High Energy QCD at NLO: from light-cone wave function to JIMWLK evolution

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DE LA RECHERCHE À L'INDUSTRIE

Low-x 2017, Bisceglie, 14 June 2017

Talk Outline

1) Quick introduction to JIMWLK at leading order.

2) Next-to-leading order JIMWLK based on the works of Balitsky, Chirilly and Grabovsky for the evolution of dipole and baryon.

3) Next-to-leading order JIMWLK based on the light-cone wave function formalism approach (direct and independent calculation).

How do scattering amplitudes depend on the collision energy $\sqrt{s}$?

At low energy hadrons consists of relatively small number of partons. As the collision energy (rapidity) increases, new partons are emitted (***Weizsacker Williams radiation***).

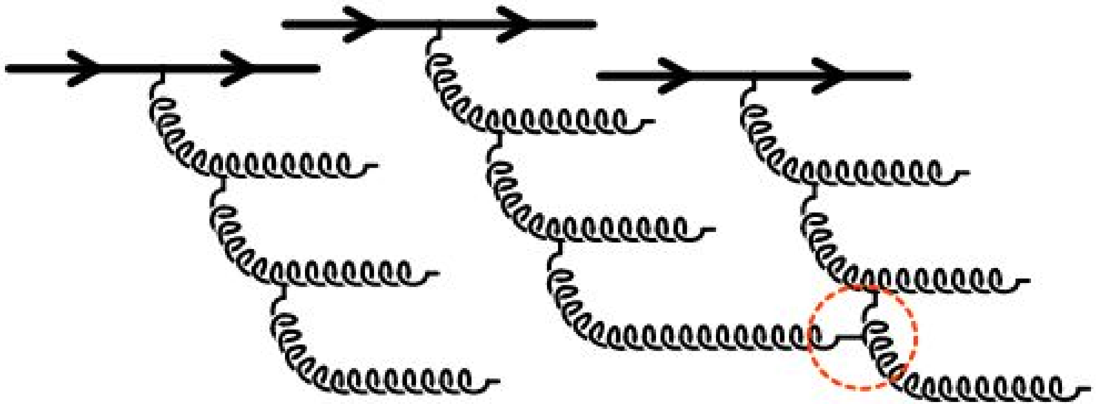
As long as the density of partons remains small, new particles are created *linearly* (BFKL): the number of new partons due to the increase in collision energy is proportional to the number of the emitting partons (so the density grows exponentially).

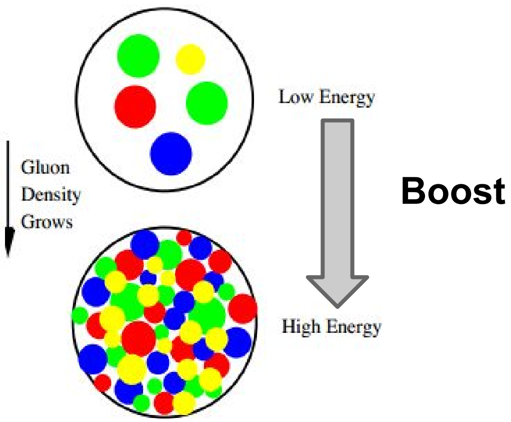
BFKL solutions suffer from three problems:

a. ***Infrared Diffusion***
b. ***Unitarity Violation***
c. ***Froissart Bound*** **-** the cross sections computed by using BFKL grow as $\sim s^{\alpha_P - 1}$ and thus violate the restriction $\sigma_{tot}(s) \leq_{s\to\infty} C\,[\ln(s/s_0)]^2$

What happens if we push to higher collision energy?

Eventually gluons start overlapping with each other and **new parton emission become a *collective process***.
Emission process becomes *non-linear* and leads to ***gluon saturation*** phenomena (also known as *Color Glass Condensate (CGC) / JIMWLK*). Gluon density grows logarithmically instead of exponentially (as in the case of BFKL).



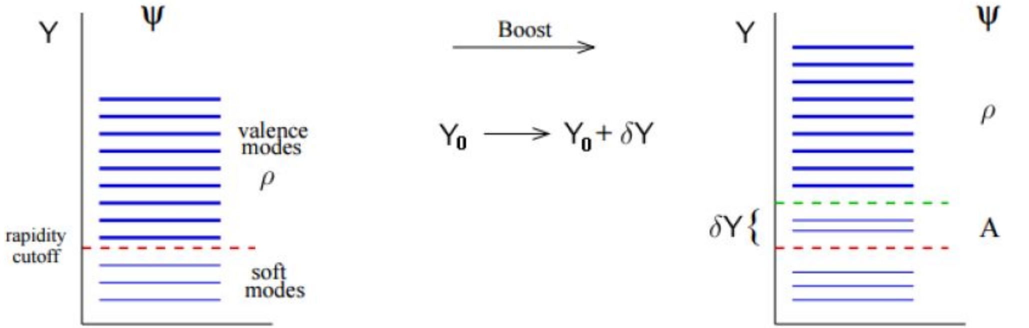


Boosting the Light-cone Hadronic Wave Function

Let us consider a fast moving projectile hadron at some rapidity Y_0. We denote its wave function by $|\Psi\rangle_{\mathsf{Y}_0}$ which is dominated by valence modes only, for which the longitudinal momentum $k^+ > \Lambda$ (modes with $k^+ \leq \Lambda$ known as soft). Under the action of boosting:

$$k^+ \longrightarrow e^{\delta \mathsf{Y}} k^+ \qquad \Lambda \longrightarrow e^{\delta \mathsf{Y}} \Lambda \qquad \mathsf{Y} = \mathsf{Y}_0 + \delta \mathsf{Y}$$

