# Odderon and twist-3 light-ray Operators

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# Outline

What it is the local operator whose anomalous dimension analytically continued to the *unphysical* point reproduces the Odderon intercept?

- Twist-2 gluon operator and anomalous dimension of light-ray operators.
- Odderon in dipole-Wilson lines formalism.
- Odderon and twist-3 operator.
- Scale dependence of Twist-3 operator and spinor formalism.
- Conclusions and Outlook.

# Anomalous dimension of local Operators

- Scale dependence of hadronic-cross section is driven by anomalous dimension of local operator
- Application of OPE in DIS: expand the moments of structure function in inverse power of the hard scale Q

$$F(j,Q^2) = \int_0^1 dx_B\, x_B^{j-2} F(x_B,Q^2) = \sum_{n=2}^{\infty} \frac{1}{Q^n} \sum_a C_n^a(j,\alpha_s(Q^2)) \langle P|\mathcal{O}_{n,j}^a(0)|P\rangle$$

## Anomalous dimension of local Operators

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After diagonalization of mixing matrix we get the multiplicatively renormalizable operators

$$Q^2 \frac{d}{Q^2} \langle P|O_{n,j}^a(0)|P\rangle = \gamma_n^a(j) \langle P|O_{n,j}^a(0)|P\rangle$$

Perurbative series of anomalous dimension:

$$\gamma_n^a(j) = \sum_{k=1}^{\infty} \gamma_n^a(k,j) \left(\frac{\alpha_s(Q^2)}{\pi}\right)^k$$

# Moments of structure function at low-x region

- Regge behavior of structure function: $F(x_B, Q^2) \sim x_B^{1-\alpha}$ ($\alpha - 1$ is the intercept)

- Scale dependence of structure function at low-$x_B$
  - for $x_B \to 0$, $F(j, Q^2)$ has poles at $j \sim 1$: *unphysical* point
  - analytically continue the anomalous dimension $\gamma_n^a(j)$ from integer $j \geq 2$ to $j \sim 1$ and then invert the moments of structure functions

- Twist $n = 2$: use DGLAP expression for $\gamma_{n=2}^a(j)$

- Twist $n \geq 3$ case is difficult already at $j \geq 2$
  - Number of operators increases with twist
  - size of corresponding matrices depends on $j$
  - $\Rightarrow$ analytical continuation to *unphysical* point of the anomalous dimension of twist $n = 3$ operator is even more difficult.

# Alternative strategy

- Calculate small-x behavior of structure functions $F(x_B, Q^2)$ within the BFKL formalism.

- and compare it with twist expansion of the moments of the structure functions $F(j, Q^2)$ (Jaroszewicz (1982))

Gluonic structure function at low-$x_B$. DIS: $\Lambda_{QCD} \ll P^2 = M^2 \ll Q^2 \ll s$

- $Q^2$ behavior of gluon structure function are driven by the anomalous dimension of twist-2 gluonic operator

$$\mu \frac{d}{\mu} F^a_{\mu +} \nabla_+^{j-2} F_+^{\mu a} = \gamma(\alpha_s, j) F^a_{\mu +} \nabla_+^{j-2} F_+^{\mu a}$$

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- Gluonic structure function obtained in the BFKL formalism

$$F(x_B, Q^2) = \frac{1}{2\pi}\int d\nu\, f(\nu)\, x_B^{-1-N(\nu)}\left(\frac{Q^2}{P^2}\right)^{\frac{1}{2}+i\nu}$$

  $N(\nu)$ pomeron intercept;

- To write the low-$x_B$ structure function in this form at NLO one has to
  - use the NLO impact factor obtained with the composite Wilson line formalism (I. Balitsky G.A.C (2012))
  - and use the NLO BFKL eigenfunctions (G.A.C, Yu. Kovchegov (2013))

# Alternative strategy

$\gamma = \frac{1}{2} + i\nu$

$$F(j,Q^2) = \int_0^1 dx_B\, x_B^{j-2} F(x_B,Q^2) = \frac{1}{2\pi i}\int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} d\gamma\, \frac{f(\gamma)}{j-2-N(\gamma)}\left(\frac{Q^2}{P^2}\right)^{\gamma}$$

