Production of a photon and two jets in the Hybrid factorization

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[T. A. , N. Armesto, A. Kovner, M. Lublinsky, E. Petreska, in preperation]

Outline

- Motivation and the ingredients
- Saturation sensitive part of the cross section
- Small dipole approximation
- Concluions

Motivation

at forward rapidity region:

- Single inclusive particle production
- two-particle correlations

are the two observables that have been analyzed in the framework of saturation physics.

Single inclusive particle production in pA collisions:

"Hybrid" formalism : Dumitru, Hayashigaki & Jalilian-Marian

- The wave function of the projectile proton is treated in the spirit of collinear factorization (an assembly of patrons with zero intrinsic transverse momenta)
- Perturbative corrections to this wave function are provided by the usual QCD perturbative splitting processes.
- Target is treated as distribution of strong color fields which during the scattering event transfer transverse momentum to the propagating partonic configuration. (CGC like treatment)

T.A., A. Kovner - 2011

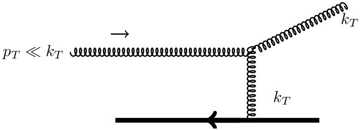
DOES LO "HYBRID" FORMULA TAKE INTO ACCOUNT ALL CONTRIBUTIONS AT HIGH $k_\perp$?

The single inclusive gluon spectrum :

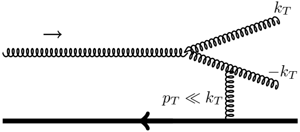
$$\frac{dN}{d^2k d\eta d^2b} \propto \left[\frac{dN}{d^2k d\eta}\right]_{elastic} + \left[\frac{dN}{d^2k d\eta}\right]_{inelastic}$$

In the limit of large transverse momentum of the produced gluon $k \gg Q_s, \Lambda_{QCD}$ there are two dominant contributions:

"Elastic Scattering" (LO)



"Inelastic Scattering" (NLO)



Inelastic contribution is sensitive to Q_s!

Particle Production at NLO within "Hybrid" formalism

G.A. Chirilli, B.W. Xiao, F. Yuan - 2012
Full NLO calculation...

A.M.Stasto, B.W.Xiao, D. Zaslavsky - 2013
Numerical analysis show strong dependence on NLO terms. Cross section becomes negative.

Possible solutions for the negativity problem:

T.A., N. Armesto, G. Beuf, A. Kovner, M. Lublinsky - 2015
Ioffe time restriction.

K. Watanabe, B.W. Xiao, F. Yuan, D. Zaslavsky - 2015
Exact kinematical constraint.

E. Iancu, A. H. Mueller and D. N. Triantafyllopoulos - 2016
New factorization scheme.

B. Ducloué, T. Lappi and Y. Zhu - 2017
Numerical analysis of the new factorization scheme.

Ioffe time restriction

The Ioffe Time Restriction provides a consistent description on what will be resolved by the target and what not!

- Only the pairs whose coherence time (Ioffe time) is greater than the propagation time through the target can be resolved by the target!

At NLO the quark splits in the projectile wave function with probability of order α_s into a quark-gluon configuration.

The dressed quark state :

$$
\begin{aligned}
|(q)\, x_B P^+, k_\perp, \alpha, s\rangle_D = \int_x e^{ik_\perp x} \Big\{ A^q |(q)\, x_B P^+, x, \alpha, s\rangle \\
+g \int_{\xi, yz} F_{(qg)}(x_B P^+, \xi, y-x, z-x)_{s\bar{s};j}\, t^a_{\alpha\beta} \\
|(q)\ y, p^+ = (1-\xi) x_B P^+, \beta, \bar{s}; (g)\ z, q^+ = \xi x_B P^+, a, j\rangle \Big\}
\end{aligned}
$$

- A^q is of order g^2 and needed to preserve the normalisation of the state at order α_s.
- $F_{(qg)}$ is the function that defines the splitting of a quark into a quark-gluon pair.

Ioffe time restriction - II

The dressed quark scatters on the target and produces final state particles.

Within "hybrid" formalism, the scattering of the qg pair is treated as a completely coherent process $\Rightarrow$ each parton picks an eikonal phase during the interaction with the target.

