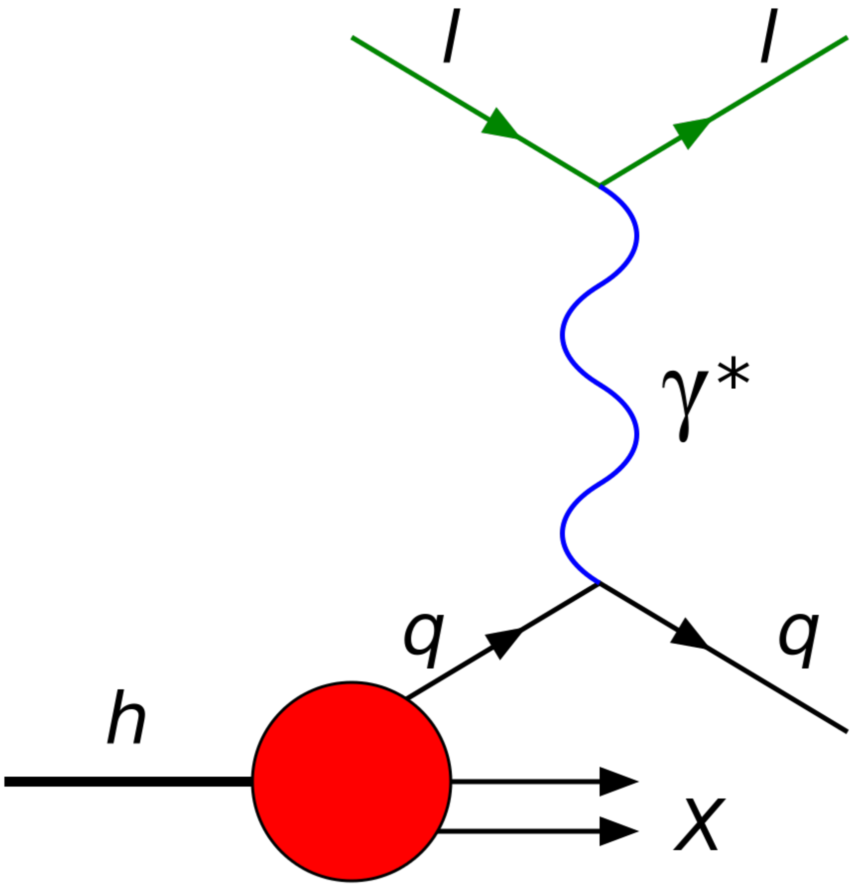
Low x Conference, Bari, Italy, June 13-17, 2017

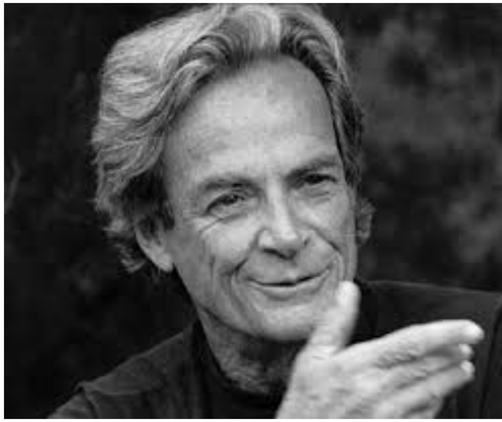
Probing quantum entanglement in hard scattering

D. Kharzeev

Based on DK, E. Levin, arXiv:1702.03489; Phys Rev D 95 (2017)

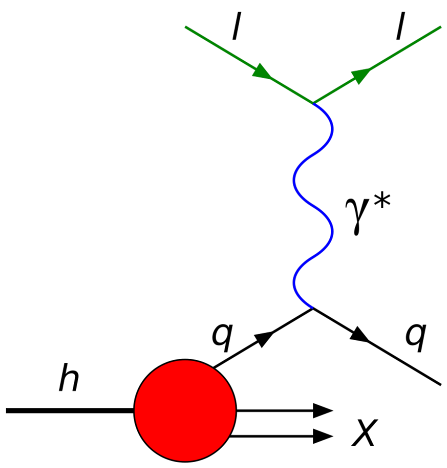
The parton model: 50 years of success





In almost fifty years that have ensued after the birth of the parton model, it has become an indispensable building block of high energy physics

The parton model: basic assumptions



In parton model, the proton is pictured as a collection of point-like quasi-free partons that are frozen in the infinite momentum frame due to Lorentz dilation.

