# 3-parton production in DIS at small x

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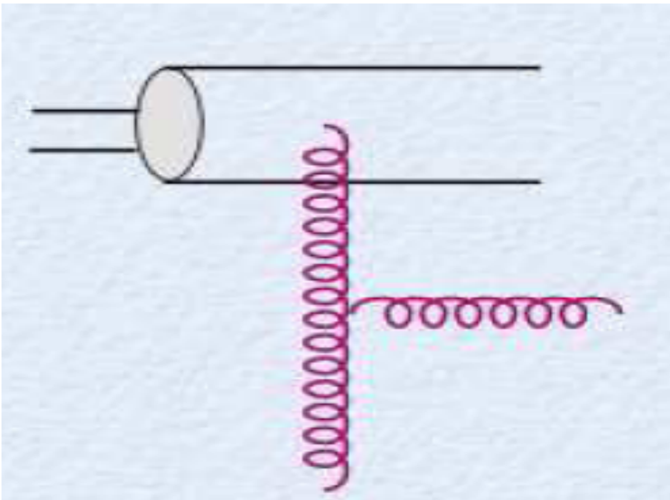
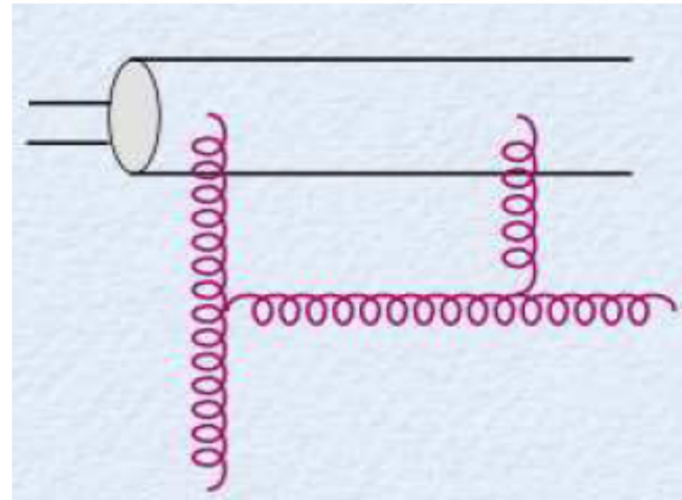
Low x 2017
June 13-17, Bari, Italy

# Perturbative QCD breaks down at small x

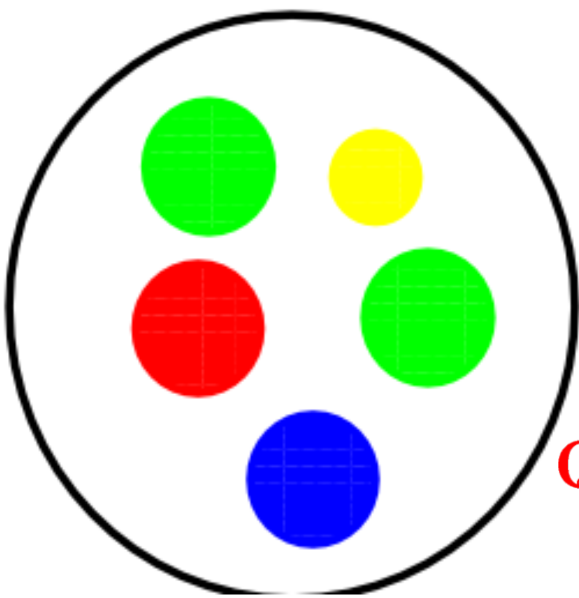
*"attractive" bremsstrahlung* *vs.* *"repulsive" recombination*



included in pQCD



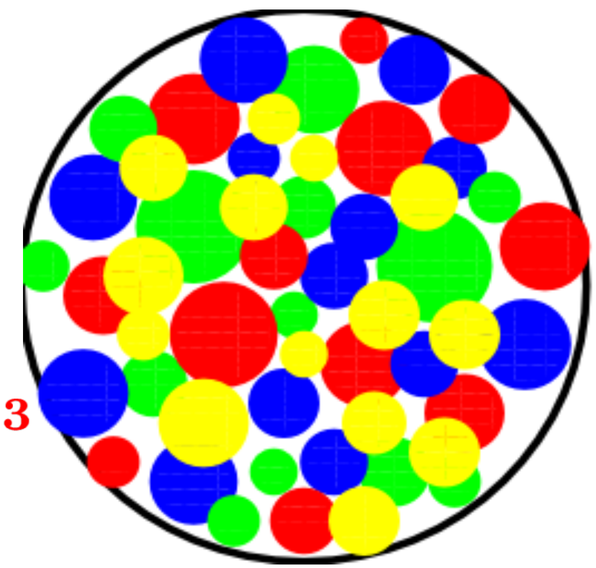
not included in pQCD
(collinear factorization)



