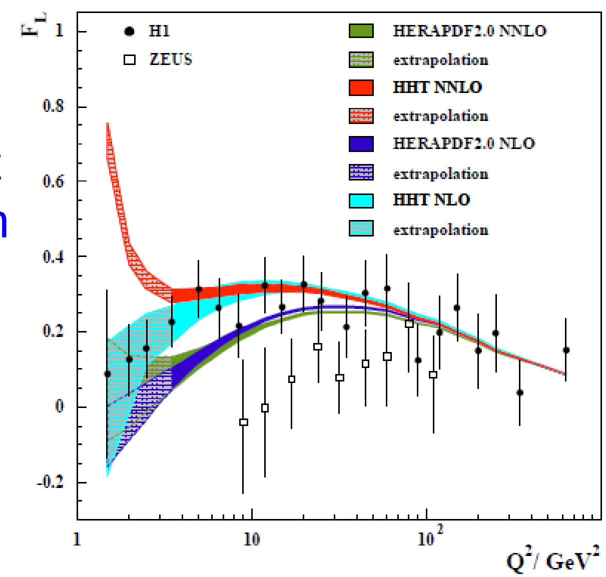
The fit is also better for low-x values above the turn over



You can also see that NNLO does better than NLO

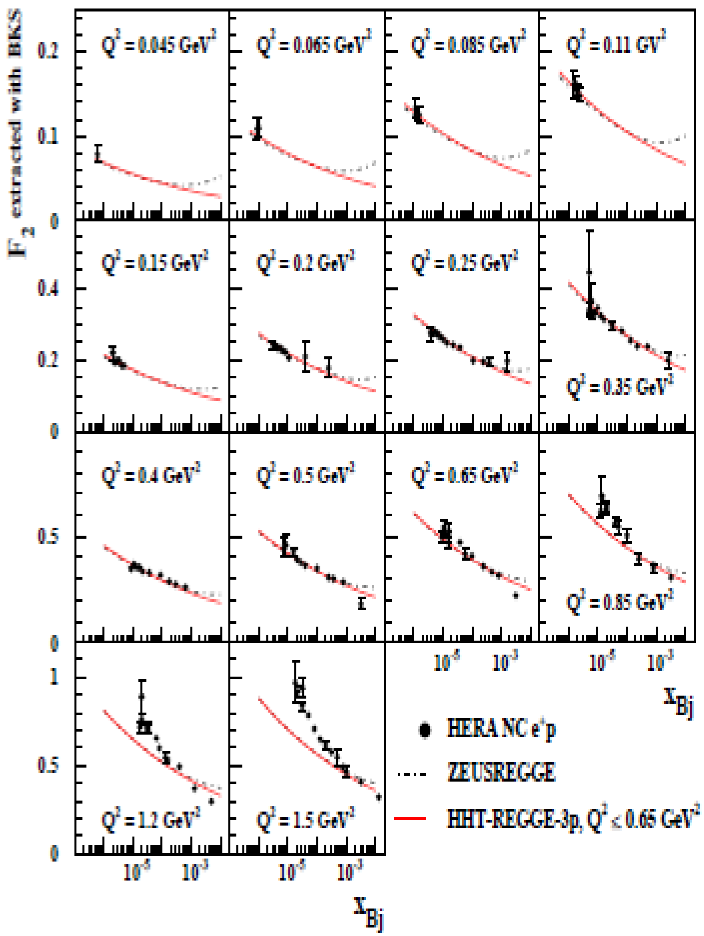


And here is what FL itself looks like. Clearly one cannot push this too low in Q^2



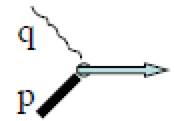
But there are many other approaches to looking for effects beyond DGLAP, consider the transition to the non-perturbative regime

Linear DGLAP evolution doesn't work for Q^2 < 1 GeV2, WHAT does? – REGGE ideas?



arXiv:1704.03187

Small x is high W^2, $x=Q^2/2p.q \quad Q^2/W^2$



$p_x^2 = W^2$

$\sigma(\gamma^* p) \sim (W^2)^{\alpha-1}$ – Regge prediction for high energy cross-sections

α is the intercept of the Regge trajectory
α=1.08 for the SOFT POMERON

Such energy dependence is well established from the SLOW RISE of all hadron-hadron cross-sections - including
$\sigma(\gamma p) \sim (W^2)^{0.08}$
for real photon- proton scattering