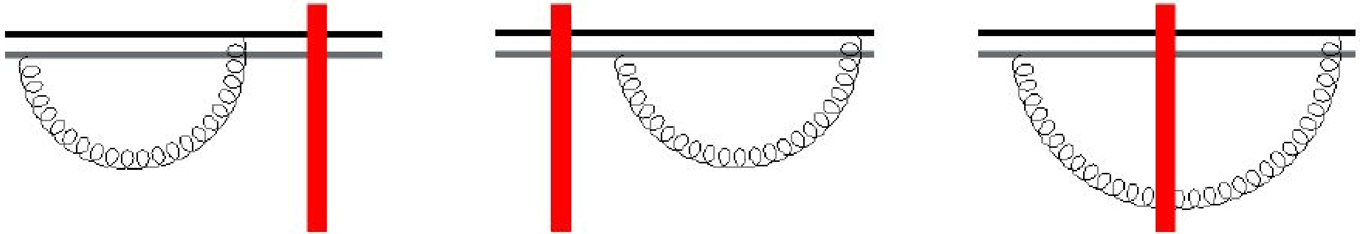
the longitudinal momenta of the partons in the LCWF get shifted up:



Therefore, new soft modes were created in the window $\Lambda < k^+ < \Lambda\, e^{\delta \mathsf{Y}}$, and the WF is given by the product $|\Psi\rangle_{\mathsf{Y}} = |soft\rangle \otimes |valence\rangle$.

Leading-Order JIMWLK

The JIMWLK equation, $\frac{d}{dY}\mathcal{O} = -H^{JIMWLK}\mathcal{O}$ describes the rapidity (denoted by Y) evolution of observables $\mathcal{O}$ in scattering process. At the leading order, it consists of three terms - interaction of the probe with the target fields and two virtual terms:



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The target (a dense nucleus) is modeled by a fixed background field A.
The S-matrix of a fast particle interacting with a target field A (the only effect of the interaction if via color change) is the Wilson line (*Eikonal Approximation*):

$$S^{ab}(x) = \left[P \exp\left\{ig \int dx^+ T_c A_c^-(x^+, x)\right\}\right]^{ab}$$

x is a two dimensional transverse coordinate (LC gauge). JIMWLK reduces to BFKL for low probability of scattering. Many phenomenological studies exist.

How to Derive the JIMWLK Hamiltonian?

1) Introduce a set of normalized states relevant for the order of expansion.

2) Find the general perturbative expression for the soft part of the hadronic wave-function up to the order of interest.

3) Find the non-vanishing matrix elements of the interactions terms (of the QCD Hamiltonian) and compute the normalized wave function in momentum space.

4) Transform the transverse part of the wave function to coordinate space.

5) Compute $\Sigma \equiv \langle \psi | \hat{S} - 1 | \psi \rangle$ and subtract the terms which are out of the resummation order. Then the JIMWLK Hamiltonian is given by:

$$H_{JIMWLK} \equiv - \left. \frac{d\Sigma}{d\delta Y} \right|_{\delta Y=0}$$

6) Rewrite the Hamiltonian with subtraction terms.

Motivations for NLO JIMWLK Equation

The LO JIMWLK is only a first term in an infinite perturbative series:

$$H_{JIMWLK} = H_{LO}(\alpha_s) + \boxed{H_{NLO}(\alpha_s^2)} + H_{NNLO}(\alpha_s^3) + \ldots$$

The NLO term is necessary because:

a. NLO Corrections are *known to be large*.

b. Built-in information on the *running coupling* - better phenomenology. The running is known to slow down the evolution.

c. To get the *region of applicability* of the leading order equation.

d. Important step towards *all order resummation*.

NLO JIMWLK reduces to NLO BFKL in linear approximation.

Towards NLO JIMWLK

The general structure of the wave-function up to g^3 (normalization up to g^4):

$$|\psi\rangle = (1 - g_s^2\,\kappa_0\,JJ - g_s^4(\delta_1\,JJ + \delta_2\,JJJ + \delta_3\,JJJJ)\,|\,no\,soft\,gluons\rangle +$$
$$+(g_s\kappa_1\,J + g_s^3\epsilon_1\,J + g_s^3\,\epsilon_2\,J\,J)\,|\,one\,soft\,gluon\rangle + g_s^2(\epsilon_3\,J + \epsilon_4\,JJ)\,|\,two\,soft\,gluons\rangle + g_s^2\,\epsilon_5\,J\,|\,q\,\bar{q}\rangle$$

Combining this wave function with:

a. ***Expected symmetries*** of the NLO kernel, SU(Nc)xSU(Nc).

b. ***Unitarity***, when S=1 we should get no evolution.