THIS IS ONLY POSSIBLE if the coherence time (Ioffe Time) of the configuration is greater than the propagation time through the target.

$$\boxed{\text{coherent scattering} \Rightarrow \frac{2(1-\xi)\xi x_B P^+}{k_\perp^2} > \tau}$$

$\tau \equiv$ a fixed time scale determined by the longitudinal size of the target.

The function $F_{(qg)}$ is written as

$$F_{(qg)} = \frac{i}{\sqrt{2\xi x_B P^+}} \Big\{ \delta_{s\bar{s}}\delta_{ij}(2-\xi) - i\epsilon_{ij}\sigma^3_{s\bar{s}}\xi \Big\} \delta^2\Big(x - [(1-\xi)y + \xi z]\Big) A^i_{\xi,x_B}(y-z)$$

Modified Weiszacker-Williams field

$$\begin{aligned} A^i_{\xi,x_B}(y-z) &= -i \int_{l_\perp^2 < 2\xi(1-\xi)x_B \frac{P^+}{\tau}} \frac{d^2 l_\perp}{(2\pi)^2} \frac{l_\perp^i}{l_\perp^2} e^{il_\perp(y-z)} \\ &= -\frac{1}{2\pi} \frac{(y-z)^i}{(y-z)^2} \left[1 - \mathrm{J}_0\left(|y-z| \sqrt{2\xi(1-\xi)\frac{x_B P^+}{\tau}} \right) \right] \end{aligned}$$

Production of a photon and two jets

Consider production of a soft photon ($\mathbf{q}_1 \lesssim Q_s$) and two hard jets ($\mathbf{q}_2, \mathbf{q}_3 \gg Q_s$).

$$\boxed{p(p_p) + A(p_A) \to \gamma(q_1) + q(q_2) + g(q_3) + X}$$

Only interested in saturation sensitive contribution (inelastic part).

Partonic cross section:

$$(2\pi)^9 \frac{d\sigma^{qA\to\gamma qgX}}{d^3q_1 d^3q_2 d^3q_3}\bigg|_{inel.} = \sum_{s\bar{s}} \sum_{\alpha\bar{\alpha}} \int \frac{d\bar{p}^+}{2\pi} \frac{dp^+}{2\pi} {}_{\text{out}}\Big\langle (\mathbf{q})[\bar{p}^+, 0]^{\bar{\alpha}}_{\bar{s}} \Big| O(q_1, q_2, q_3) \Big| (\mathbf{q})[p^+, 0]^{\alpha}_{s} \Big\rangle_{\text{out}}$$

where the number operator

$$O(q_1, q_2, q_3) = \gamma^\dagger_\theta(q_1^+, \mathbf{q}_1)\gamma_\theta(q_1^+, \mathbf{q}_1) d^{\dagger\sigma}_t(q_2^+, \mathbf{q}_2) d^\sigma_t(q_2^+, \mathbf{q}_2) a^{\dagger f}_l(q_3^+, \mathbf{q}_3) a^f_l(q_3^+, \mathbf{q}_3)$$

What is $|(\mathbf{q})[p^+, 0]^\alpha_s\rangle_{\text{out}}$?

The dressed quark state with the vanishing transverse momentum when written in terms of bare states:

$$\big|(\mathbf{q})[p^+,0]^{\alpha}_{s}\big\rangle_D = A^q \int_{\mathbf{w}} \big|(\mathbf{q})[p^+,\mathbf{w}]^{\alpha}_{s}\big\rangle_0$$

$$+ \quad g_e \sum_{s'}\sum_{\lambda} \int \frac{dk^+}{2\pi} \int_{\mathbf{w},\mathbf{v},\mathbf{x}_1} F_{(\mathbf{q}\gamma)}\left[p^+,\frac{k^+}{p^+};\mathbf{w}-\mathbf{v},\mathbf{w}-\mathbf{x}_1\right]_{ss',\lambda} \times \big|(\mathbf{q})\,[p^+-k^+,\mathbf{v}]^{\alpha}_{s'};(\gamma)[k^+,\mathbf{x}_1]_{\lambda}\big\rangle_0$$

$$+ \quad g_s \sum_{s'}\sum_{\eta} \int \frac{dm^+}{2\pi} \int_{\mathbf{w},\mathbf{v},\mathbf{x}_3} t^c_{\alpha\beta}\, F_{(\mathbf{qg})}\left[p^+,\frac{m^+}{p^+};\mathbf{w}-\mathbf{v},\mathbf{v}-\mathbf{x}_3\right]_{ss',\eta} \times \Big|(\mathbf{q})\,[p^+-m^+,\mathbf{v}]^{\beta}_{s'};(\mathbf{g})\,[m^+,\mathbf{x}_3]^{c}_{\eta}\Big\rangle_0$$