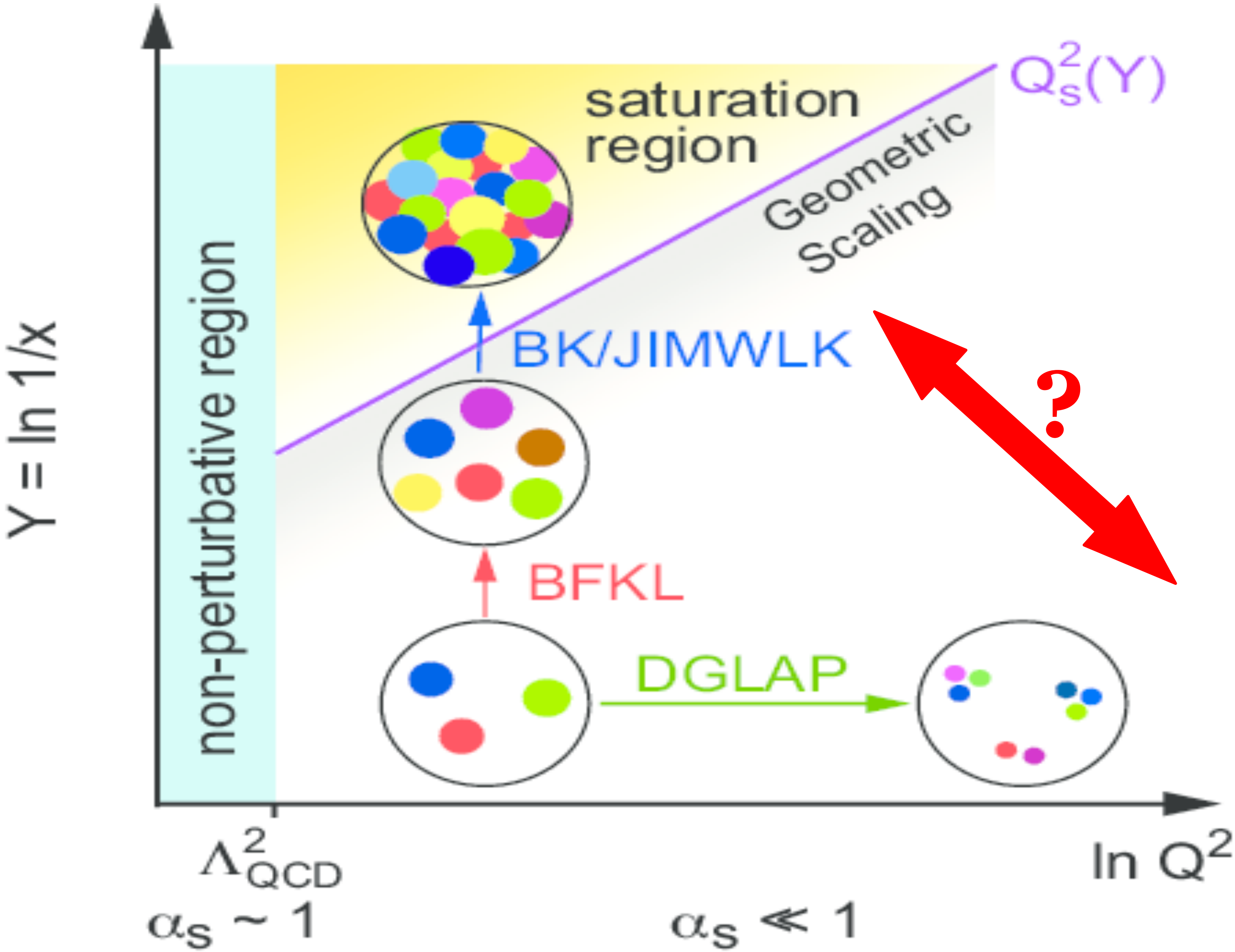
energy ~ 1/x

$$\frac{\alpha_s}{Q^2}\,\frac{xG(x,Q^2)}{\pi r^2} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3}\,(\frac{1}{x})^{0.3}$$



# QCD at high energy: *saturation*



## Probing saturation via angular correlations

polar angle (long-range rapidity correlations)

azimuthal angle (back to back)

## signatures in production spectra

multiple scattering via Wilson lines:

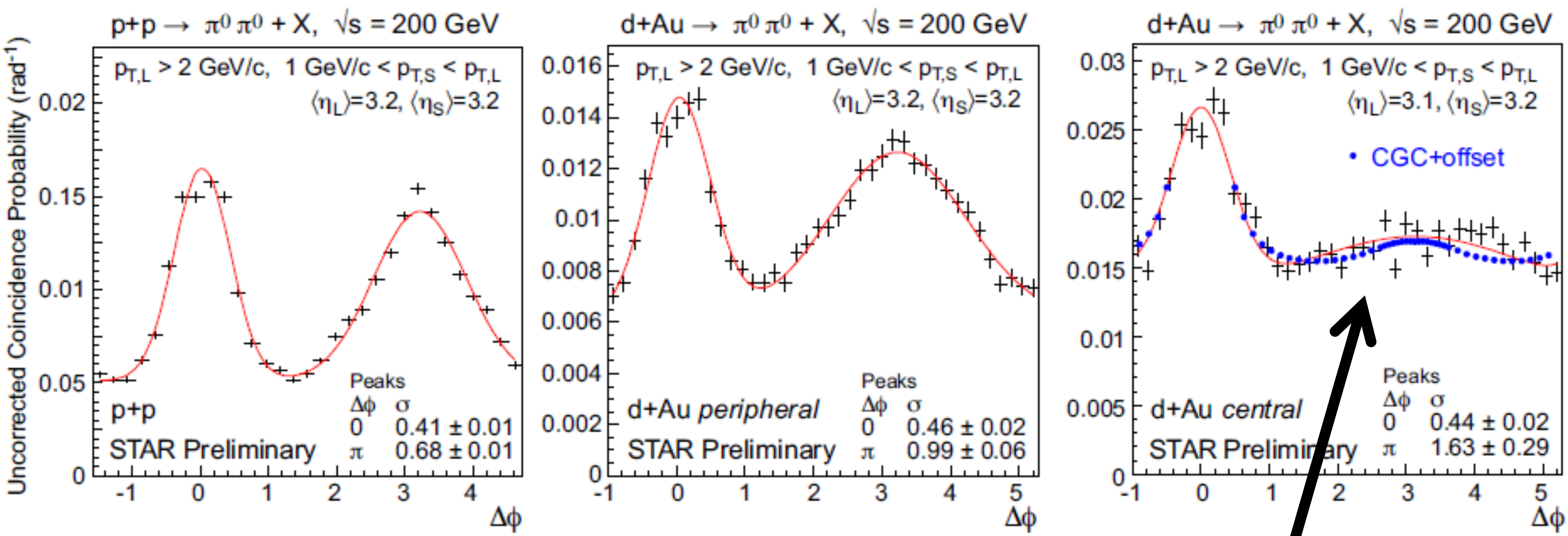
$p_t$ broadening

x-evolution via JIMWLK:

suppression of spectra/away side peaks

# di-hadron correlations in pA

Recent STAR measurement (arXiv:1008.3989v1):



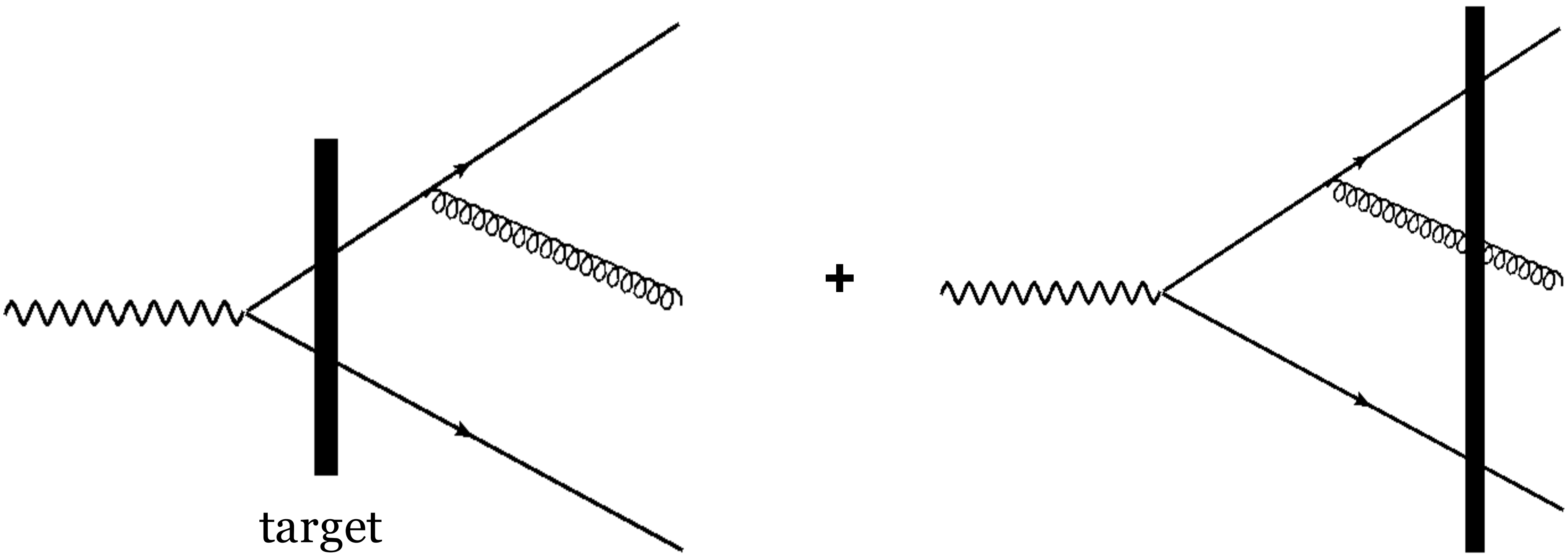

*Marquet, NPA (2007), Albacete + Marquet, PRL (2010)*
*Tuchin, NPA846 (2010)*
*A. Stasto + B-W. Xiao + F. Yuan, PLB716 (2012)*
*T. Lappi + H. Mantysaari, NPA908 (2013)*

***saturation effects de-correlate the hadrons***

shadowing+energy loss: Z. Kang, I. Vitev, H. Xing, PRD85 (2012) 054024

# 3-parton production in DIS

$\gamma^\star \mathbf{T} \to \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{g}\,\mathbf{X}$ where T is a proton or nucleus