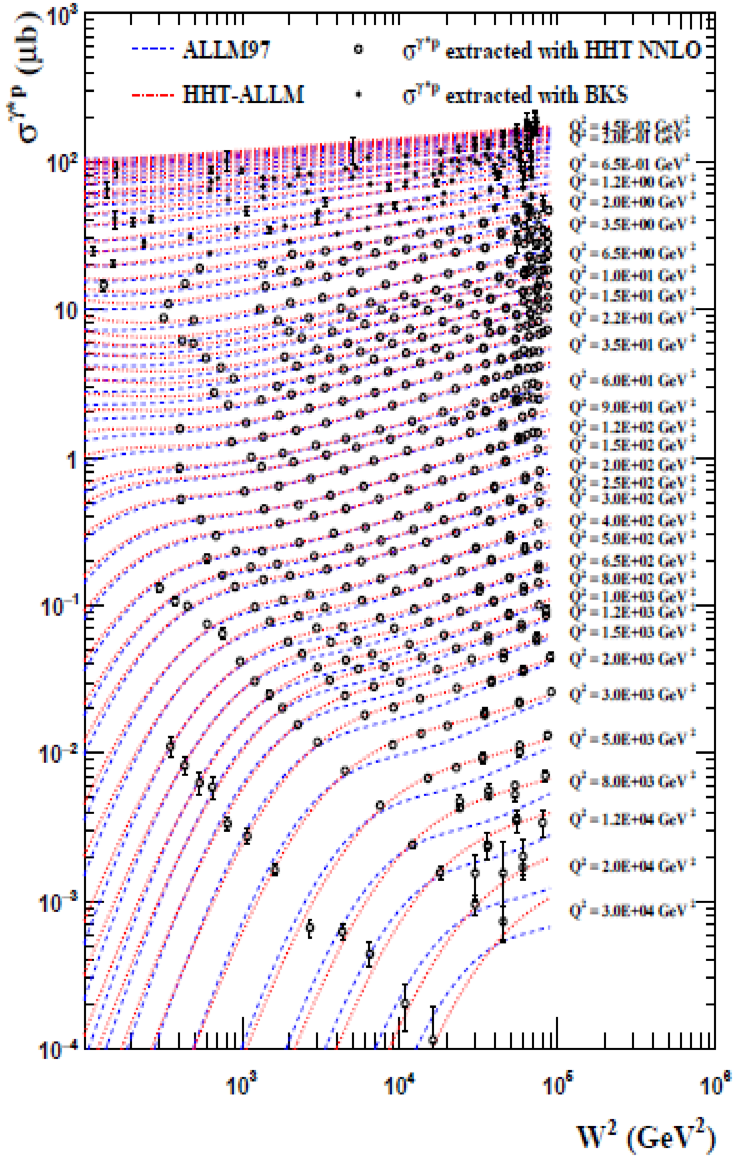
For virtual photons, at small x

$$\sigma(\gamma^* p) = \frac{4\pi^2\alpha \ F_2}{Q^2}$$

$\rightarrow \quad \sigma \sim (W^2)^{\alpha-1} \rightarrow F_2 \sim x^{1-\alpha} = x^{-\lambda}$

so a SOFT POMERON would imply
$\lambda = 0.08$ gives only a very gentle rise of F_2 at small x

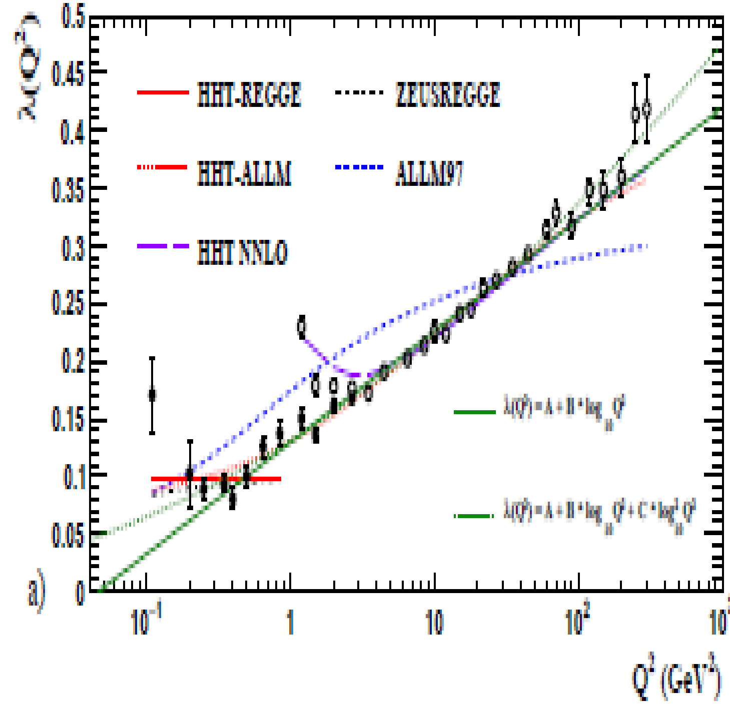
For Q^2 > 1 GeV^2 we have observed a much stronger rise…..



much steeper rise

gentle rise

The slope of F_2 at small x , $F_2 \sim x^{-\lambda}$, is equivalent to a rise of $\sigma(\gamma^*p) \sim (W^2)^\lambda$ which is only gentle for $Q^2 < 1$ GeV^2



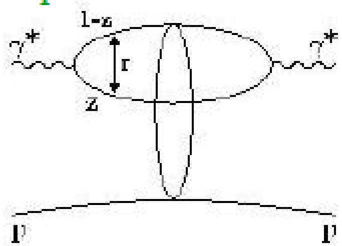
As well as the soft Pomeron, $\alpha - 1 = \lambda = 0.08$ (REGGE) should we consider

- a QCD POMERON, $\alpha(Q^2) - 1 = \lambda(Q^2)$- where this is the λ introduced on slide 14 (NNLO-DGLAP)
- a BFKL POMERON, $\alpha - 1 = \lambda \sim 0.5$
- a mixture of HARD and SOFT Pomerons to explain the transition $Q^2 = 0$ to high Q^2? (Donnachie and Landshoff mark2, or ALLM)

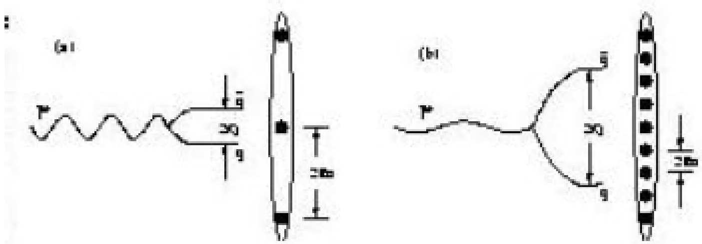
What about the Froissart bound ? – the rise MUST be tamed – non-linear effects?