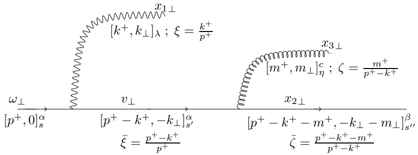
$$+ \quad g_s g_e \sum_{s's''}\sum_{\lambda}\sum_{\eta} \int \frac{dk^+}{2\pi}\frac{dm^+}{2\pi} \int_{\mathbf{w},\mathbf{v},\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3} t^c_{\alpha\beta}$$

$$\times\Bigg\{F_{(\mathbf{q}\gamma)}\left[p^+,\frac{k^+}{p^+};\mathbf{w}-\mathbf{v},\mathbf{w}-\mathbf{x}_1\right]_{ss',\lambda} F_{(\mathbf{qg})}\left[p^+-k^+,\frac{m^+}{p^+-k^+};\mathbf{v}-\mathbf{x}_2,\mathbf{v}-\mathbf{x}_3\right]_{s's'',\eta}$$

$$+F_{(\mathbf{qg})}\left[p^+,\frac{m^+}{p^+};\mathbf{w}-\mathbf{v},\mathbf{w}-\mathbf{x}_3\right]_{ss',\eta} F_{(\mathbf{q}\gamma)}\left[p^+-m^+,\frac{k^+}{p^+-m^+};\mathbf{v}-\mathbf{x}_2,\mathbf{v}-\mathbf{x}_1\right]_{s's'',\lambda}\Bigg\}$$

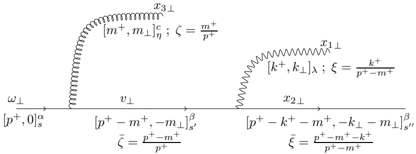
$$\times\Big|(\mathbf{q})\,[p^+-k^+-m^+,\mathbf{x}_2]^{\beta}_{s''};(\gamma)\,[k^+,\mathbf{x}_1]_{\lambda};(\mathbf{g})\,[m^+,\mathbf{x}_3]^{c}_{\eta}\Big\rangle_0$$

Photon emission before the quark gluon splitting:



$$\propto F_{(\mathbf{q}\gamma)}\left[p^+, \frac{k^+}{p^+}; \mathbf{w}-\mathbf{v}, \mathbf{w}-\mathbf{x}_1\right]_{ss',\lambda} F_{(\mathbf{qg})}\left[p^+ - k^+, \frac{m^+}{p^+ - k^+}; \mathbf{v}-\mathbf{x}_2, \mathbf{v}-x_3\right]_{s's'',\eta}$$

Photon emission after the quark gluon splitting:



$$\propto F_{(\mathbf{qg})}\left[p^+, \frac{m^+}{p^+}; \mathbf{w}-\mathbf{v}, \mathbf{w}-\mathbf{x}_3\right]_{ss',\eta} F_{(\mathbf{q}\gamma)}\left[p^+ - m^+, \frac{k^+}{p^+ - m^+}; \mathbf{v}-\mathbf{x}_2, \mathbf{v}-x_1\right]_{s's'',\lambda}$$

Out going state

The functions that define the splitting:

$$F_{(\mathbf{q}\gamma)}[p^+, \xi, \mathbf{w}-\mathbf{v}, \mathbf{w}-\mathbf{x}_1]_{ss',j} = \frac{-i}{\sqrt{2\xi p^+}}[\delta_{ss'}\delta_{ij}(2-\xi) - i\epsilon_{ij}\sigma^3_{ss'}\xi]\delta^{(2)}(\mathbf{w}-[\bar{\xi}\mathbf{v}+\xi\mathbf{x}_1])A^j(\mathbf{v}-\mathbf{x}_1)$$

with $A^j(\mathbf{v}-\mathbf{x}_1)$ is the electro-magnetic Weiszäcker-Williams field in the quark-photon splitting function

$$A^j(\mathbf{v}-\mathbf{x}_1) = -\frac{1}{2\pi}\frac{(\mathbf{v}-\mathbf{x}_1)^j}{(\mathbf{v}-\mathbf{x}_1)^2}$$