We can find the general form of the NLO JIMWLK Hamiltonian.

The General Form of the NLO JIMWLK Hamiltonian

$$
\begin{aligned}
H^{NLO\ JIMWLK} = & \int_{x,y,z} K_{JSJ}(x,y;z)\,\left[J_L^a(x)\,J_L^a(y) + J_R^a(x)\,J_R^a(y) - 2\,J_L^a(x)\,S_A^{ab}(z)\,J_R^b(y)\right] + \\
& + \int_{x\,y\,z\,z'} K_{JSSJ}(x,y;z,z')\left[f^{abc}\,f^{def}\,J_L^a(x)\,S_A^{be}(z)\,S_A^{cf}(z')\,J_R^d(y) - N_c\,J_L^a(x)\,S_A^{ab}(z)\,J_R^b(y)\right] + \\
& + \int_{x,y,z,z'} K_{q\bar{q}}(x,y;z,z')\,\left[2\,J_L^a(x)\,tr[S^\dagger(z)\,T^a\,S(z')T^b]\,J_R^b(y) - J_L^a(x)\,S_A^{ab}(z)\,J_R^b(y)\right] + \\
& + \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z')f^{acb}\left[J_L^d(x)\,J_L^e(y)\,S_A^{dc}(z)\,S_A^{eb}(z')\,J_R^a(w) - J_L^a(w)\,S_A^{cd}(z)\,S_A^{be}(z')\,J_R^d(x)\,J_R^e(y) + \right. \\
& \left. + \frac{1}{3}[\,J_L^c(x)\,J_L^b(y)\,J_L^a(w) - J_R^c(x)\,J_R^b(y)\,J_R^a(w)]\right] + \\
& + \int_{w,x,y,z} K_{JJSJ}(w;x,y;z)\,f^{bde}\left[J_L^d(x)\,J_L^e(y)\,S_A^{ba}(z)\,J_R^a(w) - J_L^a(w)\,S_A^{ab}(z)\,J_R^d(x)\,J_R^e(y) + \right. \\
& \left. + \frac{1}{3}[\,J_L^d(x)\,J_L^e(y)\,J_L^b(w) - J_R^d(x)\,J_R^e(y)\,J_R^b(w)]\right]
\end{aligned}
$$

Shortcut to the Kernels

While we obtained the general form of the Hamiltonian the 5 kernels still have to be determined. How did we manage to find them?

Smart trick - require consistency:

1) Evolution equation of ***quark dipole*** - $s(x,y) = \frac{1}{N}\mathrm{Tr}(S^\dagger(x)S(y))$

$$\frac{d}{dY}s(x,y) = -H_{NLO\ JIMWLK}\, s(x,y)$$

The NLO BK which was computed by I. Balitsky and G. A. Chirilli in hep-ph/**0710.4330 (PRD).**

2) Evolution equation of ***SU(3) Baryon*** - $B(x,y,z) = \epsilon^{ijk}\epsilon^{lmn}S^{il}(x)S^{jm}(y)S^{kn}(z)$

Connected part was computed by V. A. Grabovsky in **hep-ph/1307.5414 (JHEP):**

$$\frac{d}{dY}B(x,y,z) = -H_{NLO\ JIMWLK}\, B(x,y,z)$$

The Kernels for Gauge Invariant Operators (color singlet amplitudes)

$$K_{JSJ}(x,y;z) = -\frac{\alpha_s^2}{16\pi^3}\frac{(x-y)^2}{X^2Y^2}\Big[b\ln(x-y)^2\mu^2 - b\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2} + (\frac{67}{9}-\frac{\pi^2}{3})N_c - \frac{10}{9}n_f\Big] - \frac{N_c}{2}\int_{z'}\tilde{K}(x,y,z,z')$$

Here μ is the normalization point in the $\overline{MS}$ scheme and $b = \frac{11}{3}N_c - \frac{2}{3}n_f$ is the first coefficient of the β-function.

$$K_{JSSJ}(x,y;z,z') = \frac{\alpha_s^2}{16\,\pi^4}\Bigg[-\frac{4}{(z-z')^4} + \Big\{2\frac{X^2Y'^2 + X'^2Y^2 - 4(x-y)^2(z-z')^2}{(z-z')^4[X^2Y'^2 - X'^2Y^2]}$$
$$+ \frac{(x-y)^4}{X^2Y'^2 - X'^2Y^2}\Big[\frac{1}{X^2Y'^2} + \frac{1}{Y^2X'^2}\Big] + \frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2} - \frac{1}{X'^2Y^2}\Big]\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\Bigg] + \tilde{K}(x,y,z,z')$$

$$\tilde{K}(x,y,z,z') = \frac{i}{2}\,[K_{JJSSJ}(x;x,y;z,z') - K_{JJSSJ}(y;x,y;z,z') - K_{JJSSJ}(x;y,x;z,z') + K_{JJSSJ}(y;y,x;z,z')]$$

$$K_{q\bar{q}}(x,y;z,z') = -\frac{\alpha_s^2\,n_f}{8\,\pi^4}\Big\{\frac{X'^2Y^2 + Y'^2X^2 - (x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2 - X'^2Y^2)}\ln\frac{X^2Y'^2}{X'^2Y^2} - \frac{2}{(z-z')^4}\Big\}$$