Dipole models provide another way to model the transition $Q^2=0$ to high Q^2

At low x, γ^* to $q\bar{q}$ and the LONG LIVED ($q\bar{q}$) dipole scatters from the proton



The dipole-proton cross section depends on the relative size of the dipole $r \sim 1/Q$ to the separation of gluons in the target R_0

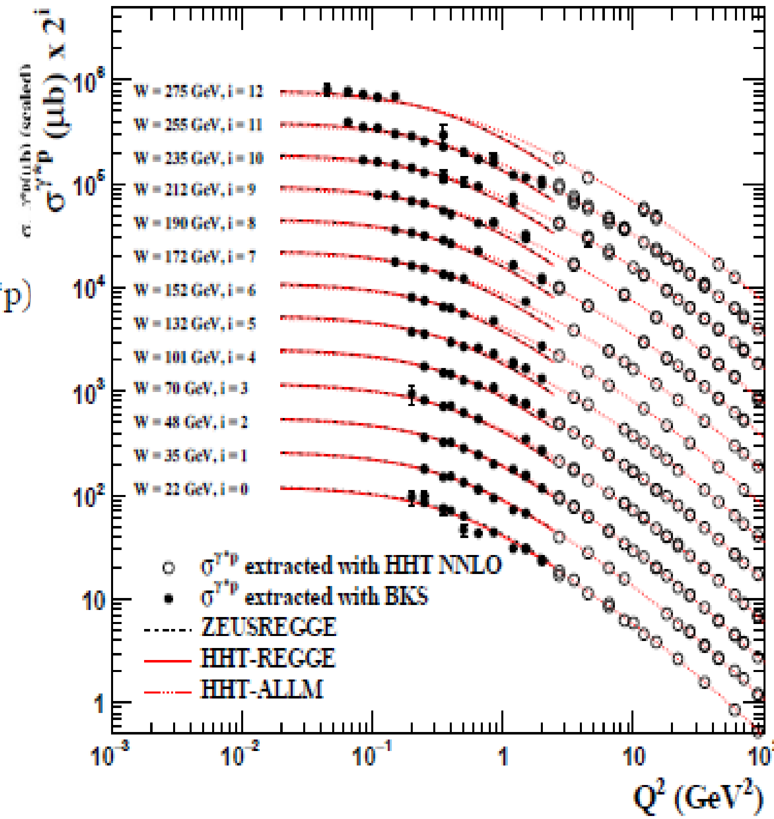


$\sigma = \sigma_0(1 - \exp(-r^2/2R_0(x)^2))$, $R_0(x)^2 \sim (x/x_0)^{\lambda} \sim 1/xg(x)$

r/R_0 small $\rightarrow$ large Q^2, x $\sigma \sim r^2 \sim 1/Q^2$, F2 flat Bjorken scaling	r/R_0 large $\rightarrow$ small Q2, x $\sigma \sim \sigma_0 \rightarrow$ saturation of the dipole cross-section

GBW dipole model

$\sigma(\gamma^* p)$



But $\sigma(\gamma^* p) = \dfrac{4\pi\alpha^2}{Q^2} F_2$ is general (at small x)

$\sigma(\gamma p)$ is finite for real photons, $Q^2=0$. At high Q^2, F_2 ~flat (weak $\ln Q^2$ breaking) and $\sigma(\gamma^* p) \sim 1/Q^2$

More sophisticated Dipole models have been developed in the context of non-linear evolution models with and without saturation. They often predict geometric scaling.

τ is a new scaling variable, applicable at small x

It can be used to define a `saturation scale' , $Q^2_s = 1/R_0^2(x) \sim \quad x^{-\lambda} \quad \sim x\, g(x)$, gluon density

- such that saturation extends to higher Q^2 as x decreases
- And INDEED, for x<0.01, $\sigma(\gamma^*p)$ depends only on τ, not on x, Q^2 separately

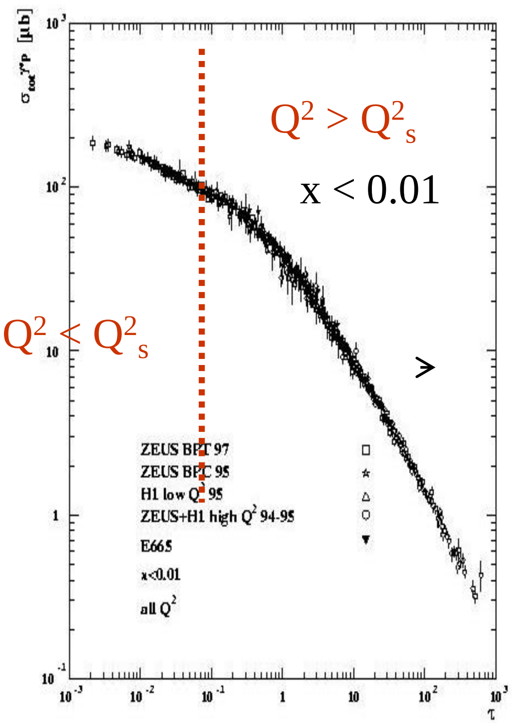
$$\sigma(\gamma^*p) = \sigma_0\,(1 - \exp(-1/\tau))$$