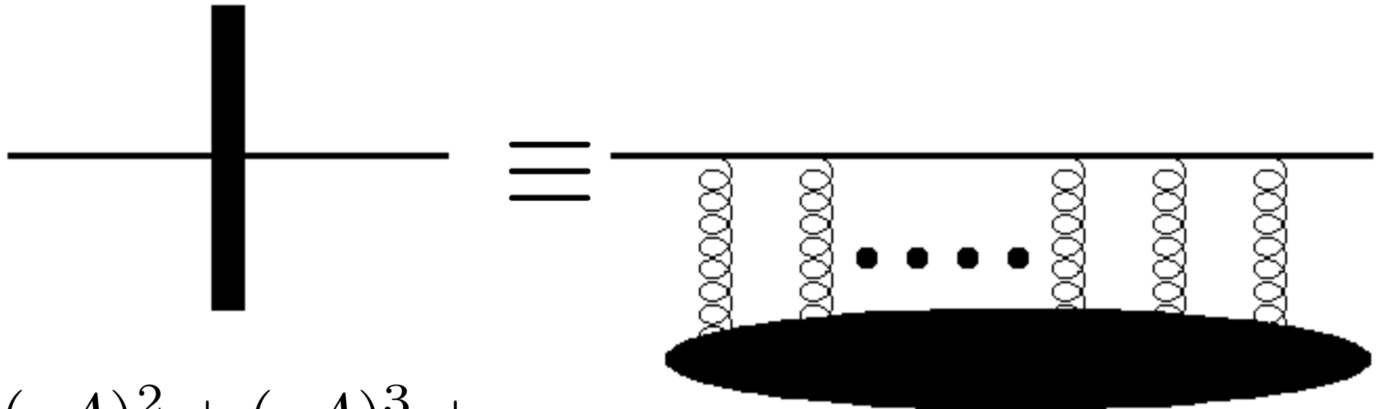



+ radiation from anti-quark

***Ayala, Hentschinski , Jalilian-Marian, Tejeda-Yeomans***
***PLB761 (2016) 229***
***NPB920 (2017) 232***

# scattering of a quark from the target

target (proton, nucleus) as a classical color field
quark propagator in the background color field: Wilson line V



$$\sim gA + (gA)^2 + (gA)^3 + \cdots$$

$$\mathrm{S}_F(q,p) \equiv \underbrace{(2\pi)^4 \delta^4(p-q)\, S_F^0(p)}_{\text{no interaction}} + \underbrace{S_F^0(q)\, \tau_f(q,p)\, S_F^0(p)}_{\text{interaction}} \qquad \text{with} \qquad S_F^0(p) = \frac{i}{\not p + i\epsilon}$$

$$\tau_f(q,p) \;\; \equiv \;\; (2\pi)\delta(p^+ \quad q^+)\,\gamma^+ \int d^2x_t\, e^{i(q_t \;\; p_t)\cdot x_t}$$

$$\left\{\theta(p^+)[V(x_t) - 1] - \theta(-p^+)[V^\dagger(x_t) - 1]\right\}$$

$$\mathrm{V}(\mathrm{x}_t) = \hat{p}\, e^{ig\int dz^+ \, A \;\; (z^+, x_t)}$$

similar for gluon propagator

# *spinor helicity methods*

<u>Review:</u>
L. Dixon, hep-ph/9601359

massless quarks: helicity eigenstates

$$u_\pm(k) \equiv \frac{1}{2}(1 \pm \gamma_5)\, u(k) \qquad \overline{u_\pm(k)} \equiv \overline{u(k)}\, \frac{1}{2}(1 \mp \gamma_5)$$

$$v_\mp(k) \equiv \frac{1}{2}(1 \pm \gamma_5)\, v(k) \qquad \overline{v_\mp(k)} \equiv \overline{v(k)}\, \frac{1}{2}(1 \mp \gamma_5)$$

helicity operator

$$\mathrm{h} \equiv \vec{\Sigma} \cdot \hat{p} = \begin{pmatrix} \vec{\sigma} \cdot \hat{p} & 0 \\ 0 & \vec{\sigma} \cdot \hat{p} \end{pmatrix}$$

$$\begin{aligned} \vec{\Sigma} \cdot \hat{p}\, u_\pm(p) &= \pm u_\pm(p) \\ -\vec{\Sigma} \cdot \hat{p}\, v_\pm(p) &= \pm v_\pm(p) \end{aligned}$$

$$u_+(k) = v_-(k) = \frac{1}{2^{1/4}} \begin{bmatrix} \sqrt{k^+} \\ \sqrt{k^-}\, e^{i\phi_k} \\ \sqrt{k^+} \\ \sqrt{k^{\ }}\, e^{i\phi_k} \end{bmatrix} \qquad u_-(k) = v_+(k) = \frac{1}{2^{1/4}} \begin{bmatrix} \sqrt{k^{\ }}\, e^{-i\phi_k} \\ -\sqrt{k^+} \\ -\sqrt{k^-}\, e^{\ \ i\phi_k} \\ \sqrt{k^+} \end{bmatrix}$$

with $e^{\pm i\phi_k} \equiv \dfrac{k_x \pm i k_y}{\sqrt{2k^+ k}} = \sqrt{2}\, \dfrac{k_t \cdot \epsilon_\pm}{k_t}$

$$\begin{aligned} n^\mu &= (n^+ = 0, n^{\ } = 1, n_\perp = 0) \\ n^\mu &= (n^+ = 1, n^{\ } = 0, n_\perp = 0) \end{aligned}$$

and

$$\begin{aligned} k^\pm &= \frac{E \pm k_z}{\sqrt{2}} \\ \epsilon_\pm &= \frac{1}{\sqrt{2}}(1, \pm i) \end{aligned}$$

