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# Saturation of the resonance spectra

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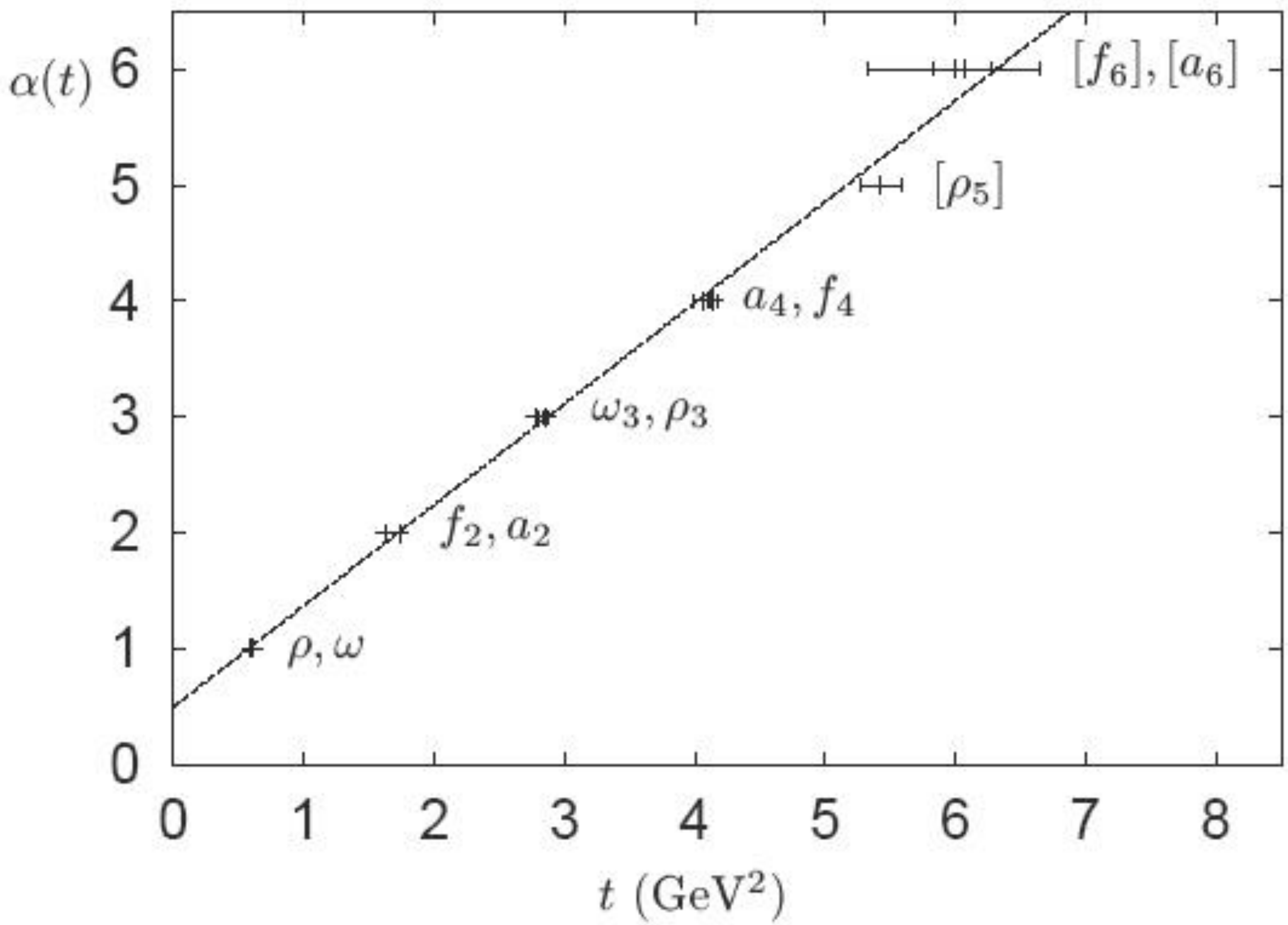
# Linear particle trajectories

Plot of spins of families of particles against their squared masses:



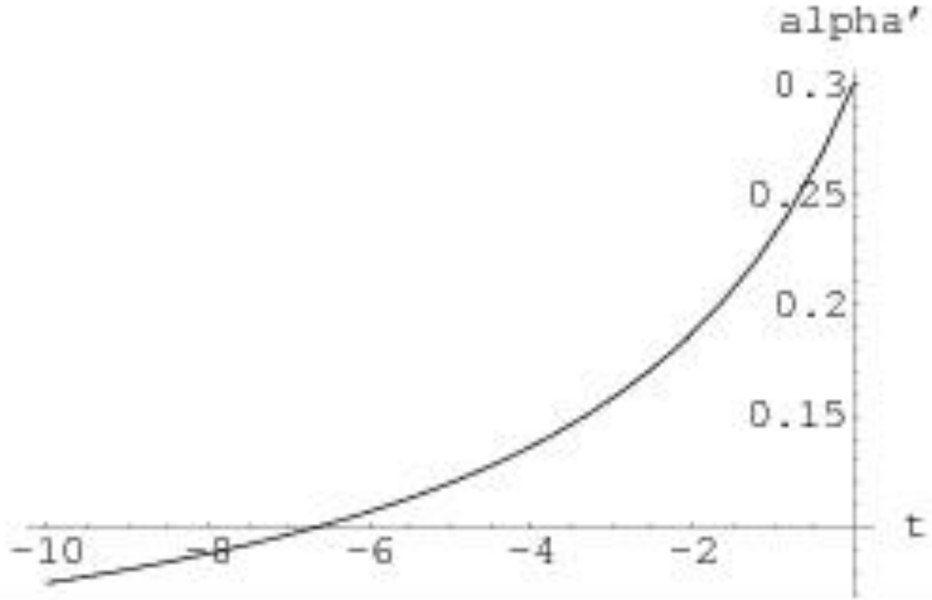
## The Pomeron trajectory

The Pomeron trajectory has threshold singularities, the lowest one being due to the two-pion exchange, required by the $t-$channel unitarity. There is a constrain (Barut, Zwanziger; Gribov) from the $t-$ channel unitarity, by which

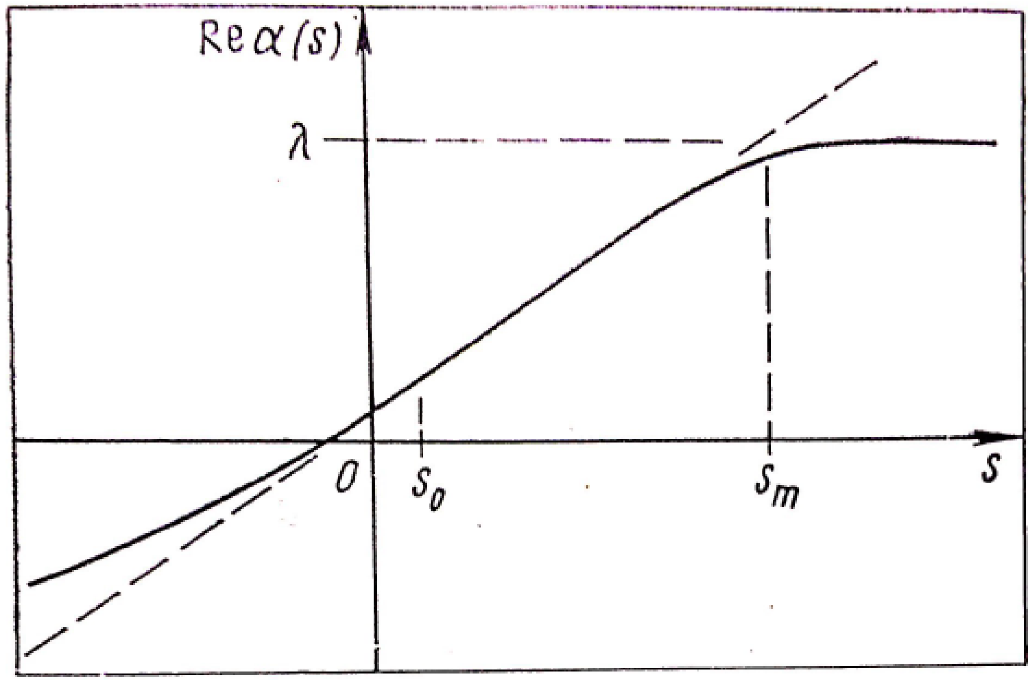
$$\Im\alpha(t) \sim (t - t_0)^{\Re\alpha(t_0)+1/2}, \quad t \to t_0,$$

where $t_0$ is the lightest threshold. For the Pomeron trajectory it is $t_0 = 4m_\pi^2$, and near the threshold:

$$\alpha(t) \sim \sqrt{4m_\pi^2 - t}. \tag{1}$$




The slope of the cone for a single pole is: $B(s,t) \sim \alpha'(t) \ln s$. The Regge residue $e^{b\alpha(t)}$ with a logarithmic trajectory $\alpha(t) = \alpha(0) - \gamma \ln(1-\beta t)$, is identical to a form factor (geometrical model).