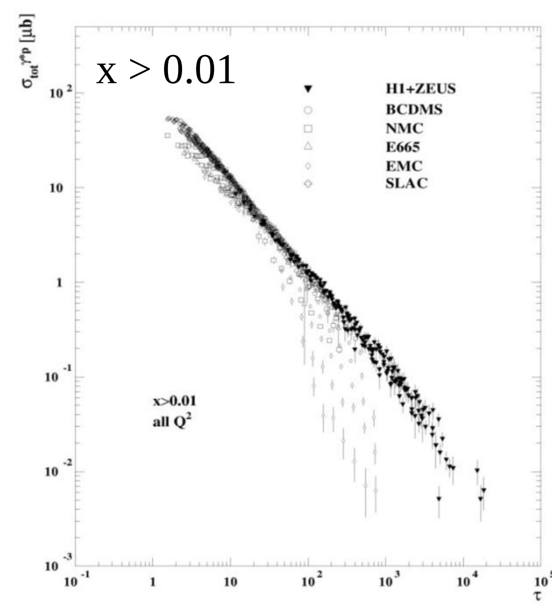
Involves only

$$\tau \quad = Q^2 R_0^2(x)$$

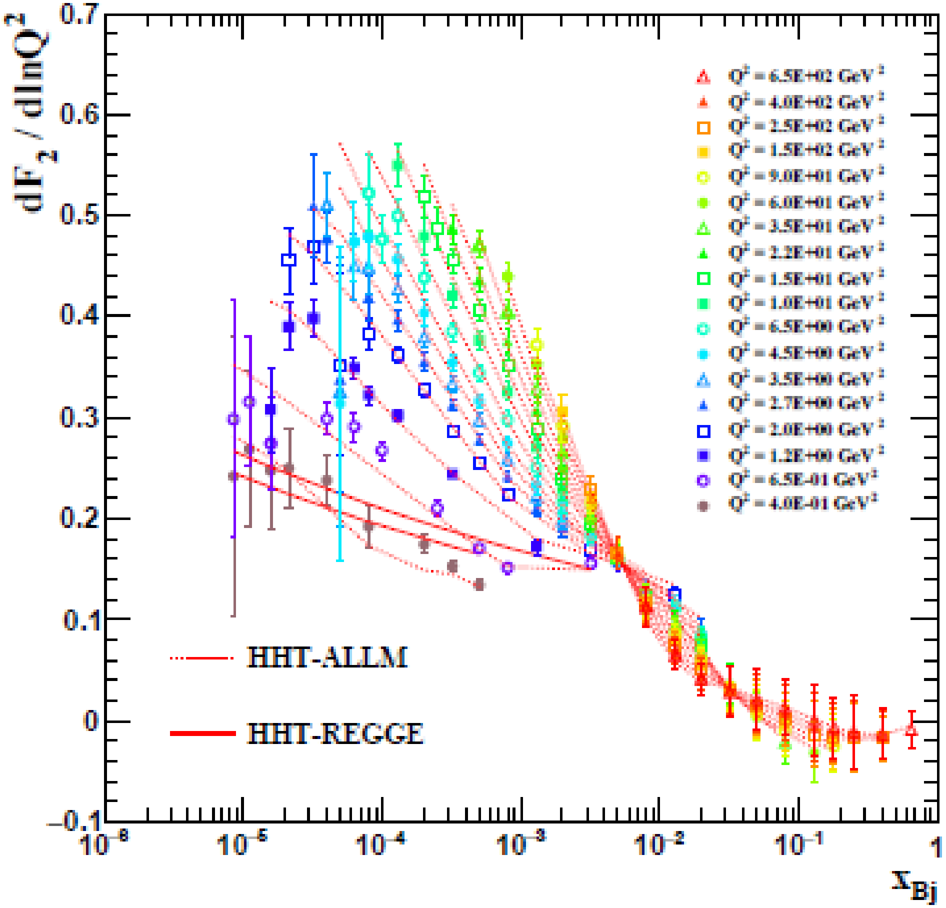
$$\tau = Q^2/Q_0^2\,(x/x_0)^{\lambda}$$

It is often said that geometric scaling has established evidence for saturation.
However it is possible to get geometric scaling over quite a large kinematic range from DGLAP/BFKL 'double asymptotic scaling'





Parting remarks



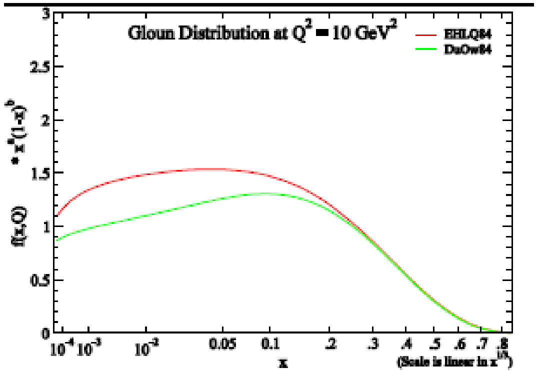
arXiv:1704.03187

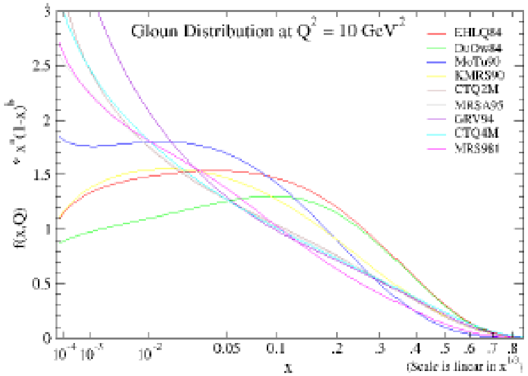
Here's a recently produced plot on $dF_2/d\ln Q^2$. At LO and x <~0.005 this quantity is directly related to the gluon PDF.
At very low x and Q^2 the turnovers could indicate saturation– a new state of high-density gluons- but one is also falling into the non-perturbative region. At HERA this is not definitive.

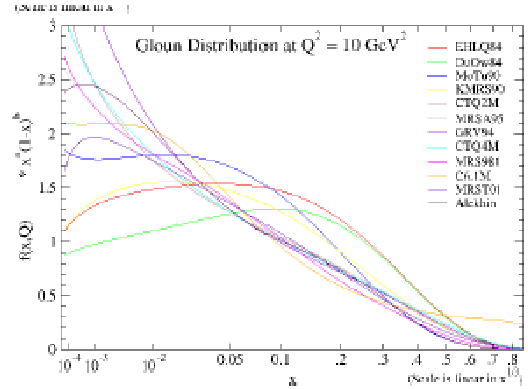
To really probe the high density region there are two ways:

- **A machine with lower x reach for higher Q^2 – the LHeC**
- **A machine with higher-density reach due to the use of nuclei -- the EIC**

Back-up





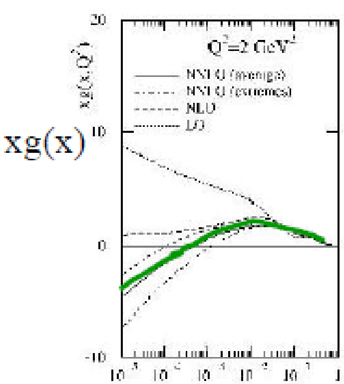


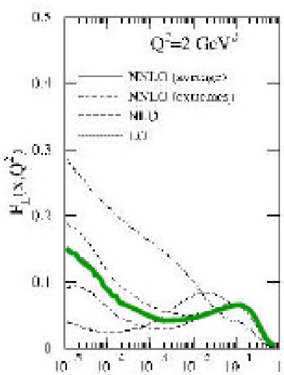
To recap what has happened…

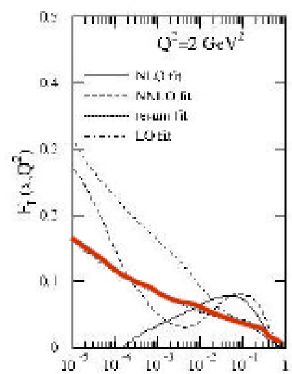
When HERA data were first published the gluon went from being flat to being steep at low-x