$F_{(\mathbf{qg})}$ is the QCD analogue of the function $F_{(\mathbf{q}\gamma)}$ and it is defined with the *modified Weiszäcker-Williams field* and includes the Ioffe time restriction.

$$\bar{A}^j_{\xi}(\mathbf{v}-\mathbf{x}) = -\frac{1}{2\pi}\frac{(\mathbf{v}-\mathbf{x})^j}{(\mathbf{v}-\mathbf{x})^2}\left[1 - J_0\left(|\mathbf{v}-\mathbf{x}|\sqrt{2\xi(1-\xi)\frac{p^+}{\tau}}\right)\right]$$

Scattering via eikonal interaction

Only the contribution comes from "Photon emission before the quark gluon splitting".

BUT it is still important to keep the Photon emission after the quark gluon splitting in order to calculate the $|\mathbf{q}\gamma\mathbf{g}\rangle_D$.

Eikonal scattering on the target leads to :

$$\Big|(\mathbf{q})[p^+,0]^{\alpha}_{s}\Big\rangle_{\rm out} \simeq g_s g_e \sum_{s's''\lambda\eta} \int \frac{dk^+}{2\pi}\frac{dm^+}{2\pi}\int_{\mathbf{w}\mathbf{v}\mathbf{x}_1\mathbf{x}_2\mathbf{x}_3}$$
$$\times F_{(\mathbf{q}\gamma)}\left[p^+,\frac{k^+}{p^+};\mathbf{w}-\mathbf{v},\mathbf{w}-\mathbf{x}_1\right]_{ss',\lambda} F_{(\mathbf{qg})}\left[p^+-k^+,\frac{m^+}{p^+-k^+};\mathbf{v}-\mathbf{x}_2,\mathbf{v}-\mathbf{x}_3\right]_{s's'',\eta}$$
$$\times\left\{t^c_{\alpha\beta}S_F^{\beta\gamma}(\mathbf{x}_2)S_A^{cd}(\mathbf{x}_3)-S_F^{\alpha\beta}(\mathbf{w})t^d_{\beta\gamma}-S_F^{\alpha\beta}(\mathbf{v})t^d_{\beta\gamma}\right\}$$
$$\times\Big|(\mathbf{q})\left[p^+-k^+-m^+,\mathbf{x}_2\right]^{\gamma}_{s''};(\gamma)\left[k^+,\mathbf{x}_1\right]_{\lambda};(\mathbf{g})\left[m^+,\mathbf{x}_3\right]^{d}_{\eta}\Big\rangle_D$$

$S_{F(A)} \equiv$ the eikonal scattering matrix of a projectile quark (gluon) in the color filed of the target.

Partonic cross section

It is now straight forward to calculate the expectation value of the number density operator in the out state:

$$
\begin{aligned}
(2\pi)^6 \frac{d\sigma^{qA\to\gamma qgX}}{d^3q_1 d^3q_2 d^3q_3} &= \frac{2}{N_c} g_s^2 g_e^2 \int \frac{dp^+}{2\pi} \int_{\mathbf{y}_1\mathbf{z}_1\mathbf{y}_2\mathbf{z}_2\mathbf{y}_3\mathbf{z}_3\mathbf{w}\mathbf{v}\bar{\mathbf{w}}\bar{\mathbf{v}}} e^{i\mathbf{q}_1\cdot(\mathbf{y}_1-\mathbf{z}_1)+i\mathbf{q}_2\cdot(\mathbf{y}_2-\mathbf{z}_2)+i\mathbf{q}_3\cdot(\mathbf{y}_3-\mathbf{z}_3)} \\
&\times\delta(p^+ - q_1^+ - q_2^+ - q_3^+)\frac{1}{\xi p^+}\left[1+(1-\xi)^2\right]\frac{1}{(1-\xi)\zeta p^+}\left[1+(1-\zeta)^2\right] \\
&\times A^i(\bar{\mathbf{v}}-\mathbf{y}_1)A^i(\mathbf{v}-\mathbf{z}_1)\bar{A}^j_\zeta(\mathbf{z}_2-\mathbf{z}_3)\bar{A}^j_\zeta(\mathbf{y}_2-\mathbf{y}_3)\delta^{(2)}\{\mathbf{w}-[(1-\xi)\mathbf{v}+\xi\mathbf{z}_1]\} \\
&\times\delta^{(2)}\{\bar{\mathbf{w}}-[(1-\xi)\bar{\mathbf{v}}+\xi\mathbf{y}_1]\}\delta^{(2)}\{\mathbf{v}-[(1-\zeta)\mathbf{z}_2+\zeta\mathbf{z}_3]\}\delta^{(2)}\{\bar{\mathbf{v}}-[(1-\zeta)\mathbf{y}_2+\zeta\mathbf{y}_3]\} \\
&\times\Bigg\{\frac{N_c^2}{2}\Big[Q(\mathbf{y}_3,\mathbf{z}_3,\mathbf{z}_2,\mathbf{y}_2)s(\mathbf{z}_3,\mathbf{y}_3) - s(\mathbf{w},\mathbf{y}_3)s(\mathbf{y}_3,\mathbf{y}_2) - s(\mathbf{v},\mathbf{y}_3)s(\mathbf{y}_3,\mathbf{y}_2) - s(\mathbf{z}_2,\mathbf{z}_3)s(\mathbf{z}_3,\bar{\mathbf{w}}) \\
&-s(\mathbf{z}_2,\mathbf{z}_3)s(\mathbf{z}_3,\bar{\mathbf{v}})\Big] - \frac{1}{2}\Big[s(\mathbf{z}_2,\mathbf{y}_2) - s(\omega,\mathbf{y}_2) - s(\mathbf{v},\mathbf{y}_2) - s(\mathbf{z}_2,\bar{\mathbf{w}}) - s(\mathbf{z}_2,\bar{\mathbf{v}})\Big] \\
&+\frac{N_c^2-1}{2}\Big[s(\mathbf{w},\bar{\mathbf{w}}) + s(\mathbf{v},\bar{\mathbf{w}}) + s(\mathbf{w},\bar{\mathbf{v}}) + s(\mathbf{v},\bar{\mathbf{v}})\Big]\Bigg\}
\end{aligned}
$$

with $s(\mathbf{x},\mathbf{y}) = \frac{1}{N_c}\mathrm{tr}\left[S_F(\mathbf{x})S_F^\dagger(\mathbf{y})\right]$ and $Q(\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{v}) = \frac{1}{N_c}\mathrm{tr}\left[S_F(\mathbf{x})S_F^\dagger(\mathbf{y})S_F(\mathbf{z})S_F^\dagger(\mathbf{v})\right]$.

- We are considering the production of two hard jets, $\mathbf{q}_2, \mathbf{q}_3 \gg Q_s$.
- Inelastic contribution $\Rightarrow$ the produced quark-gluon pair has large relative momenta, $\mathbf{q}_3 - \mathbf{q}_2 \gg Q_s$.

$\Rightarrow$ the production of small transverse size quark-gluon dipoles.

$$\mathbf{r} = \mathbf{z}_2 - \mathbf{z}_3\,, \qquad \mathbf{b} = \frac{1}{2}(\mathbf{z}_2 + \mathbf{z}_3)\,, \qquad \gamma = \mathbf{z}_1 - \mathbf{b} - (1-2\zeta)\frac{\mathbf{r}}{2}$$

$$\bar{\mathbf{r}} = \mathbf{y}_2 - \mathbf{y}_3\,, \qquad \bar{\mathbf{b}} = \frac{1}{2}(\mathbf{y}_2 + \mathbf{y}_3)\,, \qquad \bar{\gamma} = \mathbf{y}_1 - \bar{\mathbf{b}} + (1-2\zeta)\frac{\bar{\mathbf{r}}}{2}$$