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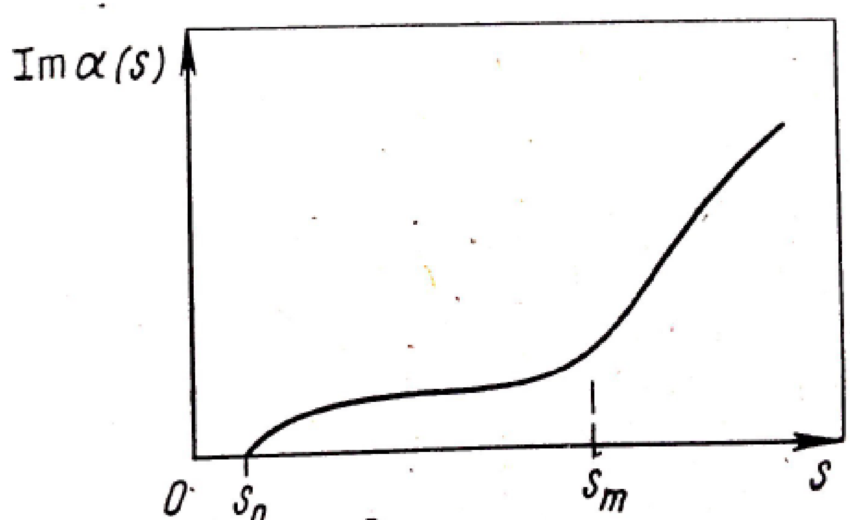



б

$$\Re e\, \alpha(s) = \alpha(0) + \frac{s}{\pi} PV \int_0^{\infty} ds' \frac{\Im m\, \alpha(s')}{s'(s'-s)}. \quad (7)$$

In Eq. (7), PV denotes the Cauchy Principal Value of the integral. The imaginary part is related to the decay width by

$$\Gamma(M_R) = \frac{\Im m\, \alpha(M_R^2)}{\alpha'\, M_R}. \quad (8)$$

The quantity $\alpha'$ in Eq. (8) denotes the derivative of the real part, $\alpha' = \frac{d\Re e\, \alpha(s)}{ds}$. The relation between $\Gamma(M)$ and $\Im m\, \alpha(\mathrm{s})$ requires $\Im m\, \alpha(s) > 0$. In a simple analytical model, the imaginary part is chosen as a sum of single threshold terms [25]

$$\Im m\, \alpha(s) = \sum_n c_n (s - s_n)^{1/2} \left(\frac{s - s_n}{s}\right)^{|\Re e\, \alpha(s_n)|} \theta(s - s_n). \quad (9)$$

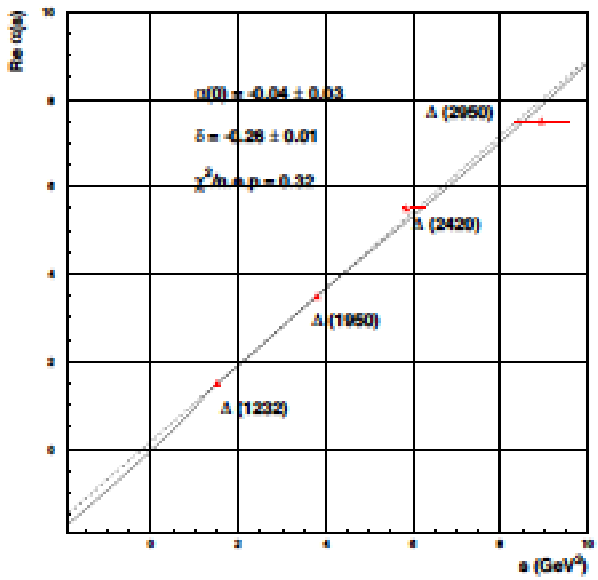



Figure 1: The real part of Δ trajectory. The dashed line corresponds to the result of a linear fit, the solid line corresponds to the final result.

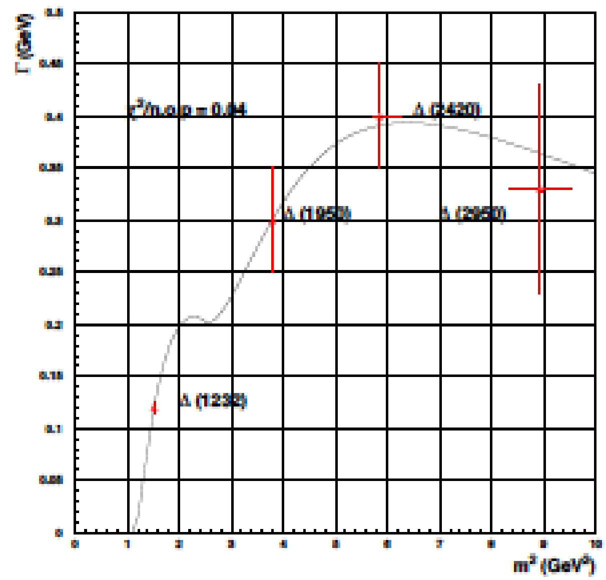



Figure 2: The width of Δ trajectory.