# spinor helicity methods

notation:

$$|i^{\pm}> \equiv \ k_i^{\pm} > \equiv u_{\pm}(k_i) = v_{\mp}(k_i) \qquad < i^{\pm}| \equiv < k_i^{\pm}| \equiv u_{\pm}(k_i) = v_{\mp}(k_i)$$

basic spinor products:

$$< i\,j > \ \equiv \ < i^-|j^+ > = \overline{u}_-(k_i)\,u_+(k_j) \ = \ \sqrt{|s_{ij}|e^{i\phi_{ij}}}$$

$$[i\,j] \ \equiv \ < i^+|j^- > = \overline{u}_+(k_i)\,u \quad (k_j) \ = \ -\sqrt{|s_{ij}|e^{-i\phi_{ij}}}$$

$$\cos\phi_{ij} \ = \ \frac{k_i^x k_j^+ \quad k_j^x k_i^+}{\sqrt{|s_{ij} \ k_i^+ k_j^+}}$$

$$\sin\phi_{ij} \ = \ \frac{k_i^y k_j^+ - k_j^y k_i^+}{\sqrt{|s_{ij}|k_i^+ k_j^+}}$$

with

$$s_{ij} \ = \ (k_i + k_j)^2 = 2k_i \cdot k_j$$
$$= \ - < ij > [ij]$$

and

$$< ii > \ = \ [ii] = 0$$
$$< ij] \ = \ [ij >= 0$$

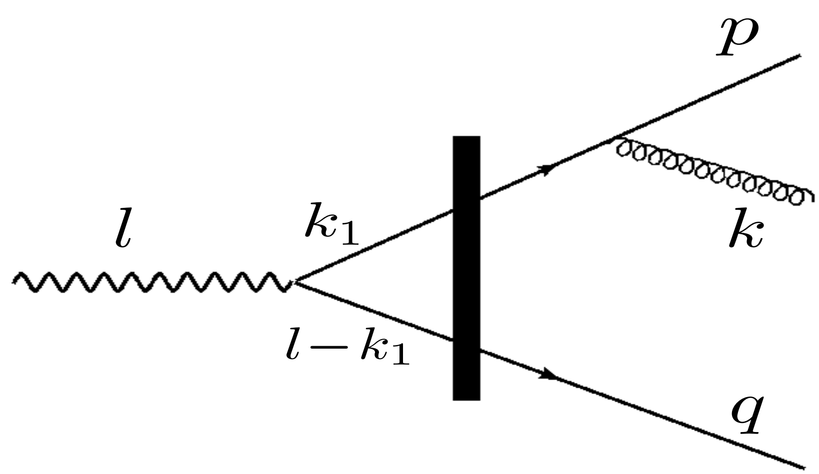
charge conjugation $< i^+|\gamma^\mu|j^+ > = < j^-|\gamma^\mu|i^- >$

Fierz identity $< i^+|\gamma^\mu|j^+ >< k^+|\gamma^\mu|l^+ > = 2[ik] \ < lj >$

any off-shell momentum $k^\mu \equiv \bar{k}^\mu + \dfrac{k^2}{2k^+}\,n^\mu$ where $\bar{k}^\mu$ is on-shell $\bar{k}^2 = 0$

any on-shell momentum $\not{p} = |p^+ >< p^+| + |p^- >< p^-|$

# Diagram A1




Numerator: Dirac Algebra

$$a_1 \equiv \overline{u}(p)\,(k)\,(\not{p}+\not{k})\;\not{k}_1\,(l)\,(\not{k}_1-\not{l})\;v(q)$$

longitudinal photons

quark anti-quark gluon helicity: + - +

$$\not{l} = l^+\not{\overline{n}} - \frac{Q^2}{2l^+}\not{n}$$

$$a_1^{L;+-+} \;=\; -\frac{\sqrt{2}}{[n\,k]}\,\frac{Q}{l^+}\,[n\,p] < k\,p > [n\,p] < n\,\overline{k}_1 > \left[n\,\overline{k}_1\right] < n\,q >$$

$$(< n\,\overline{k}_1 > \left[n\,\overline{k}_1\right] - l^+ < n\,\overline{n} > [n\,\overline{n}])$$