$$K_{JJSJ}(w;x,y;z) = -i\,\frac{\alpha_s^2}{4\,\pi^3}\Big[\frac{X\cdot W}{X^2\,W^2} - \frac{Y\cdot W}{Y^2\,W^2}\Big]\ln\frac{Y^2}{(x-y)^2}\ln\frac{X^2}{(x-y)^2}$$

$$K_{JJSSJ}(w;x,y;z,z') = -i\,\frac{\alpha_s^2}{2\,\pi^4}\Big(\frac{X_iY'_j}{X^2Y'^2}\Big)\Big(\frac{\delta_{ij}}{2(z-z')^2} + \frac{(z'-z)_iW'_j}{(z'-z)^2W'^2} + \frac{(z-z')_jW_i}{(z-z')^2W^2} - \frac{W_iW'_j}{W^2W'^2}\Big)\ln\frac{W^2}{W'^2}$$

Kernels for Color Non-Singlets Operators

Most of our interest is in gauge invariant amplitudes. For the sake of completeness it is interesting to find the kernels applicable for action on non-gauge invariant structures. This has been done by comparison of the evolution equation of one, two and three Wilson lines $S^{ij}(x)$, $S^{ij}(x)S^{kl}(y)$ and $S^{ij}(x)S^{kl}(y)S^{mn}(z)$ with a recent work of ***I. I. Balitsky*** and ***G. A. Chirilli*** **(hep-ph/1309.7644, PRD 88, 111501).**

$$K_{JSJ}(x,y,z) \to \bar{K}_{JSJ}(x,y,z) \equiv K_{JSJ}(x,y,z)+$$
$$+\frac{\alpha_s^2}{16\pi^3}\left\{\left[\frac{1}{X^2}+\frac{1}{Y^2}\right]\left[\left(\frac{67}{9}-\frac{\pi^2}{3}\right)N_c-\frac{10}{9}n_f\right]+\frac{b}{X^2}\ln X^2\mu^2+\frac{b}{Y^2}\ln Y^2\mu^2\right\}$$

$$K_{JSSJ}(x,y;z,z') \to \bar{K}_{JSSJ}(x,y;z,z') \equiv K_{JSSJ}(x,y;z,z')$$
$$+\frac{\alpha_s^2}{8\pi^4}\left[\frac{4}{(z-z')^4}-\frac{I(x,z,z')}{(z-z')^2}-\frac{I(y,z,z')}{(z-z')^2}\right]$$

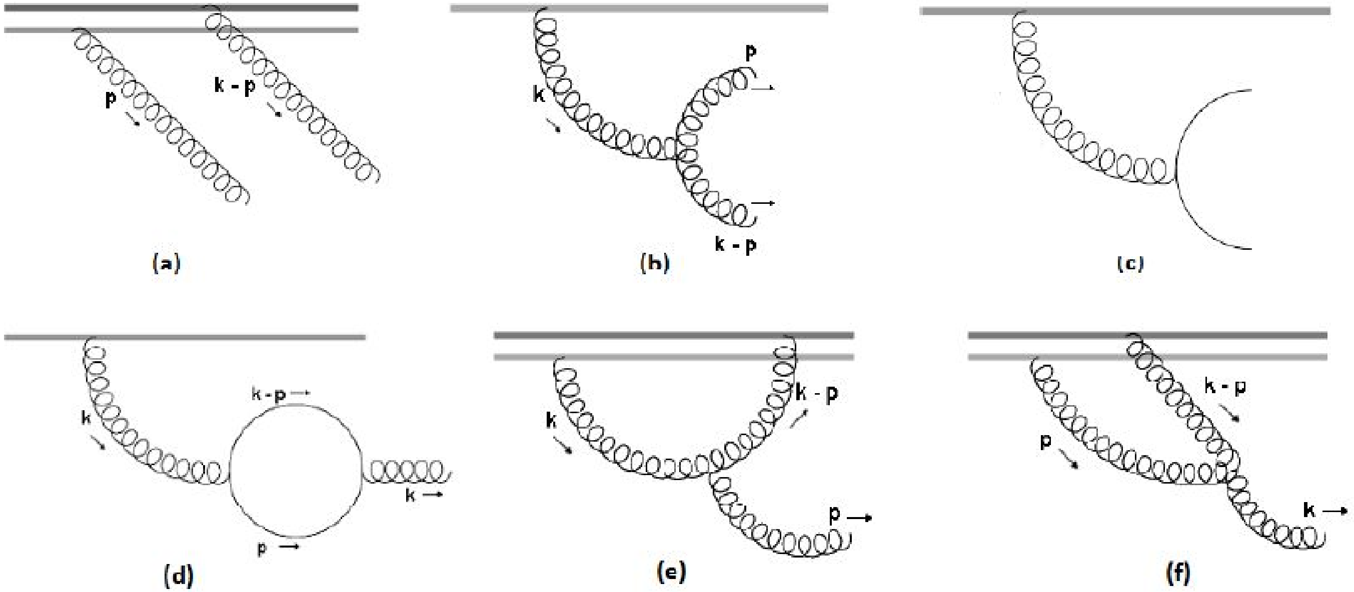
$$K_{qq}(x,y;z,z') \to \bar{K}_{qq}(x,y;z,z') \equiv K_{qq}(x,y;z,z') - \frac{\alpha_s^2 n_f}{8\pi^4}\left[\frac{I_f(x,z,z')}{(z-z')^2}+\frac{I_f(y,z,z')}{(z-z')^2}\right]$$