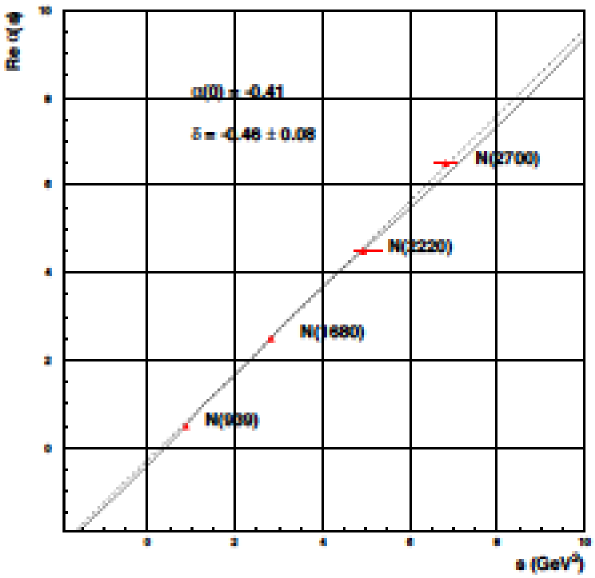



Figure 3: The real part of $N$ trajectory. The dashed line corresponds to the result of a linear fit, the solid line corresponds to the final result.

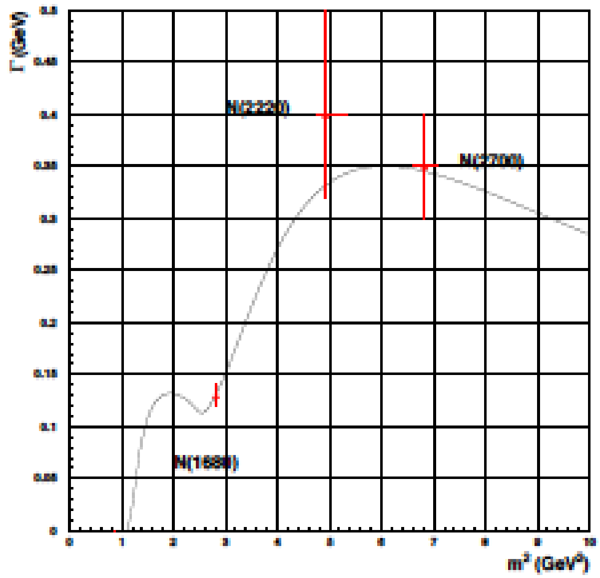



Figure 4: The width of $N$ trajectory.