BUT when HERA data proved to still be steep at very low-Q^2 the DGLAP fits produced gluons which turn over again at low-x. the gluon evolves very fast- in order to evolve fast upwards it also evolves fast downwards – and this has consequences for F_L

$Q^2 = 2\text{GeV}^2$







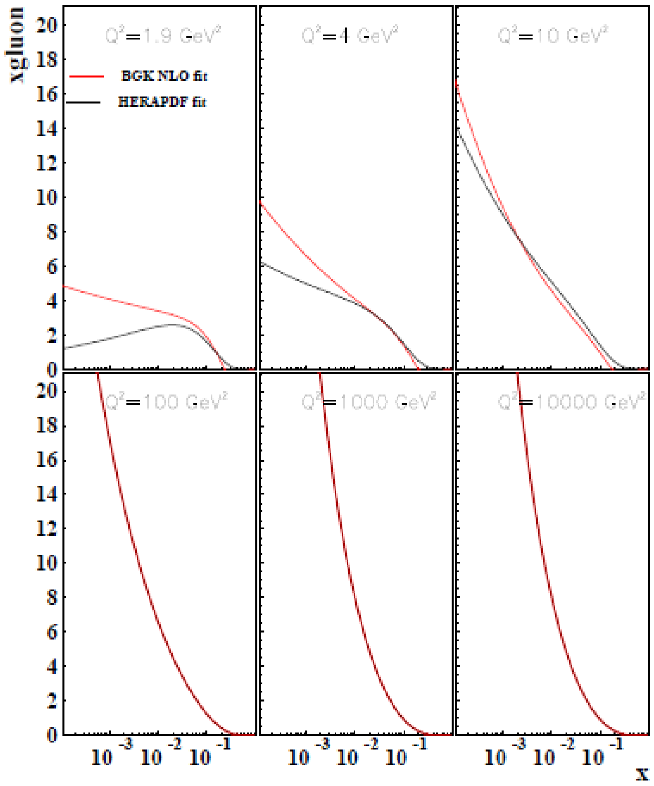
The negative gluon predicted at low x, low Q^2 from NLO DGLAP remains at NNLO (worse)

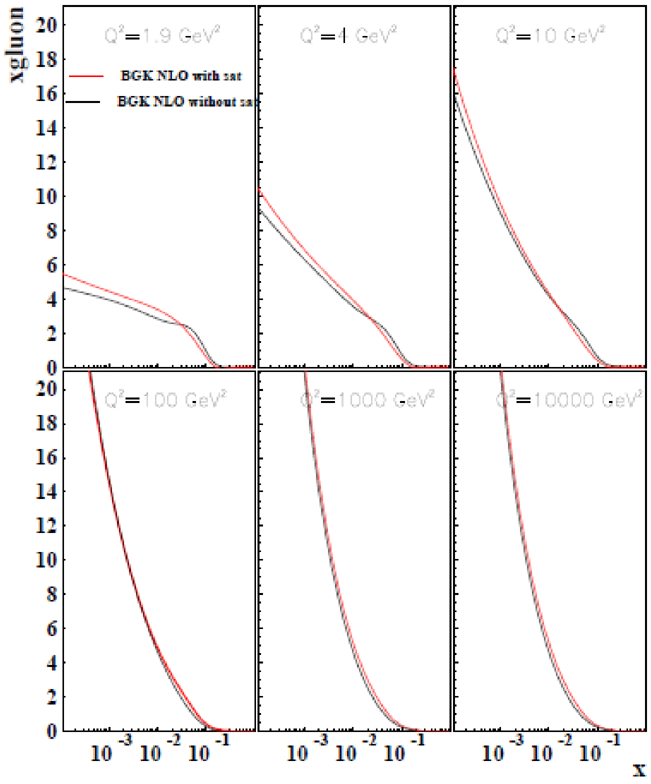
The corresponding F_L is NOT negative at $Q^2 \sim 2$ GeV^2 – but has peculiar shape

Including ln(1/x) resummation in the calculation of the splitting functions (BFKL `inspired') can improve the shape - and the χ^2 of the global fit improves

This indicates that you might want to go beyond DGLAP but it is not overwhelming

A very recent arXiV:1611.10100 attempt to establish saturation in the dipole picture uses the BGK (Bartels, Golec-Biernat, Kowalski) which combines a dipole model with DGLAP evolution of the gluon density





The BGK model gives a more reasonable shape to the low Q2 gluon, which is enhanced somewhat if saturation is included

➤ Two-staged analysis approach

- **Fit of the high-Q^2 regime: 3.5 – O(250) GeV^2**

  → Test of the validity of the dipole model in the regime still well described by DGLAP evolution

  → Test soft and hard gluon models

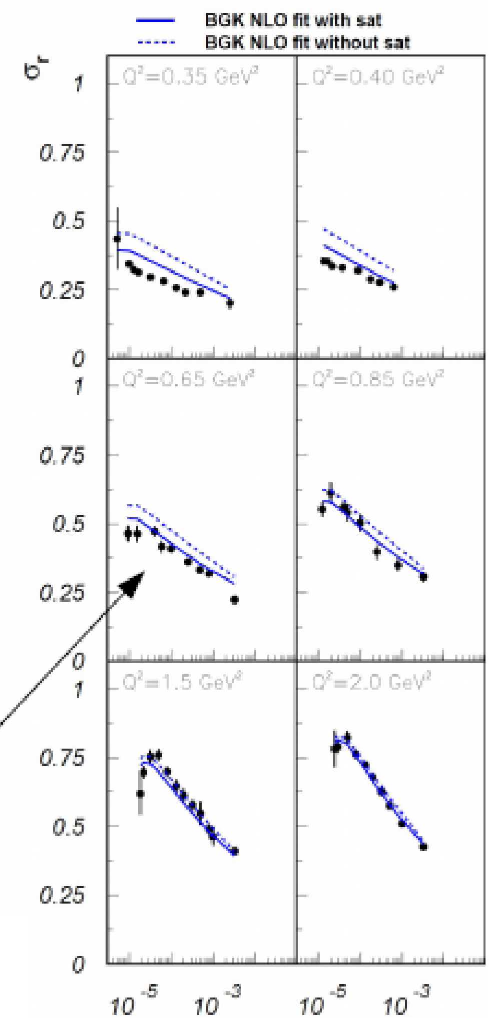
  → Test impact of valence quarks on the fits (previous data not sensitive enough)

- **Extend the fit to lower Q^2: 0.35 - 3.5 GeV^2**

  → Test whether dipole models can extend the perturbative regime also when confronted with much more precise data

  → Are there signs of saturation?