The DIS cross section is given by the incoherent sum of cross sections of scattering off individual partons.

How to reconcile this with quantum mechanics?

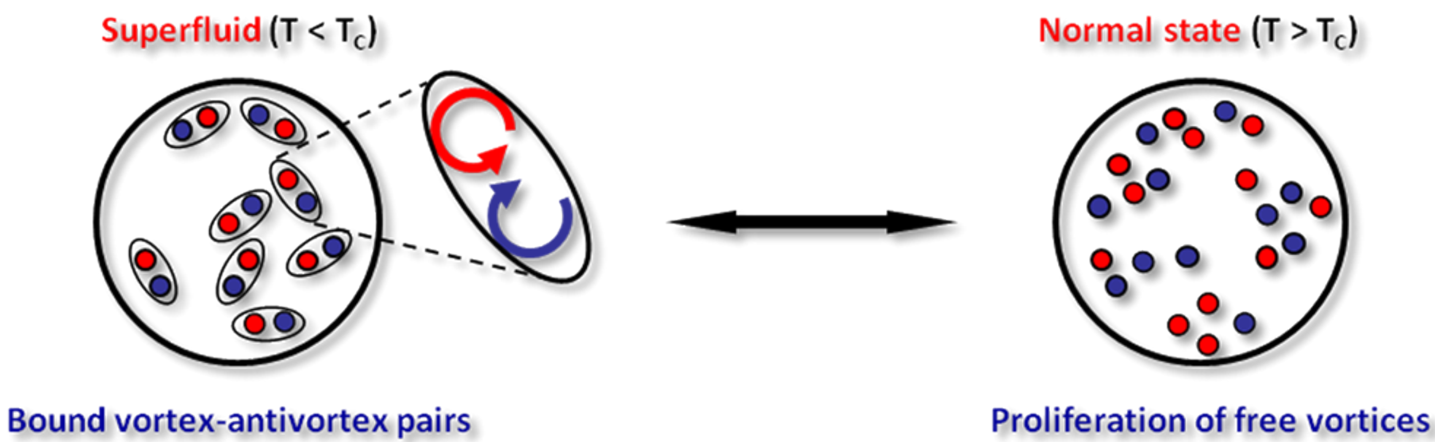
DIS and entanglement

“...we never experiment with just one electron or atom or (small) molecule. In thought experiments, we sometimes assume that we do; this invariably entails ridiculous consequences ... .”

Erwin Schrödinger, 1952

The puzzle of the parton model

In quantum mechanics, the proton is a pure state with zero entropy. Yet, a collection of free partons does possess entropy… Boosting to the infinite momentum frame does not help, as a Lorentz boost cannot transform a pure state into a mixed one.



The crucial importance of entropy in (2+1)D systems: BKT phase transition (Nobel prize 2016)

The quantum mechanics of partons and entanglement

Our proposal: the key to solving this apparent paradox is entanglement.

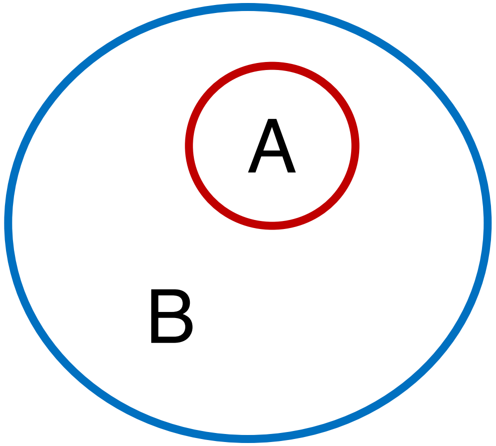
DK, E. Levin, arXiv:1702.03489, PRD

DIS probes only a part of the proton's wave function (region A). We sum over all hadronic final states; in quantum mechanics, this corresponds to accessing the density matrix of a mixed state