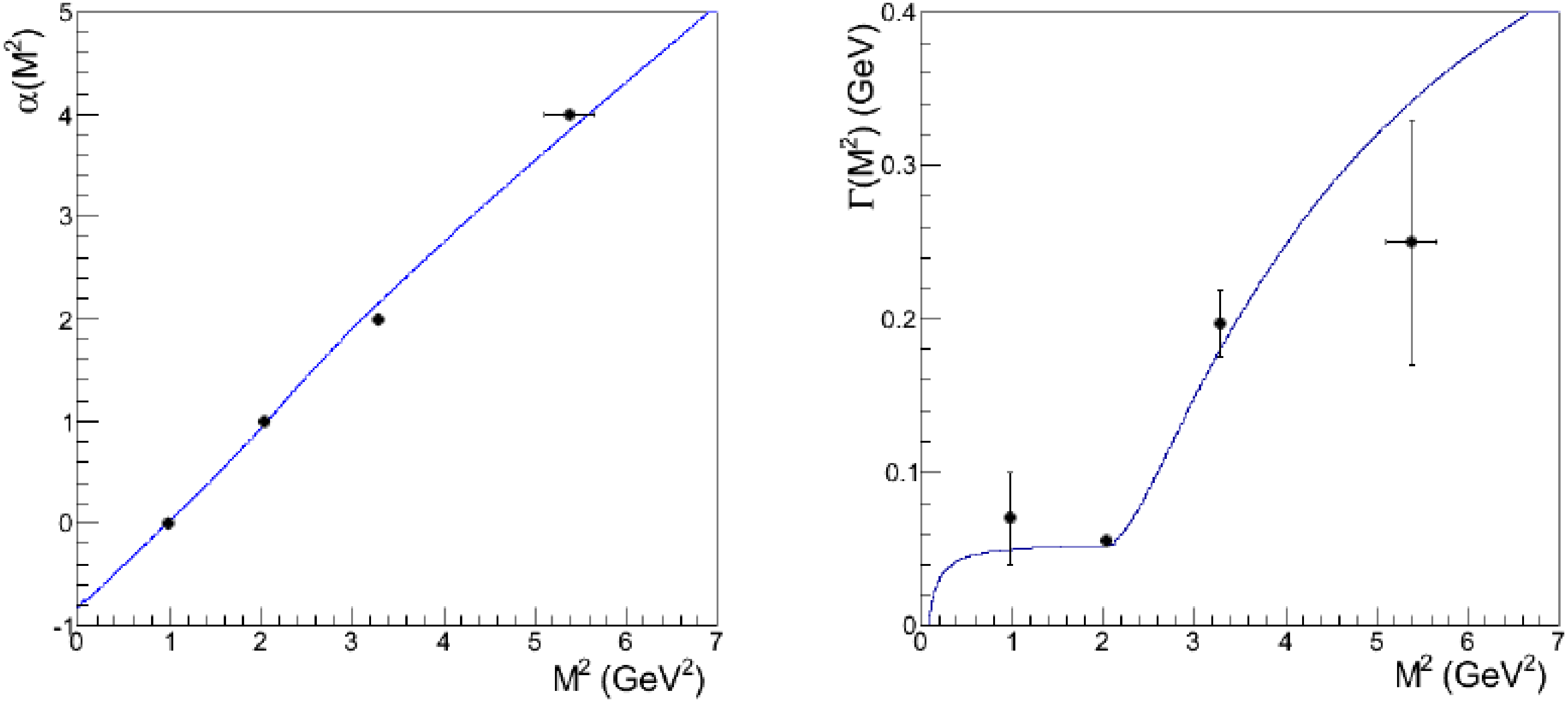



FIG. 6: Real part of $f_1$ trajectory on the left, width function $\Gamma(\mathrm{M}^2)$ on the right.