These modifications are vanishing while acting on gauge invariant structures.

$$I(x,z,z') = \frac{1}{X^2-(X')^2}\ln\frac{X^2}{(X')^2}\left[\frac{X^2+(X')^2}{(z-z')^2}-\frac{X\cdot X'}{X^2}-\frac{X\cdot X'}{(X')^2}-2\right]$$
$$I_f(x,z,z') = \frac{2}{(z-z')^2}-\frac{2X\cdot X'}{(z-z')^2(X^2-(X')^2)}\ln\frac{X^2}{(X')^2}$$

Next-to-leading Order Light Cone Diagrams

The NLO wave-function sums up the following diagrams:



About 20 different diagrams. In our first approach, we managed to fully avoid calculating all these contributions.

What Makes the Calculation of the NLO Wave Function Challenging?

1) Larger set of states enter the perturbative expansion.

1) Many non-vanishing matrix elements.

1) Complicated fourier transforms, complicated integrals.

4) Divergences.

5) Phase.

The Light Cone QCD Hamiltonian

$$H = P_+ = \int dx_+ d^2x_\perp \left((\Pi_\psi)^\mu \partial^- \psi + (\Pi_A)^\mu \partial^- A_\mu - \mathcal{L} \right) =$$

$$\int dx_+ d^2x_\perp \left(\frac{i}{2}\overline{\psi}\gamma^+\partial^-\psi - F^{\mu+}F_{\mu+} + \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \overline{\psi}(i\gamma^\mu D_\mu - m)\psi \right)$$

The Hamiltonian density that follows in LC gauge $A_a^+ = 0$:

$$H = \frac{1}{2}(\partial_i A_j)^2 - g f^{abc} A_i^b A_j^c \partial_i A_j^a + \frac{g^2}{4} f^{abc} f^{ade} A_i^b A_j^c A_i^d A_j^e - g(\partial_i A_i^a)\frac{1}{\partial^+}(f^{abc} A_b^j \partial^+ A_c^j) + 2g(\partial_i A_i^a)\frac{1}{\partial^+}(\psi_+^\dagger t^a \psi_+)$$
$$+ \frac{g^2}{2} f^{abc} f^{ade} \frac{1}{\partial^+}(A_i^b \partial^+ A_i^c)\frac{1}{\partial^+}(A_j^d \partial^+ A_j^e) + 2g^2 \frac{1}{\partial^+}(f^{abc} A_i^b \partial^+ A_i^c)\frac{1}{\partial^+}(\psi_+^\dagger t^a \psi_+) + 2g^2 \frac{1}{\partial^+}(\psi_+^\dagger t^a \psi_+)\frac{1}{\partial^+}(\psi_+^\dagger t^a \psi_+)$$
$$+ i\psi_+^\dagger(\sigma_i \partial_i)\frac{1}{\partial^+}(\sigma_j \partial_j)\psi_+ - g\psi_+^\dagger t^a (\sigma_i \partial_i)\frac{1}{\partial^+}(\sigma_j A_j^a)\psi_+ - g\psi_+^\dagger t^a (\sigma_i A_i^a)\frac{1}{\partial^+}(\sigma_j \partial_j)\psi_+ - ig^2\psi_+^\dagger t^a t^b (\sigma_i A_i^a)\frac{1}{\partial^+}(\sigma_j A_j^b)\psi_+$$

10 different interaction terms.

$$A_a^i(x) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2k^+}}\left(a_a^i(k^+, k_\perp)e^{-ik\cdot x} + a_a^{\dagger i}(k^+, k_\perp)e^{ik\cdot x} \right)$$

The NLO Wave Function

The NLO wave-function can be calculated perturbatively:

$$\left|\psi^{NLO}\right\rangle = \mathcal{N}^{NLO}\left|0\right\rangle - \left|i\right\rangle \frac{\left\langle i\left|H_{int}\right|0\right\rangle}{E_i} + \left|i\right\rangle \frac{\left\langle i\left|H_{int}\right|j\right\rangle \left\langle j\left|H_{int}\right|0\right\rangle}{E_i\, E_j}$$
$$- \left|i\right\rangle \frac{\left\langle i\left|H_{int}\right|k\right\rangle \left\langle k\left|H_{int}\right|j\right\rangle \left\langle j\left|H_{int}\right|0\right\rangle}{E_i\, E_j\, E_k} + \left|i\right\rangle \frac{\left|\left\langle j\left|H_{int}\right|0\right\rangle\right|^2 \left\langle i\left|H_{int}\right|0\right\rangle \left(E_i + 2E_j\right)}{2E_i^2\, E_j^2}$$

Where there is a summation over the different possible states:
One soft gluon, Two soft gluons or Quark anti-quark state.
H_{int} is the sum of all non-kinetic terms in the light-cone Hamiltonian (notice that in light-cone gauge we have also instantaneous interactions).