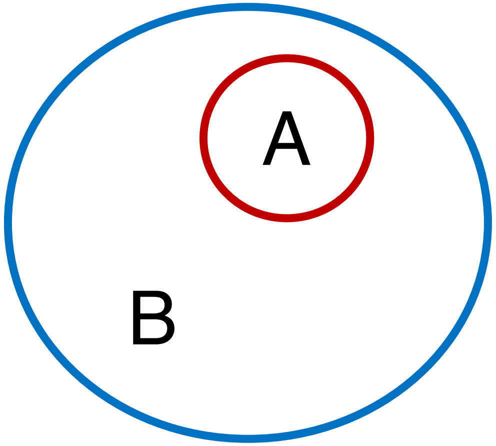
$$\hat{\rho}_A = \mathrm{tr}_{\mathrm{B}} \hat{\rho}$$

with a non-zero entanglement entropy

$$S_A = -\mathrm{tr}\left[\hat{\rho}_{\mathrm{A}} \ln \hat{\rho}_{\mathrm{A}}\right]$$



The quantum mechanics of partons and entanglement



The proton is described by a vector

$$|\Psi_{AB}\rangle = \sum_{i,j} c_{ij} \, |\varphi_i^A\rangle \otimes |\varphi_j^B\rangle$$

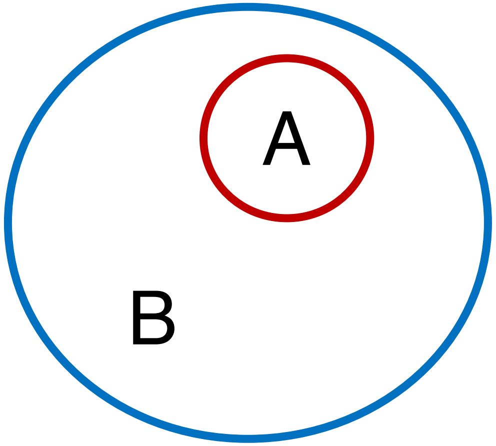
in Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$

If $|\Psi_{AB}\rangle = |\varphi^A\rangle \otimes |\varphi^B\rangle$ only one term

contributes, then the state is separable (not our case!). Otherwise, the state is **entangled**.

The quantum mechanics of partons and entanglement

The Schmidt decomposition theorem:



$$|\Psi_{AB}\rangle = \sum_n \alpha_n |\Psi_n^A\rangle |\Psi_n^B\rangle$$

There exist the orthonormal states $|\Psi_n^A\rangle$ and $|\Psi_n^B\rangle$ for which the pure state can be represented as a <u>single</u> sum with real and positive coefficients α_n

If only one term (Schmidt rank one), then the state is separable. Otherwise, it is **entangled**; but no interference between different n's.

The quantum mechanics of partons and entanglement

We assume that the Schmidt basis $|\Psi_n^A\rangle|\Psi_n^B\rangle$ corresponds to the states with different numbers of partons n; since it represents a relativistic quantum field theory (QCD), the Schmidt rank is in general infinite.

The density matrix is now given by

$$\rho_A = \mathrm{tr_B}\ \rho_{\mathrm{AB}} = \sum_{\mathrm{n}} \alpha_{\mathrm{n}}^2\ |\Psi_{\mathrm{n}}^{\mathrm{A}}\rangle\langle\Psi_{\mathrm{n}}^{\mathrm{A}}|$$

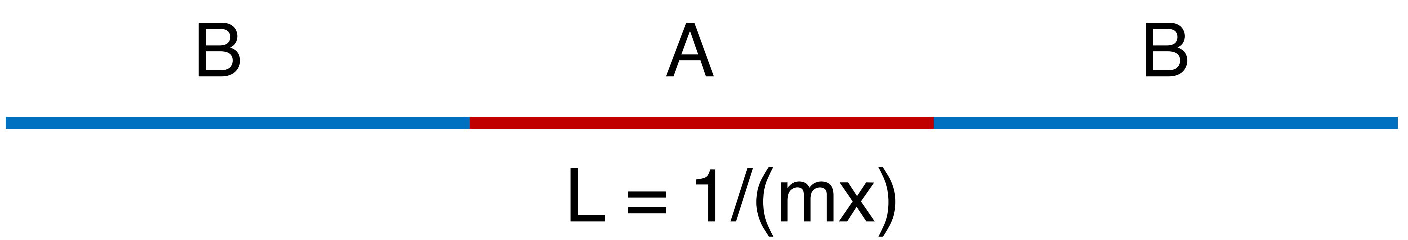
where $\alpha_n^2 \equiv p_n$ is the probability of a state with n partons