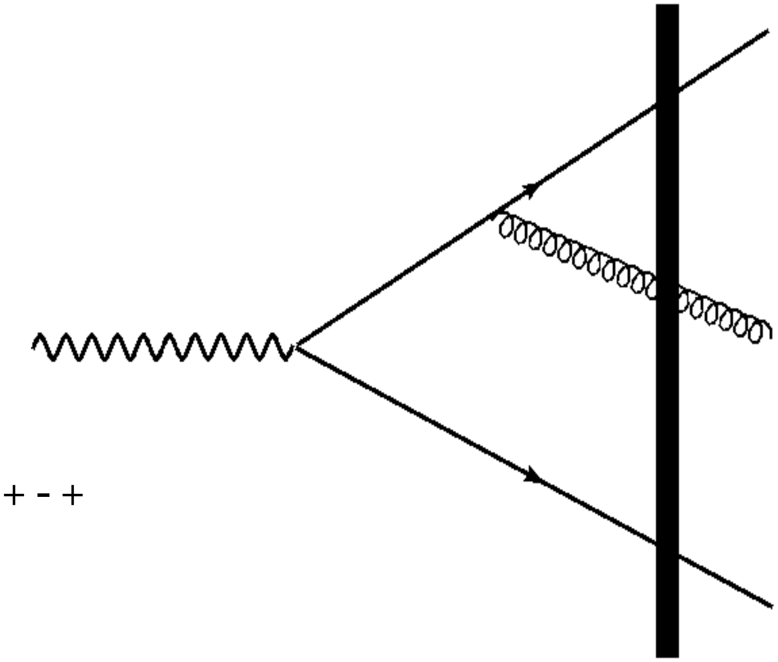
with

$$< np > = \quad [np] = \sqrt{2p^+}$$

transverse photons: +

$$a_1^{\perp=+;+-+} = -\frac{\sqrt{2}}{[nk]}[pn] < kp > [pn] < nk_1 > [k_1 n] < nk_1 > [k_1 n] < nq >$$

# Diagram A3



Numerator: Dirac Algebra

longitudinal photons

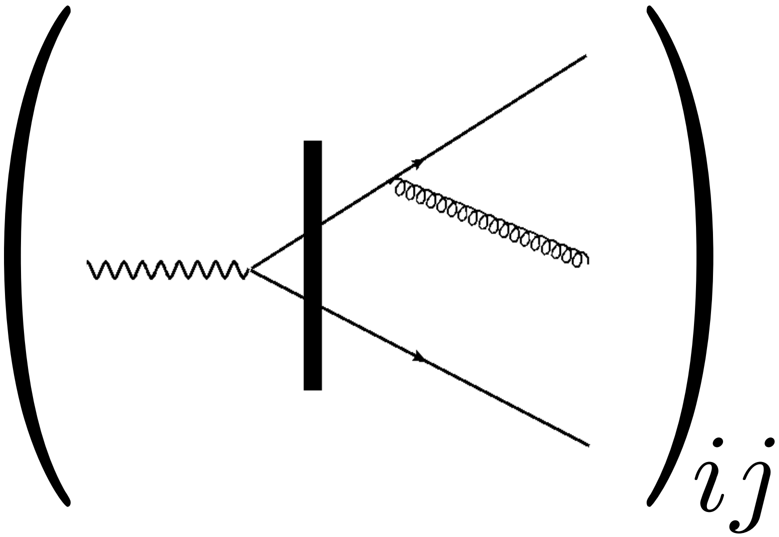
quark anti-quark gluon helicity: + - +

$$
\begin{aligned}
a_3^{L;+-+} &= \frac{\sqrt{2}Q}{l^+\,[n\bar{k}_2]}[pn]\left(< n\bar{k}_1 > [\bar{k}_1 n] - < n\bar{k}_2 > [\bar{k}_2 n]\right) < \bar{k}_2\bar{k}_1 > [\bar{k}_1 n] \\
&\quad \left(< nk_1 > [k_1 n] - l^+ < n\bar{n} > [\bar{n}n]\right) < nq > \\
&= -2^4 Q (l^+)^2 \frac{(z_1 z_2)^{3/2}}{z_3}\left[z_3\, k_{1\,t}\cdot\epsilon - (z_1+z_3)k_{2\,t}\cdot\epsilon\right]
\end{aligned}
$$

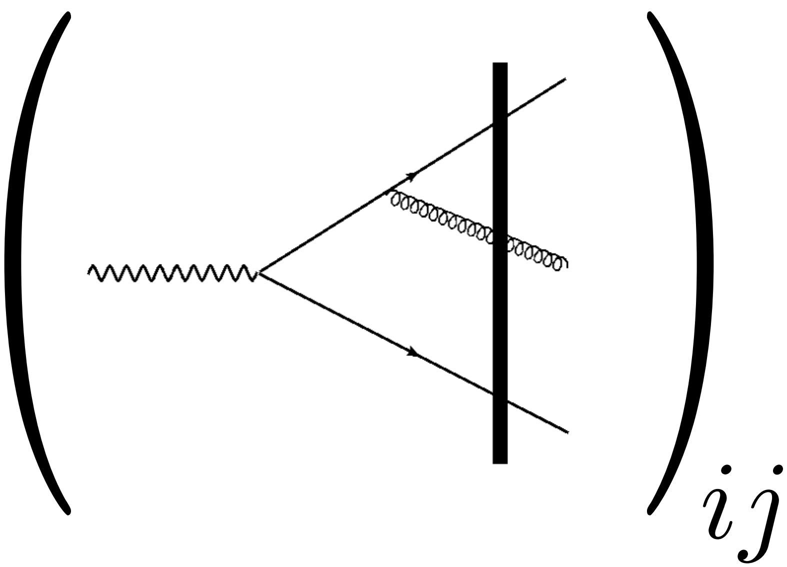
the rest is some standard integrals, <u>we know how to compute the numerators efficiently</u>

add up the amplitudes, add, square.. : **get (trace of) products of Wilson lines**