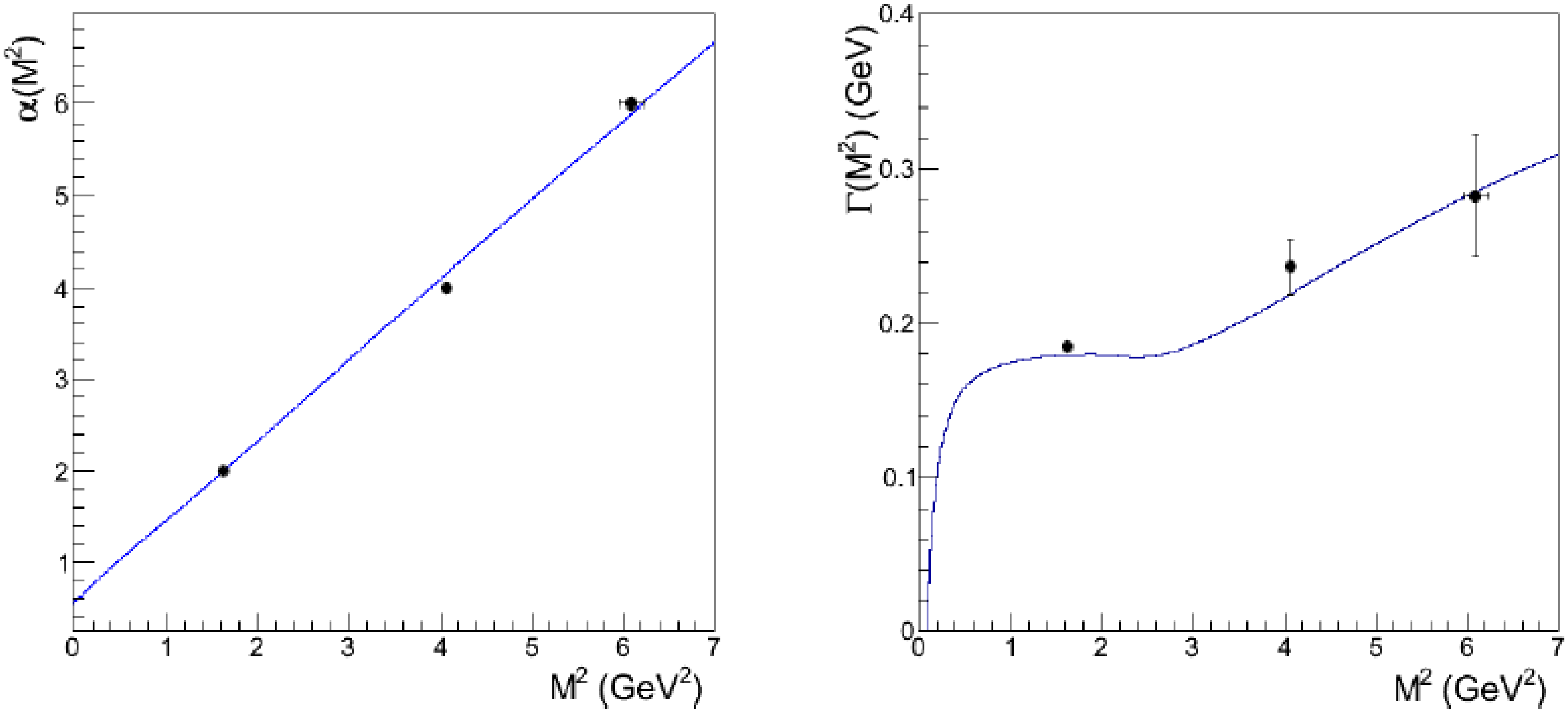
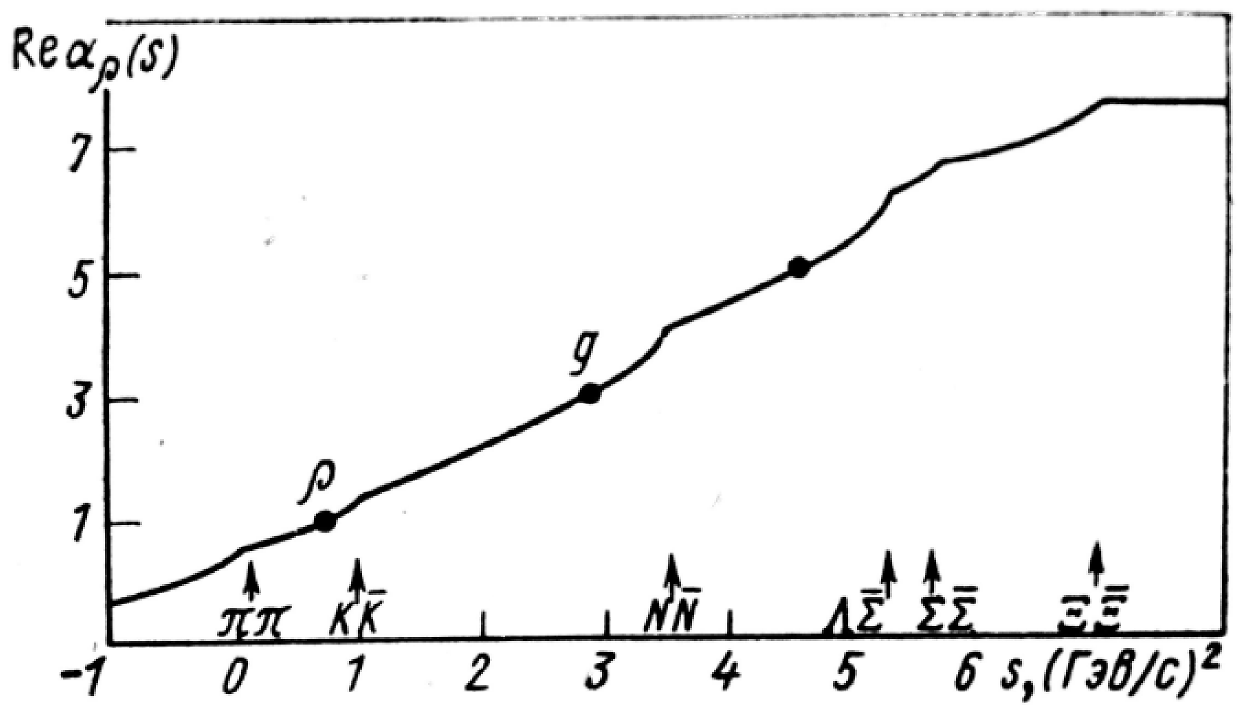



FIG. 7: Real part of $f_2$ trajectory on the left, width function $\Gamma(M^2)$ on the right.

Let us start with a toy model of a non-linear trajectory, Following [18, 31], we write a simple trajectory in which the (additive) thresholds are those made of stable particles allowed by quantum numbers. For the $\rho$ trajectory these are: $\pi\pi$, $K\bar{K}$, $N\bar{N}$, $\Lambda\bar{\Sigma}$, $\Sigma\bar{\Sigma}$, $\Xi\bar{\Xi}$. The relevant trajectory is:

$$\alpha_\rho(m) = 7.64 - 0.127\sqrt{m-0.28} - 0.093\sqrt{m-0.988} - 0.761\sqrt{m-1.88} - \text{"}\Lambda\bar{\Sigma},\ \Sigma\bar{\Sigma},\ \Xi\bar{\Xi}\text{"}, \qquad (10)$$

with the parameters of higher threshold quoterd in Ref. [18].



$$N_{theor} = \int_0^m \rho_{theor}(m')dm',$$

where

$$\rho_{theor}(m) = f(m)\exp(m/T)$$

and $f(m) \approx A/(m^2 + (500MeV)^2)^{5/4}$ (alternative choices for this slowly varying function are possible).