The entanglement entropy

The entanglement entropy is now given by
the von Neumann's formula

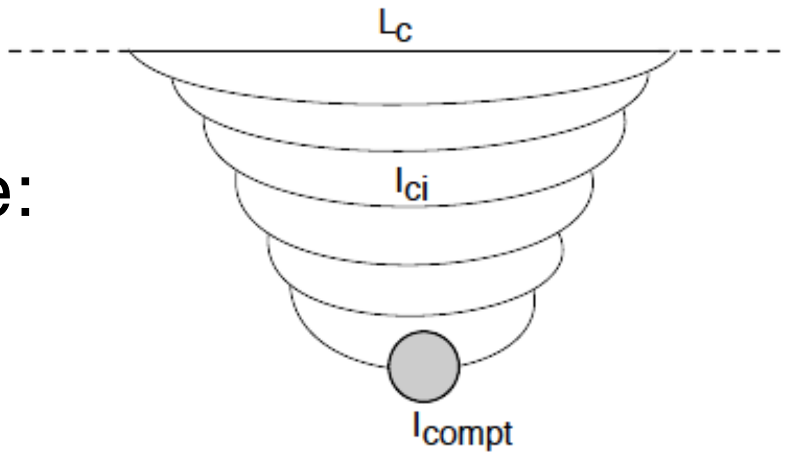
$$S = -\sum_n p_n \ln p_n$$

and can be evaluated by using the QCD evolution equations.
Let us start with a (1+1) case, followed by (3+1).



The entanglement entropy from QCD evolution

Space-time picture
in the proton's rest frame:



The evolution equation:

$$\frac{dP_n(Y)}{dY} = -\Delta\, n\, P_n(Y) + (n-1)\Delta P_{n-1}(Y)$$

The entanglement entropy from QCD evolution

$$\frac{dP_n(Y)}{dY} = -\Delta\, n\, P_n(Y) + (n-1)\Delta P_{n-1}(Y)$$

Solve by using the generating function method

(A.H.Mueller ‘94; E.Levin, M.Lublinsky ‘04):

$$Z(Y,u) = \sum_n P_n(Y) u^n.$$

Solution:

$$P_n(Y) = e^{-\Delta Y}(1 - e^{-\Delta Y})^{n-1}.$$

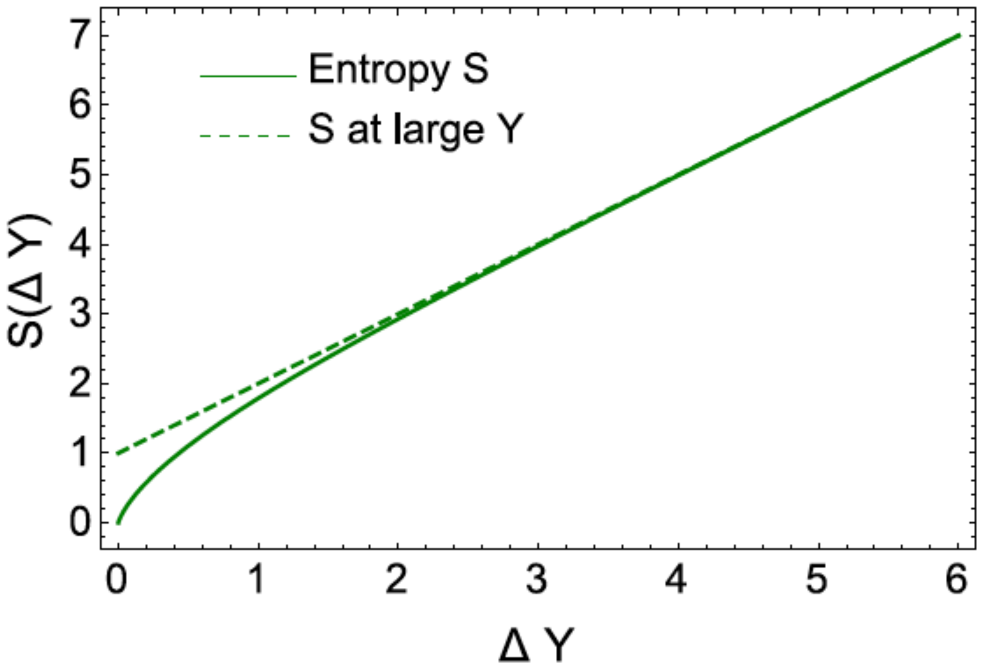
The resulting von Neumann entropy is

$$S(Y) = \ln(e^{\Delta Y} - 1) + e^{\Delta Y} \ln\left(\frac{1}{1 - e^{-\Delta Y}}\right)$$