- analytic continuation: $j-2 \to \omega$ complex continuous variable;
- Residues $\omega = N(\gamma)$; expand $N(\gamma)$ for small $\gamma$ and solve for $\gamma$

$$\gamma(\alpha_s,\omega) = \frac{\alpha_s N_c}{\pi\omega} + \mathcal{O}(\alpha_s^2)\,, \qquad\qquad F(\omega,Q^2) \sim \left(\frac{Q^2}{P^2}\right)^{\frac{\alpha_s N_c}{\pi\omega}}$$

We get the analytic continuation of anomalous dimension at the *unphysical* point $j \to 1$ of twist-2 operators: $F^a_{\mu+}\nabla_+^{-1}F_+^{\mu a}$

# Light-ray operator

- This procedure does not tell us the explicit form of the operator $F^a_{\mu+}\nabla_+^{\omega-1}F_+^{\mu a}$ at the *unphysical* point $\omega \to 0$.

- the operator is a light-ray operator (Balitsky-Kazakov-Sobko (2013-2016), see also Balitsky (2014))

$$\mathcal{F}_\omega(x_\perp) = \int_0^\infty dL_- L_-^{-\omega} \int dx_-\, F^a_{+i}(L_- + x_- + x_\perp)[L_- + x_-, x_-]^{ab} F_+^{bi}(x_- + x_\perp)$$

The anomalous dimension of this operator is the analytic continuation of the anomalous dimension of local operator $F^a_{\mu+}\nabla_+^{j-2}F_+^{\mu a}$ with j integer.

- How to see this:
  - The expansion on the light-cone of the evolution equation of non-local (light-ray) operator reproduces the anomalous dimension of local operator (Balitsky Braun (1982))
  - Due to this observation the identification of $\mathcal{F}_\omega(x_\perp)$ as the analytic continuation of local operator to the *unphysical* point is trivialized.

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- What can we say about twist-3 operators?

- What has to do Odderon with all this?

# Low-$x$ evolution of Odderon

- In the linear case the Odderon follows a dipole-BFKL type of evolution equation (Kovchegov-Szymanowski-Wallon (2003))
- To solve the evolution equation use BFKL eigenfunctions with odd n
  - $E^{n,\nu}(z_1,z_2) = \left(\frac{z_{12}}{z_1 z_2}\right)^{\frac{1+n}{2}+i\nu} \left(\frac{z^*_{12}}{z^*_1 z^*_2}\right)^{\frac{1+n}{2}-i\nu} = (-1)^n E^{n\nu}(z_1,z_1)$
  - The leading high-energy intercept is $N_{odd} - 1 = 2\frac{\alpha_s N_c}{\pi}\chi(n=1,\nu=0) = 0$ (Bartels-Lipatov-Vacca (2000))
  - The odderon intercept is equal to 1 to all loop order in the planar Limit (Caron-Huot (2015))
  - Hint in this direction is also coming from integrability and AdS-CFT Correspondence (Alfimov Gromov Kazakov (2015)).

In Wilson line formalism Odderon is understood as

$$\mathrm{tr}\{U(x_\perp)U^\dagger(y_\perp)\} - \mathrm{tr}\{U^\dagger(x_\perp)U(y_\perp)\}$$

where $U(x_\perp) = \mathrm{P}\left\{ig\int dx^+ A^-(x^+ p_1 + x_\perp)\right\}$ with $p_1^\mu$ light-con vector

# Odderon in Wilson lines formalism

- Let us find a relation between Odderon and light-ray operators
- choose $y_\perp = -x_\perp$ and expand Wilson lines for $x_\perp \to 0$ to the first non trivial order

$$\mathrm{tr}\{U(x_\perp)U^\dagger(-x_\perp)\} \sim x^i x^j x^k \int dx_1 dx_2 dx_3\, d^{abc}\, F^a_{+i}(x_1)\, F^b_{+j}(x_2)\, F^c_{+k}(x_3)$$

the resulting operator is a twist-3 operator. Find its scale dependence.

Introduce spinor notation (Braun-Manashov-Rohrwild (2009); Braun-Manashov-Pirnay (2012))

$$F_{\alpha\beta,\dot{\alpha}\dot{\beta}} = \sigma^{\mu}_{\alpha\dot{\alpha}}\sigma^{\nu}_{\beta\dot{\beta}}F_{\mu\nu} = 2\left(\epsilon_{\dot{\alpha}\dot{\beta}}f_{\alpha\beta} - \epsilon_{\alpha\beta}\bar{f}_{\dot{\alpha}\dot{\beta}}\right)$$

Here $f_{\alpha\beta}$ and $\bar{f}_{\dot{\alpha}\dot{\beta}}$ are chiral and antichiral symmetric tensors, $f^* = \bar{f}$, which belong to $(1,0)$ and $(0,1)$ representations of the Lorenz group, respectively.