According to Hagedorn's conjecture, confirmed by subsequent studies, the density of hadronic resonances increases exponentially, modulus a slowly varying function of mass, $f(m)$,

$$\rho(m) = f(m)\exp(m/T) \tag{1}$$

up to about $m = 2 \div 2.5$ MeV, whereupon the exponential rise slows down

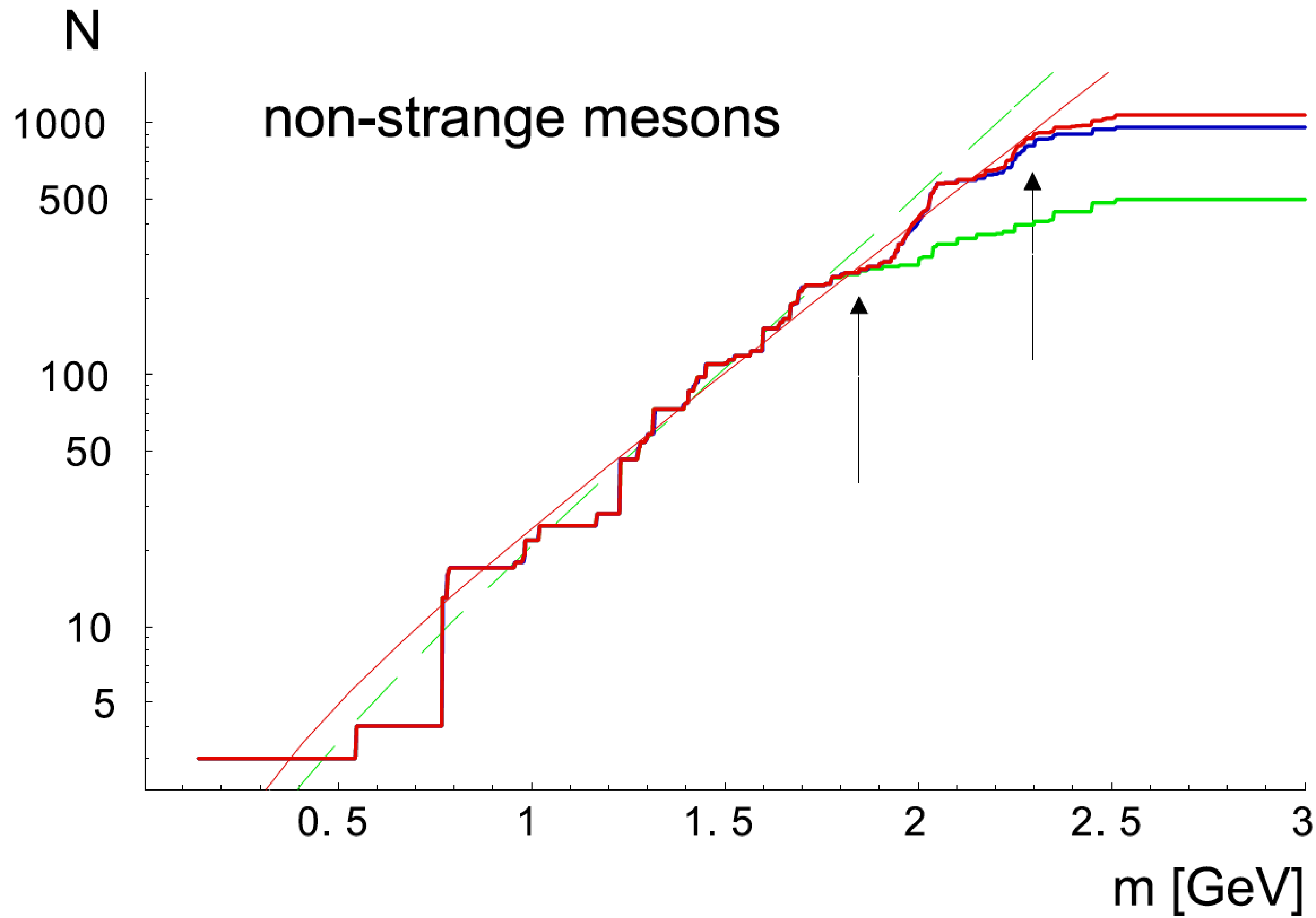
We extend the Hagedorn formula by introducing in the slope of relevant non-linear Regge trajectories. Anticipating a detailed quantitative analyses, one may observe immediately that flattening of $\Re\alpha(s = m^2)$ results in a drastic decrease of the relevant slope $\alpha'(m)$ and a corresponding change of the Hagedorn spectrum, which we parametrize as