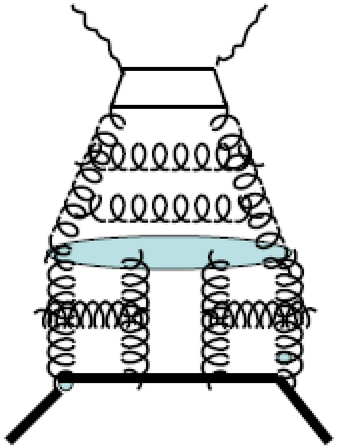
Maybe

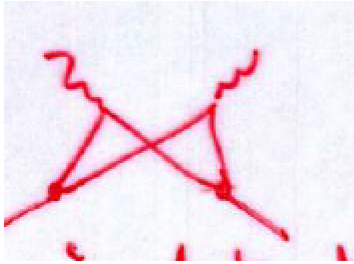


The use of non-linear evolution equations also *improves* the shape of the gluon at low x, Q^2

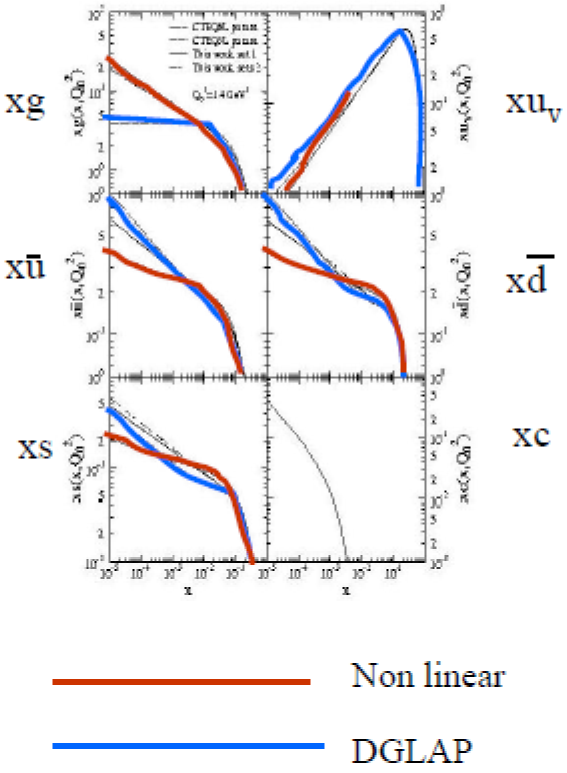
The gluon becomes steeper (high density) and the sea quarks less steep

Non-linear effects $gg \to g$ involve the summation of FAN diagrams – higher twist



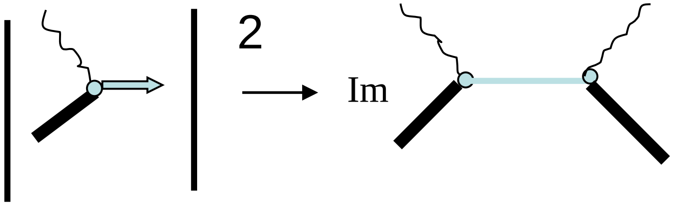


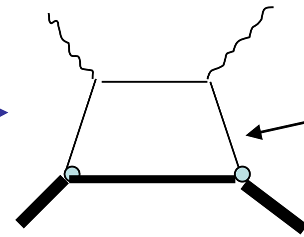
$Q^2 = 1.4$ GeV^2



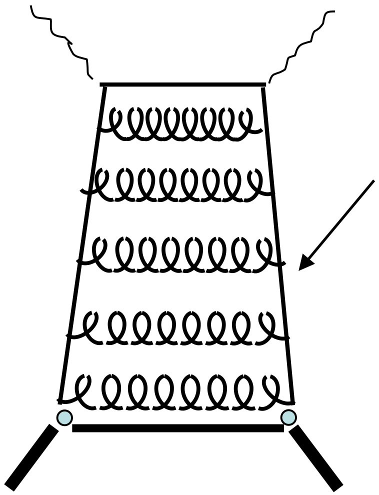
Need to extend the formalism?

Optical theorem





The handbag diagram- QPM



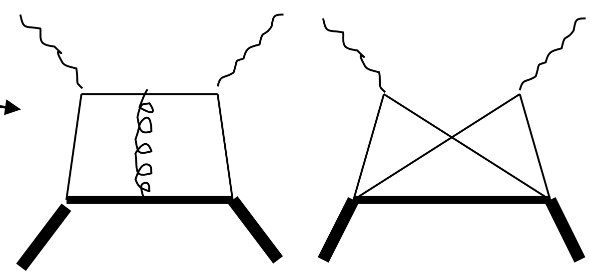
QCD at LL(Q^2)