- Consider the following twist-3 operator ($P_{ij}$ permutation operators acting on the fields coordinates)

$$\mathcal{F}^{\pm}(z) = 2gC^{abc}_{\pm}\tilde{s}^{\rho}\,(1 \mp P_{23} \pm P_{12})F_{+}{}^{\nu,a}(z_1)F^{b}_{+\rho}(z_2)F^{c}_{+\nu}(z_3)$$

  - rewrite it in spinor notation

$$\mathcal{F}^{\pm}(z) = -\frac{ig}{\sqrt{2}}C^{abc}_{\pm}\left\{s_{\mu\bar{\lambda}}\bar{f}^{a}_{++}(z_1)f^{b}_{++}(z_2)f^{c}_{++}(z_3) - s_{\lambda\bar{\mu}}f^{a}_{++}(z_1)\bar{f}^{b}_{++}(z_2)\bar{f}^{c}_{++}(z_3)\right\}$$

$C^{abc}_{+} = f^{abc}$ and $C^{abc}_{-} = d^{abc}$.

# Scale dependence of twist-3 operator

We need $d^{abc}$ since Odderon is odd under Charge conjugation.

$$\mathcal{F}^{-}(z) = -\frac{ig}{\sqrt{2}}\, d^{abc}\left\{ s_{\mu\bar{\lambda}}\bar{f}^{a}_{++}(z_1) f^{b}_{++}(z_2) f^{c}_{++}(z_3) - s_{\lambda\bar{\mu}} f^{a}_{++}(z_1)\bar{f}^{b}_{++}(z_2)\bar{f}^{c}_{++}(z_3)\right\}$$

- Observation: $f f \bar{f}$ and $\bar{f}\bar{f} f$ types of operators do not mix under renormalization.

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \frac{\alpha_s}{4\pi}\mathbb{H}^{\pm}\right)\mathcal{F}^{\pm} = 0$$

where

$$\mathbb{H}^{\pm} = \begin{pmatrix} \mathbb{H}^{\pm}_{QQ} & \mathbb{H}^{\pm}_{QF} \\ \mathbb{H}^{\pm}_{FQ} & \mathbb{H}^{\pm}_{FF} \end{pmatrix}$$

we are interested in gluodynamics so we only need ($b_0 = \frac{11}{3}N_c - \frac{2}{3}n_f$)

$$\mathbb{H}^{-}_{FF} = N_c\Big(\widehat{\mathcal{H}}_{12} + \widehat{\mathcal{H}}_{23} + \widehat{\mathcal{H}}_{31} - 6(\mathcal{H}^{-}_{12} + \mathcal{H}^{-}_{13})\Big) - b_0$$

# Evolution kernels

we also need the Bukhvostov, Frolov, Lipatov Kuraev (BFLK) kernels

$$[\widehat{\mathcal{H}}\ \varphi](z_1,z_2) = \int_0^1 \frac{d\alpha}{\alpha}\Big[2\varphi(z_1,z_2) - \bar\alpha^{2j_1-1}\varphi(z_{12}^\alpha,z_2) - \bar\alpha^{2j_2-1}\varphi(z_1,z_{21}^\alpha)\Big]\,,$$

$$[\mathcal{H}^d\varphi](z_1,z_2) = \int_0^1 d\alpha\,\bar\alpha^{2j_1-1}\alpha^{2j_2-1}\,\varphi(z_{12}^\alpha,z_{12}^\alpha)\,,$$

$$[\mathcal{H}^+\varphi](z_1,z_2) = \int_0^1 d\alpha\int_0^{\bar\alpha} d\beta\,\bar\alpha^{2j_1-2}\bar\beta^{2j_2-2}\,\varphi(z_{12}^\alpha,z_{21}^\beta)\,,$$

$$[\widetilde{\mathcal{H}}^+\varphi](z_1,z_2) = \int_0^1 d\alpha\int_0^{\bar\alpha} d\beta\,\bar\alpha^{2j_1-2}\bar\beta^{2j_2-2}\,\left(\frac{\alpha\beta}{\bar\alpha\bar\beta}\right)\varphi(z_{12}^\alpha,z_{21}^\beta)\,,$$

$$[\mathcal{H}^-\varphi](z_1,z_2) = \int_0^1 d\alpha\int_{\bar\alpha}^1 d\beta\,\bar\alpha^{2j_1-2}\bar\beta^{2j_2-2}\,\varphi(z_{12}^\alpha,z_{21}^\beta)\,,$$

$$[\mathcal{H}_{12}^{e,(k)}\varphi](z_1,z_2) = \int_0^1 d\alpha\,\bar\alpha^{2j_1-k-1}\,\alpha^{k-1}\varphi(z_{12}^\alpha,z_2),$$