$$\rho(m) \sim (\Re\alpha(m))' \exp(m/T). \tag{3}$$

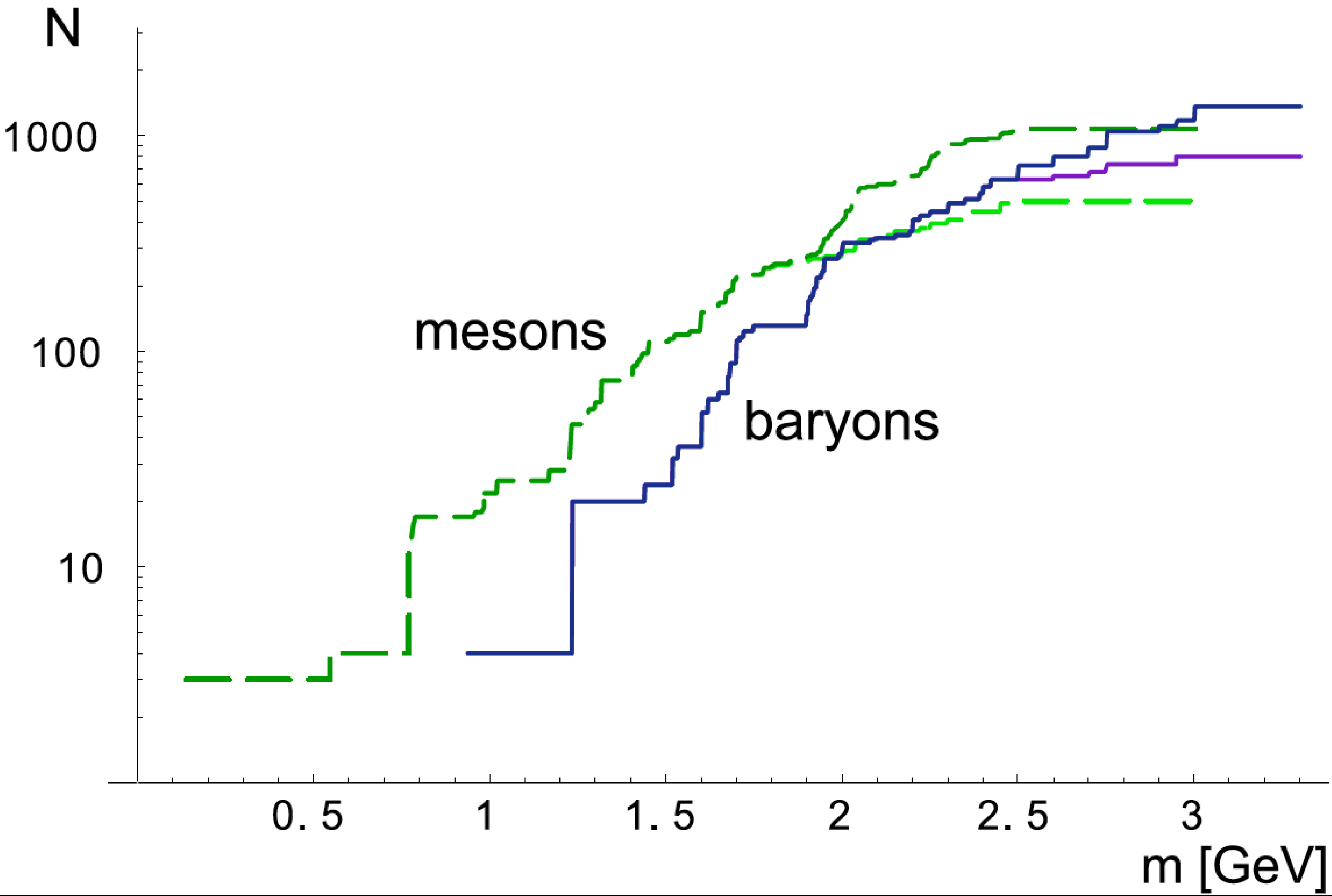
Usually, one compares the cumulants of the spectrum, defined as the number of states with mass lower than $m_i$. The experimental curve is

$$N_{exp}(m) = \sum_i g_i \Theta(m - m_i), \tag{4}$$

where $g_i = (2J_i + 1)(2I_i + 1)$ is the spin-isospin degeneracy of the $i$-th state and $m_i$ is its mass. The theoretical curve



N
non-strange mesons
1000
500
100
50
10
5
0. 5
1
1. 5
2
2. 5
3
m [GeV]



N
1000
100
10
mesons
baryons
0. 5
1
1. 5
2
2. 5
3
m [GeV]

# THANK YOU!

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