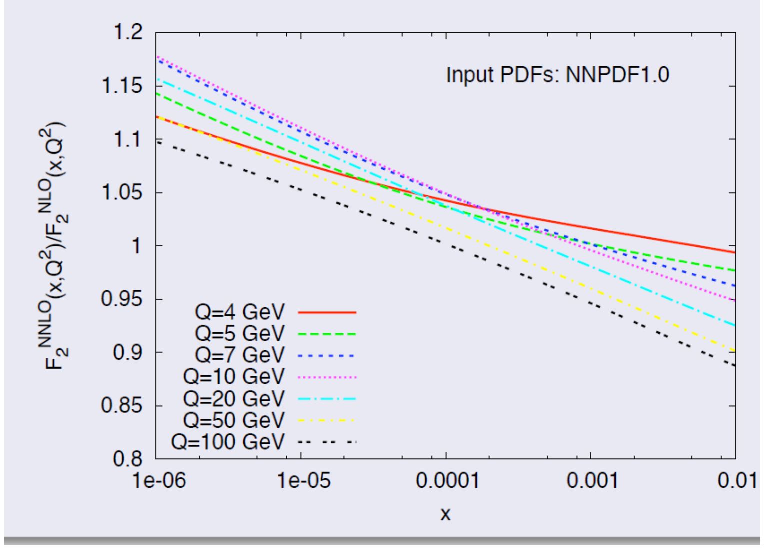
Ordered gluon ladders ($\alpha_s^n \ln Q^{2\,n}$)

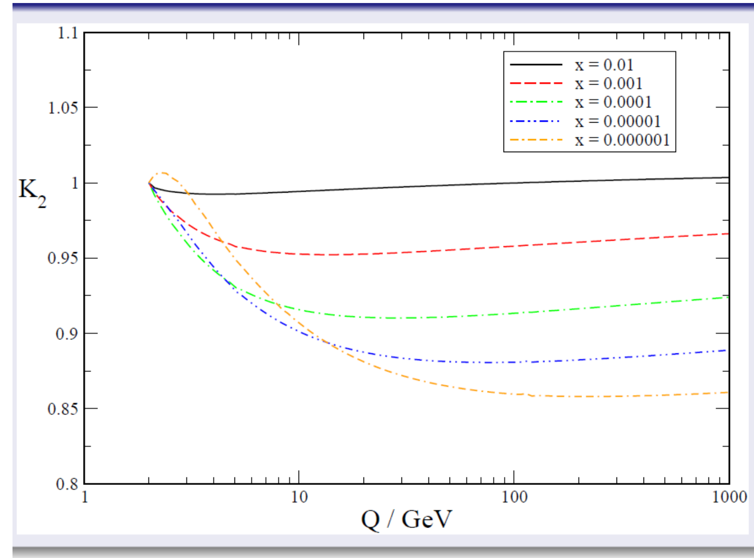
NLL(Q^2) one rung disordered $\alpha_s^n \ln Q^{2\,n-1}$

BUT what about completely disordered Ladders?
at small x there may be a need for BFKL ln(1/x) resummation?

And what about Higher twist diagrams ?
Are they always subdominant?
Important at high x, low Q^2





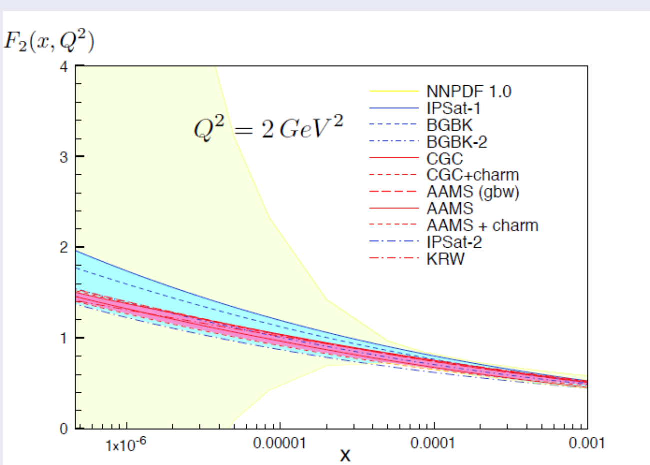


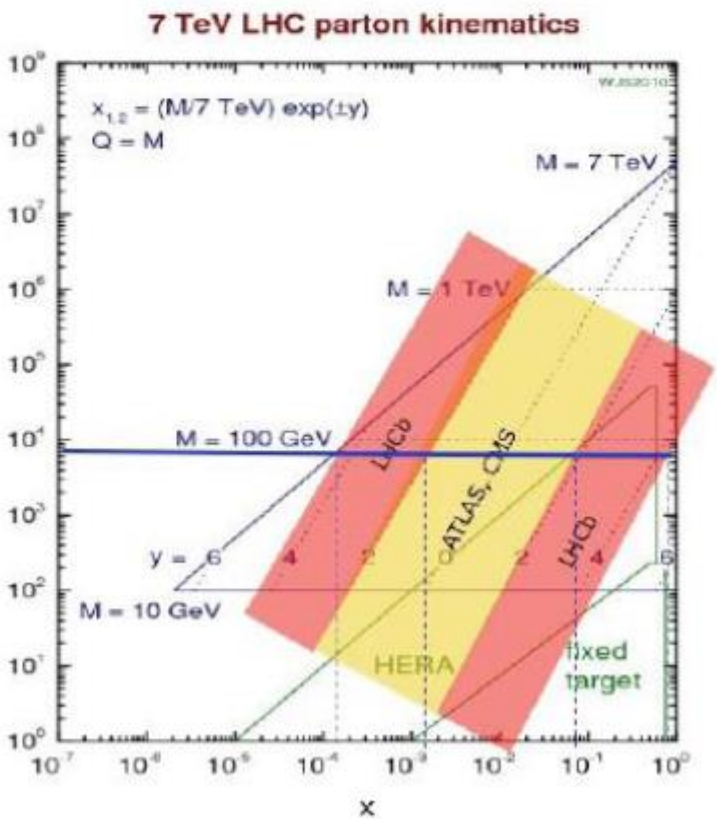
NNLO gives even more evolution, which is not what is needed

Ln(1/x) resummation gives less evolution, this could help

Another possibility: parton saturation

- Popular wisdom: saturation leads to less evolution
- Fixed scale: possible to absorb in distorted PDFs (see below
- Very difficult to compute Q^2 evolution!