Based on the above, the NLO wave function can be expressed as:

$$\left|\psi^{NLO}\right\rangle = \mathcal{N}^{NLO}\left|0\right\rangle + \left|\psi^{LO}_{g\,\rho}\right\rangle + \left|\psi_{q\bar{q}\,\rho}\right\rangle + \left|\psi_{gg\,\rho}\right\rangle + \left|\psi_{gg\,\rho\rho}\right\rangle + \left|\psi_{g\,\rho}\right\rangle + \left|\psi_{g\,\rho\rho}\right\rangle + \left|\psi_{g\,\rho\rho\rho}\right\rangle$$

Organizing the Contributions

The computation of Σ yields contributions with different structure (different number of J operators or Wilson lines).

$$\Sigma = \langle \psi^{NLO} | \hat{S} - 1 | \psi^{NLO} \rangle$$
$$= \Sigma^{LO} + \Sigma_{q\bar{q}} + \Sigma_{JJSSJ} + \Sigma_{JSSJ} + \Sigma_{JJSJ} + \Sigma_{JSJ} + \Sigma_{JJSSJJ} + \Sigma_{JJJSJ} + \Sigma_{virtual}.$$

Where:

$$\Sigma_{q\bar{q}} \equiv \langle \psi_{q\bar{q}\,\rho} | \hat{S} | \psi_{q\bar{q}\,\rho} \rangle ,$$

$$\Sigma_{JJSSJ} \equiv \langle \psi_{gg\,\rho} | \hat{S} | \psi_{gg\,\rho\rho} \rangle + \langle \psi_{gg\,\rho\rho} | \hat{S} | \psi_{gg\,\rho} \rangle ,$$

$$\Sigma_{JSSJ} \equiv \langle \psi_{gg\,\rho} | \hat{S} | \psi_{gg\,\rho} \rangle ,$$

$$\Sigma_{JJSJ} \equiv \langle \psi_{g\,\rho\rho} | \hat{S} | \psi^{LO}_{g\,\rho} \rangle + \langle \psi^{LO}_{g\,\rho} | \hat{S} | \psi_{g\,\rho\rho} \rangle ,$$

$$\Sigma_{JSJ} \equiv \langle \psi_{g\,\rho} | \hat{S} | \psi^{LO}_{g\,\rho} \rangle + \langle \psi^{LO}_{g\,\rho} | \hat{S} | \psi_{g\,\rho} \rangle ,$$

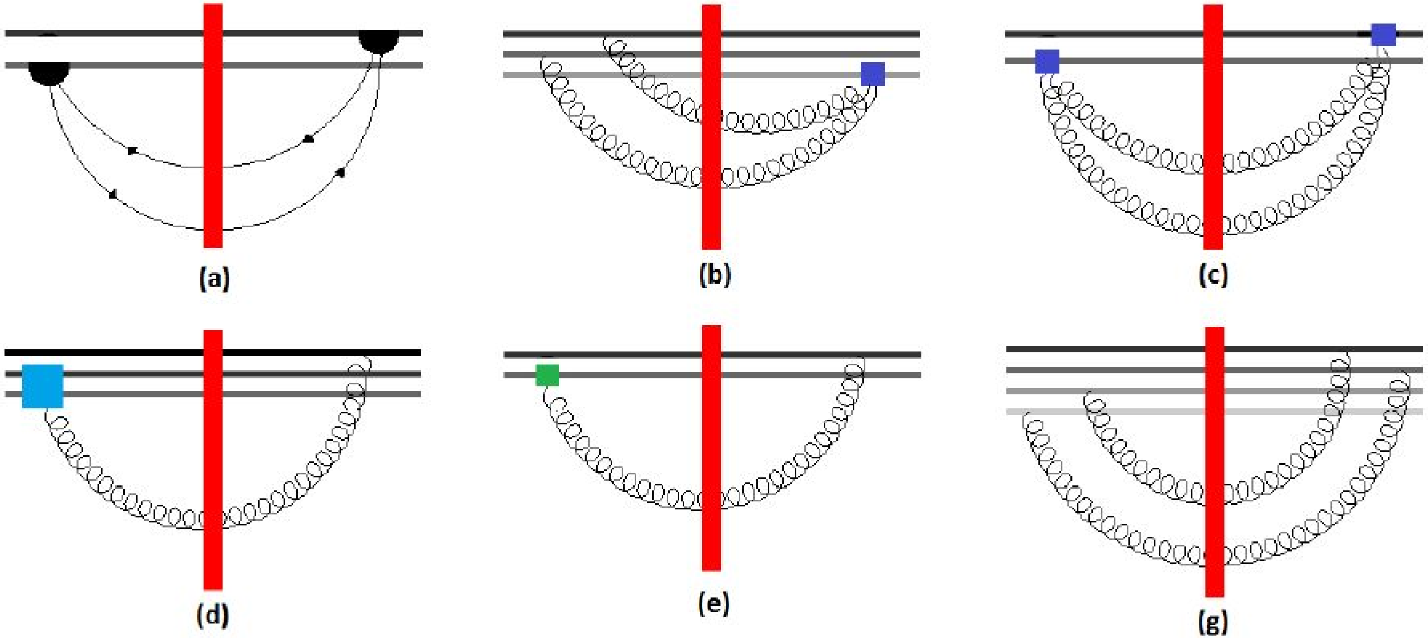
$$\Sigma_{JJSSJJ} \equiv \langle \psi_{gg\,\rho\rho} | \hat{S} | \psi_{gg\,\rho\rho} \rangle ,$$