The entanglement entropy from QCD evolution

At large ΔY , the entropy becomes

$$S(Y) \to \Delta Y$$



This "asymptotic" regime starts rather early, at

$$\Delta Y \simeq 2$$

The entanglement entropy from QCD evolution

At large ΔY (x ~ 10^{-3}) the relation between the entanglement entropy and the structure function

$$xG(x) = \langle n \rangle = \sum_n n P_n(Y) = \left(\frac{1}{x}\right)^{\Delta}$$

becomes very simple:

$$S = \ln[xG(x)]$$

The entanglement entropy from QCD evolution

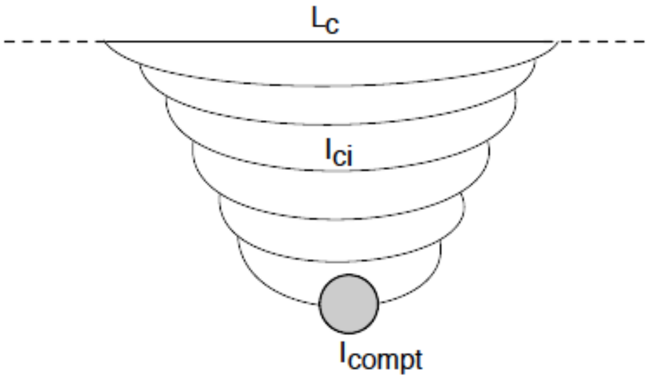
The (3+1) case is cumbersome, but the result is the same, with $\Delta = \bar{\alpha}_s \ln(r^2 Q_s^2)$

What is the physics behind this relation?

$$S = \ln[xG(x)]$$

It signals that all $\exp(\Delta Y)$ partonic states have about equal probabilities $\exp(-\Delta Y)$ – in this case the **entanglement entropy is maximal**, and the proton is a **maximally entangled state** (a new look at the parton saturation and CGC?)

Possible relation to CFT



The small x formula

$$S = \ln[xG(x)]$$

yields

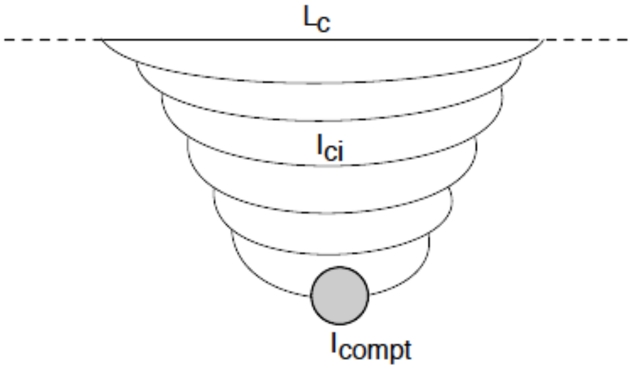
$$S(x) = \Delta \ln[1/x] = \Delta \ln \frac{L}{\epsilon}$$

where

$$L = (mx)^{-1}$$

is the longitudinal length probed in DIS (in the target rest frame), and $\epsilon \equiv 1/m$ is the proton's Compton wavelength

Possible relation to CFT



This formula

$$S(x) = \Delta \ln[1/x] = \Delta \ln \frac{L}{\epsilon}$$

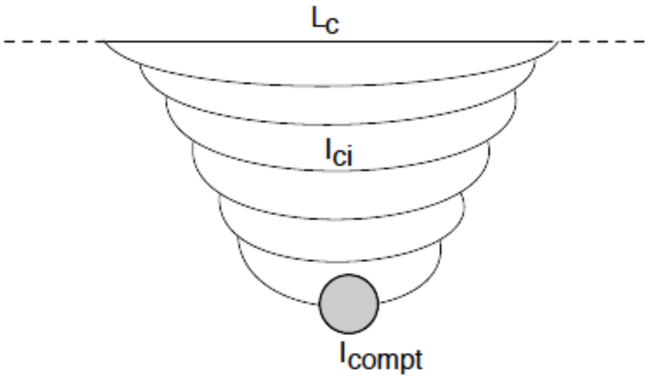
looks very similar to the well-known (1+1) CFT result:

(Holzhey, Larsen, Wilczek '94; Vidal, Latorre, Rico, Kitaev '03, Korepin '04, Calabrese, Cardy '05)

$$S_E = \frac{c}{3} \ln \frac{L}{\epsilon}$$

where c is the central charge of the CFT, and ε is the resolution scale

Possible relation to CFT



If this is not a mere coincidence:
the central charge $c \leq 1$:

$$c = 1 - \frac{6}{m(m+1)}, \quad m = 3, 4, ..., \infty$$

c=1 corresponds to free bosonic theory.
This means that $\Delta \leq 1/3$;
a bound on the growth of the structure function!

$$xG(x) \leq \text{const} \; \frac{1}{x^{1/3}}$$

The Second Law for entanglement entropy?

If the Second Law applies to entanglement entropy (EE) (a number of indications, e.g. from black hole physics), then the entropy of hadronic final state in DIS has to be equal or larger than the EE of the initial state measured through the structure function:

$$S_{\text{hadrons}} \geq S_{EE}(x)$$

Indications from holography that the entropy does not increase at strong coupling; this leads to

$S_{\text{hadrons}} \simeq S_{EE}(x)$ **parton liberation** ?

Fluctuations in hadron multiplicity

What is the relation between the parton and hadron multiplicity distributions?

Let us assume they are the same ("EbyE parton-hadron duality"); then the hadron multiplicity distribution should be given by