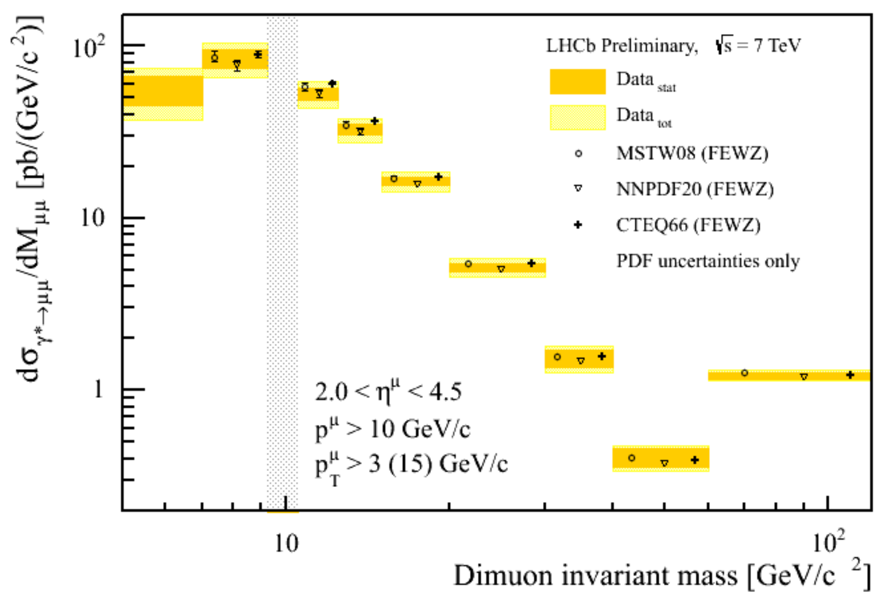


Does the inclusion of low x, Q2 data in the PDFs bias the predictions at the LHC?

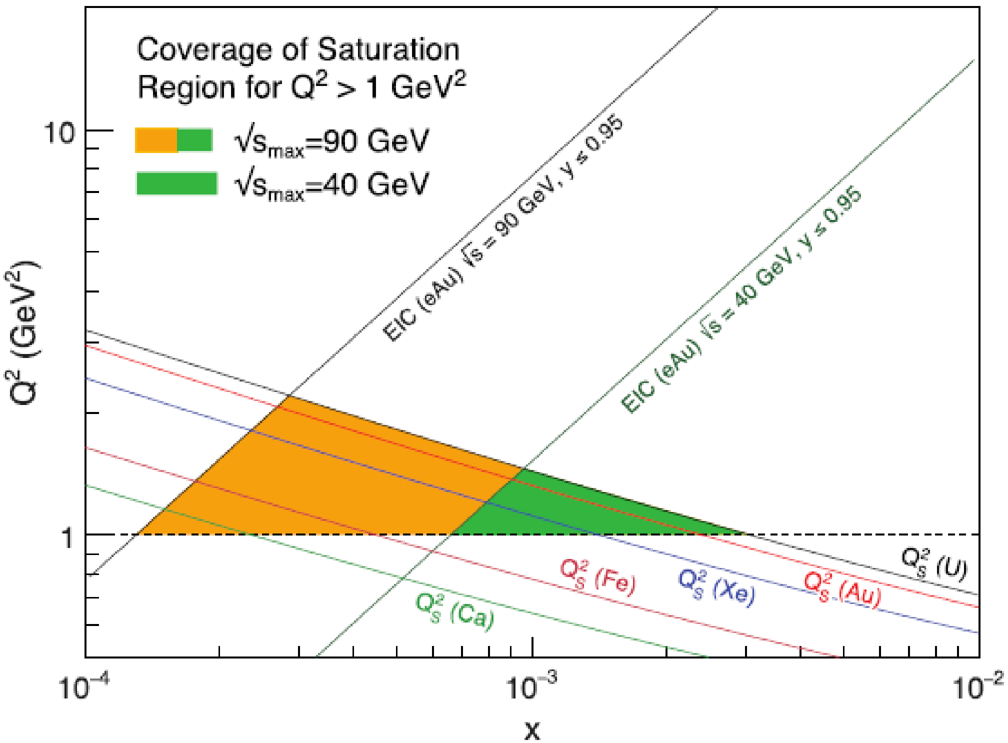
Not at high-x, Q^2 where predictions from Q^2 cut fits Q^2>10 GeV^2 agree with Q^2 cut fits Q^2>3.5 GeV^2

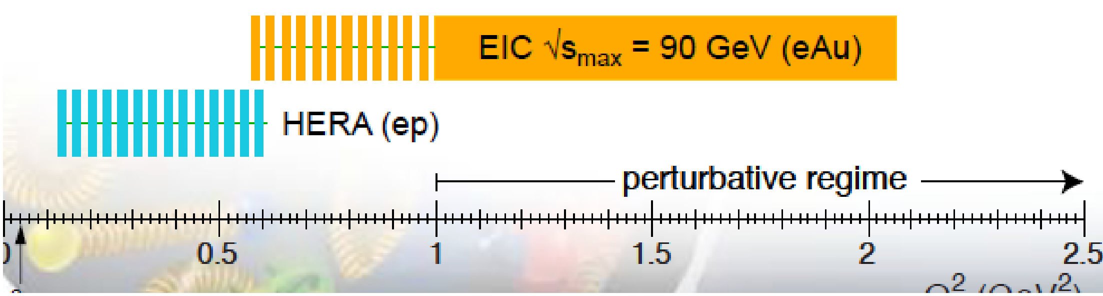
But it could matter at low-x (x < 0.0001) and moderate Q^2, Q^2 ~25-100 GeV^2.
This is the LHCb region

LHCb low-mass Drell-Yan data show no sign of deviation from DGLAP predictions– but this is a log plot, uncertainties are large



Gluon Saturation: √s and A Matter





eA at EIC:

- Need to push into regime where comparison with our current understanding can be made ⇒ $Q^2 > 1\ \mathrm{GeV/c}^2$
- Require sufficient lever arm in *Q*, *x* to study evolution
- CGC predicts characteristic *A* dependence ⇒ requires large *A* lever arm

- Need to push for highest energy and heaviest ions