Expanding the partonic cross section for small $\mathbf{r}$ and $\bar{\mathbf{r}}$:

$$(2\pi)^6 \frac{d\sigma^{qA\to\gamma q g X}}{d^3q_1 d^3q_2 d^3q_3}\bigg|_{inel.} = 2(4\pi)^2 C_F \int \frac{dp^+}{2\pi}\delta(p^+ - q_1^+ - q_2^+ - q_3^+)$$

$$\times \int_{\gamma\bar{\gamma}\mathbf{b}\bar{\mathbf{b}}\mathbf{r}\bar{\mathbf{r}}} e^{\left\{i\mathbf{q}_1\cdot(\bar{\gamma}-\gamma) + i(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3)\cdot(\bar{\mathbf{b}}-\mathbf{b}) + \frac{i}{2}[(1-2\zeta)\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{q}_3]\cdot(\bar{\mathbf{r}}-\mathbf{r})\right\}} s\left(\mathbf{b}+\xi\gamma, \bar{\mathbf{b}}+\xi\bar{\gamma}\right)$$

$$\times \frac{1}{\xi p^+}\left[1+(1-\xi)^2\right]\frac{1}{(2\pi)^2}\frac{\gamma\cdot\bar{\gamma}}{\gamma^2\bar{\gamma}^2}\frac{1}{(1-\xi)\zeta p^+}\left[1+(1-\zeta)^2\right]\frac{1}{(2\pi)^2}\frac{\mathbf{r}\cdot\bar{\mathbf{r}}}{\mathbf{r}^2\bar{\mathbf{r}}^2}$$

$$\times \left[1 - J_0\left(|\mathbf{r}|\sqrt{2\zeta(1-\zeta)\frac{(1-\xi)p^+}{\tau}}\right)\right]\left[1 - J_0\left(|\bar{\mathbf{r}}|\sqrt{2\zeta(1-\zeta)\frac{(1-\xi)p^+}{\tau}}\right)\right]$$

Small dipole approximation - II

After shifting the variables

$$\begin{aligned}\mathbf{b} + \xi\gamma &= \mathbf{b}' \\ \bar{\mathbf{b}} + \xi\bar{\gamma} &= \bar{\mathbf{b}}'\end{aligned}$$

$K_T \equiv \frac{1}{2}[(1-2\zeta)\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{q}_3]$, $P_T \equiv \mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3$, $Q_T \equiv (1-\xi)\mathbf{q}_1 - \xi(\mathbf{q}_2 + \mathbf{q}_3)$

conjugate to

$(\mathbf{r} - \bar{\mathbf{r}})$, $(\mathbf{b}' - \bar{\mathbf{b}}')$, $(\gamma - \bar{\gamma})$

Integrations over $\mathbf{r}, \bar{\mathbf{r}}, \gamma$ and $\bar{\gamma}$ can be performed and *the partonic cross section* reads:

$$(2\pi)^6 \frac{d\sigma^{qA\to\gamma qgX}}{d^3q_1 d^3q_2 d^3q_3}\bigg|_{inel.} = 2C_F(4\pi)^2\alpha_s\alpha_e \int \frac{dp^+}{2\pi}\delta(p^+ - q_1^+ - q_2^+ - q_3^+)$$
$$\times \frac{1}{\xi p^+}\left[1+(1-\xi)^2\right]\frac{1}{(1-\xi)\zeta p^+}\left[1+(1-\zeta)^2\right]\theta\left(2\zeta(1-\zeta)\frac{(1-\xi)p^+}{\tau} - K_\perp^2\right)$$
$$\times \frac{1}{Q_T^2 K_T^2}\int_{\mathbf{b}'\bar{\mathbf{b}}'} e^{iP_T\cdot(\bar{\mathbf{b}}'-\mathbf{b}')} s(\mathbf{b}', \bar{\mathbf{b}}')$$

Summary

- We have calculated the inelastic piece of the cross section for the production of a soft photon and two hard jets which is sensitive to the saturation.
- The Ioffe time constraint is implemented in this result to ensure the coherent scattering of the quark gluon pair.
- Due to the kinematics of the inelastic contribution we were able to use the small dipole approximation and in this limit our result simplifies to single dipole a opeartor.