$$\Sigma_{JJJSJ} \equiv \langle \psi_{g\,\rho\rho\rho} | \hat{S} | \psi^{LO}_{g\,\rho} \rangle + \langle \psi^{LO}_{g\,\rho} | \hat{S} | \psi_{g\,\rho\rho\rho} \rangle ,$$

$$\Sigma_{virtual} \equiv \langle 0 | (\mathcal{N}^{NLO})^{\dagger} \hat{S} \mathcal{N}^{NLO} | 0 \rangle - 1.$$

The Diagrammatic Representation

Each of the definitions in the previous slide can be interpreted as a light cone diagram:



NLO vs (LO)^2

While in the leading-order case, the result for Σ depends only on α_s and δY, at NLO the result for Σ depends on α_s^2 and either δY or $(\delta Y)^2$
Therefore, we splitted each contribution according to its arguments:

$$\Sigma_{...} = \Sigma_{...}^{NLO}(\alpha_s^2, \delta Y) + \Sigma_{...}^{(\delta Y)^2}(\alpha_s^2, (\delta Y)^2)$$

Then, we assembled together all the contributions in each case separately:

$$\Sigma^{NLO} \equiv \Sigma_{q\bar{q}}^{NLO} + \Sigma_{JJSSJ}^{NLO} + \Sigma_{JSSJ}^{NLO} + \Sigma_{JJSJ}^{NLO} + \Sigma_{JSJ}^{NLO} + \Sigma_{JJSSJJ}^{NLO} + \Sigma_{JJJSJ}^{NLO}$$

$$\Sigma^{(\delta Y)^2} = \Sigma_{JJSSJ}^{(\delta Y)^2} + \Sigma_{JSSJ}^{(\delta Y)^2} + \Sigma_{JJSJ}^{(\delta Y)^2} + \Sigma_{JSJ}^{(\delta Y)^2} + \Sigma_{JJSSJJ}^{(\delta Y)^2} + \Sigma_{JJJSJ}^{(\delta Y)^2} + \Sigma_{virtual}^{(\delta Y)^2}$$

The computation of the NLO JIMWLK Hamiltonian follows from:

$$H^{NLO} \equiv -\left.\frac{d\Sigma^{NLO}}{d\delta Y}\right|_{\delta Y=0}$$

Subtraction of the (LO)^2 Contributions

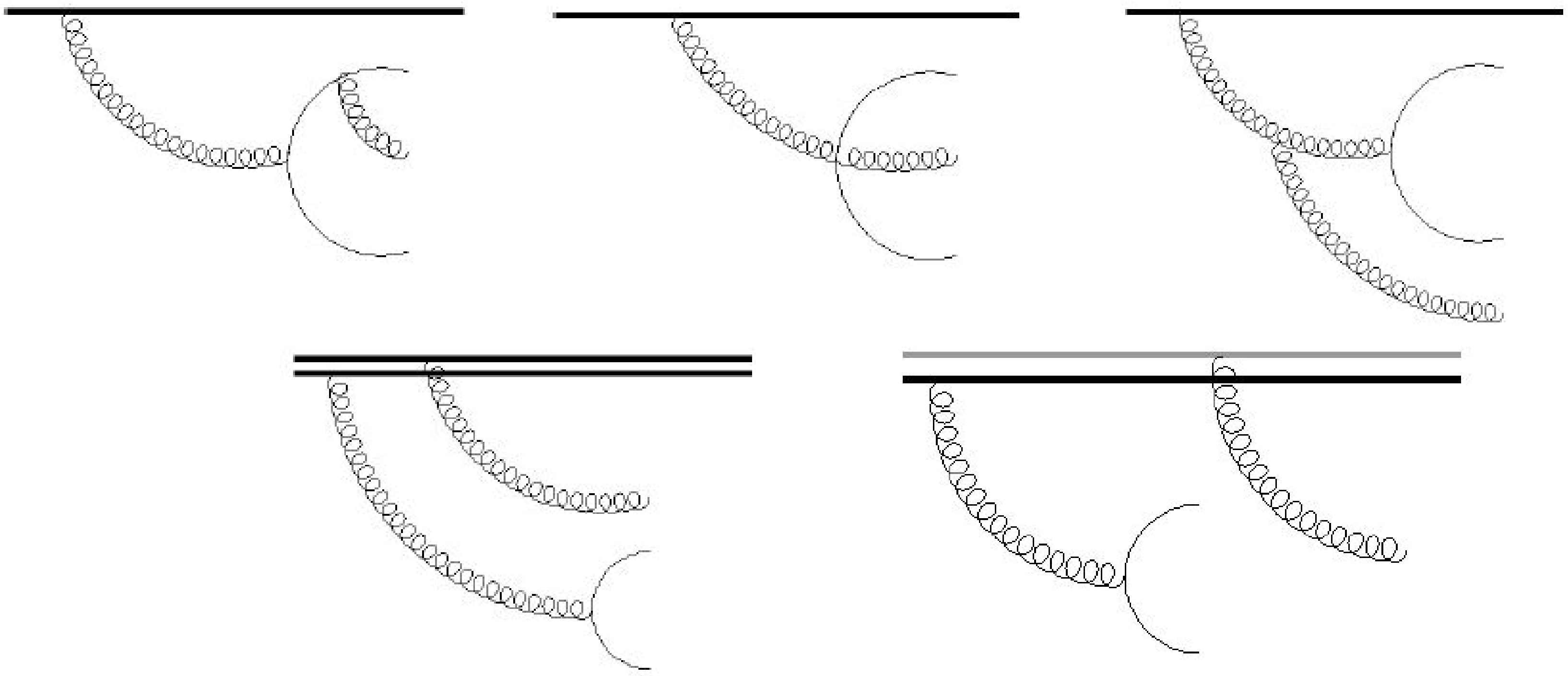
The contribution which corresponds to (LO)^2 can be written as:

$$(H^{LO\,JIMWLK})^2 = \frac{\alpha_s^2}{4\pi^4}\int_{x_\perp,y_\perp,u_\perp,v_\perp,z_\perp,z'_\perp} \frac{(X\cdot Y)(U\cdot V)}{X^2Y^2U^2V^2}\left[J_L^a(x)J_L^a(y) + J_R^a(x)J_R^a(y) - 2J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$
$$\times\left[J_L^c(u)J_L^c(v) + J_R^c(u)J_R^c(v) - 2J_L^c(u)S_A^{cd}(z')J_R^d(v)\right]$$

This contribution does not correspond to the same power of logs we are trying to resum due to double counting, therefore this contribution has to be subtracted.

A Glimpse at NNLO WF Quark Sector

The case in which quark, anti-quark and gluon are passing through the shockwave is controlled by 5 different diagrams.

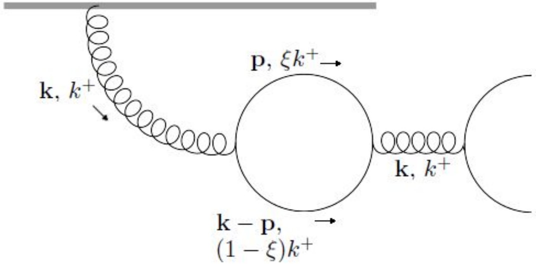


Conclusions

1) The NLO JIMWLK Hamiltonian (suitable for acting on any operator) was constructed based on symmetries and the known results for the evolution of dipole, baryon, and Wilson lines with open indices.

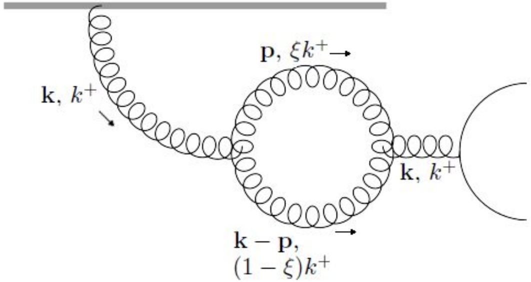
2) The light-cone wave function up to g^3(normalized up to g^4) was computed.

3) A consistent result for the NLO JIMWLK Hamiltonian was obtained by using the light-cone wave function formalism.



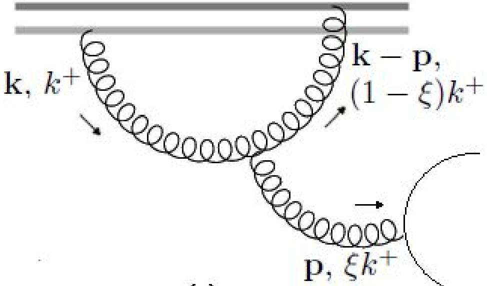
$\mathbf{p},\ \xi k^+$
$\mathbf{k},\ k^+$
$\mathbf{k},\ k^+$
$\mathbf{k}-\mathbf{p},$
$(1-\xi)k^+$

(a)



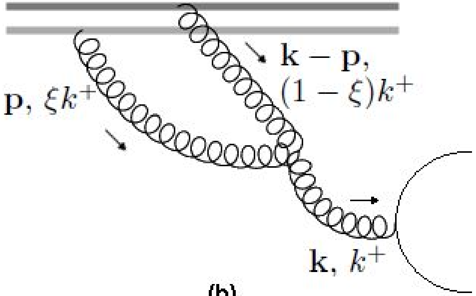
$\mathbf{p},\ \xi k^+$
$\mathbf{k},\ k^+$
$\mathbf{k},\ k^+$
$\mathbf{k}-\mathbf{p},$
$(1-\xi)k^+$

(b)



$\mathbf{k},\ k^+$
$\mathbf{k}-\mathbf{p},$
$(1-\xi)k^+$
$\mathbf{p},\ \xi k^+$

(a)



$\mathbf{p},\ \xi k^+$
$\mathbf{k}-\mathbf{p},$
$(1-\xi)k^+$
$\mathbf{k},\ k^+$

(b)