$$P_n(Y) \;=\; e^{-\Delta Y}(1 - \, e^{-\Delta Y})^{n-1}.$$

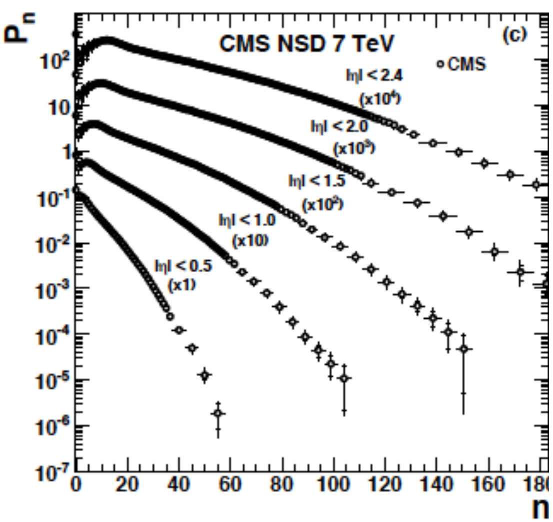
Consider cumulants

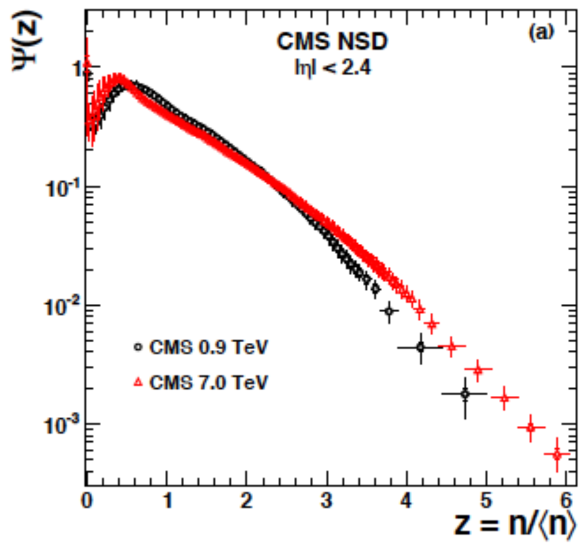
$$C_q = < n^q > / < n >^q$$

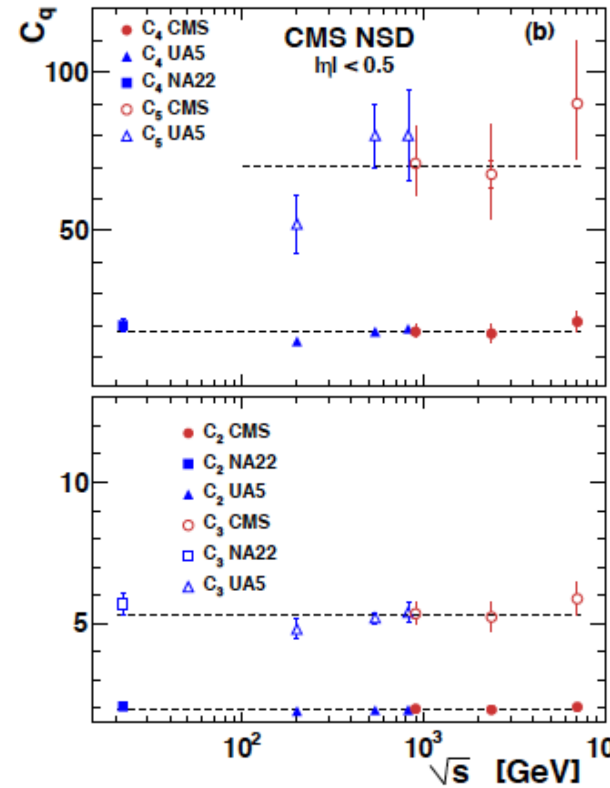
Fluctuations in hadron multiplicity

The CMS Coll., arXiv:1011.5531, JHEP01(2011)079

Charged particle multiplicities in pp interactions at $\sqrt{s} = 0.9, 2.36$, and 7 TeV







KNO scaling violated!

Fluctuations in hadron multiplicity

The cumulants can be easily computed by using the generating function

$$C_q = \left(u\frac{d}{du}\right)^q Z\left(Y,u\right)\Bigg|_{u=1}$$

We get

$$C_2 = 2 - 1/\bar{n}; \quad C_3 = \frac{6(\bar{n}-1)\bar{n}+1}{\bar{n}^2};$$

$$C_4 = \frac{(12\bar{n}(\bar{n}-1)+1)(2\bar{n}-1)}{\bar{n}^3}; \quad C_5 = \frac{(\bar{n}-1)(120\bar{n}^2(\bar{n}-1)+30\bar{n})+1}{\bar{n}^4}.$$

Fluctuations in hadron multiplicity

Numerically, for $\bar{n} = 5.8 \pm 0.1$ at |η|<0.5, E_{cm}=7 TeV we get:

theory	exp (CMS)	theory, high energy limit
$C_2 = 1.83$	C_2 = 2.0+-0.05	$C_2 = 2.0$
$C_3 = 5.0$	C_3 = 5.9+-0.6	$C_3 = 6.0$
$C_4 = 18.2$	C_4 = 21+-2	$C_4 = 24.0$
$C_5 = 83$	C_5 = 90+-19	$C_5 = 120$

It appears that the multiplicity distributions of final state hadrons are very similar to the parton multiplicity distributions – this suggests that the entropy is close to the entanglement entropy

Future directions

- The entanglement entropy (EE) can be defined and computed for any value of the coupling – generalization of parton distribution to strong coupling? holography
- The EE is a non-local order parameter in topologically ordered phases – relation of parton distributions to QCD topology?
- Elucidate connections to CFT
- Reformulate parton evolution in terms of EE (momentum space entanglement?) Kovner, Lublinsky ‘15
- A new look at thermalization

Physics at colliders:

supplement the measurements of
structure functions by the studies of hadronic final state (especially in the target fragmentation region).

Testing the Second Law for the Entanglement entropy is fundamentally important;

extracting the entropy of the final hadronic state will require event-by-event measurements.

Combine measurements of “hard” cross sections with the event-by-event measurements of associated hadron multiplicity

Summary

1. Entanglement entropy (EE) provides a viable solution to the apparent contradiction between the parton model and quantum mechanics.
2. The use of EE allows to generalize the notion of parton distribution to any value of the coupling, and possibly to elucidate the role of topology.
3. If the connection to CFT is confirmed, it will put a bound on the growth of parton distributions at small x (no other bound exists; unitarity does not provide a bound).
4. Measuring hadronic final states at colliders (RHIC,LHC,EIC,…) will allow to test the Second Law for the EE.