# structure of Wilson lines: amplitude



$$= \left[V^{\dagger}(y_t)\, V(x_t)\, t^{a}\right]_{ij}$$

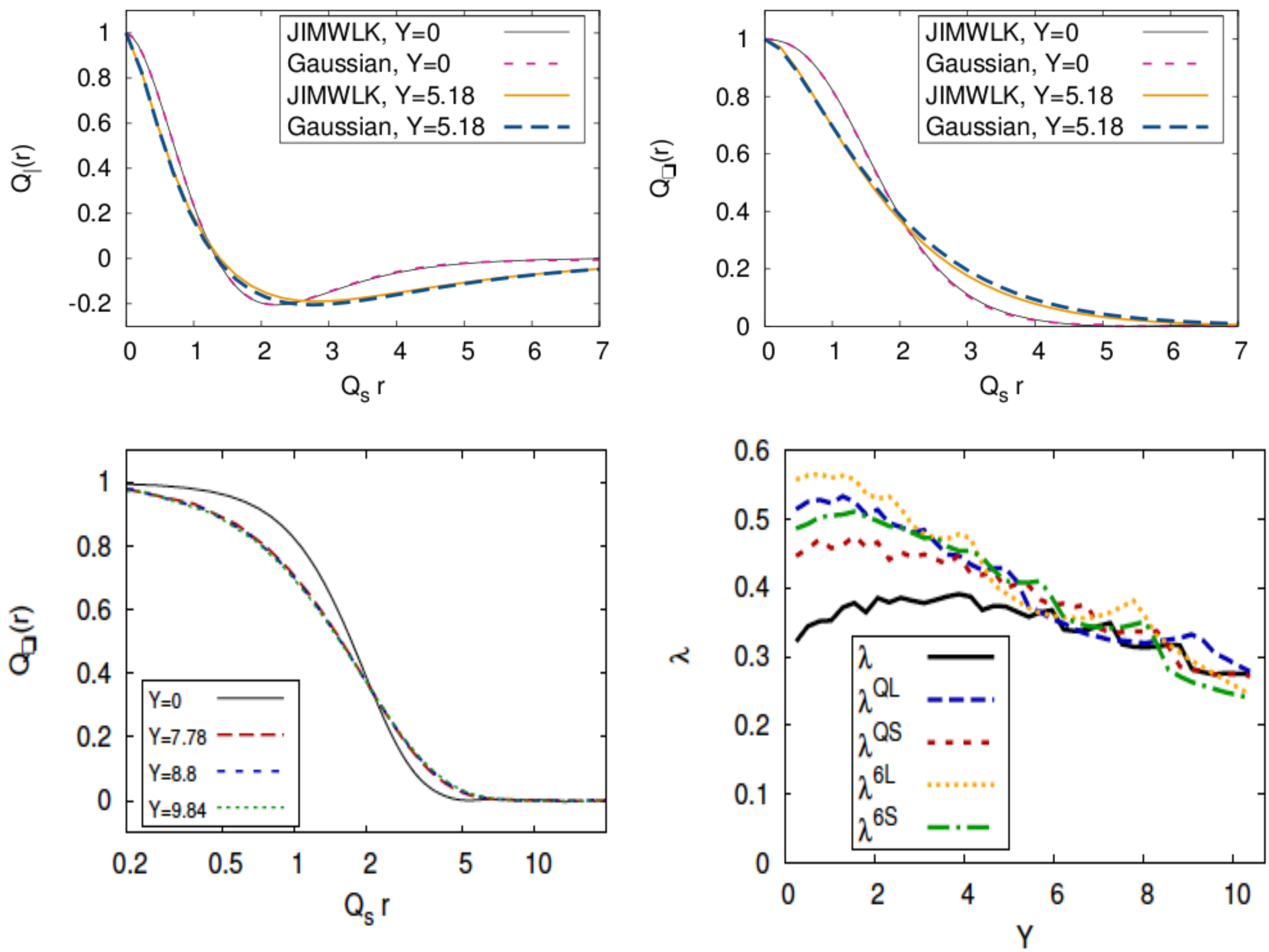


$$= \left[V^{\dagger}(y_t)\, t^{b}\, V(x_t)\right]_{ij}\; U^{ba}(z_t)$$

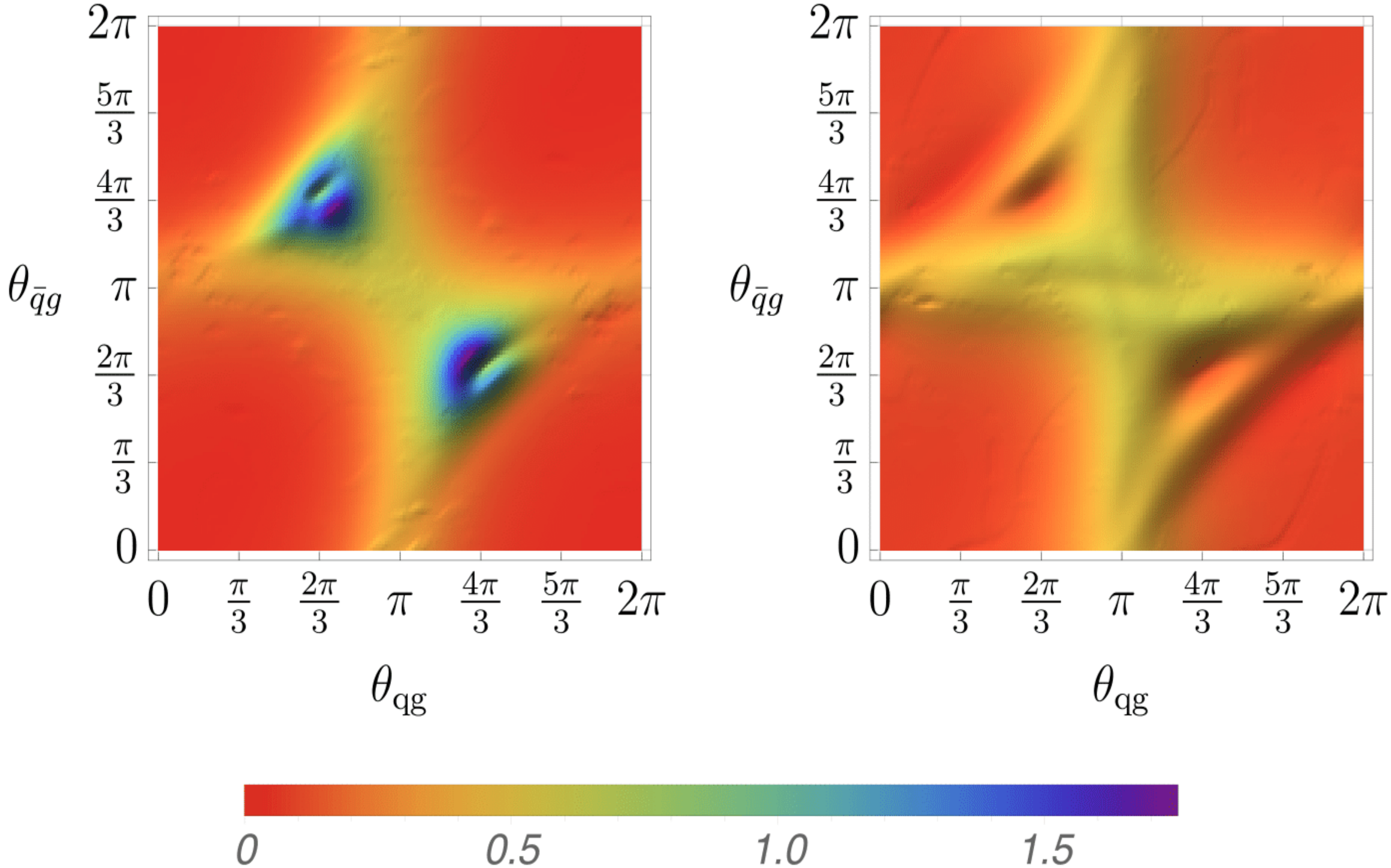
lead to dipoles and quadrupoles

# **Quadrupole:** $Q(r,r,s,s) \equiv \frac{1}{N_c} < Tr\, V(r)\, V^{\dagger}(r)\, V(s)\, V^{\dagger}(s) >$

*Dumitru-Jalilian-Marian-Lappi-Schenke-Venugopalan: PLB706 (2011) 219*



# *3-parton azimuthal angular correlations*




***multiple scattering:***
***broadening*** ***of the peak***

***x-evolution:***
***reduction*** ***of magnitude***

# some thoughts/ideas/……

## cold matter energy loss

how important is cold matter Eloss in single inclusive production in the forward rapidity region?

***cold matter energy loss?***
***Kopeliovich, Frankfurt and Strikman***
***Neufeld,Vitev,Zhang, PLB704 (2011) 590***

*Munier, Peigne, Petreska, arXiv:1603.01028*

$$z\frac{dI}{dz} \equiv \frac{\frac{d\sigma a+A\to a+g+X}{dy\,dy'\,d^2p_t}}{\frac{d\sigma a+A\to a+X}{dy\,d^2p_t}}$$

the difference between a nuclear target and a proton target is the medium induced energy loss

used to estimate the energy loss in single inclusive processes in the forward kinematics at RHIC and the LHC

**can also do this for di-jets in DIS**

(3-parton production/2-parton production)

# *SUMMARY*

***CGC is a systematic approach to high energy collisions***

***high gluon density: re-sum multiple soft scatterings***

***high energy: re-sum large logs of energy (rapidity or log 1/x)***

***Leading Log CGC works (too) well***

***it has been used to fit a wealth of data; ep, eA, pp, pA, AA***

***Precision (NLO) studies are needed***

***available for DIS, single inclusive forward production in pp, pA***

***Azimuthal angular correlations offer a unique probe of CGC***

***3-hadron/jet correlations should be even more discriminatory***