# Scale dependence of odderon twist-3 operator

- Odderon twist-3 operator in spinor notation

$$\mathrm{tr}\{U(x_\perp)U^\dagger(-x_\perp)\} \sim x^i x^j x^k \int dx_1 dx_2 dx_3\, d^{abc}\, F^a_{+i}(x_1)\, F^b_{+j}(x_2)\, F^c_{+k}(x_3)$$
$$\propto \left[f^a_{++}(x_1) - \bar{f}^a_{++}(x_1)\right]\left[f^b_{++}(x_2) - \bar{f}^b_{++}(x_2)\right]\left[f^c_{++}(x_3) - \bar{f}^c_{++}(x_3)\right]$$

- $fff$ and $\bar{f}\bar{f}\bar{f}$ are $3/2$ helicity operators and do not couple to the proton state,
- so in the forward matrix element only $ff\bar{f}$ and $\bar{f}\bar{f}f$ types of operator survive.
- It turns out that

$$x^i x^j x^k\, d^{abc}\, \langle P|F^a_{+i}(x_1)\, F^b_{+j}(x_2)\, F^c_{+k}(x_3)|P\rangle$$
$$\propto x^i d^{abc} \langle P|F^a_{+i}(u)F^b_{+j}(t)F^{c\,j}_{+}(x_3) + F^a_{+j}(x_1)F^b_{+i}(x_2)F^{c\,j}_{+}(x_3) + F^a_{+j}(x_1)F^{b\,j}_{+}(x_2)F^c_{+i}(x_3)|P\rangle$$

# Scale dependence of odderon twist-3 operator

- This operator is not multiplicative renormalizable so we cannot compute its anomalous dimension and its analytic continuation to the *unphysical* point.

- Idea:
  - We know that the Odderon intercept is $0$: $\chi(n=1,\nu=0)=0$
  - Consider the following forward light-ray operator and check whether it is $0$ in the *unphysical* point.

$$\int_0^{+\infty} du\, u^{-\omega} \int_0^u dt\, [\mathbb{H}^-_{FF} + b_0] \otimes x^\mu_\perp \Big[ F^a_{+\mu}(u) F^b_{+j}(t) F^{c\,j}_{+}(0) + F^a_{+j}(u) F^b_{+\mu}(t) F^{c\,j}_{+}(0) + F^a_{+j}(u) F^{b\,j}_{+}(t) F^c_{+\mu}(0) \Big]$$

$$\propto \int_0^{+\infty} du\, u^{-\omega} \int_0^u dt\, [\mathbb{H}^-_{FF} + b_0] \otimes \Big[ f^a_{++}(u) f^b_{++}(0) \bar{f}^c_{++}(t) + f^c_{++}(t) f^a_{++}(u) \bar{f}^b_{++}(0) + f^c_{++}(t) f^b_{++}(0) \bar{f}^a_{++}(u)$$
$$- f^c_{++}(t) \bar{f}^a_{++}(u) \bar{f}^b_{++}(0) - f^a_{++}(u) \bar{f}^c_{++}(t) \bar{f}^b_{++}(0) - f^b_{++}(0) \bar{f}^c_{++}(t) \bar{f}^a_{++}(u) \Big] \stackrel{?}{=} \mathcal{O}(\omega)$$

- next construct by analytic continuation the corresponding local operator.
  - This is opposite to the twist-2 case: there we new the local operator and constructed the by analytic continuation the operator in the *unphysical* point.

- Work in progress ...

# Conclusions and Outlook

- Light-ray operators are the analytic continuation to the *unphysical* point of twist-2 local operator.
- Expansion of the dipole-Wilson line Odderon generates twist-3 operators.
- Spinor formalism simplify the analysis of scale dependence of twist-3 operators.
- Outlook
    - Assemble the result for the scale dependence of Odderon-light ray operator.
    - Construct by analytic continuation the local operator from the Odderon-light ray operator.